The Macro-dynamics of Sorting between Workers and Firms

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Contribution

- We develop an equilibrium random on-the-job search model of the Labor market, with ex-ante heterogeneous workers and firms, and aggregate productivity shocks.
- We calibrate the model to US time-series data 1951-2007 and assess the model predictions for patterns during 2008-12 recession.
- We use the model to assess the cyclicality of sorting/mismatch between workers and jobs, both for those hired from unemployment and those who were employed the period before.
Contribution

- The model delivers rich dynamics in terms of the cyclical composition of
  - unemployed workers
  - vacancies
  - productive matches
  - transition rates
  - measured labor productivity

- The model has a recursive structure that implies that:
  - knowledge of the current aggregate shock (and the stochastic process) is a sufficient statistic for decisions regarding which worker-firm matches to form or dissolve, and who change jobs
  - the decision of which types of vacancies to create depends on the current distribution of worker-types among the unemployed and the current distribution of worker-types across job-types
Related Literature

Models of aggregate shocks with heterogeneity

- **Directed search**: Menzio & Shi (2010a,b, 2011), Kaas & Kircher (2011), Schaal (2011); **Wage posting**: Moscarini & Postel-Vinay (2011a,b), Coles & Mortensen (2011);

Cyclical behavior of labor productivity and labor market variables

- Shimer (2005), Hall (2005), Hagedorn & Manovskii (2008), Gertler & Trigari (2009), Hagedorn & Manovskii (2010), ...

Sorting between workers and firms (or unemployed and vacancies)


As far as we know, there is still very little work with double-sided worker-firm heterogeneity. Yet there is a lot of interest in understanding the evolution of match quality in recessions and booms.
Agents and Technology

- Time is discrete and indexed by $t$.
- The planning horizon for workers and firms is infinite.
- All agents are risk neutral and discount the future at rate $r$.
- Let $x$, $y$, and $z$ index worker type, firm type and the aggregate productivity level.
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- There is a continuum of workers indexed by type $x \in [0, 1]$.
  - with distribution $\ell(x)$ and home production $b(x, z)$
  - workers search both when unemployed and employed

- There is a continuum of profit maximizing firms $y \in [0, 1]$.
  - type is defined by their technology $p(x, y, z)$
  - recruit by posting vacancies $v(y)$ at increasing convex cost $c[v(y)]$
  - retain workers by responding to outside offers
Aggregate States

- $u_t(x)$: the distribution of unemployed workers at the beginning of period $t$ (prior to realization of $z_t$)
- $h_t(x, y)$: the distribution of worker-firm matches at the beginning of period $t$ (prior to realization of $z_t$)
- $z_t$ is updated from $z_{t-1}$ according to $\pi(z, z')$
- The state at the beginning of period $t$ is defined by $\{u_t(x), h_t(x, y), z_t\}$
Three Key Modeling Assumptions

1. **Transferable Utility**
   - Workers and firms value a wage change the same way.

2. **Firms make state-contingent offers and counter-offers to workers**
   - When firms contact unemployed workers, they offer them their reservation value.
   - When firms contact employed workers, they engage in Bertrand competition with current employer.

3. **Firms operate constant returns to scale production and pay flow costs to recruit new workers**
   - Hiring a new worker does not affect the productivity of existing matches, or the ability to hire more workers in the future.
Let $W_t(w, x, y)$ be the present value to a worker of type $x$ of receiving a wage $w$ when employed by a firm of type $y$. The subscript $t$ indicates that the function depends, in general, on the aggregate state at time $t$: \{u_t(x), h_t(x, y), z_t\}
Values and Match Surplus

- Let $W_t(w, x, y)$ be the present value to a worker of type $x$ of receiving a wage $w$ when employed by a firm of type $y$.
  - The subscript $t$ indicates that the function depends, in general, on the aggregate state at time $t$ : $\{u_t(x), h_t(x, y), z_t\}$

- Let $B_t(x)$ be the value of unemployment

- Let $\Pi_t(w, x, y)$ be the present value to a firm of type $y$ employing a worker of type $x$, paying a wage $w$

- The match surplus is given by
  
  $$W_t(w, x, y) - B_t(x) + \Pi_t(w, x, y) = S_t(x, y)$$
Timing

- Within a period
  1. The aggregate shock $z_t$ is realized, endogenous and exogenous separations occur
  2. Firms post vacancies and new meetings occur
  3. Production takes place
Separations (Layoffs)

- The aggregate state changes from $z_{t-1} = z$ to $z_t = z'$.
- All jobs such that $S_t(x, y) \leq 0$ are immediately destroyed,
- A fraction $\delta$ of the viable ones are also destroyed.
- Hence the stock of unemployed workers of type $x$ immediately after the realization of $z_t$ (at time $t+$) is

$$u_{t+}(x) = u_t(x) + \int [1\{S_t(x, y) \leq 0\} + \delta 1\{S_t(x, y) > 0\}] h_t(x, y) \, dy.$$

- The stock of matches of type $(x, y)$ is

$$h_{t+}(x, y) = (1 - \delta) 1\{S_t(x, y) > 0\} h_t(x, y).$$
Meeting Function

- The total measure of meeting at time $t$ is given by

$$M_t = M(L_t, V_t) = \min\{\alpha \sqrt{L_t V_t}, L_t, V_t\},$$

where $M(L_t, V_t)$ is strictly increasing in $L_t$ and $V_t$ and constant returns to scale.

- For the purposes of new meetings, the Labor force is defined by:

$$L_t = f(u_{t+}, h_{t+}) = s_0 \int u_{t+}(x) \, dx + s_1 \iint h_{t+}(x, y) \, dx \, dy$$

- Firms observe the new aggregate state and choose visibility $v_t(y)$, with aggregator:

$$V_t = g(v_t) = \int v_t(y) \, dy$$
Laws of Motion

For unemployment:

\[ u_{t+1}(x) = u_t(x) \left[ 1 - \int \lambda_{0,t} \frac{q_t v_t(y)}{M_t} 1\{S_t(x, y) > 0\} \, dy \right] \]

For employment:

\[ h_{t+1}(x, y) = h_t(x, y) + u_t(x) \lambda_{0,t} \frac{q_t v_t(y)}{M_t} 1\{S_t(x, y) > 0\} \]

\[ + \int h_t(x, y') \lambda_{1,t} \frac{q_t v_t(y)}{M_t} 1\{S_t(x, y) > S_t(x, y')\} \, dy' \]

\[ - h_t(x, y) \int \lambda_{1,t} \frac{q_t(y') v_t(y')}{M_t} 1\{S_t(x, y') > S_t(x, y)\} \, dy' \]

where \( \lambda_{0,t} \), \( \lambda_{1,t} \) and \( q_t \) are the equilibrium meeting probabilities for unemployed workers, employed workers and vacancies.
Contracting and Re-contracting
Postel-Vinay & Robin (2001) and Postel-Vinay & Turon (2010)

- An unemployed worker is offered her reservation wage:
  \[ W_t(\phi_{0,t}(x, y), x, y) - B_t(x) = 0 \]

- An employed worker is offered the minimum to outbid current (or poaching) firm,
  \[ W_t(\phi_{1,t}(x, y', y), x, y) - B_t(x) = S_t(x, y'), \]
  where \( S_t(x, y) > S_t(x, y') \)

- After an aggregate shock the current wage \( w \) may not be viable. We assume that \( w' = \phi_{2,t}(w, x, y) \) with
  - \( \phi_{2,t}(w, x, y) = \phi_{0,t}(x, y) \) if \( W_t(w, x, y) - B_t(x) < 0 \) (Worker PC binds)
  - \( \phi_{2,t}(w, x, y) = \phi_{1,t}(x, y', y) \) if \( \Pi_t(w, x, y) < 0 \) (Firm PC binds)
  - \( \phi_{2,t}(w, x, y) = w \) otherwise (status quo)
The Match Surplus and the Aggregate State

- The *value* to the worker and the value to the firm depend on $x$, $y$, aggregate productivity $z_t$, and on the distributions $v_t(y)$, $u_t(x)$, and $h_t(x, y)$ (they affect the expectations of outside offers available to the worker)
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- However, the *match surplus* depends on time only through $z$
  - Outside offers trigger a change to the transfer between firm and worker (the wage) but leave the size of the surplus unchanged
  - If the worker leaves to another firm she receives all of the current surplus
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- However, the *match surplus* depends on time only through $z$:
  - Outside offers trigger a change to the transfer between firm and worker (the wage) but leave the size of the surplus unchanged.
  - If the worker leaves to another firm she receives all of the current surplus.

- We can write the surplus as

$$S(x, y, z) = s(x, y, z) + \frac{1 - \delta}{1 + r} \int \max\{S(x, y, z'), 0\} \pi(z, z') \, dz'$$

with $s(x, y, z) = p(x, y, z) - b(x, z)$.
Vacancy Creation and the Aggregate State

Firms choose $v_t(y)$ to maximize the return to recruiting:

$$\max_{v_t(y)} \left\{ -c[v_t(y)] + q_t v_t(y) J_t(y) \right\}$$

where $J_t(y)$ is the expected value of a new match

$$J_t(y) = \int \frac{s_0 u_t(x)}{L_t} S(x, y, z)^+ \, dx + \iint \frac{s_1 h_t(x, y')}{L_t} [S(x, y, z) - S(x, y', z)]^+ \, dx \, dy'$$
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For cost function $c_0 [v(y)] = \frac{c_0}{1+c_1} v_t(y)^{1+c_1}$ and CD meeting technology: $q_t = \alpha \theta_t^{-\omega}$ we have a closed form for vacancy creation:

$$\theta_t \equiv \frac{V_t}{L_t} = \left(\frac{\alpha}{c_0}\right)^{\frac{1}{c_1+\omega}} \left(\frac{J_t}{L_t}\right)^{\frac{c_1}{c_1+\omega}},$$

$$v_t(y) = \left(\frac{q_t J_t(y)}{c_0}\right)^{\frac{1}{c_1}}.$$
1. Solve for the fixed point in $S(x, y, z)$ independently of the actual realization of aggregate productivity shocks.

2. Given an initial distribution of workers across jobs and employment states, $u_0(x)$, $h_0(x, y)$ and a realized sequence of aggregate productivity shocks $\{z_0, z_1, \ldots\}$ we can solve for the sequence of distributions of unemployed worker types, worker-firm matches, and vacancies $\{u_{t+1}(x), h_{t+1}(x, y), v_t(y)\}_{t=0}^T$. 
Parametric Specification

- Meeting function

\[ M_t = M(L_t, V_t) = \min \left\{ \alpha \sqrt{L_t V_t}, L_t, V_t \right\}, \quad \alpha > 0 \]

- Vacancy costs

\[ c[v_t(y)] = \frac{c_0 v_t(y)^{1+c_1}}{1 + c_1}, \quad c_0 > 0, \quad c_1 > 0 \]

- Value added

\[ p(x, y, z) = z \times (p_1 + p_2 x + p_3 y + p_4 x^2 + p_5 y^2 + p_6 x y) \]

- Home production

\[ b(x, z) = b_0 + z \times (b_1 x + b_2 x^2) \]

- Worker type distribution

\[ x \sim \text{Beta}(\beta_1, \beta_2) \]
Calibration

- We calibrate the model parameters by method of simulated moments.
- The model is solved at a weekly frequency and the simulated data is then aggregated (exactly as the BLS and BEA data) to form quarterly moments.
- From the data we remove a quadratic trend from log transformed data (1951-2007).
Some Comments on Identification

- $\alpha$, $s_1$, and $\delta$ (mobility) identified by the average transition rates between unemployment and employment, between jobs, and from employment to unemployment.

- $\sigma$ and $\rho$ (process for $z$) identified by standard deviation and auto-correlation of output.

- $c_0$ and $c_1$ (vacancy costs) identified by the standard deviation of vacancies and the correlation of vacancies with output.

- $\beta_i$, $b_i$, and $p_i$ (heterogeneity and match production)
  - The distribution of worker types is identified by the pattern in the number of workers unemployed 5, 15 and 27 or more weeks.
  - The contribution of firm type to value added is identified by the cross-sectional variation in value added per job, and its correlation with output.
# Model Fit to Moments

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[U]$</td>
<td>0.0562</td>
<td>0.0568</td>
<td>$sd[U]$</td>
<td>0.2140</td>
<td>0.2063</td>
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<td>$E[U^{5p}]$</td>
<td>0.0324</td>
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<td>$sd[U^{5p}]$</td>
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<td>0.2670</td>
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<td>$sd[U^{15p}]$</td>
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<td>$E[U^{27p}]$</td>
<td>0.0078</td>
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<td>$E[U^{2E}]$</td>
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<td>$sd[U^{2E}]$</td>
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<td>$E[E^{2U}]$</td>
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<td>0.0244</td>
<td>$sd[E^{2U}]$</td>
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<td>0.1267</td>
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<tr>
<td>$E[J^{2J}]$</td>
<td>0.0273</td>
<td>0.0260</td>
<td>$sd[J^{2J}]$</td>
<td>0.0924</td>
<td>0.1069</td>
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<tr>
<td>$E[\text{prod. disp.}]$</td>
<td>0.7478</td>
<td>0.6623</td>
<td>$sd[\text{prod. disp.}]$</td>
<td>0.0166</td>
<td>0.0082</td>
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<tr>
<td>$sd[V]$</td>
<td>0.2291</td>
<td>0.1860</td>
<td>corr[$U, VA$]</td>
<td>-0.7742</td>
<td>-0.9406</td>
</tr>
<tr>
<td>$sd[V/U]$</td>
<td>0.4162</td>
<td>0.3722</td>
<td>corr[$V, VA$]</td>
<td>0.6372</td>
<td>0.9159</td>
</tr>
<tr>
<td>$sd[VA]$</td>
<td>0.0363</td>
<td>0.0379</td>
<td>corr[$U^{2E}, VA$]</td>
<td>0.8143</td>
<td>0.9010</td>
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<tr>
<td>autocorr[$VA$]</td>
<td>0.9427</td>
<td>0.9553</td>
<td>corr[$E^{2U}, VA$]</td>
<td>-0.5984</td>
<td>-0.5169</td>
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<tr>
<td>corr[$V, U$]</td>
<td>-0.7642</td>
<td>-0.8005</td>
<td>corr[$\text{prod. disp}, VA$]</td>
<td>-0.3902</td>
<td>-0.4552</td>
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<tr>
<td>corr[$U^{2E}, J^{2J}$]</td>
<td>0.6333</td>
<td>0.5526</td>
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</tbody>
</table>
Parameter Estimates

Distribution of worker types

\[ M(L, V) = 1.89 \sqrt{LV} \]

Search intensity \( s_1/s_0 \) 0.022
Exogenous separation \( \delta \) 0.007

Production function

\[ c[v(y)] = 0.03v(y)^{2.12} \]

\[ b(x, z) = 0.5 + e^z(-0.1x + 4.7x^2) \]

Productivity shocks \( \sigma \) 0.049
Gaussian copula \( (\sigma, \rho) \) \( \rho \) 0.999

Complete Parameter Estimates

Effect of Heterogeneity Specification on Moments

Lise & Robin (UCL & ScPo)

The Macrodynamics of Sorting
Feasible matches with aggregate shock at median
Feasible matches with aggregate shock at 90th percentile
Feasible matches with aggregate shock at 10th percentile
Feasible matches

![Graph showing feasible matches between Worker Type and Firm Type.](image-url)
Recovering the realized shock process $z_t$

We filter out the series for $z_t$ that best matches the output series 1951q1 to 2012q4.
Simulated Unemployment

std ratio = 1.018, corr = 0.831

Simulated Vacancies

std ratio = 1.362, corr = 0.580

Simulated Unempl. to Empl.

std ratio = 1.252, corr = 0.854

Simulated Empl. to Unempl.

std ratio = 1.237, corr = 0.526

Simulated sd labor prod.

std ratio = 0.275, corr = 0.197

Simulated Unmpl 27+ Weeks

std ratio = 1.192, corr = 0.845
Labor Productivity and Output

Data - blue; Model prediction - green

Lise & Robin (UCL & ScPo)
Cyclical composition of unemployed workers

Cyclicality: low skilled 0.84, high skilled 1.23 (from regression of log unemployment rate by skill on log unemployment rate)
Relative productivity, sorting and Firms’ surplus share

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>constant $b$</th>
<th>No heterogeneity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b(x, \tilde{z})$</td>
<td>mean</td>
<td>0.9564</td>
<td>0.8350</td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>0.9040</td>
<td>0.1780</td>
</tr>
<tr>
<td></td>
<td>max</td>
<td>0.9803</td>
<td>0.9585</td>
</tr>
<tr>
<td>$p(x, y(x), \tilde{z})$</td>
<td>corr $(x, y)$</td>
<td>0.736</td>
<td>0.709</td>
</tr>
<tr>
<td>Firm share of surplus at matching</td>
<td></td>
<td>0.274</td>
<td>0.372</td>
</tr>
</tbody>
</table>
Mismatch (Sorting)

- Let \( y(x) = \text{arg max}_y S_t(x, y) \)

  absolute mismatch \( t = \frac{1}{H_t^j} \int \left[ S_t(x, y(x)) - S_t(x, y) \right] h_t^j(x, y) \, dx \, dy \)

  relative mismatch \( t = \frac{1}{H_t^j} \int \left[ \frac{S_t(x, y(x)) - S_t(x, y)}{S_t(x, y(x))} \right] h_t^j(x, y) \, dx \, dy \)

- Distribution of matches with workers hired out of unemployment

  \( h_t^0(x, y) = u_{t+}(x) \lambda_{0,t} \frac{q_tv_t(y)}{M_t} 1\{S_t(x, y) \geq 0\} \)

- Distribution of matches where the worker was employed last period

  \[
  h_t^1(x, y) = h_{t+}(x, y) \left[ 1 - \int \lambda_{1,t} \frac{q_tv_t(y')}{M_t} 1\{S_t(x, y') > S_t(x, y)\} \, dy' \right] \\
  + \int h_{t+}(x, y') \lambda_{1,t} \frac{q_tv_t(y)}{M_t} 1\{S_t(x, y) > S_t(x, y')\} \, dy'.
  \]
Cyclical Mismatch

corr new matches = 0.975, corr existing matches = 0.857

corr new matches = 0.717, corr existing matches = −0.123

- worker-job pairs where the worker was hired out of unemployment.
- worker-job pairs in which the worker was employed in the previous period.
Cyclical Mismatch

corr new matches = 0.922, corr existing matches = 0.738

corr new matches = 0.399, corr existing matches = −0.632

Absolute Mismatch

Relative Mismatch

× - worker-job pairs where the worker was hired out of unemployment.
○ - worker-job pairs in which the worker was employed in the previous period.
We develop an equilibrium random on-the-job search model of the Labor market, with ex-ante heterogeneous workers and firms, and aggregate productivity shocks. The model fits the US time-series data 1951-2007 and does quite well predicting the patterns over 2008-12. In booms, workers initially accept worse matches on average than in recessions. At the same time, once employed they move more quickly to better matches in booms than in recessions.
The Value of Unemployment

Consider a worker of type $x$ who is unemployed for the whole period $t$.

\[
B_t(x) = b(x, z) + \frac{1}{1 + r} \mathbb{E}_t \left[ (1 - \lambda_{0,t+1}) B_{t+1}(x) \right] \\
+ \lambda_{0,t+1} \int \max \left\{ W_{t+1}(\phi_{0,t+1}(x, y), x, y), B_{t+1}(x) \right\} \frac{q_{t+1}(y) v_{t+1}(y)}{M_{t+1}} \, dy
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\]

\[
+ \lambda_{0,t+1} \int \max \{ W_{t+1}(\phi_{0,t+1}(x, y), x, y), B_{t+1}(x) \} \frac{q_{t+1}(y)v_{t+1}(y)}{M_{t+1}} \, dy
\]

Since any firm the worker contacts will offer her reservation value this simplifies to

\[
B_t(x) = b(x, z) + \frac{1}{1 + r} \mathbb{E}_t B_{t+1}(x).
\]
The Value of Employment

\[ W_t(w, x, y) = w + \frac{1}{1 + r} \mathbb{E}_t \left[ 1 \{ S_{t+1}(x, y) < 0 \} + \delta 1 \{ S_{t+1}(x, y) \geq 0 \} \right] B_{t+1}(x) \]

\[ + (1 - \delta) 1 \{ S_{t+1}(x, y) \geq 0 \} \]

\[ \times \left[ \lambda_{1,t+1} \int_{y' \in \mathcal{M}_{1,t+1}(x,y)} W_{t+1}(\phi_{1,t+1}(x, y', y), x, y') \frac{q_{t+1}(y') v_{t+1}(y')}{M_{t+1}} \, dy' \right. \]

\[ + \lambda_{1,t+1} \int_{y' \in \mathcal{M}_{2,t+1}(w,x,y)} W_{t+1}(\phi_{1,t+1}(x, y, y'), x, y) \frac{q_{t+1}(y') v_{t+1}(y')}{M_{t+1}} \, dy' \]

\[ + \left. \left[ 1 - \lambda_{1,t+1} \int_{y' \in \mathcal{M}_{3,t+1}(w,x,y)} \frac{q_{t+1}(y') v_{t+1}(y')}{M_{t+1}} \, dy' \right] \times \min \{ W_{t+1}(w, x, y), \max \{ S_{t+1}(x, y) + B_{t+1}(x), B_{t+1}(x) \} \} \right] \]
Firm Value

\[
\Pi_t(w, x, y) = p(x, y, z) - w + \frac{1}{1 + r} E_t \left[ (1 - \delta) \mathbf{1}\{S_{t+1}(x, y) \geq 0\} \right]
\]

\[
\times \left[ \lambda_{1,t+1} \int_{y' \in \mathcal{M}_{2,t+1}(w,x,y)} \Pi_{t+1}(\phi_{1,t+1}(x, y, y'), x, y) \frac{q_{t+1}(y') v_{t+1}(y')}{M_{t+1}} \, dy' 
\right]
\]

\[
+ \left[ 1 - \lambda_{1,t+1} \int_{y' \in \mathcal{M}_{3,t+1}(w,x,y)} \frac{q_{t+1}(y') v_{t+1}(y')}{M_{t+1}} \, dy' \right]
\]

\[
\times \min\{\Pi_{t+1}(w, x, y), S_{t+1}(x, y)^+\} \right].
\]
Estimated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matching</td>
<td>$M = \alpha \sqrt{LV}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.894</td>
</tr>
<tr>
<td>Home production</td>
<td></td>
</tr>
<tr>
<td>$b_0$</td>
<td>0.553</td>
</tr>
<tr>
<td>Interest rate</td>
<td>$r$</td>
</tr>
<tr>
<td>$b(x, z) = b_0 + e^z$</td>
<td></td>
</tr>
<tr>
<td>$b_1$</td>
<td>-0.095</td>
</tr>
<tr>
<td>Search intensity</td>
<td>$s_1/s_0$</td>
</tr>
<tr>
<td>$b_1 x + b_2 x^2$</td>
<td></td>
</tr>
<tr>
<td>$b_2$</td>
<td>4.688</td>
</tr>
<tr>
<td>Vacancy posting costs</td>
<td>$c_0$</td>
</tr>
<tr>
<td>$p(x, y, z) = e^z$</td>
<td></td>
</tr>
<tr>
<td>$p_1$</td>
<td>0.612</td>
</tr>
<tr>
<td>Value added</td>
<td></td>
</tr>
<tr>
<td>$p_2$</td>
<td>-0.171</td>
</tr>
<tr>
<td>Exogenous separation</td>
<td>$\delta$</td>
</tr>
<tr>
<td>$p_3$</td>
<td>-1.024</td>
</tr>
<tr>
<td>Productivity shocks</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>$p_4$</td>
<td>4.650</td>
</tr>
<tr>
<td>Gaussian copula $(\sigma, \rho)$</td>
<td>$\rho$</td>
</tr>
<tr>
<td>$p_5$</td>
<td>-2.995</td>
</tr>
<tr>
<td>Worker heterogeneity</td>
<td>$\beta_1$</td>
</tr>
<tr>
<td>$p_6$</td>
<td>3.093</td>
</tr>
<tr>
<td>Beta($\beta_1, \beta_2$)</td>
<td>$\beta_2$</td>
</tr>
</tbody>
</table>

Note: $r$ is fixed at 0.05 annually.
<table>
<thead>
<tr>
<th>Fitted Moments</th>
<th>Data</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{E}[U]$</td>
<td>0.0562</td>
<td>0.0568</td>
<td>0.0573</td>
<td>0.0541</td>
<td>0.0549</td>
<td>0.0614</td>
<td>0.0615</td>
</tr>
<tr>
<td>$\mathbb{E}[U^{5p}]$</td>
<td>0.0324</td>
<td>0.0339</td>
<td>0.0348</td>
<td>0.0294</td>
<td>0.0309</td>
<td>0.0320</td>
<td>0.0312</td>
</tr>
<tr>
<td>$\mathbb{E}[U^{15p}]$</td>
<td>0.0153</td>
<td>0.0148</td>
<td>0.0155</td>
<td>0.0090</td>
<td>0.0103</td>
<td>0.0091</td>
<td>0.0089</td>
</tr>
<tr>
<td>$\mathbb{E}[U^{27p}]$</td>
<td>0.0078</td>
<td>0.0064</td>
<td>0.0067</td>
<td>0.0023</td>
<td>0.0032</td>
<td>0.0024</td>
<td>0.0029</td>
</tr>
<tr>
<td>$\mathbb{E}[U^{2E}]$</td>
<td>0.4376</td>
<td>0.4188</td>
<td>0.4090</td>
<td>0.4680</td>
<td>0.4465</td>
<td>0.4881</td>
<td>0.5109</td>
</tr>
<tr>
<td>$\mathbb{E}[E2U]$</td>
<td>0.0254</td>
<td>0.0244</td>
<td>0.0240</td>
<td>0.0262</td>
<td>0.0254</td>
<td>0.0314</td>
<td>0.0323</td>
</tr>
<tr>
<td>$\mathbb{E}[J2J]$</td>
<td>0.0273</td>
<td>0.0260</td>
<td>0.0311</td>
<td>0.0277</td>
<td>0.0276</td>
<td>0.0382</td>
<td>0.0231</td>
</tr>
<tr>
<td>$\mathbb{E}[sd \text{ labor prod}]$</td>
<td>0.7478</td>
<td>0.6623</td>
<td>0.3537</td>
<td>na</td>
<td>0.0683</td>
<td>0.1856</td>
<td>0.0953</td>
</tr>
<tr>
<td>sd[U]</td>
<td>0.2140</td>
<td>0.2063</td>
<td>0.2126</td>
<td>0.1731</td>
<td>0.1633</td>
<td>0.1678</td>
<td>0.2098</td>
</tr>
<tr>
<td>sd[U^{5p}]</td>
<td>0.3138</td>
<td>0.2670</td>
<td>0.2791</td>
<td>0.2728</td>
<td>0.2197</td>
<td>0.2238</td>
<td>0.2898</td>
</tr>
<tr>
<td>sd[U^{15p}]</td>
<td>0.4435</td>
<td>0.3699</td>
<td>0.3979</td>
<td>0.4647</td>
<td>0.3615</td>
<td>0.3344</td>
<td>0.4435</td>
</tr>
<tr>
<td>sd[U^{27p}]</td>
<td>0.5388</td>
<td>0.4740</td>
<td>0.5332</td>
<td>0.6823</td>
<td>0.5429</td>
<td>0.4601</td>
<td>0.6356</td>
</tr>
<tr>
<td>sd[U^{2E}]</td>
<td>0.1257</td>
<td>0.1509</td>
<td>0.1599</td>
<td>0.1400</td>
<td>0.1228</td>
<td>0.1130</td>
<td>0.1655</td>
</tr>
<tr>
<td>sd[E2U]</td>
<td>0.1291</td>
<td>0.1267</td>
<td>0.1300</td>
<td>0.0573</td>
<td>0.1033</td>
<td>0.1335</td>
<td>0.1374</td>
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<tr>
<td>sd[J2J]</td>
<td>0.0924</td>
<td>0.1069</td>
<td>0.1037</td>
<td>0.1899</td>
<td>0.1285</td>
<td>0.1984</td>
<td>0.1288</td>
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<tr>
<td>sd[sd labor prod]</td>
<td>0.0166</td>
<td>0.0082</td>
<td>0.0063</td>
<td>na</td>
<td>0.0042</td>
<td>0.0009</td>
<td>0.0087</td>
</tr>
<tr>
<td>sd[V]</td>
<td>0.2291</td>
<td>0.1860</td>
<td>0.1163</td>
<td>0.2349</td>
<td>0.2384</td>
<td>0.2260</td>
<td>0.1777</td>
</tr>
<tr>
<td>sd[V/U]</td>
<td>0.4162</td>
<td>0.3722</td>
<td>0.3157</td>
<td>0.3964</td>
<td>0.3223</td>
<td>0.3147</td>
<td>0.3185</td>
</tr>
<tr>
<td>sd[VA]</td>
<td>0.0363</td>
<td>0.0379</td>
<td>0.0389</td>
<td>0.0384</td>
<td>0.0379</td>
<td>0.0344</td>
<td>0.0354</td>
</tr>
<tr>
<td>autocorr[VA]</td>
<td>0.9427</td>
<td>0.9553</td>
<td>0.9557</td>
<td>0.8804</td>
<td>0.9254</td>
<td>0.7976</td>
<td>0.8754</td>
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<tr>
<td>corr[V, U]</td>
<td>-0.7642</td>
<td>-0.8005</td>
<td>-0.8272</td>
<td>-0.8846</td>
<td>-0.2614</td>
<td>-0.2608</td>
<td>-0.3463</td>
</tr>
<tr>
<td>corr[U, VA]</td>
<td>-0.7742</td>
<td>-0.9406</td>
<td>-0.9528</td>
<td>-0.9778</td>
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<td>-0.7380</td>
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<tr>
<td>corr[V, VA]</td>
<td>0.6372</td>
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<td>0.8881</td>
<td>0.9477</td>
<td>0.9315</td>
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<td>0.8604</td>
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<tr>
<td>corr[U^{2E}, VA]</td>
<td>0.8143</td>
<td>0.9010</td>
<td>0.9360</td>
<td>0.9416</td>
<td>0.2102</td>
<td>0.4501</td>
<td>0.6420</td>
</tr>
<tr>
<td>corr[E2U, VA]</td>
<td>-0.5984</td>
<td>-0.5169</td>
<td>-0.4455</td>
<td>-0.9226</td>
<td>-0.2932</td>
<td>-0.3915</td>
<td>-0.3132</td>
</tr>
<tr>
<td>corr[U^{2E}, J2J]</td>
<td>0.6333</td>
<td>0.5526</td>
<td>0.5494</td>
<td>0.9974</td>
<td>0.2857</td>
<td>0.5842</td>
<td>0.4270</td>
</tr>
<tr>
<td>corr[sd labor prod, VA]</td>
<td>-0.3902</td>
<td>-0.4552</td>
<td>-0.3910</td>
<td>na</td>
<td>0.7465</td>
<td>-0.2184</td>
<td>-0.2690</td>
</tr>
</tbody>
</table>

Model (I) baseline model; (II) home production is independent of worker type and aggregate state $b(x, z) = b$; (III) no worker or firm heterogeneity; (IV) only worker heterogeneity; (V) has no production complementarities: $p_{xy} = 0$; (VI) has production of the form $p(x, y, z) = xyz$. 