The Distribution of Wealth and the Marginal Propensity to Consume

Christopher Carroll\textsuperscript{1}  Jiri Slacalek\textsuperscript{2}  Kiichi Tokuoka\textsuperscript{3}  Matthew N. White\textsuperscript{4}

\textsuperscript{1}Johns Hopkins University and NBER
\textsuperscript{2}European Central Bank
\textsuperscript{3}Ministry of Finance, Japan
\textsuperscript{4}University of Delaware

Household Wealth Data and Public Policy
IFS/Public Economics UK Conference
Marginal Propensity to Consume

The Question:  **How Large Is the MPC (≡ κ)?**
If households receive a surprise one-off EUR 1 in income, how much more will be in aggregate spent over the next year?

**Why Do We Care?**

- MPC reflects size of economy’s response to fiscal stimulus
- Crude Keynesianism: Transitory tax cut multiplier = \( \frac{1}{1-\kappa} - 1 \)
  - If \( \kappa = 0.75 \), then multiplier is \( 4 - 1 = 3 \)
  - Some micro estimates of \( \kappa \) are this large
  - If \( \kappa = 0.05 \), then multiplier is only \( \approx 0.05 \)
  - This is about the size of \( \kappa \) in RBC models
Our Claim: Heterogeneity Is Key To Modeling the MPC

Need to Consider:

- Households are heterogeneous
- Wealth is unevenly distributed
- C function is highly concave

⇒ Distributional issues matter for aggregate C
  Giving EUR 1 to the poor \( \neq \) giving EUR 1 to the rich
Concavity of Consumption Function and Wealth Heterogeneity

Consumption/(quarterly) permanent income ratio (left scale)

Histogram: empirical (SCF1998) density of $m_t/(p_tW_t)$ (right scale)
To-Do List

1. Calibrate realistic income process
2. Match empirical wealth distribution
3. Back out optimal C and MPC out of transitory income
4. Is MPC in line with empirical estimates?
Our (Micro) Income Process

- Motivated by Friedman’s (1957) Permanent Income Hypothesis
- Idiosyncratic (household) income process:

\[ y_{t+1} = p_{t+1} \xi_{t+1} W \]
\[ p_{t+1} = p_t \psi_{t+1} \]

- \( p_t \) permanent income
- \( \psi_{t+1} \) permanent shock
- \( \xi_t \) transitory income
- \( W \) aggregate wage rate

- \( \xi_t \) incorporates unemployment insurance (Carroll (1992)):

\[ \xi_t = \mu \text{ with probability } u \]
\[ = (1 - \tau) \bar{\ell} \theta_t \text{ with probability } 1 - u \]

- \( \mu \) UI when unemployed
- \( \tau \) rate of tax collected for the unemployment benefits
Decision Problem

\[ v(m_t) = \max_{\{c_t\}} u(c_t) + \beta \mathbb{E}_t \left[ \psi_{t+1}^{1-\rho} v(m_{t+1}) \right] \]

s.t.

\[ a_t = m_t - c_t \]
\[ a_t \geq 0 \]
\[ k_{t+1} = a_t/(\mathcal{D}\psi_{t+1}) \]
\[ m_{t+1} = (\bar{\text{r}} + r)k_{t+1} + \xi_{t+1} \]
\[ r = \alpha Z(K/\bar{L})^{\alpha-1} \]

Variables normalized by \( p_t W \)
### Parameter Values

- $\beta$, $\rho$, $\alpha$, $\delta$, $\bar{\ell}$, $\mu$, and $u$ taken from JEDC special volume
- Key new parameter values:

<table>
<thead>
<tr>
<th>Description</th>
<th>Param</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob of Death per Quarter</td>
<td>$D$</td>
<td>0.00625</td>
<td>Life span of 40 years</td>
</tr>
<tr>
<td>Variance of Log $\psi_t$</td>
<td>$\sigma^2_{\psi}$</td>
<td>0.016/4</td>
<td>Carroll (1992); SCF</td>
</tr>
<tr>
<td>Variance of Log $\theta_t$</td>
<td>$\sigma^2_{\theta}$</td>
<td>0.010 $\times$ 4</td>
<td>DeBacker et al. (2013)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Carroll (1992)</td>
</tr>
</tbody>
</table>
### Annual Income, Earnings, or Wage Variances

<table>
<thead>
<tr>
<th>Study</th>
<th>$\sigma^2_\psi$</th>
<th>$\sigma^2_\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our parameters</td>
<td>0.016</td>
<td>0.010</td>
</tr>
<tr>
<td>Carroll (1992)</td>
<td>0.016</td>
<td>0.010</td>
</tr>
<tr>
<td>Storesletten, Telmer, and Yaron (2004)</td>
<td>0.008–0.026</td>
<td>0.316</td>
</tr>
<tr>
<td>Meghir and Pistaferri (2004)*</td>
<td>0.031</td>
<td>0.032</td>
</tr>
<tr>
<td>Low, Meghir, and Pistaferri (2010)</td>
<td>0.011</td>
<td>—</td>
</tr>
<tr>
<td>Blundell, Pistaferri, and Preston (2008)*</td>
<td>0.010–0.030</td>
<td>0.029–0.055</td>
</tr>
<tr>
<td>DeBacker, Heim, Panousi, Ramnath, and Vidangos (2013)</td>
<td>0.007–0.010</td>
<td>0.15–0.20</td>
</tr>
<tr>
<td>Implied by KS-JEDC</td>
<td>0.000</td>
<td>0.038</td>
</tr>
<tr>
<td>Implied by Castaneda et al. (2003)</td>
<td>0.03</td>
<td>0.005</td>
</tr>
</tbody>
</table>

*Meghir and Pistaferri (2004) and Blundell, Pistaferri, and Preston (2008) assume that the transitory component is serially correlated (an MA process), and report the variance of a subelement of the transitory component. $\sigma^2_\xi$ for these articles are calculated using their MA estimates.
1. Discount Factor $\beta$
   - ‘$\beta$-Point’ model: Single discount factor
   - ‘$\beta$-Dist’ model: Uniformly distributed discount factor

2. Empirical Wealth Variable to Match
   - Net Worth
   - Liquid Financial Assets

3. Life Cycle
   - Perpetual Youth (a la Blanchard)
   - Overlapping Generations
Dimension 1: Estimation of $\beta$-Point and $\beta$-Dist

‘$\beta$-Point’ model

- ‘Estimate’ single $\hat{\beta}$ by matching the capital–output ratio

‘$\beta$-Dist’ model—Heterogenous Impatience

- Assume uniformly distributed $\beta$ across households
- Estimate the band $[\hat{\beta} - \nabla, \hat{\beta} + \nabla]$ by minimizing distance between model $(w)$ and data $(\omega)$ net worth held by the top 20, 40, 60, 80%

$$\min_{\{\hat{\beta}, \nabla\}} \sum_{i=20, 40, 60, 80} (w_i - \omega_i)^2$$

s.t. aggregate net worth–output ratio matches steady-state value from the perfect foresight model
Results: Wealth Distribution

\[ \mathcal{F} \]

- US data (SCF)
- KS–JEDC
- \( \beta \)-Point
- \( \beta \)-Dist

Percentile

0 25 50 75 100

0 0.25 0.5 0.75 1
## Results: Wealth Distribution

<table>
<thead>
<tr>
<th>Micro Income Process</th>
<th>Friedman/Buffer Stock</th>
<th>KS-JEDC</th>
<th>KS-Orig (\diamondsuit)</th>
<th>U.S. Data*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Point Discount Factor(\dagger)</td>
<td>Uniformly Distributed Discount Factors*</td>
<td>Our solution</td>
<td>Hetero</td>
</tr>
<tr>
<td>(\beta)-Point</td>
<td>(\beta)-Dist</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top 1%</td>
<td>8.6</td>
<td>28.4</td>
<td>3.0</td>
<td>24.0</td>
</tr>
<tr>
<td>Top 20%</td>
<td>54.3</td>
<td>83.4</td>
<td>39.5</td>
<td>35.0</td>
</tr>
<tr>
<td>Top 40%</td>
<td>76.6</td>
<td>93.8</td>
<td>65.4</td>
<td></td>
</tr>
<tr>
<td>Top 60%</td>
<td>90.0</td>
<td>97.4</td>
<td>83.6</td>
<td></td>
</tr>
<tr>
<td>Top 80%</td>
<td>97.5</td>
<td>99.3</td>
<td>95.1</td>
<td></td>
</tr>
</tbody>
</table>

Notes: \(\dagger\) : \(\hat{\beta} = 0.9894\). \(\ast\) : \((\hat{\beta}, \nabla) = (0.9867, 0.0067)\). Bold points are targeted. \(K_t/Y_t = 10.3\).
Marginal Propensity to Consume and Net Worth

Most Impatient (left scale) ↓

Identical Patience (left scale) ↓

Most Patient (left scale) ↑

Representative agent's net worth →

Histogram: empirical density of net worth (right scale)
## Results: MPC (in Annual Terms)

<table>
<thead>
<tr>
<th>Micro Income Process</th>
<th>Friedman/Buffer Stock</th>
<th>KS-JEDC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta$-Point</td>
<td>$\beta$-Dist</td>
</tr>
<tr>
<td>Overall average</td>
<td>0.1</td>
<td>0.23</td>
</tr>
<tr>
<td>By wealth/permanent income ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top 1%</td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>Top 20%</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Top 40%</td>
<td>0.06</td>
<td>0.08</td>
</tr>
<tr>
<td>Top 60%</td>
<td>0.07</td>
<td>0.12</td>
</tr>
<tr>
<td>Bottom 1/2</td>
<td>0.13</td>
<td>0.35</td>
</tr>
<tr>
<td>By employment status</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employed</td>
<td>0.09</td>
<td>0.2</td>
</tr>
<tr>
<td>Unemployed</td>
<td>0.23</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Notes: Annual MPC is calculated by $1 − (1 − \text{quarterly MPC})^4$. 


Estimates of MPC in the Data: ~0.2–0.6

<table>
<thead>
<tr>
<th>Authors</th>
<th>Nondurables</th>
<th>Durables</th>
<th>Total PCE</th>
<th>Horizon</th>
<th>Event/Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blundell et al. (2008b)‡</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
<td>Estimation Sample: 1980–92</td>
</tr>
<tr>
<td>Coronado et al. (2005)</td>
<td>0.36</td>
<td></td>
<td></td>
<td>1 Year</td>
<td>2003 Tax Cut</td>
</tr>
<tr>
<td>Hausman (2012)</td>
<td>~ 0.25</td>
<td></td>
<td></td>
<td>1 Year</td>
<td>1936 Veterans’ Bonus</td>
</tr>
<tr>
<td>Johnson et al. (2009)</td>
<td>~ 0.25</td>
<td></td>
<td></td>
<td>3 Months</td>
<td>2003 Child Tax Credit</td>
</tr>
<tr>
<td>Lusardi (1996)‡</td>
<td>0.2–0.5</td>
<td></td>
<td></td>
<td></td>
<td>Estimation Sample: 1980–87</td>
</tr>
<tr>
<td>Parker (1999)</td>
<td>0.2</td>
<td></td>
<td></td>
<td>3 Months</td>
<td>Estimation Sample: 1980–93</td>
</tr>
<tr>
<td>Parker et al. (2011)</td>
<td>0.12–0.30</td>
<td>0.50–0.90</td>
<td></td>
<td>3 Months</td>
<td>2008 Economic Stimulus</td>
</tr>
<tr>
<td>Sahm et al. (2009)</td>
<td>~ 1/3</td>
<td></td>
<td></td>
<td>1 Year</td>
<td>2008 Economic Stimulus</td>
</tr>
<tr>
<td>Shapiro and Slemrod (2009)</td>
<td>~ 1/3</td>
<td></td>
<td></td>
<td>1 Year</td>
<td>2008 Economic Stimulus</td>
</tr>
<tr>
<td>Souleles (1999)</td>
<td>0.045–0.09</td>
<td>0.29–0.54</td>
<td>0.34–0.64</td>
<td>3 Months</td>
<td>Estimation Sample: 1980–91</td>
</tr>
<tr>
<td>Souleles (2002)</td>
<td>0.6–0.9</td>
<td></td>
<td></td>
<td>1 Year</td>
<td>The Reagan Tax Cuts of the Early 1980s</td>
</tr>
</tbody>
</table>

Notes: ‡: elasticity.
1. Discount Factor $\beta$
   - ‘$\beta$-Point’ model: Single discount factor
   - ‘$\beta$-Dist’ model: Uniformly distributed discount factor

2. Empirical Wealth Variable to Match
   - Net Worth
   - Liquid Financial Assets

3. Life Cycle
   - Perpetual Youth (a la Blanchard)
   - Overlapping Generations
Dimension 2: Matching Net Worth vs. Liquid Financial Assets

Liquid Assets ≡ transaction accounts, CDs, bonds, stocks, mutual funds
Matching Net Worth vs. Liquid Financial Assets

- Buffer stock saving driven by accumulation of liquidity
- May make more sense to match liquid (and retirement) assets (Hall (2011), Kaplan and Violante (2011))
- Aggregate MPC Increases Substantially: 0.23 $\uparrow$ 0.43

<table>
<thead>
<tr>
<th>$\beta$-Dist</th>
<th>Net Worth</th>
<th>Liq Fin and Ret Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall average</td>
<td>0.23</td>
<td>0.43</td>
</tr>
<tr>
<td>By wealth/permanent income ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top 1%</td>
<td>0.05</td>
<td>0.12</td>
</tr>
<tr>
<td>Top 20%</td>
<td>0.06</td>
<td>0.13</td>
</tr>
<tr>
<td>Top 40%</td>
<td>0.08</td>
<td>0.2</td>
</tr>
<tr>
<td>Top 60%</td>
<td>0.12</td>
<td>0.28</td>
</tr>
<tr>
<td>Bottom 1/2</td>
<td>0.35</td>
<td>0.59</td>
</tr>
</tbody>
</table>

Notes: Annual MPC is calculated by $1 - (1 - \text{quarterly MPC})^4$. 
Distribution of MPCs

Wealth Heterogeneity Translates into Heterogeneity in MPCs

Annual MPC

KS–JEDC
Matching net worth
Matching liquid financial + retirement assets

Percentile

0 0.25 0.5 0.75 1
0
25
50
75
100

Annual MPC
Typology of Our Models—Three Dimensions

1. Discount Factor $\beta$
   - ‘$\beta$-Point’ model: Single discount factor
   - ‘$\beta$-Dist’ model: Uniformly distributed discount factor

2. Empirical Wealth Variable to Match
   - Net Worth
   - Liquid Financial Assets

3. Life Cycle
   - Perpetual Youth (a la Blanchard)
   - Overlapping Generations
Dimension 3: Overlapping Generations

Realistic Life-Cycle Model

- Three education levels: $e \in \{D, HS, C\}$
- Age/education-specific income profiles

$$y_t = \xi_t p_t = (1 - \tau) \theta_t p_t,$$
$$p_t = \psi_t \psi_{es} p_{t-1}$$

- Age-specific variances of income shocks
- Transitory unemployment shock with prob $u$
- Household-specific mortality $D_{es}$
Household Decision Problem

\[ v_{es}(m_t) = \max_{c_t} u(c_t) + \beta \mathbb{E}_{es} \mathbb{E}_t \left[ \psi_{t+1}^{1-\rho} v_{es+1}(m_{t+1}) \right] \]

s.t.

\[
\begin{align*}
    a_t &= m_t - c_t, \\
    k_{t+1} &= a_t / \psi_{t+1}, \\
    m_{t+1} &= (\nabla + r) k_{t+1} + \xi_{t+1}, \\
    a_t &\geq 0
\end{align*}
\]
## Calibration

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient of relative risk aversion</td>
<td>$\rho$</td>
<td>1</td>
</tr>
<tr>
<td>Effective interest rate</td>
<td>$(r - \delta)$</td>
<td>0.01</td>
</tr>
<tr>
<td>Population growth rate</td>
<td>$N$</td>
<td>0.0025</td>
</tr>
<tr>
<td>Technological growth rate</td>
<td>$\Gamma$</td>
<td>0.0037</td>
</tr>
<tr>
<td>Rate of high school dropouts</td>
<td>$\theta_D$</td>
<td>0.11</td>
</tr>
<tr>
<td>Rate of high school graduates</td>
<td>$\theta_{HS}$</td>
<td>0.55</td>
</tr>
<tr>
<td>Rate of college graduates</td>
<td>$\theta_C$</td>
<td>0.34</td>
</tr>
<tr>
<td>Average initial permanent income, dropout</td>
<td>$\bar{p}_{D0}$</td>
<td>5000</td>
</tr>
<tr>
<td>Average initial permanent income, high school</td>
<td>$\bar{p}_{HS0}$</td>
<td>7500</td>
</tr>
<tr>
<td>Average initial permanent income, college</td>
<td>$\bar{p}_{C0}$</td>
<td>12000</td>
</tr>
<tr>
<td>Unemployment insurance payment</td>
<td>$\mu$</td>
<td>0.15</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>$u$</td>
<td>0.07</td>
</tr>
<tr>
<td>Labor income tax rate</td>
<td>$\tau$</td>
<td>0.0942</td>
</tr>
</tbody>
</table>
Results: Wealth Distribution in Life Cycle Model

\[ \mathbb{F} \]

- US data (SCF)
- KS–JEDC
- β–Point
- β–Dist

Percentile

0 25 50 75 100

0 0.25 0.5 0.75 1

Percentile

0 25 50 75 100
## Results: MPC (in Annual Terms)

<table>
<thead>
<tr>
<th>Wealth Measure</th>
<th>Micro Income Process</th>
<th></th>
<th>Life-Cycle Model</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>KS-JEDC</td>
<td>FBS</td>
<td>β-Point NW</td>
<td>β-Dist NW</td>
</tr>
<tr>
<td>Overall average</td>
<td>0.05</td>
<td>0.23</td>
<td>0.11</td>
<td>0.29</td>
</tr>
<tr>
<td>By wealth/permanent income ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top 1%</td>
<td>0.04</td>
<td>0.05</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>Top 20%</td>
<td>0.04</td>
<td>0.06</td>
<td>0.09</td>
<td>0.07</td>
</tr>
<tr>
<td>Top 40%</td>
<td>0.04</td>
<td>0.08</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>Top 60%</td>
<td>0.04</td>
<td>0.12</td>
<td>0.08</td>
<td>0.10</td>
</tr>
<tr>
<td>Bottom 1/2</td>
<td>0.05</td>
<td>0.35</td>
<td>0.13</td>
<td>0.49</td>
</tr>
<tr>
<td>By employment status</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employed</td>
<td>0.05</td>
<td>0.2</td>
<td>0.10</td>
<td>0.28</td>
</tr>
<tr>
<td>Unemployed</td>
<td>0.06</td>
<td>0.54</td>
<td>0.13</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Notes: Annual MPC is calculated by $1 - (1 - \text{quarterly MPC})^4$. 
Results: MPC by Age

- Initial drop in MPC: Build-up of buffer stock
- Rise while rapid income growth, fall before retirement, then increasing mortality risk
Empirical Wealth Distribution Across Countries

**Eurosystem Household Finance and Consumption Survey (HFCS)**
- Detailed wealth data from 15 euro area countries
- Ex ante harmonized, country-representative
- 62,000 households
- First wave released in April 2013, second planned for 2016
European Evidence—Wealth Distribution

Wealth-Income Ratios (Quarterly)

Austria Belgium Cyprus Germany Spain Finland France Greece Italy Luxembourg Malta Netherlands Portugal Slovenia Slovakia

excludes outside values

Net Wealth Liquid Assets
MPC Across European Countries—A bit Lower than US

Notes: The figure shows the range of aggregate MPCs spanned by the estimates based on the distribution of net wealth (lower bound) and of liquid assets (upper bound).
Wealth Inequality and MPC

- W inequality implies higher MPC, especially for liquid assets
Conclusions

What We Do

- We solve a model that matches
  - Income dynamics
  - Wealth distribution

Results

- The model produces more plausible implications about
  - Aggregate MPC
  - Distribution of MPC across households


Friedman (1957): Permanent Income Hypothesis

\[ Y_t = P_t + T_t \]
\[ C_t = P_t \]

Progress since then

- **Micro data:** Friedman description of income shocks works well
- **Math:** Friedman’s words well describe optimal solution to dynamic stochastic optimization problem of impatient consumers with geometric discounting under CRRA utility with uninsurable idiosyncratic risk calibrated using these micro income dynamics (!)
Our (Micro) Income Process

Idiosyncratic (household) income process is logarithmic Friedman:

\[ y_{t+1} = p_{t+1} \xi_{t+1} W \]
\[ p_{t+1} = p_t \psi_{t+1} \]

\( p_t \) = permanent income  
\( \xi_t \) = transitory income  
\( \psi_{t+1} \) = permanent shock  
\( W \) = aggregate wage rate
Further Details of Income Process

Modifications from Carroll (1992)
Transitory income $\xi_t$ incorporates unemployment insurance:

$$
\xi_t = \begin{cases} 
\mu & \text{with probability } u \\
(1 - \tau)\bar{\ell}\theta_t & \text{with probability } 1 - u
\end{cases}
$$

$\mu$ is UI when unemployed
$\tau$ is the rate of tax collected for the unemployment benefits
What Happens After Death?

- You are replaced by a new agent whose permanent income is equal to the population mean.
- Prevents the population distribution of permanent income from spreading out.
What Happens After Death?

- You are replaced by a new agent whose permanent income is equal to the population mean
- Prevents the population distribution of permanent income from spreading out
Ergodic Distribution of Permanent Income

Exists, if death eliminates permanent shocks:

\[ \text{DE}[\psi^2] < 1. \]

Holds.

Population mean of \( p^2 \):

\[ \text{M}[p^2] = \frac{D}{1 - \text{DE}[\psi^2]} \]
Dimension 2.a: Adding KS Aggregate Shocks

Model with KS Aggregate Shocks: Assumptions

- Only two aggregate states (good or bad)
- Aggregate productivity $Z_t = 1 \pm \triangle^Z$
- Unemployment rate $u$ depends on the state ($u^g$ or $u^b$)

Parameter values for aggregate shocks from Krusell and Smith (1998)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\triangle^Z$</td>
<td>0.01</td>
</tr>
<tr>
<td>$u^g$</td>
<td>0.04</td>
</tr>
<tr>
<td>$u^b$</td>
<td>0.10</td>
</tr>
<tr>
<td>Agg transition probability</td>
<td>0.125</td>
</tr>
</tbody>
</table>
Friedman/Buffer Stock Shocks

- **Motivation:**
  More plausible and tractable aggregate process, also simpler

- Eliminates ‘good’ and ‘bad’ aggregate state

- **Aggregate production function:** \( K_t^\alpha (L_t)^{1-\alpha} \)
  - \( L_t = P_t \Xi_t \)
  - \( P_t \) is aggregate permanent productivity
  - \( P_{t+1} = P_t \Psi_{t+1} \)
  - \( \Xi_t \) is the aggregate transitory shock.

- Parameter values estimated from U.S. data:

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance of Log ( \Psi_t )</td>
<td>( \sigma_{\Psi}^2 )</td>
<td>0.00004</td>
</tr>
<tr>
<td>Variance of Log ( \Xi_t )</td>
<td>( \sigma_{\Xi}^2 )</td>
<td>0.00001</td>
</tr>
</tbody>
</table>
Dimension 2.b: Adding FBS Aggregate Shocks

Friedman/Buffer Stock Shocks

- **Motivation:**
  - More plausible and tractable aggregate process, also simpler
- Eliminates ‘good’ and ‘bad’ aggregate state
- Aggregate production function: $K_t^\alpha (L_t)^{1-\alpha}$
  - $L_t = P_t \Xi_t$
  - $P_t$ is aggregate permanent productivity
  - $P_{t+1} = P_t \Psi_{t+1}$
  - $\Xi_t$ is the aggregate transitory shock.
- Parameter values estimated from U.S. data:

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<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance of Log $\Psi_t$</td>
<td>$\sigma^2_{\Psi}$</td>
<td>0.00004</td>
</tr>
<tr>
<td>Variance of Log $\Xi_t$</td>
<td>$\sigma^2_{\Xi}$</td>
<td>0.00001</td>
</tr>
</tbody>
</table>
Results

Our/FBS model

- A few times faster than solving KS model
- The results are similar to those under KS aggregate shocks
## Results: MPC Over the Business Cycle

<table>
<thead>
<tr>
<th>Model: $\beta$-Dist</th>
<th>Krusell–Smith (KS)</th>
<th>Friedman/Buffer Stock (FBS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario</td>
<td>Base</td>
<td>Recssn</td>
</tr>
<tr>
<td>Overall average</td>
<td>0.23</td>
<td>0.25</td>
</tr>
<tr>
<td>By wealth/permanent income ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top 1%</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Top 10%</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Top 20%</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Top 40%</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>Top 50%</td>
<td>0.09</td>
<td>0.10</td>
</tr>
<tr>
<td>Top 60%</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>Bottom 50%</td>
<td>0.35</td>
<td>0.38</td>
</tr>
<tr>
<td>By employment status</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employed</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>Unemployed</td>
<td>0.54</td>
<td>0.56</td>
</tr>
</tbody>
</table>
Results: MPC Over the Business Cycle

Krusell–Smith
- Aggregate and idiosyncratic shocks positively correlated
- Higher MPC during recessions, especially for the unemployed

Friedman/Buffer Stock
- Shocks uncorrelated
- MPC essentially doesn’t vary over BC
Macro Dynamics in Life-Cycle Model

- Population growth $N$, technological progress $\Gamma$
- **Tax rate** to finance social security and unemployment benefits: $\tau = \tau_{SS} + \tau_U$

\[
\tau_{SS} = \frac{\sum_{e \in \{D, HS, C\}} \theta_e \bar{p}_e \sum_{t=0}^{384} \left( ((1 + \Gamma)(1 + N))^{-t} \prod_{s=0}^{t} (\psi_{es} D_{es}) \right)}{\sum_{e \in \{D, HS, C\}} \sum_{t=0}^{163} \left( ((1 + \Gamma)(1 + N))^{-t} \prod_{s=0}^{t} (\psi_{es} D_{es}) \right)}
\]

\[
\tau_U = u \mu
\]