A winning formula? Elementary indices in the Retail Prices Index

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Abstract

This paper considers the case for replacing the Carli index in the Retail Prices Index for calculating price changes at the elementary aggregate level. Following Diewert (2012), we go through each of the three approaches used to select appropriate index numbers: the test, stochastic and economic approaches. In each case, we find a few areas where our conclusions differ from Diewert's. Unlike Diewert, we are not as concerned that the Carli fails the time reversibility test, but note that it fails a revised price bouncing test. We find that the stochastic approach does not clearly favour one index over another. Diewert also argues that the economic approach is inapplicable at the level of elementary aggregates, where by definition quantity weights for goods are unknown. However, we argue using insights from information theory, that the economic approach *can* be applied at this level and moreover that it favours the use of the Jevons index.

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1 Introduction

This paper aims to evaluate the different indices used to calculate elementary aggregate prices in the Retail Prices Index (RPI) and the Consumer Prices Index (CPI) in the light of recent proposals to replace the Carli index in the RPI. Following Diewert (2012), we discuss how elementary indices used in the RPI and CPI compare under the three different approaches used to select appropriate index numbers: in terms of their properties (the test approach), their statistical performance (the statistical approach) and as measures of a cost of living index (the economic approach). In doing so, we make two primary conceptual contributions to the way these indices should be compared: we introduce a revised price bouncing test in our discussion of the test approach, and we propose a constructive principle to select budget shares in cases where they are not observed - allowing us to apply the economic approach to the question of index number choice at the elementary aggregate level. Like Diewert we conclude that there is a case against the use of the Carli index. However, we differ from Diewert in a few of our conclusions. In particular we are less concerned that the Carli index fails the 'time reversal test' under the test approach but find that it fails our revised price bouncing test. Moreover while Diewert argues that the economic approach cannot be applied to the choice of index numbers at this level, we argue that it can and moreover that it justifies the use of the Jevons index.

The remainder of the paper is structured as follows. Section 2 covers the background to the issue, including a description of the different indices used in the RPI and CPI (the Carli, Dutot and Jevons indices). Section 3 discusses the mathematical relationships between these indices, and what drives the differences between them. Sections 4,5 and 6 and compare these indices according to the three different approaches. Section 7 concludes.

2 Background

The UK is currently blessed with two headline measures of consumer price inflation - the RPI and the CPI. These indices differ in a number of ways which mean they can give quite different measures of price changes from year to year. For instance, in 2011 the CPI averaged 4.5% compared to 5.2% for the RPI. The differences are a result of the data they draw from, the coverage of the indices, and the methods used to average prices. Recently, the CPI has replaced the RPI for a number of policy purposes, including the uprating of state benefits and pensions and the indexation of tax thresholds. Consequently, the large gap between the two measures, and particularly the factors that mean that the CPI tends to give a lower measure of inflation than the RPI, have increasingly come under scrutiny.

In the last few years, the largest factor contributing to the gap between the RPI and CPI has been the differences in the way price changes are aggregated in the two indices (known as the 'formula effect'). This particular cause of the long run gap between the RPI and CPI has been especially hard to defend, as it naturally leads one to ask "is one method preferable to another, and if so why is that method not used to calculate both indices?" These sorts of questions have led the Office for National Statistics (ONS) to seek to "identify, understand and eliminate unjustified causes of the formula effect gap between the CPI and RPI." (Office for National Statistics, 2012a). In October this year, this process culminated with the opening of a consultation on some proposed changes to the methods used in the RPI, which would serve to either reduce or entirely eliminate the formula effect. The suitability of these different methods and the proposed changes is the subject of this paper.

2.1 The RPI and the CPI

Both the RPI and CPI are measures of consumer price inflation. The RPI is the older of the two, dating back to an 'Interim index' that was introduced in June 1947 based on an expenditure survey carried out in 1937/38, and for most of its history was the UK's principal measure of consumer prices.

The CPI is the UK's version of a Harmonised Index of Consumer Prices (HICP), which were developed by the European Union to ensure that member states published comparable measures of inflation. These were originally to be used to assess countries' suitability to join European Monetary Union, but are now used to inform the decisions of the European Central Bank. The calculation of HICPs varies across countries but there are common regulations for their construction developed by the European statistical agency Eurostat (see Eurostat, 2004). In 2003, the CPI replaced the RPIX [1] as the index used to define the Bank of England's inflation target, and has been increasingly prominent as a general inflation measure since. Prior to this, the CPI was known simply as the HICP.

2.2 The Formula Effect

All price indices must average price changes across goods in some way to arrive at a single inflation rate. Both the RPI and CPI rely on (essentially the same) sample of prices collected across the country in each month.[2] This sample is then used to produce weighted averages of price changes relative to a base month (in the UK, January), which are calculated in successive

[1] RPIX is the RPI excluding mortgage interest payments.

[2] There a few differences. The CPI uses a different approach to gathering car prices to the RPI.

stages in a process called aggregation.[3] In the very first stage, where the ONS does not have expenditure information, an unweighted average of price changes for particular products is taken within different 'strata', defined by either region, type of shop (independent or chain retailer), or both. These give what are known as *elementary aggregate* prices. An expenditure-weighted average of these elementary aggregate is then taken to give an overall national average price index for an 'item'. These different item indices are then aggregated further through expenditure-weighted averages into 'sections' or 'classes', which are in turn aggregated into 'groups'. Finally, an overall price index is calculated from the different group indices. Some examples of the 'goods' at each stage of aggregation are given in table 1.1.

Table 1.1 Examples of goods at different levels of aggregation

Level	Price
Elementary aggregate	800g white unsliced bread sold in the south east of England
Item	800g white unsliced bread
Section/class	Bread
Group	Food

The RPI and CPI currently differ in terms of the formulae they use at the level of the elementary aggregates, and this is what gives rise to the formula effect (at subsequent stages of aggregation, the two indices are aggregated in the same way). In the RPI, one of two arithmetic averages are used: the Carli index and the Dutot.[4] The CPI by contrast makes use of the Dutot and a different index called the Jevons. The Carli is an arithmetic mean of price changes (or price relatives), while the Jevons is a geometric mean. The Dutot is the ratio of average prices in the base year and the current year. The precise formulae used to calculate these indices are as follows:

Carli:

$$P_C\left(\mathbf{p}_0, \mathbf{p}_1\right) = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{p_1^i}{p_0^i}\right)$$

Dutot:

$$P_D\left(\mathbf{p}_0, \mathbf{p}_1\right) = \frac{\frac{1}{N}\sum p_1^i}{\frac{1}{N}\sum p_0^i}$$

Jevons:

$$P_J\left(\mathbf{p}_0, \mathbf{p}_1\right) = \prod_{i=1}^{N}\left(\frac{p_1^i}{p_0^i}\right)^{\frac{1}{N}} = \frac{\prod_{i=1}^{N}\left(p_1^i\right)^{\frac{1}{N}}}{\prod_{i=1}^{N}\left(p_0^i\right)^{\frac{1}{N}}}$$

The importance of each of these formulae in the two indices is shown in table 1.2. The RPI makes use of the Carli and Dutot in roughly equal proportions, while the CPI overwhelmingly makes use of the Jevons.[5] For the remaining goods, no elementary aggregates are calculated and weights are used at every stage in the calculation of prices.

[3] For details, see section 2 of the ONS Consumer Prices Technical Manual (Office for National Statistics, 2012b).

[4] These indices are sometimes referred to as the 'average of relatives' and the 'ratio of averages'.

[5] The reason for the even split between the Carli and Dutot in the RPI is because both indices can be distorted in particular situations. The Carli can be too sensitive to situations where individual goods see large price changes (such as when a sale for some items ends). The Dutot on the other hand can be dominated by the price movements of a single good, if that good is much more expensive than others included in the calculation (see section 9.3 of Office for National Statistics, 2012b).

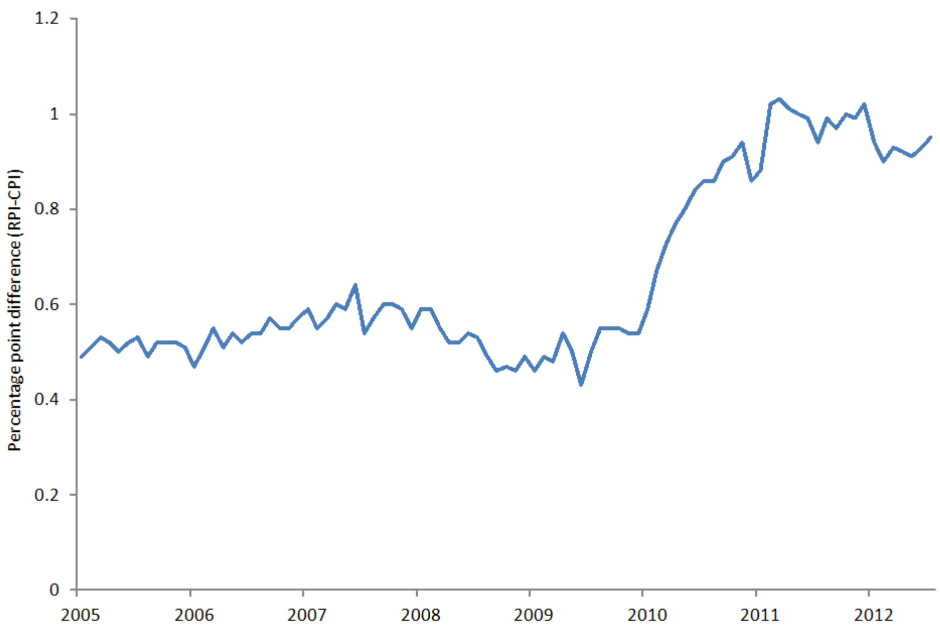
Table 1.2 Importance of different formulae used in the RPI and CPI

Index	RPI	CPI
Carli	27%	0%
Dutot	29%	5%
Jevons	0%	63%
Other (weighted) formula	43%	33%

Source: ONS (2012b)

The formula effect arises because these different formulae used to calculate elementary aggregates will give different price changes given the same data. In practice these differences have consistently worked to substantially reduce the CPI relative to the RPI. Figure 1 shows the formula effect over time from 2005-2012. The effect averaged 0.5 percenage points over the years 2005 -2009, which increased to 0.9 since 2010 (for comparison, the average annual increase in the RPI over the same period was 3.4%). The sudden increase in the formula effect can be almost entirely attributed to a change in the sampling of clothing prices that came into effect in that year (Morgan and Gooding, 2010).

Figure 1. The size of the formula effect, 2005-2012



Source: Office for National Statistics

The scale of the formula effect is a matter of concern for some. Other countries have switched from using arithmetic averages to the Jevons index, and the impact of these changes has typically been much smaller. For example, the US Bureau of Labor Statistics (BLS) estimates that between December 1998 and December 2010, the impact of a switch towards the Jevons in the US CPI averaged 0.28 percentage points per year,[6] compared to 0.53 in the UK over the same period. An ONS survey of the experience of other countries switching to the use of Jevons indicates that the impact of these changes was most often around just 0.1 - 0.2 percentage points (Evans, 2012).

There are a variety of possible reasons for the different experience of the UK in this regard. For instance, the UK uses relatively broad definitions of goods at the level of the elementary aggregates (see Fenwick, 1999). As we will see below, we would expect this broader definition

[6] Private correspondence with BLS

to widen the gap between the Carli and the Jevons. A second important factor is the UK's unusual use of the Carli index. None of the other 27 European countries that reported a HICP to Eurostat surveyed in Evans (2012) made use of the Carli index in their national price indices (see table 1.3). Indeed there seems to be have been a general move away from the Carli index historically. Evans (2012) lists some countries that have abandoned the Carli index in favour of either the Jevons and the Dutot over the last few decades including Canada (in 1978), Luxembourg (in 1996), Australia (in 1998), Italy (in 1999) and Switzerland (in 2000). Eurostat regulations also do not allow the use of the Carli in the construction of members' HICP indices except in "exceptional cases", (see section 3, pg 180 of Eurostat, 2001).

For reasons we will also discuss below, we would expect a switch from Carli to Jevons to lead to a greater formula effect than a switch away from the Dutot. This and other problems with the Carli has led some commentators to suggest that the Carli should no longer be used in the RPI (see for instance Giles, 2012). The changes proposed in the ONS consultation into the methods used in the RPI would replace Carli with either the Dutot or the Jevons indices (Office for National Statistics, 2012c), drawing on the recommendations of Diewert (2012).

Table 1.3 National elementary aggregate formulae in HICP reporting countries

Country	National index	HICP index
Austria	Jevons	Jevons
Belgium	Dutot	Dutot
Bulgaria	Jevons	Jevons
Croatia	Jevons	Jevons
Czech Republic	Dutot	Dutot
Denmark	Jevons	Jevons
Estonia	Dutot	Dutot
Finland	Jevons	Jevons
France	Jevons	Jevons
Germany	Dutot	Dutot
Greece	Jevons	Jevons
Iceland	Jevons/Dutot	Jevons/Dutot
Ireland	Jevons	Jevons
Italy	Jevons	Jevons
Lithuania	Dutot	Dutot
Luxembourg	Jevons	Jevons
Malta	Dutot	Dutot
Netherlands	Jevons/Dutot	Jevons/Dutot
Norway	Jevons	Jevons
Poland	Jevons	Jevons
Portugal	Jevons	Jevons
Romania	Jevons	Jevons
Slovakia	Dutot	Dutot
Slovenia	Dutot	Jevons
Spain	Jevons	Jevons
Sweden	Jevons	Jevons
Switzerland	Jevons	Jevons
UK	Dutot/Carli	Jevons/Dutot
USA	Jevons	Jevons

Source: Evans (2012)

3 Relationships between Formulae

In this section we will look at the mathematical relationships between the different indices in a series of propositions, and discuss their implications for the size of formula effect in the UK. Proofs are provided in the appendix to this document.

3.1 Carli and Jevons

The Jevons and the Carli - being the geometric mean and the arithmetic mean of the price relatives satisfy the classic inequality:[7]

$$P_J(\mathbf{p}_0, \mathbf{p}_1) \leq P_C(\mathbf{p}_0, \mathbf{p}_1) \tag{1}$$

with equality if $\frac{p_1^i}{p_0^i} = \pi$ for all i: that is the Jevons will *always* give either the same or a lower price increase than the Carli. It's therefore not surprising that the use of the Jevons in the CPI has consistently reduced it relative to the RPI. The extent of the difference between these indices depends crucially on the variance of the price relatives.

Proposition 1 *The difference between the Carli index and the Jevons index is bounded from below by the variance of the price-relatives:*

$$P_C(\mathbf{p}_0, \mathbf{p}_1) - P_J(\mathbf{p}_0, \mathbf{p}_1) \geq Var\left(\frac{p_1^i}{p_0^i}\right)$$

This implies that the more dispersion there is in price relatives, the lower the Jevons index will be relative to the Carli.

This observation helps us to understand both the size and growth of the formula effect in the UK seen in figure 1. The fact that UK elementary aggregates typically have broader definitions means that it is likely they will have greater variation in the size of their price relatives, which might contribute to the formula effect being so much larger in the UK. The growth in the formula effect in 2010 can be attributed to a change in the way clothing prices were collected in that year. The new sampling method was less strict in finding comparable items from one month to the next, which led to both an increase in the sample size and greater variation in the price relatives that were collected. This naturally increased the difference between the Carli used to calculate clothing price changes in the RPI and the Jevons used in the CPI.

3.2 Carli and Dutot

It is not possible to establish a similar general result for the relationship between the Dutot and the other indices. Depending on the circumstances the Dutot could be greater or less than the Carli and greater or less than the Jevons. In the case of the Carli however, it is possible to know the difference between the two indices exactly for a given set of prices. To see how, note that the Dutot can itself be rewritten as a weighted average of price relatives like the Carli, where the weights are determined by the base prices in the calculation:

$$P_D(\mathbf{p}_0, \mathbf{p}_1) = \frac{\frac{1}{N}\sum p_1^i}{\frac{1}{N}\sum p_0^i} = \frac{\frac{1}{N}\sum \left(\frac{p_1^i}{p_0^i}\right) p_0^i}{\frac{1}{N}\sum p_0^i} = \frac{1}{\frac{1}{N}\sum p_0^i}\frac{1}{N}\sum \left[p_0^i \left(\frac{p_1^i}{p_0^i}\right)\right] \tag{2}$$

From this we can show that

[7] Hardy, Littlewood and Polya (1934) p.26.

Proposition 2 *The difference between the Dutot and the Carli equals the covariance of base period prices and price relatives divided by the mean base period price*

$$P_D(\mathbf{p}_0, \mathbf{p}_1) - P_C(\mathbf{p}_0, \mathbf{p}_1) = \frac{Cov\left(p_0^i, \left(\frac{p_1^i}{p_0^i}\right)\right)}{E\left[p_0^i\right]} \tag{3}$$

Since the mean of p_0^i is always positive, this implies that the Dutot will be greater than the Carli if base prices and price relatives are positively correlated, and less than the Carli otherwise. Intuitively, this is because the Dutot effectively gives greater weight to goods which are more expensive in the base period, and so if these goods also see the fastest price increases, then the Dutot will be greater than the Carli.

In practice it seems that, at least for goods where the Carli index has been used in the RPI, these two variables have tended to be negatively correlated. We can infer this from the fact that the ONS estimates of the impact of calculating the RPI with a Dutot index where the Carli was used over the period 2007-2011, were negative over the whole period (Office for National Statistics, 2012c).

3.3 Jevons and Dutot

The results in proposition (1) and proposition (3) together imply that the Dutot will be greater than the Jevons if the covariance between price relatives and base period prices is positive. However, if the covariance is negative, then the Dutot could be greater or smaller than the Jevons. There is a useful approximation that can tell us more precisely about the relationship between the two.

Writing the price of the ith good in the tth period as a multiplicative deviation from it's expected value e_t^i

$$p_t^i = E\left(p_t^i\right)\left(1 + e_t^i\right) \tag{4}$$

where $E\left(e_t^i\right) = 0$, we can arrive at the following relationship

Proposition 3 *The difference between the Jevons and the Dutot depends on the change in the variance of the prices*

$$P_J\left(\mathbf{p}_0, \mathbf{p}_1\right) \approx P_D\left(\mathbf{p}_0, \mathbf{p}_1\right)\left(1 + \frac{1}{2}\left[Var\left(e_0^i\right) - Var\left(e_1^i\right)\right]\right)$$

This means that if the variance of prices is stable over time the Dutot can be regarded as a second order approximation to the Jevons. If the variance of prices is increasing, then the Dutot will be greater than the Jevons.

The fact that the Dutot can be greater than or less than the Jevons has two implications for our discussion of the differences between the RPI and CPI. Firstly, the use of the Dutot in both the RPI and CPI means that there is nothing that mathematically guarantees that the formula effect should always have the same sign, as in theory the Dutot in the RPI could be greater than the Jevons in the CPI (though the fact it has had the same sign in every year since the CPI began suggests that this may well continue in the future). Secondly, it helps to explain why the formula effect has been so much greater in the UK than elsewhere. Other countries have typically switched from a Dutot index to a Jevons, and so wouldn't be expected to see too great a difference in their index - while some goods would see larger price increases after the change of formula, the impact of this on overall inflation would be tempered by the fact that other goods would see smaller price increases. The Carli is however unambiguously greater than the Jevons, and so it is not surprising that the UK's experience as been rather different.

In the following three sections, we will look at different approaches to comparing the suitability of these indices as methods of aggregating price changes.

4 The Test or Axiomatic Approach

This approach posits a number of desirable properties for index numbers. These form tests (or 'axioms') against which alternative index number formulae can be ranked - with index number formulae which satisfy the most, or the most important, axioms being ranked highest. This approach does not consider any behavioural interdependence between the price and quantity data unlike the economic approach which we discuss below. The test approach has its roots in the mathematical literature on functional equations,[8] the general problem being that of determining an unknown functional form (i.e. what is the functional form for the price index?) given a set of requirements on the function. The properties ('tests') are selected to be reasonable given the context.

When we have data on prices and quantities from two periods $t \in \{0, 1\}$ the problem is to determine the forms of the price index linking the two periods

$$P(\mathbf{p}_0, \mathbf{p}_1, \mathbf{q}_0, \mathbf{q}_1)$$

and the corresponding quantity index

$$Q(\mathbf{p}_0, \mathbf{p}_1, \mathbf{q}_0, \mathbf{q}_1).$$

such that nominal growth rate is (multiplicatively) decomposable in that part reflecting price changes and that part reflecting real changes:

$$P(\mathbf{p}_0, \mathbf{p}_1, \mathbf{q}_0, \mathbf{q}_1) Q(\mathbf{p}_0, \mathbf{p}_1, \mathbf{q}_0, \mathbf{q}_1) = \frac{x_1}{x_0}.$$

This decomposition property is sometimes called *weak factor reversal* and often isn't counted as a 'test' but as a defining property of bilateral index numbers. If this holds and neither are zero then once we have chosen one index number, the other is chosen *implicitly.* For example, given a price index we can recover the quantity index implicitly:

$$Q(\mathbf{p}_0, \mathbf{p}_1, \mathbf{q}_0, \mathbf{q}_1) = \frac{x_1}{x_0} \frac{1}{P(\mathbf{p}_0, \mathbf{p}_1, \mathbf{q}_0, \mathbf{q}_1)}$$

In the case of elementary price aggregates, quantity weights are not observed. Thus the bilateral index number problem is restated slightly as that of finding a price index $P(\mathbf{p}_0, \mathbf{p}_1)$ (and implicitly a quantity index $Q(\mathbf{q}_0, \mathbf{q}_1)$), which satisfy certain tests, such that

$$P(\mathbf{p}_0, \mathbf{p}_1) Q(\mathbf{q}_0, \mathbf{q}_1) = \frac{x_1}{x_0}$$

The tests themselves have been developed over the course of well over a century mainly for the case in which prices *and* quantities are observed (for an authoritative discussion of these, see section 2 of Diewert, 1992). In most cases there are obvious analogues to these tests for the elementary aggregates case where quantities are not known.[9] In the rest of this section, we go through this set of tests, using a list in section 4 of Diewert (2012) as our starting point, but adding our own revised version of the 'price bouncing' test. Throughout we will assume that $\mathbf{p}_t \in \mathbb{R}^K_{++}$. If we want to set $\mathbf{p}_1 = \mathbf{p}_0$ we call the common vector $\mathbf{p}$.

The first three tests establish some basic properties which are satisfied by all the elementary aggregate indices we are considering.

[8]Indeed important contributors to this literature like Eichhorn (1978) and Aczél (1966) were also leading contributors to the mathematical literature.

[9]This is not always true. There is, for example, no obvious parallel to the *Tabular Standard/Basket/Constant Quantities Test* $P(\mathbf{p}_0, \mathbf{p}_1, \mathbf{q}, \mathbf{q}) = \mathbf{p}_1'\mathbf{q}/\mathbf{p}_0'\mathbf{q}$ or the *Invariance to Proportional Changes in Current Quantities Test* $P(\mathbf{p}_0, \mathbf{p}_1, \mathbf{q}_0, \lambda\mathbf{q}_1) = P(\mathbf{p}_0, \mathbf{p}_1, \mathbf{q}_0, \mathbf{q}_1)$ for $\lambda > 0$ in the context of elementary aggregates.

1. *Positivity:* the price index should always be positive

$$P(\mathbf{p}_0, \mathbf{p}_1) > 0$$

2. *Continuity:* the price index should be continuous

$$P(\mathbf{p}_0, \mathbf{p}_1) \text{ is a continuous function of its arguments}$$

3. *Identity/Constant Prices Test*: if prices are constant over the two periods being compared, then the price index should equal one

$$P(\mathbf{p}, \mathbf{p}) = 1$$

The next set of tests consider the effects of scalar transformation of the price data and can be thought of as tests concerned with the linear homogeneity of price indices. The Carli, Dutot and Jevons also satisfy all of these.

4. *Proportionality in Current Prices Test:* if all current prices are multiplied by λ then the new price index is λ times what it was originally.

$$P(\mathbf{p}_0, \lambda\mathbf{p}_1) = \lambda P(\mathbf{p}_0, \mathbf{p}_1) \text{ for } \lambda > 0$$

5. *Inverse Proportionality in Base Prices Test* if all base period prices are multiplied by λ then the new price index is the original one divided by λ.

$$P(\lambda\mathbf{p}_0, \mathbf{p}_1) = \lambda^{-1} P(\mathbf{p}_0, \mathbf{p}_1) \text{ for } \lambda > 0$$

The next pair of tests are concerned with the behaviour of the index under monotonic price changes.

6. *Monotonicity in Current Prices Test:* if any current period price increases, then the index as a whole should increase

$$P(\mathbf{p}_0, \mathbf{p}_1) < P(\mathbf{p}_0, \mathbf{p}) \quad \text{if } \mathbf{p}_1 < \mathbf{p}$$

7. *Monotonicity in Base Prices Test:* if any base period price increases, then the index as a whole should decrease

$$P(\mathbf{p}_0, \mathbf{p}_1) > P(\mathbf{p}, \mathbf{p}_1) \quad \text{if } \mathbf{p}_0 < \mathbf{p}$$

8. *Mean Value Test:* the price index should be bounded by the minimum and maximum price-relatives

$$\min_k \left\{ \frac{p_1^1}{p_0^1}, ..., \frac{p_1^K}{p_0^K} \right\} \leq P(\mathbf{p}_0, \mathbf{p}_1) \leq \max_k \left\{ \frac{p_1^1}{p_0^1}, ..., \frac{p_1^K}{p_0^K} \right\}$$

Whilst this is a fairly intuitive requirement it can be shown (Eichhorn, 1978 pg 155) that it is implied by (2), (3), (6) and (4).

The next group of tests are concerned with invariance properties of various kinds. This group of tests will help us discriminate between our three elementary indices.

9. *Commodity Reversal Test/Symmetric treatment of outlets:* rearranging the order of the components of both current and base period price vectors in the same way should have no effect on the index. That is,

$$P(\widetilde{\mathbf{p}}_0, \widetilde{\mathbf{p}}_1) = P(\mathbf{p}_0, \mathbf{p}_1)$$

where $\widetilde{\mathbf{y}}$ denotes a permutation of the elements of the vector.

This implies for instance that if we took price quotes from the same outlets in a different order (but still keeping the order the same in base and current periods), then this would have no effect on the index. It can easily be shown that all of our indices pass this test.

10. *Invariance to Changes in Units/Commensurability Test:* scaling all prices in the elementary aggregate by a common factor should not affect the index

$$P(\lambda \mathbf{p}_0, \lambda \mathbf{p}_1) = P(\mathbf{p}_0, \mathbf{p}_1)$$

This is obviously a consequence of (4) and (5).

One consequence of this is that ignoring quantity discounts and the like, a change in the units defining individual items (such as switching from single items of fruit to bunches of fruit) should not affect the index. Here we are assuming a common change in units applied to all goods which are included in the elementary aggregate. This presupposes that these goods are fairly homogeneous. All our indices satisfy this test as well, but it should be noted that the Dutot index is not in general invariant to changes in the units in which individual goods are sold. If we were to double the base and current period price of one particular item (by for instance, measuring the price of a pair of gloves rather than a single glove), then the Dutot index would change, while the Jevons and Carli would be unaffected. This comes about because the level of the Dutot index depends on the value of base period prices relative to their mean (see equation (2)). As Diewert (2012) points out, this means that the Dutot will not be appropiate for elementary aggregates where there is a great deal of heterogeneity and items are measured in different units, as in these situations "the price statistician can change the index simply by changing the units of measurement for some of the items."

11. *Time Reversal Test:* if the data for the base and current periods are interchanged, then the resulting index is the reciprocal of the original

$$P(\mathbf{p}_0, \mathbf{p}_1) = \frac{1}{P(\mathbf{p}_1, \mathbf{p}_0)}$$

This means that if prices go up one period and return to their previous level the next, a chained index should record no price increase. The Dutot and Jevons indices both satisfy this test, but the Carli does not. In fact, the Carli will record an increase in prices (unless all prices increase in the same proportion), since it can be shown that

$$P_C(\mathbf{p}_0, \mathbf{p}_1) P_C(\mathbf{p}_1, \mathbf{p}_0) \geq 1$$

If we now consider a situation in which we have more than two periods we have two further tests (see Diewert, 1993, who attributes these to Westergaard, 1890, and Walsh, 1901, respectively):

12. *Circularity Test:* The product of a chain of indices over successive periods should equal the total price change over the whole period.

$$P(\mathbf{p}_0, \mathbf{p}_1) P(\mathbf{p}_1, \mathbf{p}_2) = P(\mathbf{p}_0, \mathbf{p}_2)$$

This is a transitivity test. Combined with test (3) the circularity test implies the time reversal test.

If this test were not satisfied, then different inflation rates over a given period could be obtained by chaining the index over different subperiods. One consequence of this is that an index could go up or down even if prices had not changed. For instance, consider a case where prices increased from $\mathbf{p}_0$ to $\mathbf{p}_1$ between periods 0 and period 1, but in period 2 returned to $\mathbf{p}_0$. In this case, a chained index that didn't satisfy circularity could potentially record inflation over the three periods when there had in fact been none.

This test (and time reversal) will be satisfied by any price index that can be expressed in a form $f(\mathbf{p}_t)/f(\mathbf{p}_0)$ as it is always true that

$$\frac{f(\mathbf{x})}{f(\mathbf{y})} = \frac{f(\mathbf{x})}{f(\mathbf{z})} \times \frac{f(\mathbf{z})}{f(\mathbf{y})}$$

The Jevons and the Dutot can both be written in this form, but the Carli cannot. Indeed, the Carli does not satisfy the circularity test since the Carli is $E\left[p_1^i/p_0^i\right]$ and in general

$$E\left[\frac{x}{y}\right] \neq E\left[\frac{x}{z}\right] \times E\left[\frac{z}{y}\right]$$

As a consequence, the Carli also fails time reversal.

The fact that the Carli fails the circularity test is sometimes said to mean that a chained Carli will suffer from an upward 'bias'.[10] This is because over three periods, the difference between an overall Carli and a chained Carli is given by

$$P_c\left(\mathbf{p}_0, \mathbf{p}_2\right) - P_c\left(\mathbf{p}_0, \mathbf{p}_1\right) P_c\left(\mathbf{p}_1, \mathbf{p}_2\right) = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{p_2^i}{p_0^i}\right) - \frac{1}{N}\sum_{i=1}^{N}\left(\frac{p_1^i}{p_0^i}\right) \times \frac{1}{N}\sum_{i=1}^{N}\left(\frac{p_2^i}{p_1^i}\right)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(\frac{p_1^i}{p_0^i} \times \frac{p_2^i}{p_1^i}\right) - \frac{1}{N}\sum_{i=1}^{N}\left(\frac{p_1^i}{p_0^i}\right) \times \frac{1}{N}\sum_{i=1}^{N}\left(\frac{p_2^i}{p_1^i}\right) = Cov\left(\frac{p_1^i}{p_0^i}, \frac{p_2^i}{p_1^i}\right) \tag{5}$$

and there are reasons to believe that this expression will tend to be negative. In particular if, as seems likely, price changes tended to regress towards their mean (that is, if we thought that steep price changes were unlikely to be repeated in successive periods), then the chained Carli index would tend to increase more than it 'should' do. While this is true, it is important to remember that it only really makes sense to talk about bias with respect to some target index. The expression in equation (5) only gives the bias of the chained Carli if we think that the unchained Carli is the target. The circularity test merely says that it is desirable for the chained and unchained indices to be equal. It is not in itself informative about whether the chained Carli is larger than it should be or the unchained Carli is smaller than it should be.

13. *Multiperiod Identity Test:* The product of a chain of two indices for two periods, and an unchained index reversing those changes, should equal one

$$P\left(\mathbf{p}_0, \mathbf{p}_1\right) P\left(\mathbf{p}_1, \mathbf{p}_2\right) P\left(\mathbf{p}_2, \mathbf{p}_0\right) = 1$$

If the elementary aggregate satisfied (11) then (12) implies (13). Alternatively if the elementary aggregate satisfies (13) then it also satisfies the identity test (2), and (12).

A final test we could add to this list concerns so-called price 'bounces'. This is concerned with how an index would change if different outlets merely exchanged prices from one period to the next. One test of this property, attributed to Dalén (1992) is as follows

$$P\left(\widetilde{\mathbf{p}}_0, \widehat{\mathbf{p}}_1\right) = P\left(\mathbf{p}_0, \mathbf{p}_1\right)$$

where $\widetilde{\mathbf{y}}$ and $\widehat{\mathbf{y}}$ denote *different* permutations of the vectors $\mathbf{p}_0$ and $\mathbf{p}_1$(while in the commodity reversal test the permutations are the same for the two periods). This test has been criticised on the grounds that prices should be matched to outlets in a one to one manner across periods (that is, that $\mathbf{p}_0$ and $\mathbf{p}_1$ should not be permuted differently), for the simple reason that outlets

[10] See for instance, the pg 13 of the ONS Consumer prices technical manual (Office for National Statistics, 2012b) and also Fenwick (1999). Both documents refer to this as 'price bouncing' - a term which we use slightly differently below.

vary by quality, and so even when the same good bought in different places it ought to be considered a different product. However, as we shall see it is possible for an index to register a price increase if outlets exchanged prices with one another but then swapped *back* - a property which is somewhat harder to justify. This suggests a new test

14. *Price bouncing test:* The price index should not change if prices are rearranged and then returned to their original order

$$P\left(\mathbf{p}_0, \tilde{\mathbf{p}}_0\right) P\left(\tilde{\mathbf{p}}_0, \mathbf{p}_0\right) = 1$$

for any possible permutation of $\mathbf{p}_0$.

This test will be met by any index that satisfies a stronger property $P\left(\mathbf{p}_0, \tilde{\mathbf{p}}_0\right) = 1$. We could introduce this as a separate test but by testing how an index would respond to a change which doesn't match outlet prices across periods, it would be subject to the same criticism as Dalén's price bouncing test. Note that (11) implies (14) but the converse is not necessarily true.

Both the Jevons and the Dutot satisfy this test as both are time reversible (and both are in any case invariant to any reordering of price vectors). The Carli on the other hand fails this test, as we illustrate with a simple numerical example. Table 3.1 shows how different indices respond to price bouncing in a case with two goods sold in different stores. In period 1 we swap the period 0 prices between store A and store B, and in period 2 we swap them back. In both periods 1 and 2, the Carli index increases by 2.5%, with a cumulative increase over both periods of 5.06%. This is despite prices in period 2 being no different to what they were in period 0! In fact, the Carli will always show an increase in these sorts of situations (for the same reason that in general $P_C\left(\mathbf{p}_0, \mathbf{p}_1\right) P_C\left(\mathbf{p}_1, \mathbf{p}_0\right) \geq 1$). Both the Jevons and Dutot, on the other hand, will correctly record no price change, as they do in the example.

Table 3.1: Price Bouncing Example

	Period 0	Period 1	Period 2	
Store A price	1	1.25	1	
Store B price	1.25	1	1.25	
		Period 1 index	Period 2 index	Chained index
Carli	...	$1.25 + 0.8 = 1.025$	1.025	1.056
Dutot	...	$1.125/1.125 = 1$	1	1
Jevons	...	$\sqrt{1.25 \times 0.8} = 1$	1	1

Given this list of requirements we can now ask, how do the three elementary aggregates measure up? Table 3.2 summarises the results

Table 3.2: The Test Performance of the Elementary Aggregates

Test	Jevons	Carli	Dutot
1. *Positivity*	✓	✓	✓
2. *Continuity*	✓	✓	✓
3. *Identity/Constant Prices*	✓	✓	✓
4. *Proportionality in Current Prices*	✓	✓	✓
5. *Inverse Proportionality in Base Prices*	✓	✓	✓
6. *Monotonicity in Current Prices*	✓	✓	✓
7. *Monotonicity in Base Prices*	✓	✓	✓
8. *Mean Value*	✓	✓	✓
9. *Commodity Reversal*	✓	✓	✓
10. *Invariance to Changes in Units*	✓	✓	✓
11. *Time Reversal*	✓	×	✓
12. *Circularity*	✓	×	✓
13. *Multiperiod Identity*	✓	×	✓
14. *Price bouncing*	✓	×	✓

Not all of the tests are necessarily as important as each other and, in principle, this might present us with an aggregation problem of our own (how to weight the different tests). However, luckily the results are definitive: whatever the weights we place on the individual tests, as long as they are non-negative, the Jevons and the Dutot emerge with the strongest axiomatic backing. These indices pass all of our tests, while the Carli fails the time reversal, circularity, multiperiod identity and price bouncing tests. If we were to attach importance weights to these tests the time-reversal test (11) would seem to be essential, indeed the originator of this test (Pierson,1896) was apparently so upset when he noted that many standard index number formulae like the Paasche and Laspeyres did not satisfy this requirement that he proposed that the very idea of forming index numbers should be abandoned.[11] It was on the basis of its axiomatic failings that Diewert (2012) recommended that the Carli index should no longer be used in the RPI. However, while the Carli's failure to satisfy time reversibility is indeed a problem, it is important to realise that the index numbers into which these elementary aggregates eventually feed (the RPI and CPI) are *themselves* not time-reversible, and nor would they be even if the elementary aggregates were time-reversible.[12] This means that fixing this particular problem associated with the RPI may not be of that great a benefit.

5 The Statistical Approach

The statistical or stochastic approach to index numbers is associated with Dutot (1738), Carli (1764) and especially Jevons (1865). It aims to estimate some 'average' price change from a population price relatives $\left(p_1^i/p_0^i\right)$. One difference between this an other approaches is that it can also be used to produce standard errors and confidence intervals on the rate of inflation. The index favoured by the statistical approach is the one that is the best statistical predictor of the object of interest, which is often thought of as being the expected change in prices $E\left[p_1^i/p_0^i\right]$. A different object of interest would likely give different answers (for instance, if it were the expected log price change $E[\ln(p_1^i/p_0^i)]$). However, the first of these would seem more appropriate. In a hypothetical case where we were considering the price change of a single good, the price relative

[11] Diewert (1992)

[12] After the level of the elementary aggregates, the RPI makes use of the Young and Lowe indices to aggregate further. The Young index is not time reversible, while the Lowe index is only time reversible for some comparisons. For an explanation of these see chapter 1 of International Labor Organization (2004). Diewert (2012) also recommended that the Young index no longer be used in the RPI.

would be the more suitable estimate of the price change.[13]

This would mean that the aim of the statistical/stochastic approach to index numbers would be to identify and then to estimate $E\left(p_1^i/p_0^i\right)$ - the mean of (one plus) the rate of inflation in this population of goods. One way to approach this problem is to note that the price-relatives can always (except in some extreme cases) be described by a decomposition into their mean and an additive, mean-zero, deviation

$$\frac{p_1^i}{p_0^i} = E\left[p_1^i/p_0^i\right] + e^i \tag{6}$$

where $E\left[e^i\right] = 0$ and the variance of e^i is σ^2. We can then estimate $E\left[p_1^i/p_0^i\right]$ in an unbiased way by taking its sample analogue

$$\frac{1}{N}\sum \frac{p_1^i}{p_0^i} = P_C\left(\mathbf{p}_0, \mathbf{p}_1\right)$$

which is the Carli index.

If we wanted, we could also focus on the distribution of *log* price-relatives and describe them by their mean plus an additive mean-zero error:

$$\ln\left(p_1^i/p_0^i\right) = E\left[\ln\left(p_1^i/p_0^i\right)\right] + u^i \tag{7}$$

where $E\left[\ln\left(p_1^i/p_0^i\right)\right]$ can be estimated using its own sample analogue

$$\frac{1}{N}\sum \ln\left(p_1^i/p_0^i\right) = \ln P_J\left(\mathbf{p}_0, \mathbf{p}_1\right)$$

which is the log of the Jevons index. The International Labor Organization's consumer price manual (2004) and Diewert (2012) both use this statistical model of the evolution of price relatives to justify the Jevons index. This is because as $\frac{1}{N}\sum \ln\left(p_1^i/p_0^i\right)$ here gives the log of the inflation rate, and the anti-log of the estimate of this is the Jevons. However, we know that by Jensen's inequality $E\left[f\left(x_i\right)\right] \geq f\left(E\left[x_i\right]\right)$ when f is a convex function (with equality when all of the x_i's are the same). Here we have f as the exponential function and $x_i = \ln\left(p_1^i/p_0^i\right)$ so

$$P_J\left(\mathbf{p}_0, \mathbf{p}_1\right) = \exp\left[\ln P_J\left(\mathbf{p}_0, \mathbf{p}_1\right)\right] = \exp\left[\frac{1}{N}\sum \ln\left(p_1^i/p_0^i\right)\right] \leq E\left[\exp\left[\ln\left(p_1^i/p_0^i\right)\right]\right] = E\left[p_1^i/p_0^i\right]$$

Therefore the relationship between the Jevons and the object of interest $E\left[p_1^i/p_0^i\right]$ is an approximation rather than an equality[14] and hence, when viewed under the statistical approach to index numbers the Jevons is *biased downwards*

$$P_J\left(\mathbf{p}_0, \mathbf{p}_1\right) \leq E\left[p_1^i/p_0^i\right]$$

Notice that this is conclusion is not based on any arguments about how price relatives evolve or whether (6) is a more realistic model of the process generating price relatives than (7).[15]

This needn't mean that the statistical approach rules out the use of the Jevons however, as bias is not our only consideration here. The overall performance of an estimator can be

[13] A third alternative would be $E[p_1^i]/E[p_0^i]$ which would also seem a sensbile estimate of the price change for a one good case. This object of interest would seem to favour the Dutot. In this section however, we will set the Dutot index to one side, and focus only on comparing the statistical performance of the Carli and the Jevons.

[14] This was pointed out, in this context, by Greenlees (2001).

[15] The data generating process in (5) implies that current period prices in all outlets would be described by $p_1^i = E[p_1^i/p_0^i]p_0^i + e^i p_0^i$, which means that they would equal base period prices inflated by a common factor plus a heteroskedastic deviation. In process described by (6) however, log prices in both periods could be decomposed into mean and a homoskedastic deviation $\ln p_t^i = E[\ln p_t^i] + u_t^i$ for $t = 0, 1$ where $u^i = u_1^i - u_0^i$. The latter seems the more realistic model of the way the data is generated. However, this has no bearing on the question of what the object of interest should be, or on whether the Jevons is biased as an estimator of $E[p_1^i/p_0^i]$.

summarised by its mean squared error (MSE), which measures its expected squared deviation from the true population value of the parameter of interest (equal to the sum of its squared biased and its variance). That is, for some estimator θ

$$MSE = E[\left(\theta - \hat{\theta}\right)^2] = Var(\hat{\theta}) + Bias(\hat{\theta})^2$$

In our case, if the true population parameter of interest is $\alpha = E\left[p_1^i/p_0^i\right]$. Assuming we have a statistically random sample, then the mean squared error of the estimator provided by Carli index is simply the variance of the sample mean of price relatives

$$MSE(P_C) = E[(\alpha - P_C)^2] = \frac{\sigma^2}{N}$$

as it is unbiased. In the case of the Jevons index, on the other hand, the mean squared error (based on a variance approximation in Dalén, 1999, cited in Elliot et al., 2012) is

$$MSE(P_J) = E[(\alpha - P_J)^2] \approx \frac{\sigma^2}{N}\left(1 - \frac{\sigma^2}{P_c^2}\right) + \left(\frac{\gamma}{3} - \sigma^2\left(1 - \frac{P_c}{2}\right)\right)^2$$

where the first term represents the variance, and the second term represents the squared bias, and where

$$\gamma = \frac{1}{N}\sum\left(\frac{p_1^i}{p_0^i} - P_c\right)^3$$

or the third moment of price relatives. This determines the distribution's skewness $E\left[\left(\frac{p_1^i/p_0^i - \alpha}{\sigma}\right)^3\right]$. The MSE of the Jevons will therefore depend on the shape of the distribution of price relatives.

It is possible that $MSE(P_J) < MSE(P_C)$ in cases where the variance of the Jevons estimator is much smaller than that of the Carli. Thus, despite its bias, the Jevons can in theory perform as well or better as a statistical estimator of α than the Carli index. To what extent is this true in practice? Elliott et al compare the performance of various elementary aggregate indices using detailed data on prices for alcoholic beverages taken from the Kantar Worldpanel. They find that the lower variance of the Jevons and its bias tended to cancel out, meaning that in terms of MSE there was no "noticeable difference" between the Jevons and the Carli when $E\left[p_1^i/p_0^i\right]$ was the object of interest.[16] We can conclude therefore that on statistical grounds, current evidence doesn't suggest there is much to be gained (if anything) from replacing the Carli in the RPI. Further research could shed additional light on this question.

6 The Economic Approach

The economic approach to index number construction aims to answer the question: given prices in the base and current periods, how much would a consumer's income need to proportionally increase from one period to another such that their economic welfare remained unchanged? By saying we are interested in maintaining 'economic welfare' we mean that we will only seek to compensate the consumer for changes in the prices they face, and not for changes in other environmental factors such as air quality, or changes in the consumer's tastes, which we hold constant for the purposes of our comparison.

The question is answered conceptually by a cost of living index (COLI).[17] The COLI is defined by means of a cost function $c(\mathbf{p}_t, u)$, which tells us for any given level of prices $\mathbf{p}_t$, the

[16] Elliott et al. themselves remain agnostic as to what the object of interest should be, and consider the performance of different estimators for various different target indices.

[17] At this stage we should note that the ONS rejects the interpretation of both the CPI and RPI as attempts to measure changes in the cost of living. The ONS considers these to be rather cost of goods indices or COGIs (see Office for National Statistics, 2011).

minimum level of expenditure needed to achieve a given level of welfare or 'utility' u. The ratio of cost functions in two periods, holding the target utility constant at some level u defines the COLI or Konüs index (dating back to Konüs, 1924)

$$P_K(\mathbf{p}_0, \mathbf{p}_1, u) = \frac{c(\mathbf{p}_t, u)}{c(\mathbf{p}_0, u)}$$

Every household will have its own COLI, and in order to calculate an economy-wide inflation measure it is necessary to aggregate these in some way. Various ways of doing this are discussed in Crossley and Pendakur (2010).

Under the economic approach, the price index chosen should reflect the degree to which consumers mitigate the welfare impact of price changes by shifting their purchases away from goods and services that have become relatively more expensive and towards goods that have become relatively cheaper. This means it should explicitly take account of the interdependence of prices and quantities over time. Economists traditionally do this is by representing consumers' decision making (their preferences) with a utility function that ranks different bundles of goods and services, and more importantly is associated with particular substitution responses. For instance, a Leontief utility function assumes that consumers do not substitute between goods as prices change. Each utility function is associated with its own cost function, and if a price index coincides with the ratio of two cost functions for a particular utility function, it can be thought of as representing the COLI for those particular preferences. Two noteworthy price indices that do just this are:

1) The Laspeyres index

$$P_L(\mathbf{p}_0, \mathbf{p}_1) = \frac{\sum p_1^i q_0^i}{\sum p_0^i q_0^i} = \sum w_0^i \left(\frac{p_1^i}{p_0^i}\right)$$

where w_{i0} gives the budget shares of good i in period 0. This corresponds to the Leontief preferences referred to above.

2) The Geometric Layspeyres

$$P_{GL}(\mathbf{p}_0, \mathbf{p}_1) = \prod_{i=1}^{N} \left(\frac{p_1^i}{p_0^i}\right)^{w_{i0}} = \exp(\sum w_{i0} \ln \left(\frac{p_1^i}{p_0^i}\right))$$

This corresponds to Cobb-Douglas preferences (also known as constant shares), where a 1% increase in the price of a good results in a 1% reduction in the quantity demanded (thus keeping the good's budget share constant).

If we think that substitution between products occurs *within* certain groups but not *between* those groups and others (or if preferences within the group are described by Cobb-Douglas or Leontief 'subutility' functions), then we could calculate sub-COLIs within groups using these formulae and then combine them to get an overall index in a manner similar to the process of aggregation used to construct the RPI and CPI. For instance, a Geometric Laspeyres could be used within categories of goods where we thought substitution responses were realistically described by Cobb-Douglas preferences, and a Laspeyres index could be used if we thought that it was more appropriate to assume zero substitution.

These indices differ from the unweighted Carli and Jevons and Dutot indices that are actually used in the RPI and CPI, but their resemblance is sometimes used to justify the choice of indices used in the calculation of the elementary aggregate price changes. The Jevons for example is thought to approximate a Geometric Laspeyres within an elementary stratum. This logic has however been criticised by Diewert (2012) who writes that "...the economic approach cannot be applied at the elementary level unless price and quantity information are both available." Since at the level of elementary aggregates such information is not available, it follows that the economic approach should have nothing to say on the subject of which index is preferable.

There are two problems with applying the economic approach when quantities are unknown. The first of these is that without knowledge of the weights which should be given to each price or price relative, we will not know if elementary indices are greater than or smaller than the Laspeyres and Geometric Laspeyres - in other words the direction and scale of their bias will be unknown. It is true that there are particular assumptions about the way prices are sampled under which elementary indices will equal their COLI counterparts. These assumptions (set out in chapter 20 of International Labor Organization, 2004) are as follows:

1) If the probability of sampling item i in the base period is equal to the ratio of the purchases of item i in the base period to the total purchases of all items in the elementary stratum in the base period, then the Dutot will equal the Laspeyres. That is

$$P_L(\mathbf{p}_0, \mathbf{p}_1) = \frac{\sum p_1^i q_0^i}{\sum p_0^i q_0^i} = \frac{\sum p_1^i \rho_0^i}{\sum p_0^i \rho_0^i}$$

if

$$\rho_0^i = \frac{q_0^i}{\sum_{j=1} q_0^j}$$

where ρ_{i0} is the base period sampling probability of good i.

2) If the relative prices of each product i is sampled with a probability s_{i0} equal to its base period expenditure share within the elementary stratum, then the probability weighted Carli

$$P_c(\mathbf{p}_0, \mathbf{p}_1) = \sum s_0^i \left(\frac{p_1^i}{p_0^i}\right)$$

will equal the Laspeyres.

3) If the relative prices of items are sampled in proportion to their base period expenditure shares in the elementary stratum, then the probability weighted log of the Jevons

$$\ln P_J(\mathbf{p}_0, \mathbf{p}_1) = \sum s_0^i \ln \left(\frac{p_1^i}{p_0^i}\right)$$

will equal the log of the Geometric Laspeyres.

Of these, conditions (2) and (3) will be true under random sampling the base period provided outlets stock goods in proportion to consumers' expenditures on them. However, this assumption is unlikely to hold in practice. The ONS price sample for instance is not random, but rather consists of a list of items judged to be representive of broader categories (and the selection of these representative items is often a matter of judgement, see Gooding, 2012).[18] If conditions (1), (2) and (3) are not true, and they are essentially impossible to verify, then our elementary indices may end up calculating something rather different from what we intended. For instance, if the probability of sampling item i is equal to the ratio of the purchases of item i in the *current* period to the total purchases of all items in the elementary stratum in the *current* period, then the Dutot will not equal a Laspeyres but would instead equal the Paasche index, or

$$P_P(\mathbf{p}_0, \mathbf{p}_1) = \left(\sum w_1^i \left(\frac{p_1^i}{p_0^i}\right)^{-1}\right)^{-1} = \frac{\sum p_1^i q_1^i}{\sum p_0^i q_1^i} = \frac{\sum p_1^i \rho_1^i}{\sum p_0^i \rho_1^i}$$

[18] The list of representative items is updated annually based on a range of considerations. In 2012 walking/hiking boots replaced outdoor adventure boots as a representative item for footwear. A bag of branded chocolate also replaced candy coated chocolate as a representative item as its price had been becoming more difficult to collect (for more details see Gooding, 2012).

if

$$\rho_1^i = \frac{q_1^i}{\sum_{j=1} q_1^j}$$

which will be downward biased (and may even be lower even than the Geometric Laspeyres). Thus, unless we had some *reason* to think that the Carli or Dutot will approximate a true Laspeyres, or that the Jevons would approximate a Geometric Laspeyres, then we should be wary about economic justifications for one elementary index over another.

The second problem from a lack of quantity information at this level is that it means we will be ignorant of the nature of the interdependence of prices and quantities (consumers' substitution responses), which is necessary to understand in order to choose whether our target COLI index should be a Laspeyres or Geometric Laspeyres.

However, the problem of a lack of information does not mean that we must give up on the economic approach to questions of this sort as Diewert argues. Indeed, drawing on insights from information theory, we can show that there is in fact a constructive principle which can be used to choose appropriate weights at the level of the elementary aggregates. This is the principle of maximum entropy (PME), which we now explain.

6.1 Principle of Maximum Entropy

Our problem is that the vector of budget shares is unknown at the elementary aggregate level. However if we had some grounds for selecting one particular vector of budget shares from the infinite number of possible combinations, then we would be able to calculate indices which would arguably approximate either the Laspeyres or Geometric Lasperes. The question is: why should we choose one particular combination over another? In situations where we have limited knowledge, the PME provides a criterion which we can use to guide our choice of budget shares. This was first proposed by Jaynes in two papers (Jaynes, 1957a,b) in the context of selecting a probability distribution.

To see how this approach works, consider the following example. Suppose we have a dice that has been rolled many times. By 'many' we mean that a sufficient number for us to ignore any problems of sampling variation. Suppose that the only thing we are told about these dice rolls is the value of the average roll. What can we say about the probability of rolling a particular number given only this information? This problem would normally be considered insoluble, as Jaynes (1983) notes "on orthodox statistical theory, the problem is ill-posed and we have no basis for making any estimate at all."

Laplace's *principle of insufficient reason* provides us with a first step for assigning probabilities in situations such as these. This states that in any situation where you want to assign probabilities to different outcomes, you should set them to be equal unless you have reason to do otherwise. The maximum entropy combines the principle of insufficient reason with any information we do have, and in doing so reflects the idea that we do not want to favour any outcome unless we have adequate justification to do so.

An objective function that will achieve this outcome is the entropy function proposed by Shannon (1948).

$$H(\mathbf{p}) = -\sum_i p_i \ln p_i$$

where in the dice example p_i is the probability of rolling number i.

This function is maximised when probabilities are uniform and minimised when probabilities are degenerate on a particular outcome. In any given application, we will want to maximise entropy subject to constraints given by the knowledge we have (in the dice example, subject to knowledge of the average roll). The constrained optimum to this problem then best represents the current state of knowledge. To choose a distribution with lower entropy than the solution would be to assume information (as measured by Shannon's function) which we do not possess. To choose a distribution with higher entropy would violate the constraints provided by the information which we do possess from the data. By solving this problem, the maximum entropy approach provides us with estimates of probability distributions in cases where there is insufficient information to use standard statistical methods.

6.2 Application of Maximum Entropy to Elementary Aggregates

The PME is traditionally applied to situations where we must choose a vector of probabilities. To apply it to our case, we need only note that the budget shares $(\mathbf{w}_0, \mathbf{w}_1)$ have all of the necessary properties of probabilities so we can apply to the PME to these in the same way. In particular they conform to the Kolomogorov axioms of probability measures (so by definition we can treat them exactly like probabilities).

This suggests the following entropy measure

$$H(\mathbf{w}_0, \mathbf{w}_1) = -\sum_t \mathbf{w}_t' \ln \mathbf{w}_t$$

where the budget shares take the place of the probabilities. In the simplest case in which we have no other information (i.e. no constraints) the maximum entropy problem is

$$\max_{\mathbf{w}_0, \mathbf{w}_1} H(\mathbf{w}_0, \mathbf{w}_1) = -\sum_t \mathbf{w}_t' \ln \mathbf{w}_t$$

which is solved by choosing equal budget shares (see the proof of proposition 4 below). The intuition behind this solution is as follows. Our problem for selecting budget shares at the level of the elementary aggregates is analagous to the dice problem but in a case where we do not even know the average roll. It seems we just cannot know what the budget share of each individual good is in the same way as we couldn't know what the chances of rolling a 1 in the dice example were, which is the reason for rejecting the economic approach. However, just as we *can* assign some probabilities to dice rolls using the principle of insufficient reason, we can similarly assign weights using a budget share equivalent: if we do not have any reason to think that one good should have a greater or smaller budget share than any another, we will assign them all equal budget shares.

Suppose now that we also have available the *total* expenditure on the sum of all of the items in the elementary stratum in each period: denoted $\{x_0, x_1\}$ where $x_0 = \mathbf{p}_0'\mathbf{q}_0$ and $x_1 = \mathbf{p}_1'\mathbf{q}_1$. This is the kind of data which may be used to weight elementary aggregates at the next level up. Given this additional data the economic approach to index numbers provides constraints on the budget shares. They must satisfy certain axioms of consumer behaviour provided by GARP:[19]

$$\{\mathbf{p}_0, \mathbf{p}_1; \mathbf{w}_0, \mathbf{w}_1; x_0, x_1\} \text{ satisfies GARP}$$

GARP is a set of inequalities involving the prices, budget shares and total expenditures[20] which provide necessary and sufficient conditions for the standard economic model of consumer choice. Note that these restrictions are fully nonparametric in the sense that they do not require any knowledge of the consumer's preferences. These constraints can then be added to the maximum entropy problem which becomes:

$$\max_{\mathbf{w}_0, \mathbf{w}_1} -\sum_t \mathbf{w}_t' \ln \mathbf{w}_t \text{ subject to } \{\mathbf{p}_0, \mathbf{p}_1; \mathbf{w}_0, \mathbf{w}_1; x_0, x_1\} \text{ satisfies GARP} \tag{8}$$

[19] The Generalised Axiom of Revealed Preference. For details see Afriat (1967), Diewert (1973) and Varian (1982).

[20] Typically GARP is applied to prices and quantities but it can easily be rewritten in terms of prices and budget shares since $q_t^i = w_t^i x_t / p_t^i$.

The result will be a set of weights which satisfy economic theory and the informational content (as measured by Shannon's index) of the data. We can show that this problem is solved by equal budget shares in both periods (since this solves the unconstrained problem and it turns out that the restrictions from GARP are not binding).

Proposition 4 *The solution to the maximum entropy problem (8) is* $w_t^i = 1/N$ *for all* i, t*.*

Proof. See appendix. ■

This means that when you have no data on quantities or budget shares, the PME provides a constructive argument for equal shares across good and periods of time. These budget shares would be chosen by consumers who had equally weighted Cobb-Douglas preferences. In terms of the choice of elementary index, this would justify the Geometric Laspeyres as a COLI (since this corresponds to the COLI for Cobb-Douglas preferences), and also justify the Jevons index (since when budget shares are uniform, an unweighted index will equal the COLI). To choose different vectors of budget shares would assume information which we do not have at this level, and so would not be justified without additional evidence.

7 Conclusion

Following Diewert (2012), we have evaluated the elementary indices used in the RPI and CPI under three approaches to index numbers: the test approach, the stochastic approach and the economic approach. There are a few respects in which our conclusions differ from those of Diewert however.

Under the test approach, we agree that the Carli index fails to satisfy various properties which we would expect of a price index, including the important time reversibility test. We also find that the Carli fails a new, revised version of the price bouncing test. However, the Carli's failure to satisfy time reversibility does not provide very strong reasons to replace it in the RPI, an index which is itself not time reversible, and which would not be improved in this regard if the Carli index were replaced.

The stochastic approach depends crucially on what object of interest is being estimated. We believe that this is the expected value of price relatives. The Carli is an unbiased estimator of this, while the Jevons is not. The Jevons may still perform better as an overall estimator of our object of interest however if we judge each index by its mean squared error, as it can have a lower variance than the Carli. The evidence on which estimator performs best in practice is at present however, mixed. Further research could shed more light on this issue.

Diewert maintains that the economic approach cannot be applied at the level of elementary indices, where quantity information is by definition not available. However, we show that in the absence of additional information, the principle of maximum entropy provides a constructive argument for equal shares across goods and across periods. This approach provides justification for both the Jevons as an approximation to the Geometric Laspeyres and the Geometric Laspeyres as a target index.

The ONS rejects the interpretation of the RPI as a cost of living index, and by implication also rejects the economic approach as a means of selecting the appropriate index. Since at present the stochastic approach does not appear to offer clear guidance on this issue, this leaves the test approach. There remains a case against the Carli under this approach, but its failure to satisfy time reversal is perhaps not as serious as first appearances would suggest.

Appendix

Proof. Proof of proposition 1.

Using $x_k = \frac{p_1^i}{p_0^i}$, the classical geometric-arithmetic inequality is

$$\prod \left(x^i\right)^{1/N} \leq \sum \frac{1}{N} x^i$$

Using the change of variables

$$x^i = \left(y^i\right)^s$$

and substituting

$$\prod \left[\left(y^i\right)^s\right]^{1/N} \leq \sum \frac{1}{N} \left(y^i\right)^s$$

then taking the sth root gives

$$\prod \left(y^i\right)^{1/N} \leq \left(\sum \frac{1}{N} \left(y^i\right)^s\right)^{1/s}$$

For $s \in (0,1)$ we can use Jensen's inequality again to give

$$\left(\sum \frac{1}{N} \left(y^i\right)^s\right)^{1/s} \leq \sum \frac{1}{N} y^i$$

since z^s is concave with $s \in (0,1)$ $[\sum \frac{1}{N} \left(y^i\right)^s \leq \left(\sum \frac{1}{N} y^i\right)^s]$. Now set $s = 1/2$. Then since $Var\left(X\right) = E\left(X^2\right) - \left[E\left(X\right)\right]^2$

$$\begin{aligned} E\left(X\right) &= \sum \frac{1}{N} \left(y^i\right)^{1/2} \\ \left[E\left(X\right)\right]^2 &= \left(\sum \frac{1}{N} \left(y^i\right)^{1/2}\right)^2 \\ E\left(X^2\right) &= \sum \frac{1}{N} y^i \end{aligned}$$

$$Var\left(X\right) = \sum \frac{1}{N} y^i - \left(\sum \frac{1}{N} \left(y^i\right)^{1/2}\right)^2$$

Using the fact that $\prod \left(y_i\right)^{1/N} \leq \left(\sum \frac{1}{N} \left(y^i\right)^s\right)^{1/s}$ which we have already established with $s = 1/2$ gives

$$\prod \left(y^i\right)^{1/N} \leq \left(\sum \frac{1}{N} \left(y^i\right)^{1/2}\right)^2$$

so

$$Var\left(X\right) \leq \sum \frac{1}{N} y^i - \prod \left(y^i\right)^{1/N}$$

setting s = 1, then gives

$$P_C\left(\mathbf{p}_0, \mathbf{p}_1\right) - P_J\left(\mathbf{p}_0, \mathbf{p}_1\right) \geq Var\left(\frac{p_1^i}{p_0^i}\right)$$

■

Proof. Proof of proposition 2.

First notice that we can rewrite the Dutot as

$$P_D\left(\mathbf{p}_0,\mathbf{p}_1\right)=\frac{E\left[p_1^i\right]}{E\left[p_0^i\right]}=\frac{E\left[\left(\frac{p_1^i}{p_0^i}\right)p_0^i\right]}{E\left[p_0^i\right]}$$

Then notice that the definition of the covariance between p_{i0} and $\left(\frac{p_{i1}}{p_{i0}}\right)$ is

$$Cov\left(p_0^i,\left(\frac{p_1^i}{p_0^i}\right)\right)=E\left[p_0^i\left(\frac{p_1^i}{p_0^i}\right)\right]-E\left[p_0^i\right].E\left[\left(\frac{p_1^i}{p_0^i}\right)\right]$$

$$\Rightarrow Cov\left(p_{i0},\left(\frac{p_1^i}{p_0^i}\right)\right)/E\left[p_0^i\right]=E\left[p_0^i\left(\frac{p_1^i}{p_0^i}\right)\right]/E\left[p_0^i\right]-E\left[\left(\frac{p_1^i}{p_0^i}\right)\right]$$

This is just the difference between the Dutot and the Carli, so we have that

$$\Rightarrow P_D(\mathbf{p}_0,\mathbf{p}_1)-P_C(\mathbf{p}_0,\mathbf{p}_1)=\frac{Cov\left(p_0^i,\left(\frac{p_1^i}{p_0^i}\right)\right)}{E\left[p_0^i\right]}$$

■

Proof. Proof of proposition 3.

Writing prices as in equation (4), we can think of the Dutot as the empirical counterpart of

$$P_D\left(\mathbf{p}_0,\mathbf{p}_1\right)=\frac{E\left(p_1^i\right)}{E\left(p_0^i\right)}$$

and the Jevons is the counterpart of

$$P_J\left(\mathbf{p}_0,\mathbf{p}_1\right)=\prod\left(\frac{E\left(p_1^i\right)\left(1+e_1^i\right)}{E\left(p_0^i\right)\left(1+e_0^i\right)}\right)^{1/N}=\frac{E\left(p_1^i\right)}{E\left(p_0^i\right)}\prod\left(\frac{1+e_1^i}{1+e_0^i}\right)^{1/N}$$

Rearranging gives

$$P_J\left(\mathbf{p}_0,\mathbf{p}_1\right)=P_D\left(\mathbf{p}_0,\mathbf{p}_1\right)\prod\left(\frac{1+e_1^i}{1+e_0^i}\right)^{1/N}$$

The Jevons is equal to the Dutot multiplied by a function of the deviations in each period. We can approximate the value of $\prod\left(\frac{1+e_1^i}{1+e_0^i}\right)^{1/N}$ by taking a second order Maclaurin expansion.

Let

$$\prod\left(\frac{1+e_1^i}{1+e_0^i}\right)^{1/N}=f(\mathbf{e}_1,\mathbf{e}_0)$$

then our approximation is

$$f(\mathbf{e}_1,\mathbf{e}_0)\approx f(\mathbf{0},\mathbf{0})+\left[\begin{array}{cc}\left.\frac{\partial f(\mathbf{e}_1,\mathbf{e}_0)}{\partial\mathbf{e}_1}\right|_{\mathbf{e}_1,\mathbf{e}_0=0} & \left.\frac{\partial f(\mathbf{e}_1,\mathbf{e}_0)}{\partial\mathbf{e}_0}\right|_{\mathbf{e}_1,\mathbf{e}_0=0}\end{array}\right]\left[\begin{array}{c}\mathbf{e}_1\\ \mathbf{e}_0\end{array}\right]+\frac{1}{2}\left[\begin{array}{cc}\mathbf{e}_1' & \mathbf{e}_0'\end{array}\right]\left[\begin{array}{cc}\frac{\partial^2 f(\mathbf{e}_1,\mathbf{e}_0)}{\partial\mathbf{e}_1\partial\mathbf{e}_1'} & \frac{\partial^2 f(\mathbf{e}_1,\mathbf{e}_0)}{\partial\mathbf{e}_0\mathbf{e}_1'}\\ \frac{\partial^2 f(\mathbf{e}_1,\mathbf{e}_0)}{\partial\mathbf{e}_1\partial\mathbf{e}_0'} & \frac{\partial^2 f(\mathbf{e}_1,\mathbf{e}_0)}{\partial\mathbf{e}_1\partial\mathbf{e}_1'}\end{array}\right]\left[\begin{array}{c}\mathbf{e}_1\\ \mathbf{e}_0\end{array}\right]$$

The derivatives of $f(\mathbf{e}_1,\mathbf{e}_0)$ are the following

$$\left.\frac{\partial f(\mathbf{e}_1,\mathbf{e}_0)}{\partial e_1^i}\right|_{\mathbf{e}_1,\mathbf{e}_0=0}=1/N,\forall i$$

$$\left.\frac{\partial f(\mathbf{e}_1,\mathbf{e}_0)}{\partial e_0^i}\right|_{\mathbf{e}_1,\mathbf{e}_0=0}=-1/N,\forall i$$

$$\left.\frac{\partial^2 f(\mathbf{e}_1,\mathbf{e}_0)}{\partial e_1^i \partial e_0^j}\right|_{\mathbf{e}_1,\mathbf{e}_0=0} = -(1/N)^2, \forall i,j$$

$$\left.\frac{\partial^2 f(\mathbf{e}_1,\mathbf{e}_0)}{\partial e_1^i \partial e_1^j}\right|_{\mathbf{e}_1,\mathbf{e}_0=0} = \left.\frac{\partial^2 f(\mathbf{e}_1,\mathbf{e}_0)}{\partial e_0^i \partial e_0^j}\right|_{\mathbf{e}_1,\mathbf{e}_0=0} = (1/N)^2, \forall i \neq j$$

$$\left.\frac{\partial^2 f(\mathbf{e}_1,\mathbf{e}_0)}{\partial (e_1^i)^2}\right|_{\mathbf{e}_1,\mathbf{e}_0=0} = \left.\frac{\partial^2 f(\mathbf{e}_1,\mathbf{e}_0)}{\partial (e_0^i)^2}\right|_{\mathbf{e}_1,\mathbf{e}_0=0} = \frac{1}{N}\left(\frac{1}{N}-1\right), \forall i$$

So our approximation evaluates to

$$= 1+\frac{1}{2}\left[\left(\frac{1}{N}\sum e_1^i\right)^2 - \left(\frac{1}{N}\sum (e_1^i)^2\right) - 2\left(\frac{1}{N}\sum e_1^i\right)\left(\frac{1}{N}\sum e_0^i\right) - \left(\frac{1}{N}\sum e_0^i\right)^2 + \left(\frac{1}{N}\sum (e_0^i)^2\right)\right]$$

$$= 1+\frac{1}{2}\left[\left(\frac{1}{N}\sum (e_0^i)^2\right) - \left(\frac{1}{N}\sum (e_1^i)^2\right)\right]$$

so we have that

$$P_J(\mathbf{p}_0,\mathbf{p}_1) \approx P_D(\mathbf{p}_0,\mathbf{p}_1)\left(1+\frac{1}{2}\left[Var\left(e_0^i\right) - Var\left(e_1^i\right)\right]\right)$$

■

Proof. Proof of proposition 4.

This proof consists of two stages. First we show that the solution to the maximum entropy problem (8) subject to the constraints that preferences satisfy GARP is $w_t^i = 1/N$ for all i, t. Then we show that constant and equal budget shares are consistent
with GARP, and so the additional constraint in this problem is not binding.

The solution to $\max_{w_0,w_1} -\sum_t w_t' \ln w_t$subject to $\sum_t w_t = 1$ is

$$\max_{w_0,w_1} -\sum_t w_t' \ln w_t - \lambda(\sum_t w_t - 1)$$

where λ is the Lagrange multiplier.

$$\implies w_t^i \frac{1}{w_t^i} + \ln w_1^i - \lambda = 0$$

$$\implies \ln w_t^i = \lambda - 1, \forall i, t$$

$$\implies w_t^i = \exp(\lambda - 1), \forall i, t$$

which implies that budget shares are constant across i and t. Combining this with constraint tells us that the entropy maximising budget shares will be $1/N$. This completes the first stage of our proof.

To prove the second stage our strategy will be to show that a violation of GARP is impossible with equal budget shares. A violation of GARP implies that there exist two periods t and s, when the consumer chooses quantities q_t and q_s such that:

$$\mathbf{p}_t^{'}\mathbf{q}_s \leq x_t$$

and

$$\mathbf{p}_s^{'}\mathbf{q}_t < x_s$$

or equivalently

$$\sum p_t^i \left[\frac{w_s^i x_s}{p_s^i}\right] \leq x_t$$

$$\Rightarrow \sum \left(\frac{p_t^i}{p_s^i}\right) w_s^i \leq \frac{x_t}{x_s}$$

and

$$\sum \left(\frac{p_s^i}{p_t^i}\right) w_t^i < \frac{x_s}{x_t}$$

Our working assumption is that budget shares are constant $w_t^i = w_s^i = \frac{1}{N}$ for all i,t. So these conditions imply that

$$\frac{1}{N}\sum \left(\frac{p_t^i}{p_s^i}\right) \leq \frac{x_t}{x_s}$$

and

$$\frac{1}{N}\sum \left(\frac{p_s^i}{p_t^i}\right) < \frac{x_s}{x_t}$$

Now we know from Jensen's inequality that

$$\frac{1}{\frac{1}{N}\sum \left(\frac{p_t^i}{p_s^i}\right)} \leq \frac{1}{N}\sum \left(\frac{p_s^i}{p_t^i}\right)$$

since the reciprocal is a convex function. However since for positive prices and expenditures

$$\frac{1}{\frac{1}{N}\sum \left(\frac{p_t^i}{p_s^i}\right)} \geq \frac{x_s}{x_t}$$

then this implies that

$$\frac{x_s}{x_t} < \frac{x_s}{x_t}$$

which is a contradiction. It follows that equal budget shares satisfy GARP. ■

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