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The Right to Buy Public Housing in Britain: A Welfare Analysis

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Abstract

We investigate the impact on social welfare of the UK policy introduced in 1980 by which public housing tenants (council housing in UK parlance) had the right to purchase their houses at heavily discounted prices. This was known as the Right to Buy (RTB) policy. Although this internationally-unique policy was the largest source of public privatization revenue in the UK and raised home ownership as a share of housing tenure by around 15 percentage points, the policy has been little analyzed by economists. We analyze the equilibrium housing policy of the public authority in terms of quality and quantity of publicly-provided housing both in the absence and presence of a RTB policy. We examine the incentives to purchase using RTB for households with different wealth trajectories and differing qualities of public housing. We investigate the welfare effects of various adjustments to the policy, in particular (i) tighter restrictions on resale; (ii) reduced discounts on RTB sales; and (iii) returning the proceeds from RTB sale to local authorities to replace part of the public properties sold.

JEL Classification Codes: I38; R38

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1 Introduction

This paper undertakes what we believe to be the first welfare analysis of the Right to Buy (RTB) policy for publicly-owned housing that was developed in the late 1970s in the United Kingdom (UK) and which became a flagship policy of the Thatcher government. RTB allowed tenants in publicly-owned council housing to buy their rented accommodation at a heavily subsidized price (albeit with subsidies that varied both over time and geographically across local jurisdictions). Overall, RTB was largely responsible for an increase in the share of home ownership among householders in the UK from 55% in 1979 to over 70% in the early 2000s, thereby inducing a large-scale change in asset ownership among UK households in a relatively short period. Despite being an innovative and internationally-unique policy, RTB has been little analyzed by economists in Britain and elsewhere.

In this paper, we examine the incentives implied by the RTB regime in the context of a model of optimal housing choice among heterogeneous households. A standard result in housing models is that households with more risky income profiles would, *ceteris paribus*, prefer to rent rather than buy.¹ We show how the effect of RTB was to induce households with risky incomes to utilize RTB. We examine the intrinsic incentives contained within the program and show that these could potentially conflict with the underlying objectives of the policy. We predict a change in the composition of tenants in public housing over time as a result of RTB, and construct a formal model of who buys under RTB.

The paper is structured as follows. In the remainder of this section, we explain the origins and development of the RTB policy in greater detail. Section 2 constructs a standard model of the choice between home ownership and private renting where agents are risk-averse, face a down payment constraint on house purchase, and differ in their degree of income volatility. Section 3 introduces a public housing (council house) sector, providing subsidized rentals targeted at low income families. We show that when the authorities only observe council housing applicants' current incomes (verification of past income volatility is costly), it may be optimal to house all applicants by varying the quality of housing within the public budget constraint. Alternatively, a waiting list system may reduce the demand for council housing by giving the authorities greater information on the income trajectory of applicants. Given an exogenous budget constraint for construction and renovation, the ratio of private to public sector rents, and preferences over housing quality, we derive an equilibrium solution whereby the local authority matches the supply and average quality of the public housing stock to demand. The 'mix' of council houses and tenants thereby generated are therefore the pre-set variables when we consider sales of council houses.

Section 4 then introduces a RTB policy without resale restrictions on RTB purchases. We show that, so long as the quality of council houses is not too low, all households with at least some prospect of high incomes exercise their right to purchase. Under reasonable assumptions, there is likely to be a secondary market in which some RTB properties are resold and some RTB purchasers revert to private renting. Once a large proportion of the stock is sold off, however, there are welfare costs from the policy to would-be council house renters, as well as a change in the composition of residual occupants of council houses. Again, however, the pre-existing average quality of council houses is an important determinant of how the policy evolves. Section 5 investigates the implications for social welfare of three adjustments

¹See Ioannides (1979), Rosen et al (1984). This result finds empirical support in, inter alia, Robst et al (1999). Among a number of subsequent contributions, Ortalo-Magne and Rady (2002) argue that the standard result hinges on the nature of the covariance between volatility in income and asset prices.

to the RTB policy: (i) imposing resale restrictions on RTB properties; (ii) reducing the discounts on RTB sales; and (iii) using the resulting proceeds to build new public properties. We show that some of these policies are only effective under certain conditions (primarily, again, the quality of the existing council housing stock). Section 6 concludes the paper. Graphs of evidences and proofs of propositions are provided in appendices.

1.1 Background: Council housing and the development of ‘Right to Buy’

‘Council housing’ (the term for public housing constructed by local government authorities in the United Kingdom), was a policy developed from the 1920s onwards in order to re-house low income families in subsidized housing. The need to re-house families arose from an ongoing policy of slum clearance in the 1920s and 1930s; a need heightened from the 1940s onwards because 4 million UK houses were seriously damaged or destroyed by bombing between 1939 and 1945. Aided by the 1946 Land Acquisitions Act, which allowed local authorities to acquire land for construction, and by direct subsidies from central government, council house building accelerated after 1945 and continued well into the 1970s. By 1979, around 32% of all dwellings in Britain were council houses, totaling some 6.5 million properties - a far greater share of the total housing stock than that of public housing in, for example, the United States.²

Council housing is normally allocated to families by a simple queuing mechanism with priority given to families with special housing needs (state of existing accommodation, severe health problems, eviction unrelated to personal behavior, homelessness, etc) but also more generally to those with low incomes and/or observed proxies for low income such as family size and employment status. For those without priority needs, waiting times could be as long as several years or indeed indefinite, although in the heyday of council housing a local authority could usually offer a property of some kind within a shorter period. For tenants, the attractiveness of council housing is that rents are usually significantly lower than those for comparative-sized privately-owned properties. Maintenance costs are in general covered by local authorities. This was made possible in part by direct subsidies for construction from central government and the development of low-cost system building methods, which, however, subsequently sometimes led to higher maintenance costs falling on the local authority.

As a relatively cheap solution to a major social problem, council housing was generally regarded as a major success story in public housing policy in the United Kingdom. However, the policy had obvious costs. Rents were subsidized and did not cover full economic cost, so local authorities bore a burden that was partly subsidized by block grants from central government. Because council housing was subsidized, generally excess demand prevailed. Families who got on a council house waiting list had no incentive to remove themselves from their list; indeed, even if families’ economic conditions improved, there was no incentive to exit the council house sector at all. This, it was suggested, reduced the flexibility of the labour market (e.g. Hughes and McCormick, 1981). And as the criteria for council house priority shifted from slum clearance to family ‘need’, there were strong incentives for families to assume the characteristics (lack of job, large number of children, partnership dissolution) that increased eligibility.

As real incomes increased, the need for further council house building seemed less clear

²Jones and Murie (2006) Table 2.2. and p.52.

cut. Although significant council house building continued into the late 1970s, the building program peaked in the early 1950s. A succession of Conservative governments began to emphasize home ownership as a means of distributing wealth more widely; hence some Conservative local authorities began to see their council housing stock both as a burden on local taxpayers (subsidized rents and maintenance costs) but also as a potential means of widening asset ownership in their local communities with the potential for externalities arising from a better social mix of residents. There was strong pressure on national government from these local authorities to permit a policy of subsidized selling of the existing council house stock (albeit with equal resistance from the Labour Party and the architects of post-war housing policy). It was the arrival of the Conservative central government under Margaret Thatcher with a large majority in Parliament in 1979, however, that provided the key impetus for a national Right to Buy policy.

1.2 Evolution of Right to Buy

The Housing Act of 1980 introduced a statutory right to buy for council tenants with at least three years' tenure in their council house - 'statutory' in the sense that the policy should be implemented in all local authorities and not just voluntarily by a few Conservative councils. Discounts on the sale price relative to the market value of the property ranged from 33% for council house tenants with three years' residence through to a maximum of 50% after twenty years' residence. Local authorities were also required to make mortgages available to would-be purchasers albeit subject to standard age limits and income multiples. The discount would be repayable if the property was sold within five years of a RTB purchase and there was a floor price such that recently constructed properties should not be sold at less than the cost of construction. Subsequent legislation in the 1980s relaxed the conditions still further – for example by increasing the discounts for owners of apartments and relaxing the cost floor condition.

Graph 1 shows clear peaks in council house sales arising from the 1980 Act and the liberalization of eligibility conditions and increased discounts in the mid-1980s. Note that new council house builds almost disappeared after the introduction of RTB although construction of other forms of social housing (around 20,000 new builds a year) continued. Nevertheless, although sales were at a lower level after these spikes associated with policy changes and the general decumulation of the council housing stock, they continued into the mid-2000s at a significant level for two reasons. First, new tenants became eligible by attaining residency requirements and increasingly took advantage of the relatively low interest rates through much of the later period. Second, because of residual opposition to the policy from some Labour councils, the process and speed of sales varied from local authority to local authority. For example, some councils opposed to the policy initially attempted to evade the statutory provisions to facilitate sales by handing all or most of their council house stock over to community or social housing associations where conditions for sale were considerably stricter until these were liberalised in the mid-2000s – hence the further upsurge in RTB sales in the early-2000s. These policies delayed the peak of sales in some localities.

By the mid-2000s, around 2.8 million council houses had been sold in the United Kingdom – almost half the total stock and indeed greater than half in areas such as Scotland which had seen high rates of council house sales in the previous period (Jones and Murie, 2006). However the rate of RTB sales has slowed considerably for reasons to be described shortly.

Who were the gainers and losers from this policy? The clear winners from the policy were

those tenants who had been able to purchase relatively desirable properties at subsidized prices. Central government also gained because the bulk of receipts from council house sales were offset by a reduction in central grants to local authorities - that is, receipts were mostly transferred to central government. After 1990, local authorities normally had access to 25% of the capital receipts with the remaining 75% treated as 'reserved receipts' which could not be utilized for general funding of local services (including replacement house building). The sale of council houses was the 'largest privatization' undertaken in Britain in this period in terms of raising revenue for central government, exceeding the proceeds from the sale of any of the major public utilities throughout the late 1980s and early 1990s (House of Commons, 1999, p.11).

In contrast, the losers were, arguably, those tenants of public housing who were unable or unwilling to buy their council house or apartment because they had low incomes and/or a less desirable property. Apart from any sense of 'relative deprivation', there was an increasing risk of the public housing sector becoming over time a residual of the lowest quality housing inhabited by the lowest income groups - a process of 'ghettoization' at odds with the ideals of some founders of 'garden city' public housing built for the working poor in 1930s. For local authorities, too, sales were a mixed blessing. Whilst reducing the need for covering maintenance costs and subsidized rents, they had received little of the receipts from the council house sale programme and still had a statutory duty to rehouse the homeless and those with pressing housing needs. Malpass and Murie (1999) point out that subsidies paid by local authorities to cover council house tenants had fallen (in nominal terms) from £2.1 billion in 1980 to £1.2 billion by 1990 - a clear saving. Nevertheless, as those authors note, the total number of homeless families more than doubled over the same period, from 63,000 to 146,000. By being forced to house homeless families in unsuitable or more expensive housing (e.g. private tenancies or even hotels), the savings from council house sales were rapidly eroded.

The incoming Labour government in 1997 decided to tighten up the rules for selling council houses. A series of measures between 1998 and 2004 were designed to tighten eligibility, reduce discounts, to limit access to public mortgages and to reduce the capacity of RTB owners to resell their properties within a short period at a profit. It should be noted, however, that this was a period when private mortgages were relatively easily accessed, and when house prices and private house building were both increasing. Of particular interest were changes that came into effect in 1999. Until this point, there was a national maximum discount of £50,000. Clearly, since some regions (notably London) had higher house prices (and house price increases) than others, the effective subsidy was thereby of different value across the country. In 1999, ceilings were reduced and applied differentially across regions, with the highest ceiling in London (£38,000) and the lowest in northern England (£22,000). Subsequently, in 2003, the policy was changed such that ceilings could be reduced where there was clear evidence of public housing shortages (as evidenced by homelessness and queues for public housing). This led to further reductions in ceilings - not in the low housing cost areas such as the north, but in areas of greatest pressure on available public housing - namely London and the South East. In nine local authorities in the south east and all but two London boroughs, the maximum ceiling was reduced to £16,000 (Jones and Murie, 2006, p.40; House of Commons, 2012). Subsequently, for similar reasons, the national government in Scotland ceased council house sales altogether. However, in the same period the Right to Acquire (a policy similar to RTB that allows housing association tenants to buy their home at a discount) was introduced for tenants of many housing associations, thus ending the

means by which local councils could hand their housing stock over to subsidized community housing associations in order to evade the statutory RTB rules.

The consequences of this regional and time variation in subsidies is illustrated in Graphs 2A and 2B for selected regions from the financial year 1998-99 onwards. Note the faster average fall in the discount as a fraction of the sale price in London than in the highlighted regions and in England as a whole in Graph 2A. This stems from two factors: the reduction in ceilings described above, but also the fastest rate of growth in the assessed house value of properties sold in London than elsewhere. Graph 2B then shows the differential time path of RTB sales in different regions of England – not simply linked to differential subsidies, although this factor obviously played a role in the very sharp decline in sales in London after 2004-05.

This is not quite the end of the story of the RTB policy in Britain, however. In 2012 the incoming Coalition government between the Conservatives and the Liberal Democrats announced that they wished to ‘reinvigorate’ the RTB policy. The central policy proposal was to raise the maximum discount to £75,000 across all local authorities irrespective of local house prices and pressure on public housing outside London, and indeed to £100,000 in London (House of Commons, 2012). The discussion document also proposed that local authorities might be able to retain a greater fraction of the proceeds from council house sales, so long as these were used to build replacement affordable (i.e. relatively low quality) homes. Only a small fraction of the publicly-owned housing stock now remains outside the Right to Buy/Right to Acquire provisions, this being primarily purpose-built accommodation for the elderly and communally-owned social housing.

2 A Model of Housing Tenure

To examine choice of housing tenure, consider a discrete-time economy with two goods: a nondurable good, for example food, and a durable good: housing. Suppose that the nondurable good is produced and consumed within a single period and cannot be stored i.e. it fully depreciates over the period. And suppose that the durable good, housing, does not depreciate but requires maintenance costs: the costs of fully utilizing a house in each period are borne by the owner – whether the private homeowner or the private or public landlord. The cost equals a fraction $\delta \in (0, 1)$ of the end-of-period housing value.

There are different types of houses – for example by size, quality of build, amenity value etc, in both the private and public housing sector. Given this menu of choice, houses in the private sector are broadly allocated among families by such factors as household composition and household income. Public housing is also heterogeneous and will typically be matched to eligible households of given composition by an administrative formula (e.g relating numbers of rooms to size of family). In general we assume that public housing is distributed over a dimension of quality $\eta \in (0, 1)$ with the quality of any type of public housing at best equal to (i.e. normalized to 1) the equivalent privately-owned house and at worse of some lower minimum quality, 0.

2.1 Households

Following Kiyotaki, Michaelides, and Nikolov (2011), suppose that households *ceteris paribus* obtain higher utility from owner occupation relative to renting - whether private or public -

since ownership allows the household greater freedom in terms of home improvement, interior decoration etc. Therefore define $\phi \in (0, 1)$ as the parameter of disutility of renting versus owning a house. Households live forever and houses can be inherited. They generate utility from nondurable consumption, C , and durable housing, H . All households have the same constant relative risk aversion in total expenditure:

$$u(C, H, \mathbb{I}_{[rent]}) = C^\alpha \left((1 - \phi \mathbb{I}_{[rent]}) H \right)^{1-\alpha}, \quad 0 < \alpha < 1, \quad H \in \{1, \eta\}, \quad (1)$$

where α is the share of consumption in a household's total expenditure, $\mathbb{I}_{[rent]} = 1$ if the household is a tenant, and $\mathbb{I}_{[rent]} = 0$ if the household is a housing owner. Assume no household savings or borrowing except for mortgages.

Consider a very simple income process whereby there are two possible outcomes of household income in any period: a 'high' income Y^H and a 'low' income Y^L . Define λ as the probability of getting the high income; hence, $1 - \lambda$ is the probability of receiving the low income. Households differ in their lifetime wealth according to their cumulated probabilities of receiving Y^H and Y^L . We make no assumption as to the evolution of these probabilities over time for individual households, although it is likely that income 'shocks' (and hence draws from the income process) are likely to be positively correlated in the short run. As to the continuum of household lifetime wealth allocations arising from the income process, for expositional purposes we discretize this distribution into three wealth categories: 'high' wealth households who almost always receive 'high' income draws; 'middle' wealth households who receive 'high' and 'low' income outcomes with almost equal probability (such as self-employed or skilled manual workers), and 'low' wealth households who almost always receive 'low' income draws. Denote these three types of households as, respectively, λ^h (where $\lambda^h \cong 1$), λ^m (where $\lambda^m \cong 0.5$) and λ^l (where $\lambda^l \cong 0$) households.

We assume that the proportions of households in the three categories remain fixed over time. Hence define N^h as the number of high wealth households, N^m as the number of middle wealth households and N^l as the number of low wealth households, where $N^l, N^m, N^h > 1$ (i.e. there exists a positive number of each type of household). We also assume that the income 'draw' of any household in each period is observable, for example to the local public authority. However, the cumulated process of income draws ('wealth') is private information to the household. The primary focus of this paper is the decision to exercise the Right-to-Buy option on public housing, and (as we shall see) individuals with consistent draws of Y^H will not be eligible to enter the public housing sector. Hence, for much of the paper we will abstract from 'high' wealth households and focus on the tenure choices of 'low' and 'middle' wealth households as defined above. However, before establishing the equilibrium distribution of housing tenures with a public (council) housing sector and incentives to exercise Right-to-Buy, we examine briefly equilibrium in the private housing market, where the existence of high wealth households is important.

2.2 Equilibrium in the Private Housing Market

In this subsection we briefly derive a standard model of equilibrium tenure and pricing of owned and rented houses in the private sector. This derivation is required in the context of our subsequent 'Right-to-Buy' model because, as we shall see, council house tenants exercising Right-to-Buy may then have the option to sell their purchased council house and to choose to rent in the private sector.

The private housing market is competitive. The profit of constructing a property and selling it is zero. The housing price equals the construction cost of the house, which is assumed fixed over time for simplicity. Let P be the per unit housing price in the private housing market. There are sufficient exogenous funds available to finance households' home ownership. The gross interest rate on mortgages equals $1+r$. There is a fixed adjustment cost (FC) of purchasing a house, which is incurred when a tenant switches to home ownership. Mortgages are paid at the beginning of each period and amortized to infinity. Define M as the per period mortgage payment per quality of housing, then $M = \frac{r}{1+r}P$.

In the private housing rental market, suppose that there are competitive landlords that offer private rental properties. Define R as the equilibrium housing rent. By assumption there is a maintenance cost, δ , already defined. The housing rent is determined by the equivalence of the current cost and discounted payoff of leasing a house, then $R = \frac{\delta+r}{1+r}P$.

Having paid the fixed cost (FC) at the moment of purchasing a property, a homeowner pays the mortgage plus the discounted maintenance cost of the house. The per-period payment equals the equilibrium rent, Then the discounted cost of the owned property equals the mortgage payment plus maintenance cost, which equals the competitive private rent, R . The homeowner loses the house if he or she fails to pay R . He or she will however prefer to own rather than rent so long as the payment is made because of the disutility of renting, ϕ .

To ensure that every household lives in a property, assume that household incomes (either low or high) are greater than the equilibrium housing rent. The lump sum fixed cost of purchasing a property for new homeowners is large such that low income households cannot afford to purchase a private property. Only high income households can afford private properties.

Assumption 1 *The equilibrium housing rent, housing price, lump sum fixed cost for new homeowners, and household incomes satisfy: $R < Y^L < R + FC < Y^H < P$.*

In the private housing market, households either rent, purchase (new ownership), or own (retained ownership) private properties. If a household rents or owns a house at time t , it pays the housing rent each period, so its consumption is $C_t = Y_t - R$. A household can purchase a house only if it receives the high income. Its consumption at the time of a new purchase is $C_t = Y^H - R - FC$. Because of the disutility of renting relative to owning houses, private housing owners always prefer continuing living in their own properties to rented properties. The value of owning a property for a low or middle wealth household is:

$$\begin{aligned} V_o^i(Y_t) &= C_t^\alpha + (1+r)^{-1} E_t [V_o^i(Y_{t+1})], \quad i \in \{l, m\}, \\ &= (Y_t - R)^\alpha + r^{-1} (\lambda^i (Y^H - R)^\alpha + (1 - \lambda^i) (Y^L - R)^\alpha), \end{aligned} \quad (2)$$

where $Y_t \in \{Y^H, Y^L\}$. Only high income households can afford the fixed cost of purchasing houses. The state variables of a household's optimization problem at time t are its current income, Y_t , and whether it purchased or owned a house in the previous period, $\psi_{t-1} = \mathbb{I}_{[\text{own a house at time } t-1]}$. If the household owns a house, its value function equals the value of continuing living in the house; otherwise, it decides whether to purchase a house whenever it receives the high income. Then a household's value function at time t is:

$$V^i(Y_t, \psi_{t-1}) = \max_{\psi_t \in \{0,1\}} \{ \psi_{t-1} V_o^i(Y_t) + (1 - \psi_{t-1}) (V_r^i(Y_t) + \psi_t \mathbb{I}_{[Y_t=Y^H]} (V_p^i - V_r^i(Y^H))) \}.$$

where V_r^i and V_p^i are the household's values of renting and purchasing a private house, respectively. When the probability of receiving the high income is not too large, households

prefer renting to owning private houses when they receive the high income. To obtain the cutoff point, we solve the households' optimization problem recursively as described in Appendix B.1, to derive the following proposition:

Proposition 1 *A high income household prefers renting to owning a private house if and only if its probability of getting the high income is not too large such that $\lambda^i < \bar{\lambda}$, where:*

$$\bar{\lambda} = \frac{r \left((1 - \phi)^{1-\alpha} - \left(1 - \frac{FC}{Y^H - R}\right)^\alpha \right) - \left(1 - (1 - \phi)^{1-\alpha}\right) \left(\frac{Y^L - R}{Y^H - R}\right)^\alpha}{\left(1 - (1 - \phi)^{1-\alpha}\right) \left(1 - \left(\frac{Y^L - R}{Y^H - R}\right)^\alpha\right)}. \quad (3)$$

Our analysis focuses on the households which are interested in public housing. They usually have relatively low wealth. They prefer renting to purchasing private houses even if they receive the high income occasionally. We now make the following assumption:

Assumption 2 *Middle wealth households' probability of receiving the high income is not too high such that: $\lambda^m < \bar{\lambda}$. However, it is large relative to either the disutility of renting houses or low wealth households' probability of receiving the high income, and the number of middle wealth households is relatively small compared to that of low wealth households such that:*

$$\frac{\lambda^m - \left(1 - (1 - \phi)^{1-\alpha}\right)}{\lambda^m - \lambda^l} > \left(1 + \frac{(1 - \lambda^m) N^m}{(1 - \lambda^l) N^l}\right)^{-1} > 1 - \lambda^l.$$

Then $\lambda^l < \lambda^m < \bar{\lambda}$. In these circumstance, both low and middle wealth households prefer renting to purchasing private properties even when they receive the high income on occasions. The intuition is as follows. Low wealth households have a high probability of receiving low income. Even when they receive the high income, the high probability of later low income outcomes raises the possibility of costly future housing tenure changes. So they are not likely to purchase a private property even if they receive a high income for a period sufficient to overcome the initial fixed cost of purchasing a property.

The same story is likely to hold for middle wealth households' incomes. Although their incomes are more volatile, meaning they are often in a position to buy a property at market prices, risk-averse households may prefer not to buy when receiving a high income because they are trading the disutility of renting (versus owning) against the potential transactions and search costs arising from selling properties and finding alternative accommodation in the face of future income volatility. This accords with the existing literature on choice of housing tenure. As we shall see, however, subsidizing purchase through RTB may changes the value function of middle wealth households considerably.

We now extend the model to an additional form of tenure: public ('council') housing. In the following sections we first set up the optimal council housing system without RTB policy, then compare low and middle wealth households' welfare before and after the RTB policy (without resale restriction) was introduced. After that, we investigate how imposing resale restrictions on RTB in recent decades changes the social welfare.

3 Equilibrium of Housing Tenure with Council Housing

We now incorporate a council house sector (public housing) into the model. The rationale for, and mechanisms for allocating, council housing were described in Section 1. In this

section we examine the individual household’s decision as to whether or not to apply for a council house. We also examine the local authority’s decision as to what quantity of council houses to provide, and of what quality.

Initially, we examine a setting where there is no ‘Right to Buy’. We assume that the local authority’s key observable of ‘need’, other than obvious factors such as family composition, is current income. The council housing rental system comprises two components: existing tenancies and a waiting list. The local authority can verify that an applicant applying to join the waiting list for a council house has a low income. Note our presumption that a high wealth household (i.e. a household with a sequence of high incomes) will never be on the waiting list and is unlikely currently to be living in a council house. We assumed that lifetime permanent income or wealth is private information to the individual applicant i.e. the local authority only observes ‘high’ income Y^H or ‘low’ income Y^L . Thus the local authority will be faced with two kinds of applicants in our model: those with almost permanently low incomes (low wealth households), and those with low outcomes from a more volatile income process (middle wealth households). It is reasonable to assume that the local authorities’ first priority would be to house those with almost permanently low incomes, although they will have a degree of legal responsibility, resources permitting, to house anyone who would otherwise be homeless. Local authorities might try to differentiate between the two types of households by further investigation: for example the simplest device for revealing information on income trajectories is through a queuing system: households may have several income realizations before they reach the top of the waiting list, and some of those with only temporary low incomes may be sifted out by requiring households that receive a high income are required to leave the council housing queue. However, a certain fraction of those with volatile incomes may thereby still obtain council houses, and will want to do so insofar as risk aversion (and subsidized rentals) leads them to prefer council house tenancy to either private ownership or private renting.

However, another solution to this rationing and selection problem is to limit the demand for council housing by ensuring that the council house stock is of lower quality than equivalently-sized privately-owned properties, thereby reducing the demand for council houses and increasing the supply available within a fixed budget. Consequently, when the share of expenditure on high quality private housing rental is large for low income households, the local authorities are able to provide council housing of lower quality than private houses such that low income households are better off renting council houses, but high income (middle wealth) households are not so interested in public rentals even when they have low income realizations.

Assumption 3 *Low income households have too low income to benefit from renting private housing: $Y^L < \bar{Y}$, where \bar{Y} solves $\bar{Y} - \frac{r}{1+r}FC - \left(\frac{\bar{Y}-R}{\alpha}\right)^\alpha \left(\frac{R}{1-\alpha}\right)^{1-\alpha} = 0$.*

The supply of council houses is determined as follows: local authorities borrow money up to a given constraint from the central government in order to build houses. Over time, the local authorities can endogenize the quality of council housing to optimize their objective function subject to their initial capital budget constraint for housing, defined as B . As mentioned in Section 1, when RTB was introduced, the central government did not allow the local authorities to keep the bulk of their proceeds from council house sales to build replacement dwellings. Thus, B is assumed to remain fixed over time. Combining the budget constraint with the measure of quality chosen by the local authority gives the stock

of council houses available and their average quality in each period. Local authorities can build new council houses by borrowing from central government at a risk-free interest rate, so that the costs of a council house equal the amortized fixed cost, $\frac{r}{1+r}FC$, plus the council housing rent subsidies. Define τ as the proportion of private market rent payment required for a council housing tenant, where $0 < \tau \leq 1$. With η defined as the quality of council houses, council housing tenants pay rent that equals $\tau R\eta$ to the local authorities. Then the council housing rent subsidies equal $(1 - \tau)R\eta$. Assume that the local authorities' initial budget is sufficient to offer low quality council houses to all low income households (whether of low or middle wealth). However, higher quality builds will involve a longer waiting list both by reducing the quantity available and increasing demand.

Assumption 4 *The local authorities' initial budget on providing council housing for rent, B , is sufficient to cover the fixed cost of council housing for all low income households, but it is limited such that the quality of council housing is lower than private housing:*

$$0 \leq \frac{B}{\bar{N}} - \frac{r}{1+r}FC \leq \min \left\{ \frac{R}{1-\alpha}, \frac{Y^L}{\alpha} \right\} - Y^L,$$

where the total number of low income households is $\bar{N} = (1 - \lambda^l) N^l + (1 - \lambda^m) N^m$.

In such a case, the budget allows the local authority to offer council houses to all low and middle wealth households currently receiving a low income. However, there are underlying constraints on the decision of the local authority to provide council houses of a given quality relative to private rentals. The quality of council houses cannot be so low such that low income households are not interested in renting them. Nor can the quality be so high as to exceed the local authorities' budget. And in addition, it takes time to adjust the quality of the housing stock by new builds, renovations and conversions. Within these constraints, let N be the stock of council houses provided by local authorities. Define W_c as the aggregate welfare of low income households under council housing rental system. Then the local authorities' optimization problem is equivalent to:

$$\begin{aligned} \frac{rW_c}{(1-\phi)^{1-\alpha}} = \max_{\eta \in (0,1), \tau \in (0,1), N \leq \bar{N}} & N (Y^L - \tau R\eta)^\alpha \eta^{1-\alpha} + (\bar{N} - N) (Y^L - R)^\alpha \\ \text{subject to} & (Y^L - \tau R\eta)^\alpha \eta^{1-\alpha} > (Y^L - R)^\alpha; \quad (\text{Participation}) \quad (4) \\ & \frac{r}{1+r}FC + (1 - \tau)R\eta \leq \frac{B}{\bar{N}}. \quad (\text{Budget Constraint}) \end{aligned}$$

Since middle and low wealth households can choose to rent either council houses of lower quality or private houses with higher rents, an obvious solution to this problem is that the optimal number of council houses provided equals the number of middle and low wealth households with low incomes that choose to apply for council houses given the rent differential and the differences in quality between the public and private sectors. The optimal council house rental policy and the aggregate welfare of households in the council house sector can be summarized in the following proposition (a detailed derivation is provided in Appendix 2):

Proposition 2 *Under Assumption 1-4, the local authorities provide council houses of lower quality than private ones to all low income households. The number of council houses is:*

$$\bar{N} = (1 - \lambda^l) N^l + (1 - \lambda^m) N^m. \quad (5)$$

The optimal council housing quality is:

$$\eta = \frac{1 - \alpha}{R} \left(Y^L + \frac{B}{\bar{N}} - \frac{r}{1+r} FC \right); \quad (6)$$

and the optimal required payment on council housing rent is:

$$\tau = \frac{Y^L}{R\eta} - \frac{\alpha}{1 - \alpha} = \frac{Y^L - \alpha \left(Y^L + \frac{B}{\bar{N}} - \frac{r}{1+r} FC \right)}{(1 - \alpha) \left(Y^L + \frac{B}{\bar{N}} - \frac{r}{1+r} FC \right)}. \quad (7)$$

The aggregate welfare of the low income households becomes

$$rW_c = \bar{N} (1 - \phi)^{1-\alpha} \left(\frac{\alpha}{1 - \alpha} R \right)^\alpha \eta.$$

4 Unrestricted Right-to-Buy

In this section, we examine the impact on the welfare of low income households of applying the RTB policy to the council housing rental system. To focus on how RTB affects the aggregate welfare of the low income households, throughout the analysis we fix the budget of the local authorities, B , the quality of council housing, η , the discount on council houses, τ , and the number of council houses, N . We assume these parameters are determined by the equilibrium outlined in Proposition 2 in the previous section. We investigate whether RTB improves the welfare of low income households within these fixed parameters.

This section demonstrates that (i) the RTB policy improves the welfare of current low income households, but if and only if the council housing is of sufficiently low quality; (ii) that the RTB policy improves current public housing tenants' welfare at the expense of future low income households; and (iii) that RTB policy is likely to provide greater benefit to middle wealth households than to low wealth households. We show that given the inequalities in utility across tenure types underpinning tenure choice, those at the front of the queue among council house tenants to take advantage of RTB will be those for whom the 'excess return' to home ownership relative to continued renting is greatest. We also focus on the implications of whether RTB purchasers can resell their council houses without any time limit or other constraints (we term this 'unrestricted' Right-to-Buy) because a resale restriction has implications both for which tenants opt to exercise RTB and also for the quality of council houses sold under the policy.

4.1 Purchase through Right-to-Buy

Under RTB, council housing tenants are allowed to buy the council houses they are living in at a discounted market price. Once a council house is sold, it is permanently moved out of the public stock of housing. This affects future potential council tenants. The council housing assignment procedure from the local authorities' point of view therefore now contains three rather than two components: (i) a decision on the optimal stock of council houses; (ii) a waiting list policy; and (iii) a RTB policy. Clearly, these components interact.

The RTB works as follows. First, RTB gives an exogenously set discount on the council

house sale price given the council housing quality, η .³ For simplicity, assume that the annualized discount on the sale price (and therefore mortgage reduction) is equal to the original discount of rental payment. In addition, the RTB policy may alleviate the initial payment constraint because tenants may now be given access to mortgages (or greater mortgages) than they would have otherwise had. By this simplifying assumption, the required mortgage payment for a RTB-purchased house is the same as the cost of renting a council house but the flow of utility generated by the property increases because the disutility of renting relative to owning houses is removed.

RTB is therefore attractive to council tenants for two reasons. First, it reduces the price of home ownership by discounting the sale price. This is especially attractive to volatile income (middle wealth) households for several reasons: (i) if they subsequently have a series of low income, their recurrent cost is no higher than renting a council house; (ii) if they have a subsequent sequence of high incomes and have held the house for a minimum period, they can rent the RTB house out to a private tenant and rent elsewhere; or (iii) they have the possibility of selling the RTB house at market price and purchasing a different house. Thus, even the most risk averse middle wealth households might want to exploit RTB. In theory, the relaxation of the initial payment restriction and the lower sale price also potentially makes RTB attractive to low income households although accumulating enough capital to afford the down payment on even the subsidized purchase price may prove an insuperable problem.

RTB is in principle attractive to low income tenants whether they are middle or low wealth households, since their utility is increased by owning instead of renting a council house. Indeed, without restrictions on RTB eligibility, households prefer to purchase council houses to renting either council or private houses. This has an important consequence, however. Low and middle wealth households that do not have council houses but rent privately have an incentive to join the waiting list for council housing with a view to purchasing a council house at a later date. In the short run, therefore, the demand for council housing may even increase. The effect of this additional incentive on the size of the queue for council houses and, therefore, the potential effect on the quality of council housing in the future (given the fixed budget for council house construction) will depend on the priority criteria for RTB purchase established by the RTB policy (for example, priority or higher subsidies for long-duration tenants). While it is costless for private tenants to join the queue for council houses, if offered a council house, they are trading the lower quality of the public rented property relative to the private rental for the future possibility of obtaining a subsidized house purchase.

4.2 Equilibrium with Unrestricted RTB

We now consider the issue of whether a homeowner who has exercised RTB will resell their properties if permitted, which we term ‘unrestricted RTB’. We show that this decision will depend on the quality of the RTB-purchased property (hence we shall also show that any restriction on resale may affect the quality of properties sold). The next issue that arises is as to who would repurchase a property originally obtained under RTB. An important

³The discount rate may vary across local authorities but is set by central government, albeit sometimes amended as a result of lobbying by individual authorities (see the discussion in Section 1 of the period from 1997 onwards). However, to our knowledge, the actual discount rates and ceilings on subsidies chosen by the central government were not based on any explicit welfare analysis.

effect of the RTB policy, by our earlier assumptions, is to put properties with a different distribution of qualities onto the private housing market, whether for outright ownership or rental. Hence there will be some, primarily lower quality, properties on the private market as a result of RTB that might prove attractive to, for example, low wealth households which, for whatever reason, have not been in the council rental sector at the time that the RTB policy was implemented. Depending on these factors, we will demonstrate that resale restrictions may or may not increase overall welfare.

We assume that the average quality of council houses within any local authority available for RTB purchase is determined by the budget constraint, as illustrated previously, and varies across local authorities. If the average quality of council house properties available for RTB purchase in a particular locality is ‘high’, then middle wealth households will continue to wish to live in their RTB-purchased properties in the future. However, if the quality of council properties available for RTB is relatively low, middle wealth households who have exercised RTB might prefer to resell the RTB purchase and rent privately. Post-RTB tenure transitions may then arise since some lower wealth households who were not in the council house sector but were renting privately are able to purchase the lower quality property privately.

To consider these post-RTB tenure transitions, we can consider a simple model along the lines of Wheaton (1990). Renting or reselling RTB properties is subject to search frictions and delays in finding a suitable match. The search cost arises because in general RTB properties are of different (on average, lower) qualities than private houses and also the supply of RTB properties of given quality may be spatially distributed unevenly in the housing market as a whole. Hence we assume that homeowners continue to live in their RTB properties until a match is formed. The match surplus is then shared by both parties in relation to the bargaining power of the initial homeowner. Assume this bargaining power equals β where $0 \leq \beta \leq 1$ and that the matching function between RTB owners and ‘searchers’ follows a standard functional form such as Cobb-Douglas.

RTB rental or resale contracts are signed at the beginning of each period before household incomes are realized. The duration of RTB rental contract is one period. When a match of renting RTB housing is formed, the searcher rents the RTB house for one period and has to return the house to its initial owner afterward. He/she has to search again if (s)he wants to continue renting a RTB house. As the quality of council houses is lower than private ones, the marginal utility of renting a RTB house is strictly decreasing in the household’s income. Because low wealth households have a higher probability of getting the low income, they have higher utility from renting RTB houses than middle wealth households. Moreover, because of the disutility of renting relative to owning RTB properties, households having the same level of wealth will not form a profitable match. Hence the only profitable match on renting RTB houses is that middle wealth RTB homeowners rent out their RTB properties to low wealth households.

In contrast, when a match of reselling RTB housing is formed, the searcher purchases a RTB house and can continue living in it indefinitely. Low wealth households generate greater total welfare from owning RTB houses than middle wealth households, and middle wealth RTB homeowners may be interested in selling their RTB houses to low wealth searchers. Whether middle wealth RTB purchasers continue living in their RTB properties or sell them and rent (or ultimately purchase other) private housing therefore depends on the quality of their RTB property. If the quality of RTB properties is high, middle wealth RTB purchasers continue living in their properties whatever income they receive in the future. However, if the quality of their RTB properties is low, they will seek to resell their houses to the households

that have low wealth.

The disutility of renting relative to owning houses also affects the RTB rental and resale decision. If the disutility of renting is extremely high, all RTB homeowners will prefer living in their properties than renting out or selling their houses to private parties. Then there is no second market for RTB houses. At the other extreme, if the disutility of renting is extremely low, no initial council housing tenant is interested in purchasing the house he lives in under RTB. The equilibrium is then the same as that with the option of renting council houses discussed in the previous section. It seems highly likely that preferences are such that an intermediate setting, which generates both a RTB market and resale market, will exist in practice. Hence the issues of resale and resale restrictions are important components of the RTB policy.

Assumption 5 *The disutility of renting relative to owning a house is moderate such that:*

$$\frac{(Y^H - (1 - \alpha)Y^L)^\alpha - (\alpha Y^L)^\alpha}{(Y^H - R)^\alpha - (Y^L - R)^\alpha} < \left(\frac{(1 - \phi)R}{(1 - \alpha)Y^L} \right)^{1-\alpha} < \frac{(Y^H - (1 - \alpha)Y^L)^\alpha}{\frac{(Y^H - R)^\alpha - (Y^L - R)^\alpha}{\left(\frac{(1 - \alpha)Y^L}{R}\right)^{1-\alpha}} + (\alpha Y^L)^\alpha}.$$

The total surplus of either renting or reselling a RTB property from middle wealth households to low wealth households is strictly decreasing in the quality of initial council houses sold under RTB. Under Assumptions 4 and 5, middle wealth RTB homeowners sell instead of renting out their properties to low wealth households when they prefer renting private housing to owning RTB housing. Further, the difference between the increment in utility of renting RTB housing relative to private housing between the low and high income households is negative. It is greater than the surplus of a match on renting a RTB house, which equals the searcher's gain minus the provider's loss in current utility. Middle wealth RTB homeowners are not interested in renting out their houses to low wealth households because the surplus from the match is negative when the quality of council housing is less than that of private one. The surplus of a RTB housing resale at equilibrium, S , equals the searcher's gain minus the provider's cost in utility, which satisfies:

$$\begin{aligned} \frac{rS(\eta)}{\lambda^m - \lambda^l} &= \left(\frac{\alpha}{1 - \alpha} R \right)^\alpha \eta - (1 - \phi)^{1-\alpha} (Y^L - R)^\alpha \\ &\quad - \left(\left(\frac{Y^H - Y^L}{\eta} + \frac{\alpha}{1 - \alpha} R \right)^\alpha \eta - (1 - \phi)^{1-\alpha} (Y^H - R)^\alpha \right). \end{aligned} \quad (8)$$

At equilibrium, the value of owning a RTB house, V_{RTB}^i , and renting a private house, V_r^i , for a household of type i , $i \in \{l, m\}$ (low or middle wealth), satisfy, respectively:

$$rV_{RTB}^i = \left(\lambda^i \left(\frac{Y^H - Y^L}{\eta} + \frac{\alpha}{1 - \alpha} R \right)^\alpha + (1 - \lambda^i) \left(\frac{\alpha}{1 - \alpha} R \right)^\alpha \right) \eta; \quad (9)$$

$$rV_r^i = (1 - \phi)^{1-\alpha} (\lambda^i (Y^H - R)^\alpha + (1 - \lambda^i) (Y^L - R)^\alpha). \quad (10)$$

The effect of Right-to-Buy on the welfare of eligible council house tenants, when the quality of council houses is low, can be summarized in the following proposition, the proof of which is given in Appendix B.3.

Proposition 3 *When the quality of RTB houses is high such that $\eta > \bar{\eta}$, middle wealth RTB homeowners continue living in their houses. However, when the quality of RTB houses is low such that $\eta < \bar{\eta}$, middle wealth RTB homeowners sell their houses to low wealth households, where $\bar{\eta}$ solves equation (8) when the surplus of RTB housing resale is zero.*

RTB without restriction on resale improve the aggregate welfare of the low income households if and only if the quality of RTB houses is not too high. When the cutoff quality of positive surplus from RTB resale is low such that $\bar{\eta} < \underline{\eta}$, where:

$$\underline{\eta} \equiv \frac{\lambda^l + \Lambda}{1 - \frac{1-\lambda^l-\Lambda}{(1-\phi)^{1-\alpha}}} \left(\frac{Y^L - R}{\frac{\alpha}{1-\alpha} R} \right)^\alpha, \quad \Lambda \equiv (\lambda^m - \lambda^l) (1 - \lambda^m) \frac{N^m}{\bar{N}}, \quad (11)$$

the welfare of low income households is improved if the quality of RTB houses is low such that $\eta < \underline{\eta}$. When $\bar{\eta} > \underline{\eta}$, the welfare of low income households is improved if $\eta < \hat{\eta}$, where:

$$\hat{\eta} \in (\underline{\eta}, \bar{\eta}) \text{ solves } \hat{\eta} - \left(\frac{\frac{1-\lambda^l}{\lambda^m-\lambda^l} - \beta}{1 - \frac{1-\lambda^l-\Lambda}{(1-\phi)^{1-\alpha}}} \right) rS(\hat{\eta}) = \frac{\lambda^l + \Lambda}{1 - \frac{1-\lambda^l-\Lambda}{(1-\phi)^{1-\alpha}}} \left(\frac{Y^L - R}{\frac{\alpha}{1-\alpha} R} \right)^\alpha. \quad (12)$$

Otherwise, the RTB reduces the aggregate welfare of low income households.

This proposition arises from two features of our model: First, the RTB policy increases the aggregate welfare of households eligible for the policy by eliminating the disutility of renting houses of low quality from the local authorities. Second, the utility obtained by owners of RTB properties from continuing to live in their properties is strictly increasing in the quality of the property. If there are few restrictions on resale of RTB properties, as was broadly the case when the policy restrictions were relaxed during the 1980s, then all households who exercise their RTB are better off so long as reselling costs are small. Irrespective of whether RTB purchasers retain their RTB properties or sell them if they receive a high income in the future, they retain the surplus derived from the policy.

However the allocation of this surplus between low and middle wealth households depends on the quality of the stock of RTB properties. If the cutoff quality whereby positive surplus from RTB resale is obtained is low, such that $\bar{\eta} < \underline{\eta}$, RTB without restriction on resale improves the aggregate welfare of low income households if and only if the quality of RTB houses is not too high such that $\eta < \underline{\eta}$. Middle wealth RTB homeowners continue living in their RTB houses if the quality of RTB houses is relatively high such that $\eta > \bar{\eta}$ but resell their houses to low wealth households otherwise. However, if $\bar{\eta} > \underline{\eta}$, the RTB increases the welfare of low income households if and only if $\eta < \hat{\eta}$. In this case, the surplus from RTB resale is always positive and any middle wealth RTB homeowner resells his house to a low wealth household at equilibrium.

Of course, although this policy improves the welfare of existing council house tenants eligible for RTB, there is a welfare cost. When the quality of council houses is high such that $\eta > \underline{\eta} \mathbb{I}_{[\eta > \bar{\eta}]} + \hat{\eta} \mathbb{I}_{[\eta < \bar{\eta}]}$, house sales through RTB transfer too much benefit to the middle wealth households that are temporarily receiving low income at the cost of a welfare loss arising from future low wealth households being unable to get council houses. Future new council housing applicants with low income and therefore potentially eligible for council housing may have to join a waiting list for council housing and rent private housing in the interim. Moreover, not only is the benefit that the local authorities transfer to the middle wealth households greater when council house quality is higher, but the impact of RTB is to

generate a longer waiting list for new council housing applicants in the subsequent periods when the council housing quality is high such that $\eta > \bar{\eta}$. This arises simply because the construction budget of the local authority, B , is fixed in the short term and councils cannot therefore replenish their stock of council houses or social housing in the short run, even by diluting the quality of the residual stock (e.g. by conversion of houses to smaller housing units).

It should not however be assumed that the incidence of the RTB policy is borne simply by new applicants for council houses. Of course, if the cost of the depleted public housing stock is directly borne by household on the council waiting list in the form of higher private sector rents, then given our utility function in terms of consumption and housing, the welfare loss to these households (which may contain both low and middle wealth households) is likely to outweigh the benefit of RTB (which is likely to disproportionately benefit middle wealth over low wealth households). However if the cost is borne by the local authorities in the form of paying private sector rents on behalf of homeless households in higher quality private rentals rather than lower quality public housing, then there is a financial loss to the local authorities since RTB receipts are not recovered by the local authorities (see Section 1). This may reduce the local authorities' capacity to build new council houses or may lead them to raise rents among residual council house tenants.

Alternatively, the welfare cost may be borne by central government, and hence taxpayers more generally, by providing subsidies to housing costs for low income households that are unable to access council houses when the stock of RTB houses is diminishing. Since private rentals are of higher quality than public rentals, there may not be a welfare loss to future low income households. However, assuming the central government has a positive shadow cost of public funds (see Section 3), there remains an opportunity cost to the RTB policy, since low income households on the waiting list would have been prepared to live in a council house of lower quality and lower rent.

5 Adjustments of RTB Policy

The previous section showed that introducing the RTB policy without restrictions must transfer too much benefit to middle wealth households exercising their right to RTB that are temporarily in poverty at the cost of either prospective future low wealth council housing applicants on the waiting list or taxpayers more generally (if the cost of private sector rentals by would-be council house tenants was borne by either the local authority or central government). However these transfers of social welfare depended on two factors: (i) the disutility of renting rather than owning a property and (ii) the average quality of the council house stock. In this section, we study three adjustments of the RTB policy which might better target the policy on low wealth households rather than disproportionately benefit middle wealth households which happened to be in possession of a council house by virtue of previous low income. These policies are: (i) resale restrictions; (ii) reducing the discounts on RTB sales; and (iii) returning the proceeds from RTB sales to local authorities for new council housing construction.

5.1 Right-to-Buy with Resale Restrictions

A natural policy reform to consider is to impose some form of resale restriction (indeed just such a policy reform was introduced to the RTB policy in its later years – see Section 1). However, the welfare effect of such a policy depends on the parameters described previously since, as we described, the probability of a right to resale being exercised depends on several factors: the wealth of the RTB household, the quality of the RTB house, and the household's disutility of renting. We pursue further in this section by considering a variant of a resale restriction policy: one in which owners of RTB properties have to sell their properties back to the local authority from which they purchased it at the original (real) price if they move to another private property. We also assume, to avoid gaming the system, that RTB purchasers are not eligible for either renting council houses or exercising RTB in the future. The effect of these restrictions is that RTB property owners receive no future benefit from selling the property as they have to rent privately afterwards irrespective of whether they receive a high or low income.

Comparing the difference in utility between owning a RTB property and renting privately, the cutoff quality of RTB properties that attracts a middle or low wealth homeowner to continue living in the property, $\tilde{\eta}^i$, solves:

$$\left(\frac{\alpha}{1-\alpha}R\right)^\alpha \tilde{\eta}^i - \frac{\lambda^i r S(\tilde{\eta}^i)}{\lambda^m - \lambda^l} = (1-\phi)^{1-\alpha} (Y^L - R)^\alpha, \quad \text{where } i \in \{m, l\}. \quad (13)$$

With resale restrictions, the cutoff quality of RTB properties above which middle or low homeowners prefer continue living in the property rather than renting privately satisfies:

$$\tilde{\eta}^l < \tilde{\eta}^m < \bar{\eta},$$

where $\bar{\eta}$ is the cutoff quality of RTB houses above which middle wealth homeowners prefer continue living in their properties to resale without restrictions, that is, the surplus of RTB resale without restrictions is zero, $S(\bar{\eta}) = 0$, where S is given in equation (8).

In contrast, low wealth households are more interested in owning RTB houses than middle wealth households with resale restrictions. The quality of council housing at equilibrium is always larger than low wealth households' reserve quality of participation, that is, $\eta > \tilde{\eta}^l$. Middle wealth RTB homeowners continue living in their houses if the quality of council housing is high such that $\eta > \tilde{\eta}^m$; otherwise, they sell their houses back to local authorities and rent privately. With resale restrictions imposed, the change in welfare only happens when the quality of RTB housing is low such that middle wealth RTB homeowners want but are not able to sell their houses to low wealth households: $\eta < \bar{\eta}$. Then they either hold on to their houses (when $\eta > \tilde{\eta}^m$) or sell them back to the local authorities (when $\eta < \tilde{\eta}^m$). The results are given in the following proposition, the detailed proof of which is provided in Appendix B.4.

Proposition 4 *Imposing resale restrictions on RTB houses improve the aggregate welfare of low income households if and only if the quality of RTB houses is low such that $\eta < \tilde{\eta}^m$. It can decrease the welfare if the quality of RTB houses is relatively high such that $\eta \in (\tilde{\eta}^m, \bar{\eta})$.*

Resale restrictions have no effect on the choice of whether or not to exercise RTB when middle wealth households prefer owning council houses to reselling their houses to low wealth households. This happens when the quality of council houses is high, or equivalently, the

original government budget on council housing is relatively unconstrained. In these circumstances, introducing RTB generates a waiting list for new council housing applicants. Low wealth households have a greater welfare loss than middle wealth households when they are not able to get council houses and if they bear some of the burden from renting privately at higher cost. Over time, however, the composition of tenants of low quality council houses shifts from both middle and low wealth households that receive low income to predominantly low wealth households.

Resale restrictions on RTB could even have a negative net welfare effect when the quality of RTB houses is not too high but middle wealth households prefer owning council houses to renting private houses. Relatively high quality council houses are more attractive to middle wealth households that temporarily have low incomes, since they are better off owning high quality council housing than renting a private house even when they fall back into poverty in the future. The local authorities lose more potential benefit to low wealth households from imposing resale restrictions on relatively high quality RTB houses because middle wealth RTB homeowners retain their houses. The potential redistribution of surplus arising from RTB housing resale from middle to low wealth households vanishes.

In contrast, when the quality of RTB houses is low such that middle wealth RTB homeowners prefer renting private housing to continuing living in their own RTB purchases, resale restrictions can help to improve the housing condition of low wealth households without giving too much benefit to the medium wealth households. Middle wealth households sell their RTB houses back to the local authorities, which can then provide these properties to future council housing applicants either for rent or for sale under RTB. Middle wealth households are not interested in purchasing these low quality RTB houses even if they rent them when they receive the low income. They will move out of the council house sector and rent private housing once they receive a high income in the future. In contrast low wealth council housing tenants are interested in purchasing the low quality council houses that they live in under RTB. So at equilibrium, all council houses that are purchased are occupied by low wealth households. Hence, resale restrictions are practical reforms where potential RTB properties are of low quality.

Overall, imposing resale restrictions on RTB houses works for a low quality council housing stock but not for a high quality stock, leaving more low quality council houses to the low wealth households that need help in the future. The original losses from sale of high quality council housing under RTB cannot be recouped from imposing resale restrictions. The local authorities should not have get rid of high quality council houses through selling them under RTB.

Finally, since the quality of council houses is strictly increasing in the local authorities' budget, B , introducing RTB helps to improve the aggregate welfare of households in poverty while reducing the budget only if the original budget under council housing rental system is low. When the local authorities' budget under the original council housing rental system is high, council housing quality is high. RTB on high quality houses is not optimal for the local authorities. Otherwise, the budget deficit is reduced at the cost of a welfare loss.

5.2 RTB with Reduced Discount

In the previous analysis, we assumed that the discounts on RTB sales given by local authorities equalled the rent subsidies implicit in the council house rent. In this subsection, we study whether social welfare is increased when the discounts that local authorities can

offer on RTB sales are reduced. The required mortgage payment for a RTB house increases as the subsidy on purchasing a council house decreases. The utility of owning a council house is strictly decreasing in the required payment rate. It decreases faster for low income households than for high income households. A low income household prefers owning a RTB property than renting it under the initial rental system if and only if the required payment under RTB is less than or equal to $\hat{\tau}$, where:

$$\hat{\tau} = \tau + \left(1 - (1 - \phi)^{\frac{1-\alpha}{\alpha}}\right) \frac{\alpha}{1 - \alpha}, \quad (14)$$

and the required payment rate for renting council housing, τ , is given by (7). To insure that RTB policies improve the welfare of low income households relative to the initial council housing rental system, a necessary condition is that the discount on RTB sales is not too low such that the required payment is less than or equal to $\hat{\tau}$.

The surplus of a RTB housing resale at equilibrium is strictly decreasing in the required payment on the property. For a high quality council property with quality greater than $\bar{\eta}$, where $\bar{\eta}$ solves $S(\bar{\eta}) = 0$ with $S(\bar{\eta})$ given in (8), the government can reduce the discount so that high income households are not interested in living in the property. The required payment is set such that the surplus from RTB resale is nonnegative. Then at equilibrium, any low income household can own a council property, purchasing from either the local authority, or from a high income RTB homeowner, who sold his/her property because (s)he preferred renting private housing to owning the council property given the lower discount.

However, because of the necessary condition on the upper bound of the required payment, $\hat{\tau}$, the reduced-discount adjustment works only when the quality of council housing is not too high such that $\eta \leq \bar{\eta}$, where $\bar{\eta}$ solves $S(\bar{\eta}, \hat{\tau}) = 0$, where $S(\eta, \hat{\tau})$ increases in either η or $\hat{\tau}$ and is given by:

$$\begin{aligned} \frac{rS(\eta, \hat{\tau})}{\lambda^m - \lambda^l} &= (Y^L - \hat{\tau}R\eta)^\alpha \eta^{1-\alpha} - (1 - \phi)^{1-\alpha} (Y^L - R)^\alpha \\ &\quad - ((Y^H - \hat{\tau}R\eta)^\alpha \eta^{1-\alpha} - (1 - \phi)^{1-\alpha} (Y^H - R)^\alpha). \end{aligned} \quad (15)$$

Thus, introducing RTB to relatively high quality council housing ($\eta > \bar{\eta}$) with reduced discounts relative to renting council housing can help to improve the welfare of the low income households only if the quality of the RTB houses is not too high ($\eta \leq \bar{\eta}$).

5.3 RTB with New Construction

Until now, we assumed no new council housing is constructed using the receipts of RTB. This accords with the evidence presented in the discussion in Section 1 and the evidence in Graph 1. In this subsection, we investigate how social welfare changes if the local authorities use the proceeds of high quality ($\eta > \bar{\eta}$, where $\bar{\eta}$ solves $S(\bar{\eta}) = 0$ with $S(\bar{\eta})$ given in (8)) RTB houses to build new council houses. This is done within a balanced budget setting; hence the government reduces discounts on RTB sales and the local authority increases the number of council properties within the fixed budget on council housing, B .

We focus on high quality RTB housing in our investigation here. As shown in the results of the previous subsection, reducing the discount on RTB sales of high quality council housing can improve the welfare of the low income households only if the quality of RTB properties is not too high such that $\eta \leq \bar{\eta}$, where $\eta \leq \bar{\eta}$ solves $S(\bar{\eta}, \hat{\tau}) = 0$ with $S(\eta, \hat{\tau})$ given in (15). We will explore in the following analysis whether using the proceeds from RTB sales

to replace part of the council housing stock is welfare improving and how this depends on council house average quality. For simplicity, we fix the discount on RTB sales at its lower bound (below which low income households prefer renting to owning council housing), so the required payment is set at $\hat{\tau}$, where $\hat{\tau}$ is given in (14).

When the council housing quality is high ($\eta \leq \bar{\eta}$), middle wealth RTB owners prefer continuing living in their RTB properties than selling them to low income households. The local authorities have to use the resulting proceeds to build new public properties to satisfy the housing need of future low income households who were not qualified for purchasing RTB properties when the policy was introduced. The proceeds from RTB sales increase in the required payment for a RTB property, which equals the government's budget on council housing, B , minus the discount on RTB properties, $(1 - \hat{\tau}) R\eta\bar{N}$, where \bar{N} is given in (5). The problem becomes whether the government can improve the welfare of the low income households within the budget $B - (1 - \hat{\tau}) R\eta\bar{N}$ when there are $\lambda^l N^l$ low wealth households and $\lambda^m N^m$ middle wealth households that are interested in council housing but could not access RTB housing when the policy was introduced. Our previous results apply for the new construction problem when we accordingly replace the budget and the numbers of low and middle wealth households who need council housing when receiving the low income.

Therefore, using RTB proceeds to build new council housing helps to maintain the local authorities' budget on council housing without reducing the welfare of the low income households if and only if the proceeds of RTB sales are large enough to cover the minimum cost of providing new council housing to future low income households at equilibrium. However, if the government is replacing the RTB sales with high quality council housing, the aggregate welfare of the low income households is not higher unless local authorities only rent out rather than sell these new properties. This would involve further restrictions on the RTB policy. That is, the new construction refinement of using RTB proceeds to build new council housing for future RTB sales works (in the sense of increasing the available stock of council houses) only if:

- (i) The quality of the new RTB houses is low such that $\eta \leq \bar{\eta}$;
- (ii) The initial number of RTB houses, \bar{N} , is large, so the proceeds from the RTB sales are large enough to cover the cost of providing council housing for all future low income households;
- (iii) The number of low wealth households, N^l , is small relative to that of middle wealth households, N^m , so the low income households that are not able to get council houses (on the waiting list) under the initial RTB policy is small; and
- (iv) The disutility of renting relative to owning houses, ϕ , is large.

The new revised RTB policy of the current government (2012) is to increase the availability of funds from RTB sales to local councils in order to allow them to build new properties. The government's requirement is that the funds be used to replace the sold-by-RTB house one for one, but at lower quality. The implication of our examination of the new construction refinement of RTB provides some evidence in support of this revised government policy.

6 Conclusion

This paper has provided the first analysis from a theoretical perspective of the innovative national ‘Right-to-Buy’ (RTB) policy for selling public housing to public tenants in the UK. The policy was the largest single privatisation in the period from 1980 to the mid-2000s, raising considerable sums for central government and increasing the share of owner occupation in the UK by almost 15 percentage points. Having described the growth and original rationale for public housing, the paper examines the development and evolution of the RTB policy over time, and its implications for the stock of public (council) housing.

To provide a theoretical underpinning for policy evaluation, the model assumes that council house tenants - and therefore potential RTB purchasers - are heterogeneous, and that not all characteristics are observable to local authorities (council house suppliers). The policy tools available to the local authority are the interrelated decisions (within a fixed construction budget) of what quality of council houses to build and whether to house all applicants for council house tenancy or to create a waiting list for council houses, which thereby reveals greater information concerning individual potential tenants, but at the cost of not being able to house all potentially deserving applicants. Crucially, we demonstrate that the RTB policy itself may affect the subsequent composition of applicants for council houses and therefore the composition of residual council house tenants as some tenants choose to exercise their preference for RTB.

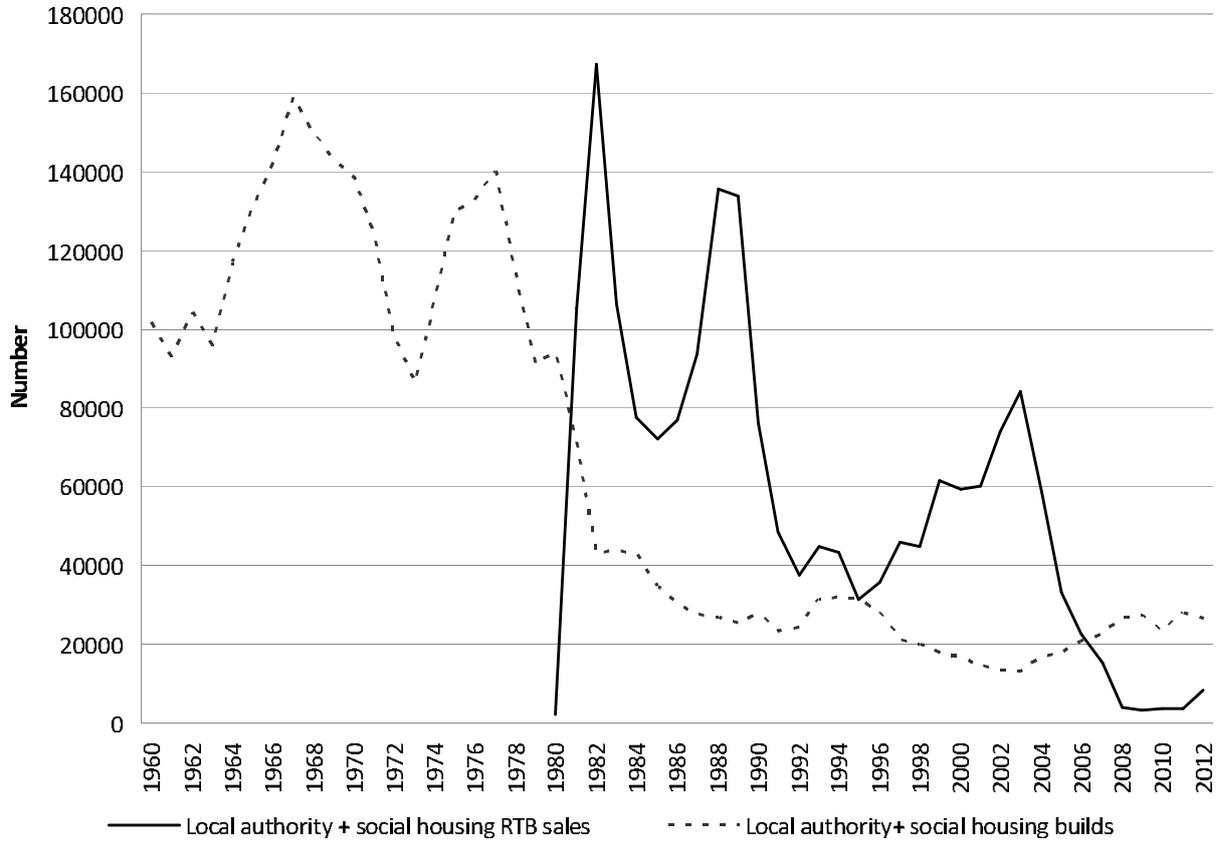
As in standard housing tenure models, the decision of council house tenants to exercise RTB (and to retain ownership of the property having exercised their RTB) hinges on the household’s preference for ownership over renting. However additional factors specific to the policy context are the average quality of RTB properties, a policy decision of the local authority as described above, and the magnitude of the discount given to would-be RTB purchasers. We explore the implications of these parameters on the household’s decision to exercise RTB, and on the evolving composition of tenants in the public sector versus owners. We then considered three potential reforms of the RTB programme: restrictions on RTB resale; enhanced minimum tenure requirements for RTB eligibility; and allowing local public authorities to retain RTB receipts for further construction of low quality builds. We demonstrate that whether these policies have any impact on RTB decisions hinges crucially on assumptions concerning the parameters described above. For example, resale restrictions may improve, or worsen, social welfare depending on the quality of council houses sold under the policy. In similar vein, we suggest that returning receipts from sales to local authorities (a reform constantly suggested by critics of the policy) is most effective with the proviso that receipts are used primarily and explicitly to subsidize low quality (‘affordable’) builds. It is of interest to note that this last policy refinement was introduced as this paper was being written.

A Graphs

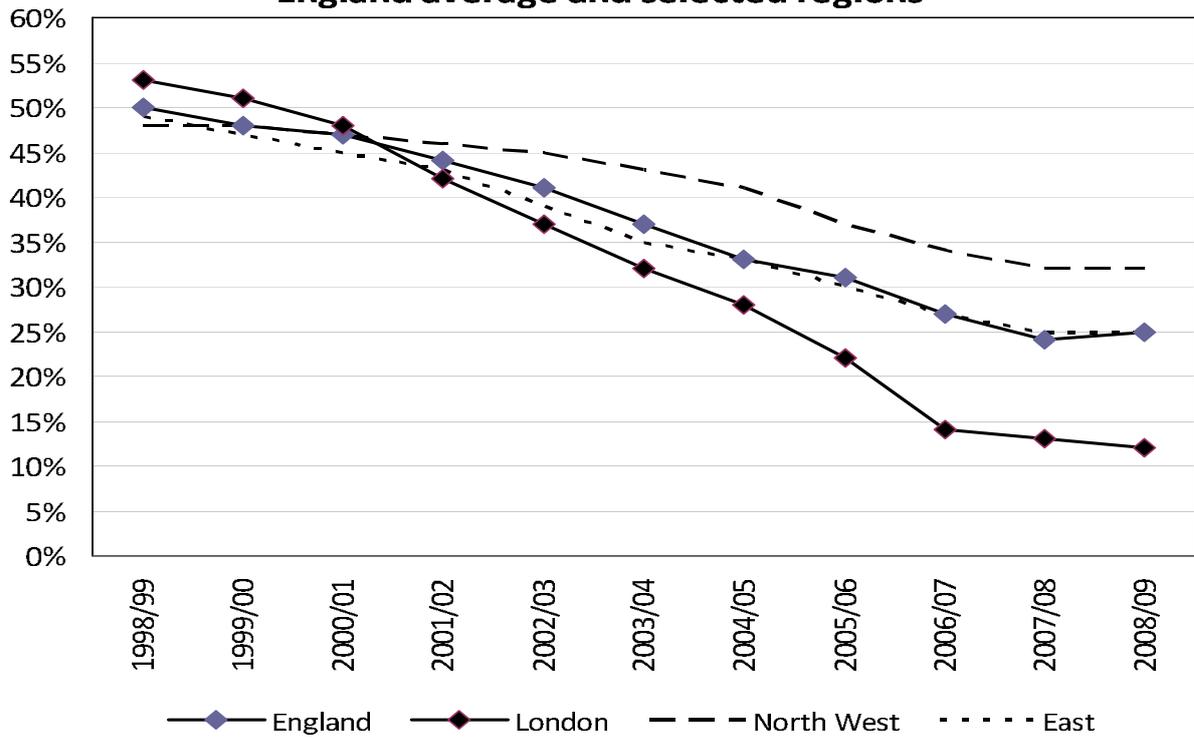
Graph 1:

New builds and RTB sales of public housing 1980-81 to 2012-13

Source: Department of Communities and Local Government *Housing Statistics Tables 244, 678*

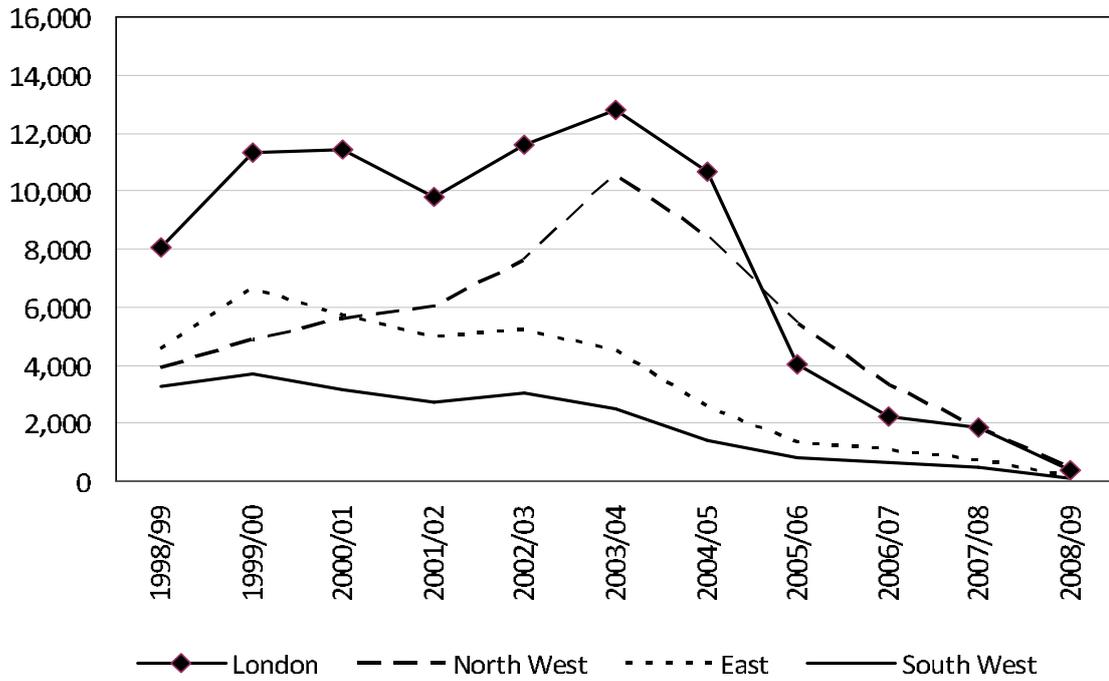


**Graph 2A:
Average discount on RTB sales as % of purchase price,
England average and selected regions**



Source: DCLG: Table 643, Social Housing Sales: Financial Statistics

**Graphs 2B:
Number of RTB sales by selected regions in England: 1998-99
to 2008-09**



Source: DCLG: Table 643, Social Housing Sales: Financial Statistics

B Proofs of Propositions

B.1 Proof of Proposition 1

In the private housing market, any household can rent a house whether it receives the high or low income. The value of renting a house is:

$$V_r^i(Y_t) = (1 - \phi)^{1-\alpha} (Y_t - R)^\alpha + (1 + r)^{-1} E_t [V^i(Y_{t+1}, 0)], \quad Y_t \in \{Y^H, Y^L\}.$$

Only the high income households are able to afford the initial fixed cost of purchasing a house. When the household purchase a house, it can continue living in the house until sold. The value of purchasing a house for a high income household is:

$$V_p^i = (Y^H - R - FC)^\alpha + (1 + r)^{-1} E_t [V^i(Y_{t+1}, 1)], \quad \text{where } Y_t = Y^H.$$

Homeowners prefer living in their own houses than selling their houses and rent houses, so: $V^i(Y_t, 1) = V_o^i(Y_t)$. A high income household of type i prefers renting to purchasing a private house if and only if $V_r^i(Y^H) > V_p^i$ at equilibrium, which is equivalent to:

$$(1 - \phi)^{1-\alpha} (Y^H - R)^\alpha - (Y^H - R - FC)^\alpha + (1 + r)^{-1} E_t [V_r^i(Y_{t+1}) - V_o^i(Y_{t+1})] > 0.$$

Together with the value of owning a house as shown in equation (2), the difference in the expected future value of renting relative to purchasing a house is:

$$E_t [V_r^i(Y_{t+1}) - V_o^i(Y_{t+1})] = -\frac{1+r}{r} (1 - (1 - \phi)^{1-\alpha}) (\lambda^i (Y^H - R)^\alpha + (1 - \lambda^i) (Y^L - R)^\alpha).$$

Thus, a high income household prefer renting to purchasing a private house when its probability of getting the high income is not too large such that $\lambda^i < \bar{\lambda}$, $i \in \{l, m\}$, where:

$$\bar{\lambda} = \frac{r \left((1 - \phi)^{1-\alpha} - \left(1 - \frac{FC}{Y^H - R}\right)^\alpha \right) - (1 - (1 - \phi)^{1-\alpha}) \left(\frac{Y^L - R}{Y^H - R} \right)^\alpha}{(1 - (1 - \phi)^{1-\alpha}) \left(1 - \left(\frac{Y^L - R}{Y^H - R} \right)^\alpha \right)}.$$

B.2 Proof of Proposition 2

Take the first order conditions of the local authorities' optimization problem with respect to the council housing quality η , required rental payment τ , and the number of council houses provided, N , respectively, we get:

$$\begin{aligned} -N\alpha\tau R (Y^L - \tau R\eta)^{\alpha-1} \eta^{1-\alpha} + N(1 - \alpha) (Y^L - \tau R\eta)^\alpha \eta^{-\alpha} &= \theta(1 - \tau)R; \\ -N\alpha R\eta (Y^L - \tau R\eta)^{\alpha-1} \eta^{1-\alpha} &= -\theta R\eta; \\ (Y^L - \tau R\eta)^\alpha \eta^{1-\alpha} - (Y^L - R)^\alpha &= \frac{\theta}{N^2} (B + \nu); \end{aligned}$$

where θ is the local authorities' shadow price of providing council housing and ν is the shadow cost of creating a waiting list of council housing rental, where $\theta \geq 0$ and $\nu \geq 0$. $\nu > 0$ if and only if it is optimal for the local authorities to provide council housing to every low income household, $N = \bar{N}$, that is, no waiting list. We obtain the optimal council housing quality

and rental rate subsidies from the local authorities' maximization problem. Note that low income households' participation constraint is satisfied since $\frac{\theta}{N^2}(B + \nu) > 0$.

Rearranging the first order conditions of the local authorities maximization problem with council housing rental system, we get the following conditions:

$$R = \frac{1 - \alpha}{\alpha} \left(\frac{Y^L}{\eta} - \tau R \right); \quad \theta = N\alpha \left(\frac{Y^L}{\eta} - \tau R \right)^{\alpha-1};$$

$$\eta - \left(\frac{Y^L - R}{\frac{Y^L}{\eta} - \tau R} \right)^\alpha = \frac{\alpha}{N} \left(\frac{Y^L}{\eta} - \tau R \right)^{-1} (B + \nu) \geq 0.$$

Under Assumptions 1, the low income households can afford the private housing rent, so:

$$\frac{Y^L}{\eta} - \tau R \geq Y^L - R > 0 \Rightarrow \theta > 0.$$

Thus, the local authorities' budget constraint holds with equality:

$$\frac{r}{1+r} FC + (1 - \tau)R\eta = \frac{B}{N}.$$

From the local authorities' budget constraint, the optimal council housing quality satisfies:

$$\eta = \frac{1 - \alpha}{R} \left(Y^L + \frac{B}{N} - \frac{r}{1+r} FC \right).$$

And the optimal required payment on council housing rent is:

$$\tau = \frac{Y^L}{R\eta} - \frac{\alpha}{1 - \alpha} = \frac{Y^L - \alpha \left(Y^L + \frac{B}{N} - \frac{r}{1+r} FC \right)}{(1 - \alpha) \left(Y^L + \frac{B}{N} - \frac{r}{1+r} FC \right)}.$$

By Assumption 3 and 4, $\eta, \tau \in (0, 1)$. The shadow price of creating a waiting list for council houses satisfies:

$$\frac{\nu}{N} = Y^L - \frac{r}{1+r} FC - \left(\frac{Y^L - R}{\alpha} \right)^\alpha \left(\frac{R}{1 - \alpha} \right)^{1-\alpha}$$

$$\Rightarrow \frac{d}{dY^L} \frac{\nu}{N} = 1 - \left(\frac{Y^L - R}{\alpha} \right)^{\alpha-1} \left(\frac{R}{1 - \alpha} \right)^{1-\alpha} < 0, \quad \forall Y^L < \frac{R}{1 - \alpha}.$$

Then ν is strictly decreasing in Y^L on $(R, \frac{R}{1-\alpha})$. When $Y^L = \frac{R}{1-\alpha}$, $\frac{\nu}{N} = -\frac{r}{1+r} FC < 0$. By Assumption 3, the fixed cost of purchasing a house is less than the housing price, then $R = \frac{r}{1+r} P > \frac{r}{1+r} FC$. The housing rent is greater than the local authorities' amortized fixed cost payment each period. Then $\frac{\nu}{N} = R - \frac{r}{1+r} FC > 0$ when $Y^L = R$. So by Assumption 3:

$$\frac{\nu}{N} > 0 \quad \forall Y^L < \bar{Y}.$$

Creating a waiting list for council housing rental system is costly. So it is optimal for the local authorities to provide council housing to all council housing applicants (households that receive the low income). Households' optimal decision on housing choices becomes the

same as with private housing except that they rent council houses instead of private ones when they receive the low income. The optimal number of council houses offered is:

$$N = \bar{N} = (1 - \lambda^l) N^l + (1 - \lambda^m) N^m,$$

where N^l is the number of low wealth households (with probability $1 - \lambda^l$ in poverty) and N^m is the number of middle wealth households (with probability $1 - \lambda^m$ in poverty). Therefore, the aggregate welfare of all low income households (either of low or middle wealth) is

$$rW_c = \bar{N} (Y^L - \tau R \eta)^\alpha \eta^{1-\alpha} (1 - \phi)^{1-\alpha} = \bar{N} (1 - \phi)^{1-\alpha} \left(\frac{\alpha}{1 - \alpha} R \right)^\alpha \eta.$$

B.3 Proof of Proposition 3

Take the derivative of the surplus of RTB housing resale as shown in equation (8) with respect to the RTB housing quality, η , get $\frac{dS}{d\eta}$ has the same sign as:

$$\left(\frac{1}{\kappa + 1} \right)^\alpha - 1 + \frac{\alpha \kappa}{\kappa + 1}, \quad \text{where} \quad \kappa \equiv \frac{(1 - \alpha) (Y^H - Y^L)}{\alpha R \eta} > \frac{Y^H - Y^L}{\alpha Y^L}.$$

The derivative of it is $\alpha \left(\frac{1}{\kappa + 1} \right)^\alpha - \alpha \left(\frac{1}{\kappa + 1} \right)^{\alpha+1}$, which is negative. So the expression is strictly decreasing in κ . Also $\frac{Y^H - Y^L}{\alpha Y^L}$ is strictly increasing in Y^H and $Y^H > Y^L$. Therefore, $\frac{dS}{d\eta} < 0$. The surplus of RTB housing resale is positive if and only if $\eta > \bar{\eta}$, where $\bar{\eta}$ solves $S = 0$.

Middle wealth RTB homeowners continue living in their houses if the quality of the RTB houses is high such that $\eta > \bar{\eta}$; otherwise, they sell their houses to low wealth households and rent private housing. In the first case, no further council housing is available for future generations. A RTB homeowner's value equals that when he lives in the RTB house forever. From Proposition 2, among the initial RTB housing owners, there are $(1 - \lambda^l) N^l$ low wealth households and $(1 - \lambda^m) N^m$ middle wealth households. When the quality of RTB houses is high such that $\eta > \bar{\eta}$, RTB homeowners live in their houses and other households rent private houses. The aggregate welfare of low income households when $\eta > \bar{\eta}$ is:

$$\begin{aligned} rW(\eta) &= \left((1 - \lambda^l)^2 N^l + (1 - \lambda^m)^2 N^m \right) \left(\frac{\alpha}{1 - \alpha} R \right)^\alpha \eta \\ &\quad + \left((1 - \lambda^l) \lambda^l N^l + (1 - \lambda^m) \lambda^m N^m \right) (1 - \phi)^{1-\alpha} (Y^L - R)^\alpha. \end{aligned}$$

The second case is when the quality of council houses is low, which happens when the local authorities' budget on council housing is tightly limited. Without resale restriction on RTB houses, house tenants will choose to exercise their right-to-buy but then sell the RTB house and rent privately when they receive the high income in the future. Under Assumption 2, $N^l > \bar{N}$. That is, there are more low wealth households than RTB houses. Then all RTB houses are occupied by all low wealth households and some low wealth households have to rent private housing. The aggregate welfare of low income households when $\eta < \bar{\eta}$ is:

$$\begin{aligned} rW(\eta) &= \left((1 - \lambda^l)^2 N^l + (1 - \lambda^m)^2 N^m \right) \left(\frac{\alpha}{1 - \alpha} R \right)^\alpha \eta \\ &\quad + \left((1 - \lambda^l) \lambda^l N^l + (1 - \lambda^m) \lambda^m N^m \right) (1 - \phi)^{1-\alpha} (Y^L - R)^\alpha \\ &\quad + \left((1 - \beta) (1 - \lambda^l) + \beta (1 - \lambda^m) \right) (1 - \lambda^m) N^m rS(\eta). \end{aligned}$$

Comparing the aggregate welfare of low income households with and that without RTB as shown in Proposition 2, the difference between the two satisfies:

$$\begin{aligned} \frac{rW(\eta) - rW_c(\eta)}{\bar{N}(1-\phi)^{1-\alpha} \left(\frac{\alpha}{1-\alpha}R\right)^\alpha} &= \left(\frac{1-\lambda^l - \Lambda}{(1-\phi)^{1-\alpha}} - 1\right)\eta + (\lambda^l + \Lambda) \left(\frac{Y^L - R}{\frac{\alpha}{1-\alpha}R}\right)^\alpha \\ &\quad + \left(\frac{1-\lambda^l}{\lambda^m - \lambda^l} - \beta\right) rS(\eta)\mathbb{I}_{[\eta < \bar{\eta}]}. \end{aligned}$$

where Λ is the difference in the fraction of high income RTB homeowners between whether the RTB houses initially owned by middle wealth households are resold to low wealth households:

$$\Lambda \equiv (\lambda^m - \lambda^l)(1 - \lambda^m) \frac{N^m}{\bar{N}} = (\lambda^m - \lambda^l) \left(1 - \left(1 + \frac{(1 - \lambda^m)N^m}{(1 - \lambda^l)N^l}\right)^{-1}\right).$$

Under Assumption 2, the difference between the aggregate welfare of low income households with and without RTB is strictly decreasing in the quality of RTB houses η . Let:

$$\underline{\eta} \equiv \left(1 - \frac{1 - \lambda^l - \Lambda}{(1 - \phi)^{1-\alpha}}\right)^{-1} (\lambda^l + \Lambda) \left(\frac{Y^L - R}{\frac{\alpha}{1-\alpha}R}\right)^\alpha.$$

If the cutoff quality of positive surplus from RTB resale is low such that $\bar{\eta} < \underline{\eta}$, RTB without restriction on resale improve the aggregate welfare of the low income households if and only if the quality of RTB houses is not too high such that $\eta < \underline{\eta}$. Middle wealth RTB homeowners continue living in their houses when the quality is high such that $\eta \in (\bar{\eta}, \underline{\eta})$ but resell their houses to low wealth households when the quality is low such that $\eta < \bar{\eta}$.

Otherwise, if the cutoff quality of positive surplus from RTB resale is high such that $\bar{\eta} > \underline{\eta}$, RTB without restriction on resale improve the aggregate welfare of the low income households if and only if the quality of RTB houses is not too high such that $\eta < \hat{\eta}$, where:

$$\left(1 - \frac{1 - \lambda^l - \Lambda}{(1 - \phi)^{1-\alpha}}\right) \hat{\eta} - \left(\frac{1 - \lambda^l}{\lambda^m - \lambda^l} - \beta\right) rS(\hat{\eta}) = (\lambda^l + \Lambda) \left(\frac{Y^L - R}{\frac{\alpha}{1-\alpha}R}\right)^\alpha, \quad \underline{\eta} < \hat{\eta} < \bar{\eta}.$$

Middle wealth RTB homeowners resell their houses to low wealth households at equilibrium.

B.4 Proof of Proposition 4

If the quality of council houses is high such that $\eta > \bar{\eta}$, all RTB homeowners continue living in their houses either with or without restrictions on resale. The aggregate welfare of low income households stays the same when the resale restrictions are imposed on RTB houses.

If the quality of council houses is not too high such that $\tilde{\eta}^m < \eta < \bar{\eta}$, middle wealth RTB homeowners who would resell their houses to low wealth households without resale restriction now continue living in their houses. The aggregate welfare of low income households becomes:

$$\begin{aligned} r\widetilde{W}(\eta) &= \left((1 - \lambda^l)^2 N^l + (1 - \lambda^m)^2 N^m\right) \left(\frac{\alpha}{1 - \alpha}R\right)^\alpha \eta \\ &\quad + \left((1 - \lambda^l) \lambda^l N^l + (1 - \lambda^m) \lambda^m N^m\right) (1 - \phi)^{1-\alpha} (Y^L - R)^\alpha. \end{aligned}$$

If the quality of council houses is not too high such that $\eta < \tilde{\eta}^m$, middle wealth RTB homeowners who would resell their houses to low wealth households without resale restriction now sell their houses back to local authorities and rent privately. The local authorities either rent out these houses or sell them again under RTB to future council housing applicants. At equilibrium, middle wealth households are not interested in purchasing these low quality council houses under RTB, but low wealth households are. So at equilibrium, all RTB houses are resold to low wealth households. The final outcome is the same as a special case without resale restrictions when low wealth households have bargaining power equal to one, $1 - \beta = 1$. Thus, the aggregate welfare of low income households becomes:

$$\begin{aligned} r\tilde{W}(\eta) &= \left((1 - \lambda^l)^2 N^l + (1 - \lambda^m)^2 N^m \right) \left(\frac{\alpha}{1 - \alpha} R \right)^\alpha \eta + (1 - \lambda^l) (1 - \lambda^m) N^m rS(\eta) \\ &\quad + \left((1 - \lambda^l) \lambda^l N^l + (1 - \lambda^m) \lambda^m N^m \right) (1 - \phi)^{1 - \alpha} (Y^L - R)^\alpha. \end{aligned}$$

Comparing the aggregate welfare of low income households under RTB with and that without resale restrictions as shown in Proposition 3, the difference between the two satisfies:

$$\frac{r\tilde{W}(\eta) - rW(\eta)}{\bar{N} (1 - \phi)^{1 - \alpha} \left(\frac{\alpha}{1 - \alpha} R \right)^\alpha} = \beta rS(\eta) \mathbb{I}_{[\eta < \tilde{\eta}^m]} - \left(\frac{1 - \lambda^l}{\lambda^m - \lambda^l} - \beta \right) rS(\eta) \mathbb{I}_{[\tilde{\eta}^m < \eta < \bar{\eta}]}.$$

Therefore, imposing resale restrictions on RTB houses improve the aggregate welfare of low income households if and only if the quality of the RTB houses is low such that $\eta < \tilde{\eta}^m$, where $\tilde{\eta}^m$ solves equation (13) and $\tilde{\eta}^m < \bar{\eta}$. It even has negative effect on the welfare if the quality of RTB houses is relatively high such that $\eta \in (\tilde{\eta}^m, \bar{\eta})$.

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