Marriage Market, Labor Supply and Education Choice

Pierre-Andre Chiappori, Monica Costa-Dias and Costas Meghir

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How does tax and welfare policy affect key life-cycle decisions, such as marriage, labor supply and education?

- Policies changing incentives to work can also affect education and marital choices in the long-run
- These feedback into labor supply and the allocation of resources within the family
- And have distributional and intergenerational impacts
- All these may bias our understanding of the long-term effects of policy reforms
Here we look at the interactions between marriage, labor supply and education

1. What is the role of education for marital sorting and the intra-household allocation of time and resources?

2. In turn, how do marital prospects affect education choices?

Two fundamental insights from Becker offer the elements needed to understand these interactions

- **Human Capital**: education as an investment
- **Matching**: marriage as a matching game in which the allocation of marital surplus to spouses is determined in equilibrium
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Two fundamental insights from Becker offer the elements needed to understand these interactions

- **Human Capital:** education as an investment

- **Matching:** marriage as a matching game in which the allocation of marital surplus to spouses is determined in equilibrium
But how do matching and human capital interact and what are the implications of these interactions for the intrahousehold allocation of resources?

Specifically, is matching on human capital assortative?

- Key to understand the origins of inequality, its transmission across generations and the long-run effects of policy
- Ambiguous predictions from economic theory
  - Public goods tend to generate PAM
  - Domestic production may push in the opposite direction (especially with specialization)
  - Risk sharing: more HC means both higher wages and possibly more wage volatility
The approach we take

- Structural life-cycle model of education, marriage, labor supply and savings
  - Intertemporal family labor supply and savings
  - Marital sorting on human capital
  - Marriage market equilibrium determines sharing rule
  - Education has returns in the labor and marriage markets

- Estimate the model on data from UK
  - Combine information on matching patterns and life-cycle labor supply and earnings to identify marital surplus and the Pareto weights

- Test assortative matching

- Use model for policy evaluation (in progress)
Collective models (Chiappori, 1988, 1992; Blundell, Chiappori and Meghir, 2005)
- Exogenous Pareto weights, static framework, no pre-marital investments
- We relax these but assume Transferable Utility

Dynamic individual labor supply (Attanasio, Low and Sanchez-Marcos, 2008; Low, Meghir and Pistaferri, 2010; Blundell et al., 2015)
- Marriage and often education are exogenous
- We model family labor supply with endogenous education and marriage

Dynamic family labor supply with limited commitment (Mazzocco, 2007; Voena, 2015)
- We impose full commitment but characterize equilibrium and model pre-marital HC investment and choice of spouse
Pre-marital investments and marriage (Chiappori, Iyigun and Weiss, 2009)
- Post-marital behaviour not modelled

Draw from Choo and Siow (2006) and Chiappori, Salanie and Weiss (2014), who develop a stochastic matching framework
- Size of the surplus \textit{and} its distribution are identified from matching patterns alone
- We add information from post-marital behaviour to identify values of marriage and Pareto weights
  - Life-cycle labor supply offers information on preferences and the marital surplus
  - Equilibrium in the marriage market then identifies the Pareto weights from the matching patterns
Life-cycle model: life in three stages

- Education happens before entering the matching game
  - Information: idiosyncratic ability, costs of education
  - Depends on expected returns in marriage and labour markets
  - HC is a function of education and ability

- Marriage in frictionless matching market, by HC
  - Determines who marries whom by HC, the intra-household allocations and the returns to education
  - Value of marriage: public good and risk sharing

- Married/single life in T periods
  - Wage rate is a function of HC and productivity shocks
  - Efficient choice of (private and public) consumptions and labour supply defines marital surplus
  - Full commitment and no divorce: sharing rule depends on marriage market conditions at time of marriage
  - Transferable utility: like unitary model

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Marriage Market
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• **Human Capital** \( (H) \): individual \( i \) of gender \( G \), education \( (s) \) and ability \( (\theta) \)

\[
H_i = H_G(s_i, \theta_i)
\]

• **Wages**: gender specific functions education, ability, and age \( (t) \), and subject to persistent AR(1) shocks \( (e) \)

\[
w_{it} = W_G(\theta_i, s_i, t)e_{it}
\]

where

\[
\ln W_G(\theta_i, s_i, t) = \ln \tilde{W}_G(\theta_i) + \delta_{1G}(s_i)t + \delta_{2G}(s_i)t^2 + \delta_{3G}(s_i)t^3
\]
Individual lifetime utility has three components

- Expected discounted economic utilities from stage 3
- Subjective utility of marriage
- Utility cost of education

Preferences at time $t$ of stage 3 over private consumption ($C_{it}$), public consumption ($Q_t$) and Labor Supply ($L_{it} \in \{0, 1\}$)

$$u_i (Q_t, C_{it}, L_{it}) = \ln (C_{it}Q_t + \alpha_{it}L_{it}Q_t)$$

where $\alpha_{it}$ is an education, gender and marital status specific poly in $t$ subject to unobserved heterogeneity transitory preference shocks
Two remarks on these utilities

- Belong to Generalised Quasi-Linear family ⇒ satisfy the TU property *ex-post* ⇒ Pareto efficient choice of \((C, Q, L)\) independent of \(PW\) (=1 for specific transformation)

  - For log utilities, use the exponential
  - In any period, conditional on labor supply and savings, problem is static and couple chooses consumptions to maximise (subject to budget constraint)

\[
\exp u_i + \exp u_j = Q_t (C_t + \alpha_{it}L_{it} + \alpha_{jt}L_{jt})
\]

- Belong to ISHARA class ⇒ TU property *ex-ante*, in expectations: there exists a cardinalization such that the household max the sum of discounted utilities of its members
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The Model: TU *ex-ante* I

- Take couple \((H_m, H_f)\) at time \(t\), with expected values \((V_{mt}, V_{ft})\)

- There exists \(\Upsilon_t (H_m, H_f)\) such that any Pareto efficient allocation satisfies (\(\delta\): discount factor)

\[
\exp \left\{ \frac{1 - \delta}{1 - \delta T - t} V_{mt} \right\} + \exp \left\{ \frac{1 - \delta}{1 - \delta T - t} V_{ft} \right\} = \exp \left\{ \frac{1 - \delta}{1 - \delta T - t} \Upsilon_t (H_m, H_f) \right\}
\]

- \((V_{mt}, V_{ft})\) depend on the sharing rule, \(\Upsilon_t (H_m, H_f)\) does not

- Choose \(\bar{U}_{it} = \exp \left\{ \frac{1 - \delta}{1 - \delta T - t} V_i \right\}\). TU applies *ex-ante*

\[
\bar{U}_{mt} + \bar{U}_{ft} = \exp \left\{ \frac{1 - \delta}{1 - \delta T - t} \Upsilon_t (H_m, H_f) \right\}
\]
Thus couples behave throughout stage 3 as a single decision maker, maximizing \( \Upsilon(H_m, H_f) \).

Conditional on labor supply, optimal savings satisfy the Euler equation derived using this parameterization.

The labor supply of both spouses can then be selected to maximise discounted utility of the couple.
Apply a similar transformation at the time of marriage, to define the economic value of match \((m, f)\):

\[
\bar{U}_m + \bar{U}_f = \exp \left\{ \frac{1 - \delta}{1 - \delta^T} \Upsilon (H_m, H_f) \right\} \equiv g (H_m, H_f)
\]

Use similar transformation to represent value of remaining single

\[
\bar{U}^S_i (H_i) = \exp \left\{ \frac{1 - \delta}{1 - \delta^T} V^S_i (H_i) \right\} = g^G (H_i)
\]

Then the economic surplus generated by marrying man and woman \((m, f)\) is

\[
\Sigma (H_m, H_f) = g (H_m, H_f) - g^M (H_m) - g^F (H_f)
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\]
Assume HC is defined in a finite domain and add subjective preferences for marrying a spouse $H$ (Choo and Siow, 2006)

The total gain generated by matching $(m, f)$ is

$$g_{mf} = g(H_m, H_f) + \beta^H_m + \beta^H_f$$

And total marital surplus is

$$\Sigma_{mf} = \Sigma(H_m, H_f) + (\beta^H_m - \beta^0_m) + (\beta^H_f - \beta^0_f)$$

Idiosyncratic preferences only depend on the spouse’s HC: relaxing this assumption in a large, frictionless market implies that the number of singles tends to zero and lifetime utility to infinity
Agents are characterised by their HC and draw preferences for marriage before entering the market.

The value of matching man $m$ with female $f$ was defined before

$$g_{mf} = g(H_m, H_f) + \beta^H_f + \beta^H_m$$

Matching stability requires

$$U_m + U_f \geq g_{mf} \quad \text{for all } (m, f)$$
$$U_m + U_f = g_{mf} \quad \text{if } m \text{ married to } f$$

$(U_m, U_f)$ describe how the gain $g_{mf}$ is divided between spouses if $(m, f)$ match.
Separability result (Chiappori, Salanie and Weiss, 2015): There exists a set of numbers $\bar{U}_M(H_m, H_f)$ and $\bar{U}_F(H_m, H_f)$ such that

$$\bar{U}_M(H_m, H_f) + \bar{U}_F(H_m, H_f) = g(H_m, H_f)$$

and, for any couple $(m, f)$:

$$U_m = \bar{U}_M(H_m, H_f) + \beta_{m}^{H_f}$$
$$U_f = \bar{U}_F(H_m, H_f) + \beta_{f}^{H_m}$$

This means that the value of marrying $f$ for man $m$ is the sum of a deterministic component, that depends on the HC of both spouses, and an idiosyncratic preference for spouses of HC $H_f$. 
Corollary: for every man (woman) the optimal match satisfies

\[ \bar{U}_M(H_m, H_f) + \beta^H_m \geq \bar{U}_M(H_m, H) + \beta^H_m, \quad \forall H \]

- \( H \) discrete: multinomial discrete choice model identifies \( \bar{U}_G \)
- Since we know \( \bar{U}_G \) up to the Pareto weight (sharing rule) from the stage 3 problem, this identifies the Pareto weight
- Because the surplus is identified by lifecycle behavior, we have overidentifying restrictions
Education Choices made in anticipation of the marriage market

Individuals do not know their marital preferences (we are working to relax this assumption)

Costs of education depend on observables and a random shock

We assume that, conditional on parental background, parental income at the time of education choice affects education but not preferences
Returning to the original cardinalization, the lifetime value of $H_m$ for the man (resp. female) is

$$\bar{V}_m(H_m) = \frac{1 - \delta^T}{1 - \delta} \mathbb{E} \ln(\max_H [\bar{U}_M(H_m, H) + \beta_m^H])$$

Education Choice is then driven by

for man $m$: \[ s_m = \arg \max_{s \in S} \{ \bar{V}_m(H_m) - c_{sm}(X_m, \nu_{sm}) \} \]

for woman $f$: \[ s_f = \arg \max_{s \in S} \{ \bar{V}_f(H_f) - c_{sf}(X_f, \nu_{sf}) \} \]
Within period allocations to consumption and public goods \textit{conditional} on savings and labor supply are obtained by solving

$$\max_{Q_t, C_t} \quad Q_t \left( C_t + \alpha_{mt} L_{mt} + \alpha_{ft} L_{ft} \right)$$

under the budget constraint

$$w_{mt} + w_{ft} + y_t^C + RK_{t-1} = K_t + C_t + w_{mt} L_{mt} + w_{ft} L_{ft} + pQ_t$$
Allocation of consumption ex ante follows the optimal risk sharing rule

\[
\frac{\partial u_{mt}(Q_t, C_{mt}, L_{mt})}{\partial C_{mt}} = \mu \frac{\partial u_{ft}(Q_t, C_{ft}, L_{ft})}{\partial C_{ft}}
\]

This is conditional on aggregate consumption, savings and labor supply (that do not depend on the sharing rule)

The allocation in our case is

\[
C_{mt} = \frac{1}{1 + \mu}pQ_t - \alpha_{mt}L_{mt}
\]

\[
C_{ft} = \frac{\mu}{1 + \mu}pQ_t - \alpha_{ft}L_{ft}.
\]
Household savings/consumption satisfy the usual Euler equation.

Labor supply is a dynamic discrete choice problem.

By backwards induction and using the result on optimal risk sharing we can show that

\[ E_{t-1} V_{ft} (e_t, \alpha_t, K_{t-1}, H, \mu) = \Upsilon_{t-1} (e_{t-1}, \alpha_{t-1}, K_{t-1}, H) + \ln \left( \frac{\mu}{1 + \mu} \right) \sum_{\tau=t}^{T} \beta^{\tau-t} \]

where \( \Upsilon_t \) is defined recursively by

\[ \Upsilon_{t-1} (e_{t-1}, \alpha_{t-1}, K_{t-1}, H) = \max_{L_t} E_{t-1} \left[ \max_{K_t} \{ 2 \ln Q_t (L_t, K_t) + \ln p + \delta I_t (e_t, \alpha_t, K_t, H) \} \right] \]
The Data

- Data: British Household Panel Survey for the years 1991-2008
- 4,300 Couples, 1131 Single men and 937 single women aged 23-50
- Marital status assessed at age 30 or above
  - Singles: those never married
  - Those married followed for the entire marital spell
  - We ignore divorce since we do not include it in the models
- Three education levels (statutory, high school, college)
- Employment defined as working 5 hours or more per week
- We net out aggregate growth from wages
Estimation I

- In our model education and marriage are endogenous
- One way of estimating is to
  - Solve the model
  - For each set of trial parameters generate moments
  - Match these moments with their observed data counterparts
  - This involves solving for equilibrium in the marriage market
We use a simplified procedure

- Estimate the earnings process outside the model
  - instrument education with parental income, controlling for parental background
  - instrument participation using income out of work

- Conditional on wages, solve the post marriage problem to estimate
  - preferences for working
  - the distribution of ability conditional on education and marriage

- Then back-out the Pareto weights - more later

- Then estimate the parameters driving cost of education - important for counterfactuals
\[
\ln w_{it} = \ln \tilde{W}_G(\theta_i) + \delta_1 G(s_i)t + \delta_2 G(s_i)t^2 + \delta_3 G(s_i)t^+e_{it} + \epsilon_{it} \\
\epsilon_{it} = \rho_s G e_{it-1} + \zeta_{it}
\]
## Earnings process II

<table>
<thead>
<tr>
<th></th>
<th>Men</th>
<th></th>
<th></th>
<th>Women</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Stat</td>
<td>HS</td>
<td>Univ</td>
<td>Stat</td>
<td>HS</td>
</tr>
<tr>
<td>(7)</td>
<td>Autocorr coeff ($\rho$)</td>
<td>0.502</td>
<td>0.594</td>
<td>0.416</td>
<td>0.811</td>
<td>0.820</td>
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<td></td>
<td></td>
<td>(.115)</td>
<td>(.131)</td>
<td>(.226)</td>
<td>(.104)</td>
<td>(.067)</td>
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<td>(8)</td>
<td>Var innov in prod ($\sigma^2_\xi$)</td>
<td>0.024</td>
<td>0.012</td>
<td>0.026</td>
<td>0.030</td>
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<td></td>
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<td>(.005)</td>
<td>(.005)</td>
<td>(.049)</td>
<td>(.006)</td>
<td>(.004)</td>
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<tr>
<td>(9)</td>
<td>Var ME ($\sigma^2_\epsilon$)</td>
<td>0.004</td>
<td>0.007</td>
<td>0.001</td>
<td>0.012</td>
<td>0.005</td>
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<td></td>
<td></td>
<td>(.004)</td>
<td>(.005)</td>
<td>(.049)</td>
<td>(.004)</td>
<td>(.002)</td>
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<tr>
<td>N</td>
<td></td>
<td>9,116</td>
<td>11,990</td>
<td>4,291</td>
<td>8,432</td>
<td>7,469</td>
</tr>
</tbody>
</table>

Chiappori, Costa-Dias, Meghir

Marriage Market
Table: Earnings levels by gender and education

<table>
<thead>
<tr>
<th>Ability Type</th>
<th>Stat</th>
<th>HS</th>
<th>Univ</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Men</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ability type 1</td>
<td>2.61</td>
<td>2.74</td>
<td>2.74</td>
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<tr>
<td></td>
<td>(.01)</td>
<td>(.01)</td>
<td>(.03)</td>
</tr>
<tr>
<td>ability type 2</td>
<td>3.13</td>
<td>3.25</td>
<td>3.24</td>
</tr>
<tr>
<td></td>
<td>(.00)</td>
<td>(.00)</td>
<td>(.00)</td>
</tr>
<tr>
<td><strong>Women</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ability type 1</td>
<td>1.95</td>
<td>2.00</td>
<td>2.82</td>
</tr>
<tr>
<td></td>
<td>(.03)</td>
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<td>(.04)</td>
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<tr>
<td>ability type 2</td>
<td>2.95</td>
<td>2.95</td>
<td>3.10</td>
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<tr>
<td></td>
<td>(.01)</td>
<td>(.00)</td>
<td>(.02)</td>
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</table>
### Table: Ability conditional on Education among couples

<table>
<thead>
<tr>
<th></th>
<th>Men</th>
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<tbody>
<tr>
<td></td>
<td>Stat Ed</td>
<td>HS</td>
<td>Univ</td>
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<tr>
<td></td>
<td>ab 1</td>
<td>ab 2</td>
<td>ab 1</td>
<td>ab 2</td>
<td>ab 1</td>
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<tr>
<td>Stat Ed</td>
<td>ab 1</td>
<td>0.216</td>
<td>0.429</td>
<td>0.213</td>
<td>0.270</td>
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<tr>
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<td>(.02)</td>
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<td>(.07)</td>
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<td></td>
<td>ab 2</td>
<td>0.124</td>
<td>0.231</td>
<td>0.209</td>
<td>0.308</td>
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<tr>
<td></td>
<td>(.02)</td>
<td>(.02)</td>
<td>(.02)</td>
<td>(.02)</td>
<td>(.05)</td>
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<tr>
<td>HS</td>
<td>ab 1</td>
<td>0.117</td>
<td>0.247</td>
<td>0.132</td>
<td>0.156</td>
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<tr>
<td></td>
<td>(.01)</td>
<td>(.02)</td>
<td>(.01)</td>
<td>(.02)</td>
<td>(.04)</td>
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<tr>
<td></td>
<td>ab 2</td>
<td>0.153</td>
<td>0.483</td>
<td>0.262</td>
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<td></td>
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<td>(.02)</td>
<td>(.01)</td>
<td>(.01)</td>
<td>(.04)</td>
</tr>
<tr>
<td>Univ</td>
<td>ab 1</td>
<td>0.148</td>
<td>0.287</td>
<td>0.276</td>
<td>0.071</td>
</tr>
<tr>
<td></td>
<td>(.05)</td>
<td>(.40)</td>
<td>(.08)</td>
<td>(.06)</td>
<td>(.04)</td>
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<tr>
<td></td>
<td>ab 2</td>
<td>0.093</td>
<td>0.472</td>
<td>0.051</td>
<td>0.603</td>
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<tr>
<td></td>
<td>(.08)</td>
<td>(.09)</td>
<td>(.09)</td>
<td>(.03)</td>
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</table>
Table: Proportion of ability type 1 among singles by gender and education

<table>
<thead>
<tr>
<th></th>
<th>secondary</th>
<th>high school</th>
<th>university</th>
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<tbody>
<tr>
<td>men</td>
<td>0.753</td>
<td>0.705</td>
<td>0.505</td>
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<tr>
<td></td>
<td>(.03)</td>
<td>(.02)</td>
<td>(.03)</td>
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<tr>
<td>women</td>
<td>0.439</td>
<td>0.171</td>
<td>0.109</td>
</tr>
<tr>
<td></td>
<td>(.04)</td>
<td>(.06)</td>
<td>(.08)</td>
</tr>
</tbody>
</table>
The Economic Surplus from marriage

- Given the estimates of all parameters in wages and preferences, we can compute the economic Surplus.
- We classify it on the basis increasing human capital
- Positive assortative matching

Table: Economic surplus from marriage by human capital of both spouses

<table>
<thead>
<tr>
<th>Men H: (s, θ)</th>
<th>Women</th>
<th>Low HC</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>High</th>
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</thead>
<tbody>
<tr>
<td>Low H: (1, 1)</td>
<td></td>
<td>30</td>
<td>61</td>
<td>121</td>
<td>156</td>
<td>232</td>
<td>289</td>
</tr>
<tr>
<td>2: (2, 1)</td>
<td></td>
<td>47</td>
<td>84</td>
<td>151</td>
<td>193</td>
<td>279</td>
<td>343</td>
</tr>
<tr>
<td>3: (3, 1)</td>
<td></td>
<td>53</td>
<td>95</td>
<td>171</td>
<td>218</td>
<td>312</td>
<td>384</td>
</tr>
<tr>
<td>4: (1, 2)</td>
<td></td>
<td>59</td>
<td>107</td>
<td>193</td>
<td>248</td>
<td>354</td>
<td>437</td>
</tr>
<tr>
<td>5: (2, 2)</td>
<td></td>
<td>90</td>
<td>146</td>
<td>248</td>
<td>314</td>
<td>434</td>
<td>530</td>
</tr>
<tr>
<td>High H: (3, 2)</td>
<td></td>
<td>89</td>
<td>150</td>
<td>262</td>
<td>333</td>
<td>462</td>
<td>565</td>
</tr>
</tbody>
</table>
Figure: BHPS data and model predictions: employment of men and women over the lifecycle

Notes: Full lines are for BHPS data and the dashed lines are for model simulations.
Figure: BHPS data and model predictions: log annual earnings for men and women over the lifecycle

Notes: Full lines are for BHPS data and the dashed lines are for model simulations. Annual earnings in real terms (GBP 1,000, 2008 prices).
Intrahousehold allocations - The Pareto weights I

- Probability of man with HC $H_m$ marrying a woman of HC $H$:

$$\Pr(H | H_m) = \Pr \left( \bar{U}_M(H_m, H) + \beta_m^H \geq \bar{U}_M(H_m, H') + \beta_m^{H'} \right)$$

$$= \Pr \left( \beta_m^H - \beta_m^{H'} \geq \bar{U}_M(H_m, H') - \bar{U}_M(H_m, H) \right)$$

- Now recall that

$$\bar{U}_M(H_m, H) = \exp \left\{ \frac{1 - \delta}{1 - \delta_T} V_m(H_m, H, \mu) \right\}$$

where

$$V_m(H_m, H, \mu) = \Upsilon(H_m, H) + \frac{1 - \delta_T}{1 - \delta} \ln(\mu(H_m, H))$$
If the $\beta$’s are extreme value rv, we can recover $\mu$ from $Pr(H|H_m)$

Since we do not observe ability, hence HC, we do not directly observe $Pr(H|H_m)$

However, it can be recovered from the estimated conditional distribution of $(\theta_m, \theta | s_m, s)$ and the observed distribution of $(s_m, s)$
### Table: Women’s Pareto weights by spouses’ human capital

<table>
<thead>
<tr>
<th>Men $(s, \theta)$</th>
<th>Women $(s, \theta)$</th>
<th>LowH $(1, 1)$</th>
<th>2 $(2, 1)$</th>
<th>3 $(1, 2)$</th>
<th>4 $(2, 2)$</th>
<th>5 $(3, 1)$</th>
<th>High H $(3, 2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low H: $(1, 1)$</td>
<td>0.19</td>
<td>0.27</td>
<td>0.45</td>
<td>0.67</td>
<td>1.62</td>
<td>2.60</td>
<td></td>
</tr>
<tr>
<td>2: $(2, 1)$</td>
<td>0.13</td>
<td>0.19</td>
<td>0.31</td>
<td>0.45</td>
<td>1.81</td>
<td>1.49</td>
<td></td>
</tr>
<tr>
<td>3: $(3, 1)$</td>
<td>0.10</td>
<td>0.16</td>
<td>0.29</td>
<td>0.44</td>
<td>1.39</td>
<td>2.57</td>
<td></td>
</tr>
<tr>
<td>4: $(1, 2)$</td>
<td>0.09</td>
<td>0.14</td>
<td>0.24</td>
<td>0.37</td>
<td>2.04</td>
<td>1.95</td>
<td></td>
</tr>
<tr>
<td>5: $(2, 2)$</td>
<td>0.05</td>
<td>0.08</td>
<td>0.15</td>
<td>0.22</td>
<td>0.64</td>
<td>1.31</td>
<td></td>
</tr>
<tr>
<td>HighH: $(3, 2)$</td>
<td>0.05</td>
<td>0.08</td>
<td>0.15</td>
<td>0.15</td>
<td>0.63</td>
<td>1.04</td>
<td></td>
</tr>
</tbody>
</table>
These are estimated Pareto weights based on observed behavior.

The model is over identified since one can use females or males to estimate the weights - in principle the model is testable.

For the purposes of counterfactual simulations one needs to solve the model to find a new equilibrium in the marriage market.

This involves computing the new surplus function; Then finding the set of stable matches given education; Then iterating on the education choice.
Table: Sorting patterns and the sharing of consumption and welfare

<table>
<thead>
<tr>
<th></th>
<th>Women’s HC</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lowest</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>highest</td>
</tr>
<tr>
<td>Men’s HC: lowest</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% matches in cell</td>
<td>0.070</td>
<td>0.014</td>
<td>0.040</td>
<td>0.018</td>
<td>0.003</td>
<td>0.002</td>
</tr>
<tr>
<td>man’s share in welfare</td>
<td>0.60</td>
<td>0.57</td>
<td>0.54</td>
<td>0.51</td>
<td>0.47</td>
<td>0.45</td>
</tr>
<tr>
<td>man’s share in consumption</td>
<td>0.93</td>
<td>0.90</td>
<td>0.61</td>
<td>0.53</td>
<td>0.38</td>
<td>0.23</td>
</tr>
<tr>
<td>Men’s HC: 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% matches in cell</td>
<td>0.041</td>
<td>0.020</td>
<td>0.040</td>
<td>0.040</td>
<td>0.012</td>
<td>0.002</td>
</tr>
<tr>
<td>man’s share in welfare</td>
<td>0.62</td>
<td>0.59</td>
<td>0.55</td>
<td>0.53</td>
<td>0.47</td>
<td>0.48</td>
</tr>
<tr>
<td>man’s share in consumption</td>
<td>0.96</td>
<td>0.98</td>
<td>0.72</td>
<td>0.66</td>
<td>0.30</td>
<td>0.31</td>
</tr>
<tr>
<td>Men’s HC: 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% matches in cell</td>
<td>0.005</td>
<td>0.004</td>
<td>0.002</td>
<td>0.008</td>
<td>0.008</td>
<td>0.006</td>
</tr>
<tr>
<td>man’s share in welfare</td>
<td>0.63</td>
<td>0.60</td>
<td>0.56</td>
<td>0.53</td>
<td>0.48</td>
<td>0.45</td>
</tr>
<tr>
<td>man’s share in consumption</td>
<td>1.01</td>
<td>0.99</td>
<td>0.74</td>
<td>0.68</td>
<td>0.41</td>
<td>0.23</td>
</tr>
<tr>
<td>Men’s HC: 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% matches in cell</td>
<td>0.139</td>
<td>0.030</td>
<td>0.075</td>
<td>0.058</td>
<td>0.005</td>
<td>0.008</td>
</tr>
<tr>
<td>man’s share in welfare</td>
<td>0.63</td>
<td>0.60</td>
<td>0.56</td>
<td>0.54</td>
<td>0.46</td>
<td>0.47</td>
</tr>
<tr>
<td>man’s share in consumption</td>
<td>0.99</td>
<td>0.99</td>
<td>0.79</td>
<td>0.73</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>Men’s HC: 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% matches in cell</td>
<td>0.052</td>
<td>0.024</td>
<td>0.059</td>
<td>0.069</td>
<td>0.003</td>
<td>0.027</td>
</tr>
<tr>
<td>man’s share in welfare</td>
<td>0.66</td>
<td>0.63</td>
<td>0.59</td>
<td>0.56</td>
<td>0.51</td>
<td>0.48</td>
</tr>
<tr>
<td>man’s share in consumption</td>
<td>1.02</td>
<td>1.03</td>
<td>0.86</td>
<td>0.83</td>
<td>0.67</td>
<td>0.44</td>
</tr>
<tr>
<td>Men’s HC: highest</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% matches in cell</td>
<td>0.006</td>
<td>0.008</td>
<td>0.011</td>
<td>0.032</td>
<td>0.009</td>
<td>0.050</td>
</tr>
<tr>
<td>man’s share in welfare</td>
<td>0.66</td>
<td>0.62</td>
<td>0.58</td>
<td>0.58</td>
<td>0.51</td>
<td>0.49</td>
</tr>
<tr>
<td>man’s share in consumption</td>
<td>1.02</td>
<td>1.02</td>
<td>0.86</td>
<td>0.89</td>
<td>0.69</td>
<td>0.52</td>
</tr>
</tbody>
</table>
The Surplus is supermodular (the sum of the diagonal in any two sub-matrices is higher than the sum of the off diagonals).

The Gradient of the surplus is much steeper with respect to female human capital than it is with respect to male.

This is because education has a much stronger effect on earnings and employment of women than it does on men.

We can test whether we obtain PAM.
Graham, 2013: Take $H_m > H'_m$ and $H_f > H'_f$. Suppose

$$g(H_m, H_f) + g(H'_m, H'_f) - g(H'_m, H_f) - g(H_m, H'_f) > 0$$

This means that this subpopulation has more assortative matching than predicted by pure random allocations.

In our case, this is true almost always.

The prediction comes from the structure of the estimated surplus, which does not involve using matching patterns.
The final element of estimation is the education choice.

This is an important part of the story because it defines the endogenous supply of human capital.

Policies that change educational choices have the potential of affecting the marriage market and intrahousehold allocations.

**Table: Cost of education**

<table>
<thead>
<tr>
<th></th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HS</td>
<td>Univ</td>
</tr>
<tr>
<td>constant</td>
<td>0.702</td>
<td>2.656</td>
</tr>
<tr>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>background factor 1</td>
<td>0.073</td>
<td>-0.285</td>
</tr>
<tr>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>background factor 2</td>
<td>-0.014</td>
<td>0.002</td>
</tr>
<tr>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>log parental income</td>
<td>-0.084</td>
<td>-0.212</td>
</tr>
<tr>
<td>(0.01)</td>
<td>(0.03)</td>
<td>(0.01)</td>
</tr>
</tbody>
</table>
Concluding Remarks

- This a rich equilibrium framework for considering life-cycle decisions in a unified way

- It allows us to analyze the long term effects of policies in a systematic way

- It also provides an empirical framework for understanding marital patterns

- On the agenda (shorter and longer run):
  - Counterfactual policy simulations
  - Divorce
  - Imperfectly transferable utility
  - Limited commitment in equilibrium