

# Testing for intertemporal nonseparability

IFS Working Paper W13/19

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# Testing for Intertemporal Nonseparability\*

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July 18, 2013

**Abstract:** This paper presents a nonparametric analysis of a common class of intertemporal models of consumer choice that relax consumption independence. Within this class and in the absence of any functional form restrictions on instantaneous preferences, we compare the revealed preference conditions for rational habit formation and rational anticipation. We show that these models are nonparametrically equivalent in the presence of finite data sets composed of prices, interest rates, and consumption choices.

**JEL Classifications:** D11, D12, D91.

**Keywords:** anticipation, habits, revealed preference, time separability.

## 1. INTRODUCTION

The discounted utility model is the standard framework for thinking about dynamic consumer behaviour.<sup>1</sup> The model typically supposes that an agent's preferences over consumption profiles can be represented by  $\sum_t \beta^{t-1} u(x_t)$ , where  $u$  denotes a time-invariant, cardinal, and concave instantaneous utility function defined over the period  $t$  consumption vector  $x_t$ , and where  $\beta$  is the discount factor defined as  $1/(1 + \rho)$ , with  $\rho$  denoting the discount rate. A key feature of the discounted utility model is that it explicitly assumes time separability, or consumption independence.

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\* We are grateful to John Quah and to seminar participants at Leicester for useful conversations. Crawford acknowledges financial support from the Leverhulme Trust, reference number F/08 519/E, as well as the Centre for the Microeconomic Analysis of Public Policy at the Institute for Fiscal Studies; Polisson acknowledges funding from the ESRC/MRC/NIHR, reference number G0802297.

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<sup>1</sup> For the origins of this literature, see Rae (1834), Böhm-Bawerk (1889), Fisher (1930), and Samuelson (1937).

This embodies the assumption that an individual's preferences over consumption in any period are independent of consumption in any other period.

That intertemporal separability is a strong assumption has of course long been recognised. Samuelson (1952) famously expressed the view that 'the amount of wine I drank yesterday and will drink tomorrow can be expected to have effects upon my today's indifference slope between wine and milk'. Koopmans (1960) argued that 'there is no clear reason why complementarity of goods could not extend over more than one time period'. Despite the manifest implausibility of this assumption, it remains popular, mainly because it greatly simplifies the analysis of intertemporal choice.

The two most obvious and straightforward approaches that incorporate intertemporal *nonseparability*, i.e., that allow preferences at a point in time to depend upon consumption choices at others, are rational habit formation and rational anticipation. Rae (1834) was perhaps the first to propose the idea that utility from current consumption can be affected by past consumption. The notion that a knowledge of future consumption can affect present decision making goes back as far as Jevons (1871). Both nonseparable approaches have delivered meaningful insights into consumer behaviour, and both are able to explain empirical consumption 'puzzles' where the time separable benchmark falls short.

Models of habit formation have been developed and applied with some enthusiasm,<sup>2</sup> while models of anticipation have been slower to advance.<sup>3</sup> Nonetheless, the suggestion that anticipation and habit formation may be equally effective in explaining consumer behaviour is at the core of this paper. While habits and anticipation

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<sup>2</sup> Prominent applications include Becker and Murphy (1988) on the price-responsiveness of addictive behaviour, Meghir and Weber (1996) on intertemporal nonseparabilities and liquidity constraints, and Abel (1990), Constantinides (1990), and Campbell and Cochrane (1999) on asset-pricing anomalies, including the equity premium puzzle. Macroeconomists have appealed to habit formation to better explain movements in asset prices (Jermann, 1998; Boldrin, Christiano, and Fisher, 2001), to investigate the relationship between economic growth and savings (Carroll, Overland, and Weil, 2000), and to explain the responsiveness of aggregate spending to shocks (Fuhrer, 2000).

<sup>3</sup> Quiggin (1982) axiomatised a theory of anticipated utility—more commonly known as rank-dependent expected utility theory—which generalised the expected utility model in order to explain prominent behavioural anomalies, including the Allais paradox. Loewenstein (1987) proposed that instantaneous utility is equal to utility from current consumption plus some function of consumption in future periods. Incorporating future consumption in this way allows the consumer to have a preference for improvements over time and for suffering unpleasant outcomes quickly rather than delaying them. More recently, Caplin and Leahy (2001) have shown that anticipatory utility can explain the equity premium puzzle just as effectively as habit formation.

certainly come in many flavours, in general the literature treats them as though they are *distinct*. In the absence of specific parametric restrictions on instantaneous preferences, we show that this is not the case within a common class of intertemporally nonseparable models. We derive the empirical implications of these models in the revealed preference tradition of Samuelson (1948), Houthakker (1950), Afriat (1967), Diewert (1973), and Varian (1982), and demonstrate an equivalence in the presence of finite data sets composed of prices, interest rates, and consumption choices.

The paper is organised as follows. Section 2 introduces the framework within which we investigate the observable implications of intertemporal nonseparability. Section 3 outlines the revealed preference conditions for models of rational habit formation and rational anticipation within this framework. Section 4 contains the main equivalence result of the paper. Section 5 provides some brief concluding remarks.

## 2. FRAMEWORK

In order to isolate intertemporal nonseparability, we adhere to many of the principal assumptions of the benchmark discounted utility model—only consumption independence is relaxed.<sup>4</sup> Note therefore that we continue to assume instantaneous preferences that are stable over some horizon, separable aggregation,<sup>5</sup> perfect foresight, exponential discounting, and perfect liquidity.

We let  $x_t \in \mathbb{R}_+^K$  be a vector of consumption goods (where each good is indexed by  $k \in \kappa = \{1, \dots, K\}$ ) purchased at corresponding spot prices  $p_t \in \mathbb{R}_{++}^K$  in period  $t \in \tau = \{1, \dots, T\}$ , where  $\tau$  denotes the set of contiguous periods *observed* by the econometrician. In order to allow for lags and leads of consumption, we also make use of two augmented sets of periods. More specifically, we allow for  $N \in \{1, \dots, T - 1\}$  lags or leads, and we denote the augmented sets by  $\underline{\tau} = \{1 - N, \dots, T\}$  and  $\bar{\tau} = \{1, \dots, T + N\}$ . Discounted prices are given by  $\hat{p}_t \in \mathbb{R}_{++}^K$ .<sup>6</sup> Finally, we let  $B = \{y_t \in \mathbb{R}_+^K \text{ for all } t \in \tau : \sum_{t \in \tau} \hat{p}_t \cdot y_t \leq \sum_{t \in \tau} \hat{p}_t \cdot x_t\}$  denote the lifetime budget set. We assume that the econometrician observes a data set of discounted prices and consumption choices  $\{(\hat{p}_t, x_t)\}_{t \in \tau}$ . Given these observables, we ask whether

<sup>4</sup> By this, we mean that instantaneous preferences are allowed to depend upon lags and leads of consumption.

<sup>5</sup> As shown in Kubler (2004), the nonseparable representation in Kreps and Porteus (1978) fails to deliver any meaningful empirical content whatsoever unless the intertemporal aggregator is weakly separable. Within our framework, intertemporal aggregation remains additive.

<sup>6</sup> Prices are discounted throughout according to  $\hat{p}_t = p_t / \prod_{s=1}^{s=t-1} (1 + r_s)$  for all  $t \in \tau \setminus \{1\}$  and  $\hat{p}_1 = p_1$ , where  $r_t \geq 0$  denotes the rate of interest between period  $t$  and  $t + 1$  for all  $t \in \tau \setminus \{T\}$ .

there are necessary and sufficient conditions which guarantee the existence of some instantaneous utility functions  $u : (\mathbb{R}_+^K)^{N+1} \rightarrow \mathbb{R}$  and  $v : (\mathbb{R}_+^K)^{N+1} \rightarrow \mathbb{R}$ , as well as a discount factor  $\beta \in (0, 1]$ , such that a consumer could have been solving either

$$\max_{\{x_t\}_{t \in \tau}} \sum_{t \in \bar{\tau}} \beta^{t-1} u(x_t, x_{t-1}, x_{t-2}, \dots, x_{t-N}) \quad (1)$$

or

$$\max_{\{x_t\}_{t \in \tau}} \sum_{t \in \mathcal{T}} \beta^{t-1} v(x_t, x_{t+1}, x_{t+2}, \dots, x_{t+N}) \quad (2)$$

subject to the lifetime budget constraint, where (1) corresponds to habit formation and (2) to anticipation. We also ask whether the utility functions  $u$  and  $v$  are necessarily distinct. We formalise this approach in the following section.

### 3. REVEALED PREFERENCE ANALYSIS

#### 3.1 Rational Habit Formation

We begin with an examination of the revealed preference conditions for a standard model of rational habit formation.

**Definition 1** *The data set  $\{(\hat{p}_t, x_t)\}_{t \in \tau}$  is consistent with rational habit formation if there exist a non-satiated, concave, and differentiable<sup>7</sup> utility function  $u : (\mathbb{R}_+^K)^{N+1} \rightarrow \mathbb{R}$ , a discount factor  $\beta \in (0, 1]$ , and unobserved consumption  $x_t = y_t \in \mathbb{R}_+^K$  for each  $t \notin \tau$ , such that  $\sum_{t \in \bar{\tau}} \beta^{t-1} u(x_t, \dots, x_{t-N}) \geq \sum_{t \in \bar{\tau}} \beta^{t-1} u(y_t, \dots, y_{t-N})$  for all  $\{y_t\}_{t \in \tau} \in B$ .*

This definition simply states that a data set is rationalisable by rational habit formation if the observed consumption profile delivers weakly greater lifetime utility than any other consumption profile satisfying the lifetime budget constraint. We now establish the revealed preference conditions for this model.

**Lemma 1** *The following statements are equivalent:*

1. *The data set  $\{(\hat{p}_t, x_t)\}_{t \in \tau}$  is consistent with the model of rational habit formation in Definition 1.*

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<sup>7</sup> Note that differentiability is without loss of generality throughout.

2. There exist  $(u_t, \rho_t^0, \dots, \rho_t^N) \in \mathbb{R} \times (\mathbb{R}^K)^{N+1}$  for each  $t \in \bar{\tau}$ ,  $x_t \in \mathbb{R}_+^K$  for each  $t \notin \tau$ , and  $\beta \in (0, 1]$ , such that

$$u_{t'} \leq u_t + \begin{pmatrix} \rho_t^0 \\ \vdots \\ \rho_t^N \end{pmatrix} \cdot \begin{pmatrix} x_{t'} - x_t \\ \vdots \\ x_{t'-N} - x_{t-N} \end{pmatrix} \quad (\text{H.1})$$

for all  $(t, t') \in \bar{\tau} \times \bar{\tau}$ ,

$$\beta^{t-1} \rho_{kt}^0 + \dots + \beta^{t-1+N} \rho_{k(t+N)}^N = \hat{p}_{kt} \quad (\text{H.2})$$

for all  $(k, t) \in \kappa \times \tau$  with  $x_{kt} > 0$ , and

$$\beta^{t-1} \rho_{kt}^0 + \dots + \beta^{t-1+N} \rho_{k(t+N)}^N \leq \hat{p}_{kt} \quad (\text{H.3})$$

for all  $(k, t) \in \kappa \times \tau$  with  $x_{kt} = 0$ .

**Proof: Necessity.** Suppose that the data set  $\{(\hat{p}_t, x_t)\}_{t \in \tau}$  is consistent with the model of rational habit formation in Definition 1. Since  $u$  is non-satiated, with  $\tilde{x}_t = x_t$  for all  $t \notin \tau$ ,  $\{x_t\}_{t \in \tau}$  solves  $\max_{\{\tilde{x}_t\}_{t \in \tau} \in B} \sum_{t \in \bar{\tau}} \beta^{t-1} u(\tilde{x}_t, \dots, \tilde{x}_{t-N})$ , so that there exists  $\lambda > 0$  such that  $\beta^{t-1} \partial u(x_t, \dots, x_{t-N}) / \partial x_{kt} + \dots + \beta^{t-1+N} \partial u(x_{t+N}, \dots, x_t) / \partial x_{kt} \leq \lambda \hat{p}_{kt}$  for all  $(k, t) \in \kappa \times \tau$ . Note that the inequality is binding for any  $(k, t) \in \kappa \times \tau$  with  $x_{kt} > 0$ . Concavity of  $u$  implies that

$$u(x_{t'}, \dots, x_{t'-N}) \leq u(x_t, \dots, x_{t-N}) + \begin{pmatrix} \partial u(x_t, \dots, x_{t-N}) / \partial x_t \\ \vdots \\ \partial u(x_t, \dots, x_{t-N}) / \partial x_{t-N} \end{pmatrix} \cdot \begin{pmatrix} x_{t'} - x_t \\ \vdots \\ x_{t'-N} - x_{t-N} \end{pmatrix}$$

for all  $(t, t') \in \bar{\tau} \times \bar{\tau}$ . Now let  $u(x_t, \dots, x_{t-N}) = \lambda u_t$  and  $\partial u(x_t, \dots, x_{t-N}) / \partial x_t = \lambda \rho_t^0, \dots, \partial u(x_t, \dots, x_{t-N}) / \partial x_{t-N} = \lambda \rho_t^N$  for all  $t \in \bar{\tau}$ .

**Sufficiency.** Suppose that there exist  $(u_t, \rho_t^0, \dots, \rho_t^N) \in \mathbb{R} \times (\mathbb{R}^K)^{N+1}$  for each  $t \in \bar{\tau}$ ,  $x_t \in \mathbb{R}_+^K$  for each  $t \notin \tau$ , and  $\beta \in (0, 1]$ , such that (H.1)–(H.3) are satisfied. Define  $u : (\mathbb{R}_+^K)^{N+1} \rightarrow \mathbb{R}$  as follows:

$$u(x^0, \dots, x^N) = \min_{t \in \bar{\tau}} \left( u_t + \begin{pmatrix} \rho_t^0 \\ \vdots \\ \rho_t^N \end{pmatrix} \cdot \begin{pmatrix} x^0 - x_t \\ \vdots \\ x^N - x_{t-N} \end{pmatrix} \right).$$

Notice that  $u$  is non-satiated,<sup>8</sup> concave, and differentiable. By the definition of  $u$ ,  $u(x_t, \dots, x_{t-N}) \leq u_t$  for all  $t \in \bar{\tau}$ . Since

$$u(x_t, \dots, x_{t-N}) = u_{t'} + \begin{pmatrix} \rho_{t'}^0 \\ \vdots \\ \rho_{t'}^N \end{pmatrix} \cdot \begin{pmatrix} x_t - x_{t'} \\ \vdots \\ x_{t-N} - x_{t'-N} \end{pmatrix} \leq u_t$$

for some  $t' \in \bar{\tau}$ , it must be that  $u(x_t, \dots, x_{t-N}) = u_t$  for all  $t \in \bar{\tau}$  in order to satisfy (H.1). Lastly, consider any  $\{y_t\}_{t \in \tau} \in B$  with  $y_t = x_t$  for any  $t \notin \tau$ . By the definition of  $u$ , it must be that

$$u(y_t, \dots, y_{t-N}) \leq u_t + \begin{pmatrix} \rho_t^0 \\ \vdots \\ \rho_t^N \end{pmatrix} \cdot \begin{pmatrix} y_t - x_t \\ \vdots \\ y_{t-N} - x_{t-N} \end{pmatrix}$$

for all  $t \in \bar{\tau}$ , which implies that for some  $\beta \in (0, 1]$ ,

$$\sum_{t \in \bar{\tau}} \beta^{t-1} u(y_t, \dots, y_{t-N}) \leq \sum_{t \in \bar{\tau}} \beta^{t-1} u_t + \sum_{t \in \tau} \hat{p}_t \cdot (y_t - x_t) \quad (3)$$

$$\leq \sum_{t \in \bar{\tau}} \beta^{t-1} u_t \quad (4)$$

$$= \sum_{t \in \bar{\tau}} \beta^{t-1} u(x_t, \dots, x_{t-N}). \quad (5)$$

Inequality (3) follows from (H.2) and (H.3), since  $\beta^{t-1} \rho_t^0 + \dots + \beta^{t-1+N} \rho_{t+N}^N \leq \hat{p}_t$  for all  $t \in \tau$ ; inequality (4) follows since  $\sum_{t \in \tau} \hat{p}_t \cdot y_t \leq \sum_{t \in \tau} \hat{p}_t \cdot x_t$ ; and equality (5) follows since  $u(x_t, \dots, x_{t-N}) = u_t$  for all  $t \in \bar{\tau}$ .  $\square$

The restrictions in (H.1)–(H.3) exhaust the pure empirical implications of the model of rational habit formation in Definition 1. In other words, if we observe a data set that satisfies these conditions, then the observed consumption choices are *consistent* with the model. The converse of this statement is also true, implying that data which do not satisfy the restrictions are *inconsistent*. Note that for each  $t \in \bar{\tau}$ , the parameters  $(\rho_t^0, \dots, \rho_t^N)$  are not completely free to vary within  $(\mathbb{R}^K)^{N+1}$  due to the sign restrictions imposed by (H.2) and (H.3). However, as long as we observe some strictly positive consumption, some of these parameters must also be strictly positive, which guarantees non-satiation. Further note that rational habit formation

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<sup>8</sup> Non-satiation of  $u$  is given by the sign restrictions on  $(\rho_t^0, \dots, \rho_t^N)$  for each  $t \in \bar{\tau}$  imposed by (H.2) and (H.3).

contains the classical life-cycle model as a special case. To see this, let  $\rho_t^l = 0$  for all  $l \neq 0$  and  $t \in \bar{\tau}$ .<sup>9</sup> Notice that Lemma 1 has an equivalent cyclical monotonicity representation, which is first proven in Theorem 1 of Crawford (2010). However, the formulation presented here is much more computationally convenient. This is because cyclical monotonicity requires that we check *every* possible subset of the data—an enormous number of calculations even for a data set of moderate size—whereas the conditions in Lemma 1 can be implemented very efficiently using a simple grid or random search and standard linear programming techniques.

### 3.2 Rational Anticipation

We next consider a standard model of rational anticipation.

**Definition 2** *The data set  $\{(\hat{p}_t, x_t)\}_{t \in \tau}$  is consistent with rational anticipation if there exist a non-satiated, concave, and differentiable utility function  $v : (\mathbb{R}_+^K)^{N+1} \rightarrow \mathbb{R}$ , a discount factor  $\beta \in (0, 1]$ , and unobserved consumption  $x_t = y_t \in \mathbb{R}_+^K$  for each  $t \notin \tau$ , such that  $\sum_{t \in \mathcal{I}} \beta^{t-1} v(x_t, \dots, x_{t+N}) \geq \sum_{t \in \mathcal{I}} \beta^{t-1} v(y_t, \dots, y_{t+N})$  for all  $\{y_t\}_{t \in \tau} \in B$ .*

As we saw earlier, this definition again embodies the principle of revealed preference, i.e., the data are consistent with rational anticipation if the observed consumption profile delivers weakly greater lifetime utility than any other consumption profile satisfying the lifetime budget constraint. We now establish the revealed preference conditions for this model.

**Lemma 2** *The following statements are equivalent:*

1. *The data set  $\{(\hat{p}_t, x_t)\}_{t \in \tau}$  is consistent with the model of rational anticipation in Definition 2.*
2. *There exist  $(v_t, \pi_t^0, \dots, \pi_t^N) \in \mathbb{R} \times (\mathbb{R}^K)^{N+1}$  for each  $t \in \mathcal{I}$ ,  $x_t \in \mathbb{R}_+^K$  for each  $t \notin \tau$ , and  $\beta \in (0, 1]$ , such that*

$$v_{t'} \leq v_t + \begin{pmatrix} \pi_t^0 \\ \vdots \\ \pi_t^N \end{pmatrix} \cdot \begin{pmatrix} x_{t'} - x_t \\ \vdots \\ x_{t'+N} - x_{t+N} \end{pmatrix} \quad (\text{A.1})$$

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<sup>9</sup> If we further impose that  $\beta = 1$ , the restrictions are equivalent to cyclical monotonicity in Browning (1989).



for all  $(t, t') \in \underline{\tau} \times \underline{\tau}$ ,

$$\beta^{t-1-N} \pi_{k(t-N)}^N + \cdots + \beta^{t-1} \pi_{kt}^0 = \hat{p}_{kt} \quad (\text{A.2})$$

for all  $(k, t) \in \kappa \times \tau$  with  $x_{kt} > 0$ , and

$$\beta^{t-1-N} \pi_{k(t-N)}^N + \cdots + \beta^{t-1} \pi_{kt}^0 \leq \hat{p}_{kt} \quad (\text{A.3})$$

for all  $(k, t) \in \kappa \times \tau$  with  $x_{kt} = 0$ .

**Proof:** The proof of Lemma 2 is analogous to the earlier proof of Lemma 1. Necessity makes use of concavity in the instantaneous utility function  $v$  as well as standard optimality conditions for convex problems. Sufficiency constructs a piecewise linear instantaneous utility function to rationalise the data, using the lower envelopes of the hyperplanes in (A.1).  $\square$

As in Lemma 1, the restrictions in (A.1)–(A.3) exhaust the pure empirical implications of the model of rational anticipation in Definition 2. Once again note that for each  $t \in \underline{\tau}$ , the parameters  $(\pi_t^0, \dots, \pi_t^N)$  are not completely free to vary within  $(\mathbb{R}^K)^{N+1}$  due to the sign restrictions imposed by (A.2) and (A.3). Like habit formation, rational anticipation contains the life-cycle model as a special case. To see this, let  $\pi_t^l = 0$  for all  $l \neq 0$  and  $t \in \underline{\tau}$ .

#### 4. EQUIVALENCE

The following proposition gives the main result of the paper.

**Proposition 1** *The dataset  $\{(\hat{p}_t, x_t)\}_{t \in \tau}$  is consistent with the model of rational habit formation in Definition 1 if and only if it is consistent with the model of rational anticipation in Definition 2.*

**Proof:** *Necessity.* Suppose that there exist  $(u_t, \rho_t^0, \dots, \rho_t^N) \in \mathbb{R} \times (\mathbb{R}^K)^{N+1}$  for each  $t \in \bar{\tau}$ ,  $x_t \in \mathbb{R}_+^K$  for each  $t \notin \tau$ , and  $\beta \in (0, 1]$ , such that (H.1)–(H.3) are satisfied. Define  $(\pi_t^0, \dots, \pi_t^N) \in (\mathbb{R}^K)^{N+1}$  according to

$$\begin{pmatrix} \pi_t^0 \\ \vdots \\ \pi_t^N \end{pmatrix} = \beta^N \begin{pmatrix} \rho_{t+N}^N \\ \vdots \\ \rho_{t+N}^0 \end{pmatrix}$$

for all  $t \in \underline{\tau}$ , and  $v_t \in \mathbb{R}$  according to

$$v_t = \beta^N u_{t+N}$$

for all  $t \in \underline{\tau}$ , such that (A.1)–(A.3) are satisfied.

*Sufficiency.* Suppose that there exist  $(v_t, \pi_t^0, \dots, \pi_t^N) \in \mathbb{R} \times (\mathbb{R}^K)^{N+1}$  for each  $t \in \underline{\tau}$ ,  $x_t \in \mathbb{R}_+^K$  for each  $t \notin \tau$ , and  $\beta \in (0, 1]$ , such that (A.1)–(A.3) are satisfied. Define  $(\rho_t^0, \dots, \rho_t^N) \in (\mathbb{R}^K)^{N+1}$  according to

$$\begin{pmatrix} \rho_t^0 \\ \vdots \\ \rho_t^N \end{pmatrix} = (1/\beta^N) \begin{pmatrix} \pi_{t-N}^N \\ \vdots \\ \pi_{t-N}^0 \end{pmatrix}$$

for all  $t \in \bar{\tau}$ , and  $u_t \in \mathbb{R}$  according to

$$u_t = v_{t-N}/\beta^N$$

for all  $t \in \bar{\tau}$ , such that (H.1)–(H.3) are satisfied.  $\square$

Within this particular class of intertemporally nonseparable models (stable instantaneous preferences, separable aggregation, perfect foresight, exponential discounting, perfect liquidity), this nonparametric equivalence arises for a number reasons: (1) we only observe a finite subset of the consumer's choices; (2) we only require non-satiation in the instantaneous utility functions; (3) unobserved consumption is the same in both models; and (4) we do not allow for a durable habit-forming or anticipatory good. As a result, intertemporal marginal rates of substitution between any two periods are observationally equivalent across these models. In other words, given a finite data set, we cannot reject that they are the same. Lastly, note that without separable aggregation, intertemporal choice delivers no meaningful empirical content as in Kubler (2004), in which case a trivial equivalence arises.

It is easy to see that by imposing further structure on the problem, the equivalence no longer holds. For example, with stronger assumptions on the shapes of the utility functions  $u$  and  $v$ , we obtain further sign restrictions on  $(\rho_t^0, \dots, \rho_t^N)$  and  $(\pi_t^0, \dots, \pi_t^N)$  that can potentially differ. Furthermore, we could assume the observed subset contains the boundaries of the consumer's problem. A related modification would impose restrictions on unobserved consumption that vary across models. Lastly, if we treat habits/futures as durables (i.e., represent them by an unobservable stock variable

which includes the entire history/future of consumption), as in Demuynck and Verriest (2013), then the observational equivalence no longer obtains.

## 5. CONCLUDING REMARKS

In the absence of functional parametric restrictions on instantaneous preferences, we have shown that data on prices, interest rates, and consumption profiles do not allow the econometrician to distinguish between the models of rational habit formation in Definition 1 and rational anticipation in Definition 2. The finding suggests that functional forms may drive empirical differences within this common class of intertemporally nonseparable models.

## REFERENCES

- (1) Abel, A. B. (1990), “Asset Prices under Habit Formation and Catching up with the Joneses”, *American Economic Review*, 80(2), 38–42.
- (2) Afriat, S. N. (1967), “The Construction of Utility Functions from Expenditure Data”, *International Economic Review*, 8(1), 67–77.
- (3) Becker, G. S., and Murphy, K. M. (1988), “A Theory of Rational Addiction”, *Journal of Political Economy*, 96(4), 675–700.
- (4) Böhm-Bawerk, E. (1889), *The Positive Theory of Capital* (Macmillan, London).
- (5) Boldrin, M., Christiano, L. J., and Fisher, J. D. M. (2001), “Habit Persistence, Asset Returns, and the Business Cycle”, *American Economic Review*, 91(1), 149–166.
- (6) Browning, M. (1989), “A Nonparametric Test of the Life-Cycle Rational Expectations Hypothesis”, *International Economic Review*, 30(4), 979–992.
- (7) Campbell, J. Y., and Cochrane, J. H. (1999), “By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior”, *Journal of Political Economy*, 107(2), 205–251.
- (8) Caplin, A., and Leahy, J. (2001), “Psychological Expected Utility Theory and Anticipatory Feelings”, *Quarterly Journal of Economics*, 116(1), 55–79.
- (9) Carroll, C. D., Overland, J., and Weil, D. N. (2000), “Saving and Growth with Habit Formation”, *American Economic Review*, 90(3), 341–355.
- (10) Constantinides, G. M. (1990), “Habit Formation: A Resolution of the Equity Premium Puzzle”, *Journal of Political Economy*, 98(3), 519–543.
- (11) Crawford, I. (2010), “Habits Revealed”, *Review of Economic Studies*, 77(4), 1382–1402.
- (12) Demuynck, T., and Verriest, E. (2013), “I’ll Never Forget my First Cigarette: A Revealed Preference Analysis of the ‘Habits as Durables’ Model”, *International Economic Review*, forthcoming.

- (13) Diewert, W. E. (1973), “Afriat and Revealed Preference Theory”, *Review of Economic Studies*, 40(3), 419–425.
- (14) Fisher, I. (1930), *The Theory of Interest* (MacMillan, New York).
- (15) Fuhrer, J. C. (2000), “Habit Formation in Consumption and Its Implications for Monetary-Policy Models”, *American Economic Review*, 90(3), 367–390.
- (16) Houthakker, H. S. (1950), “Revealed Preference and the Utility Function”, *Economica*, 17(66), 159–174.
- (17) Jermann, U. J. (1998), “Asset Pricing in Production Economies”, *Journal of Monetary Economics*, 41, 257–275.
- (18) Jevons, W. S. (1871), *The Theory of Political Economy* (London: Macmillan).
- (19) Koopmans, T. C. (1960), “Stationary Ordinal Utility and Impatience”, *Econometrica*, 28(2), 287–309.
- (20) Kreps, D.M., and Porteus, E.L. (1978), “Temporal Resolution of Uncertainty and Dynamic Choice Theory”, *Econometrica*, 46(1), 185–200.
- (21) Kubler, F. (2004), “Is Intertemporal Choice Theory Testable?”, *Journal of Mathematical Economics*, 40(1–2), 177–189.
- (22) Loewenstein, G. (1987), “Anticipation and the Valuation of Delayed Consumption”, *Economic Journal*, 97(387), 666–684.
- (23) Meghir, C., and Weber, G. (1996), “Intertemporal Nonseparability or Borrowing Restrictions? A Disaggregate Analysis Using a U.S. Consumption Panel”, *Econometrica*, 64(5), 1151–1181.
- (24) Quiggin, K. (1982), “A Theory of Anticipated Utility”, *Journal of Economic Behavior and Organization*, 3(4), 323–343.
- (25) Rae, J. (1834), *The Sociological Theory of Capital* (Macmillan, London).
- (26) Samuelson, P. A. (1937), “A Note on Measurement of Utility”, *Review of Economic Studies*, 4(2), 155–161.
- (27) Samuelson, P. A. (1948), “Consumption Theory in Terms of Revealed Preference”, *Economica*, 15(60), 243–253.
- (28) Samuelson, P. A. (1952), “Probability, Utility, and the Independence Axiom”, *Econometrica*, 20(4), 670–678.
- (29) Varian, H. R. (1982), “The Nonparametric Approach to Demand Analysis”, *Econometrica*, 50(4), 945–973.