Marriage Market, Labor Supply and Education Choice

Pierre-Andre Chiappori, Monica Costa-Dias and Costas Meghir

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The basic motivation for this project is to understand how policy affects individual life-cycle decisions.

Long term effects will change education choices and the Marriage market.

In turn this will have effects on labor supply and will have intergenerational impacts.

Two fundamental, Beckerian insights: Notion of Human Capital and Matching as an equilibrium phenomenon.

It is extraordinary how Becker has offered us all the elements for understanding such interactions.
Matching on human capital: is it assortative?
Public goods (under TU) tend to generate PAM
However, domestic production pushes in the opposite direction (especially if there is specialization)
Risk sharing is still another dimension:
More HC means both higher expected wages and more wage volatility
Especially since PAM depends on ‘supermodularity’
Background

- This paper draws together different strands of the literature
- Standard labor supply models take marriage and education as exogenous
- Here we model family labor supply where both education and marriage is endogenous
- Collective Models (Chiappori, 1988, 1992 and Blundell, Chiappori and Meghir, 2005)
- Pareto weights are exogenous - here they are endogenously determined
- Transferable Utility framework.
Background

- Here we have full commitment but characterize equilibrium
- In standard models Identity (and HC) of the spouse exogenous
- We recognize endogeneity of both pre-marital HC investment and choice of a spouse
Background

- Intra-household allocations are important for understanding pre-marital investments (Chiappori, Iyigun and Weiss, AER 2009)
- We draw from Choo and Siow (2006) and Chiappori, Salanie and Weiss do develop a stochastic matching framework
- In that context the values for each partner are just identified from matching patterns
- Our model adds identifying information by using information on the entire lifecycle to identify values of marriage and Pareto weights.
- Life-cycle Labor supply offers information on preferences and marital surplus
- Equilibrium in the marriage market identifies the Pareto weights and the matching patterns
The framework

Basic features:

- Agents invest in education *before entering the matching game*
- Human Capital: education, ability and random shocks
- Human Capital stock determines the wage
- Risk: shocks affecting HC and wages, multiplicative
- Efficient risk sharing within the household and efficient labor supply
- Preferences: leisure, one private and one public good
- Transferable utility context
Framework

- Three stages of life
- Stage 3 - includes T periods
- Under TU this is like a unitary dynamic model
- Defines total expected surplus at the household level
- Intra-household allocation not determined
- Then stage 2: determines
  - Matching patterns (who marries whom by Human Capital)
  - (Future, contingent) intra-household allocations
    - ultimately, the returns to education
- Finally stage 1: education decisions
The Model - Wages

- Human Capital
  \[ H = H(g, s, \theta) \]

- Wages are gender specific and subject to persistent shocks
  \[ w_{it}^{sg} = W_t^{sg} H_i f(g, s, \text{age}) u_{it} \]

- Wages grow with age - proxy for experience
The Model - Preferences

- Within period references over private consumption (C), public consumption (Q), Labor Supply ($L_i \in \{0, 1\}$).

- Gender and education specific

  $$u_i(Q_t, C_{i,t}, L_{i,t}) = \ln(C_{i,t}Q_t + \alpha_i(age, g, s)L_{i,t}Q_t)$$

- Individuals/Households Maximize lifetime expected utility based on Pareto efficiency
Solving the Model - Within Period

- Total household consumption and labor supply do not depend on the Pareto weight.
- This is because of transferable utility makes the Pareto frontier ex-post linear.
- This is consistent with risk averse behavior.
- To see this note that within period allocations are invariant to monotonic transformations and we can write the Pareto problem (conditional on savings) as

$$\exp u_i + \mu \exp u_j = Q_t \left( C_{1,t} + \mu C_{2,t} + \alpha_{1t} L_{1,t} + \mu \alpha_{2t} L_{2,t} \right)$$

under a within-period budget constraint.

- The first order conditions for the private consumptions are

$$Q_t = \lambda_t = \mu Q_t$$

implying the ex-post Pareto weight is $\mu = 1$. 
The Model

- With our preference structure we can apply a result by Schulhofer-Wohl (2006)
- TU property obtains ex-ante in expectation
- Take the individual expected value functions $V_m, V_f$
- Then there exists a function $\Upsilon(H)$ such that any Pareto efficient allocation satisfies

$$\exp\left\{ \frac{1 - \delta}{1 - \delta^T} V_m \right\} + \exp\left\{ \frac{1 - \delta}{1 - \delta^T} V_f \right\} = \exp\left\{ \frac{1 - \delta}{1 - \delta^T} \Upsilon(H) \right\}$$

- Where the households in the third stage maximize $\Upsilon(H)$
The Model

- Now define $\bar{U}_m(H_m, H)$ to be the (deterministic part of) utility of a male with human capital $H_m$ for a female with human capital $H$.
- Equivalently, the female has her own preferences.
- These are a monotonic transformation of the individual value functions.

$$\bar{U}_i = \exp \left( \frac{1 - \delta}{1 - \delta^T} V_i \right)$$

- The economic value of a match is

$$\bar{U}_m + \bar{U}_f = \exp \left\{ \frac{1 - \delta}{1 - \delta^T} \Upsilon (H) \right\} \equiv g(H_m, H_f)$$
Similarly we can represent the problem of singles as maximizing

$$\bar{U}_i^S = \exp \left\{ \frac{1 - \delta}{1 - \delta^T} V^S (H_i) \right\}$$  \hspace{1cm} (1)

Define the economic surplus from marriage

$$\Sigma (H_m, H_f) = g (H_m, H_f) - g^M (H_m) - g^F (H_f)$$
The Model - Marital Preferences

- Individuals have a random preference \( (\beta^H_i) \) for marrying a person of human capital type \( H \)
- Taking this into account the total surplus from marriage is

\[
\Sigma_{mf} = \Sigma (H_m, H_f) + (\beta_m^{Hf} - \beta_m^0) + (\beta_f^{Hm} - \beta_f^0)
\]

- Allowing for match specific effects with large support in a frictionless environment is problematic:
- It implies the number of singles tends to zero and their utility to infinity
The Model - Matching

- Define the value of matching two people with human capital $H_m$, $H_f$ as

\[ g_{mf} = g(H_m, H_f) + \beta_H^{Hf} + \beta_H^{Hm} \] (2)

- If $U_m$, $U_f$ relate to the optimal match, stability requires that

\[ U_m + U_f = g_{mf} \]

- For individuals not matched to each other it must be that

\[ U_m + U_f \geq g_{mf} \]
Chiappori, Salanie and Weiss prove a separability result:

\[ \bar{U}_M (H_m, H_f) + \bar{U}_F (H_m, H_f) = g (H_m, H_f) \]

where the random elements of the utility do not enter

Individual utility can then be written as

\[
\begin{align*}
U_m &= \bar{U}_M (H_m, H_f) + \beta^H_m \\
U_f &= \bar{U}_F (H_m, H_f) + \beta^H_f
\end{align*}
\]
The Model - Matching

- The Corollary of the above is that for every man (woman) the optimal match satisfies

\[ \bar{U}_M(H_m, H_f) + \beta^H_f \geq \bar{U}_M(H_m, H) + \beta^H_m \quad \forall H \]

- Since \( H \) is discrete, this leads to a multinomial discrete choice model that can identify \( \bar{U}_j, j = m, f \)

- Since we know \( \bar{U}_j, j = m, f \) up to the Pareto weight (share) this identifies the Pareto weight.
The Model - Education Choice

- Education Choices are made in anticipation of the marriage market
- Individuals do not know their marital preferences
- Costs of education depend on observables and a random cost shock
- We assume that conditional on parental background, parental income at the time of education choice affects education but not preferences
The Model - Education Choice

- Returning to the original cardinalization, expected utility for the male (resp. female) conditional on his (her) human capital is given by

$$\bar{V}_m(H_m) = E\delta^* \ln(\max_{H_f} [\bar{U}_m(H_m, H_f) + \beta^{H_f}])$$

with $$\delta^* = (1 - \delta^T)/(1 - \delta)$$

- Education Choice is then driven by

$$s_m = \arg \max_{s \in S} \{ \bar{V}_m(H_m) - c_{sm}(X_m, \upsilon_{sm}) \}$$

for man $$m$$:

$$s_f = \arg \max_{s \in S} \{ \bar{V}_f(H_f) - c_{sf}(X_f, \upsilon_{sf}) \}$$

for woman $$f$$:
Solving the Model

Within period allocations to consumption and public goods conditional on savings and labor supply are obtained by solving

$$
\max_{Q_t, C_t} Q_t \left( C_t + \alpha_{mt} L_{mt} + \alpha_{ft} L_{ft} \right)
$$

under the budget constraint

$$
\begin{align*}
    w_{mt} + w_{ft} + y_t^C + RK_{t-1} &= K_t + C_t + w_{mt} L_{mt} + w_{ft} L_{ft} + pQ_t
\end{align*}
$$
Solving the Model - Efficient risk sharing

- Allocation of consumption ex ante follows the optimal risk sharing rule

\[
\frac{\partial u_{mt}(Q_t, C_{mt}, L_{mt})}{\partial C_{mt}} = \mu \frac{\partial u_{ft}(Q_t, C_{ft}, L_{ft})}{\partial C_{ft}}
\]

- This is conditional on aggregate consumption, savings and labor supply (that do not depend on Pareto weights)
- depends on the original cardinalization (here logs)
- is not identified by observing labor market and savings behavior
- The allocation in our case is

\[
C_{mt} = \frac{1}{1 + \mu} pQ_t - \alpha_{mt} L_{mt}
\]
\[
C_{ft} = \frac{\mu}{1 + \mu} pQ_t - \alpha_{ft} L_{ft}.
\]
Solving the Model - The value functions

- Household savings/consumption satisfy the usual Euler equation
- Labor supply is a dynamic discrete choice problem.
- By backwards induction and using the result on optimal risk sharing we can show that

\[
E_{t-1} V_{t} (e_{t}, \alpha_{t}, K_{t-1}, H, \mu) = \\
I_{t-1} (e_{t-1}, \alpha_{t-1}, K_{t-1}, H) + \ln \left( \frac{\mu}{1 + \mu} \right) \sum_{\tau=t}^{T} \beta^{\tau-t}
\]

where \( I_t \), is defined recursively by

\[
I_{t-1} (e_{t-1}, \alpha_{t-1}, K_{t-1}, H) = \max_{L_t} E_{t-1} \left[ \max_{K_t} \left\{ 2 \ln Q_t (L_t, K_t) + \ln p + \delta I_t (e_t, \alpha_t, \right. \right. \\
\left. \left. K_t) \right\} \right]
\]
The Data

- Data: British Household Panel Survey for the years 1991-2008
- 4,300 Couples, 1131 Single men and 937 single women aged 23-50
- Marital status assessed is assessed at age 30 or above
- Singles: those never married
- Those married followed for the entire marital spell
- We ignore divorce since we do not include it in the models
The Data

- Three education levels (statutory, high school, college)
- We net out aggregate growth
- Employment defined as working 5 hours or more per week
Earnings process

\[
\ln w_{i,t}^g = \ln W_{t}^{g,s} + \ln H_{i}^{g} + m^{g,s}(age_{it}) + \theta_i + e_{i,t}^{g,s}, \quad g = m, f, \quad s = 1, 2, 3
\]

\[
e_{i,t}^{g,s} = \rho^{g,s} e_{i,t-1}^{g,s} + \epsilon_{it}^{g,s}
\]
Estimation

- We use method of moments
- A key element of the model is the endogenous nature of education and ability
- We thus need to account for and know the joint distribution of ability and education among couples and singles.
- We use earnings and employment to recover the distribution of ability given education and marital status
- We can then construct the distribution of human capital which depends on education and ability
- Moments by education, gender and marital status include:
  - means variances and quantiles of earnings,
  - Regression coefficients of employment on age polynomial
  - Intensity of employment (over five years or more) to capture labor market attachment
  - Covariance of earnings within couples
Table: Earnings equation by gender and education

<table>
<thead>
<tr>
<th></th>
<th>Men</th>
<th>Women</th>
<th></th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stat ((\delta_1))</td>
<td>HS ((\delta_2))</td>
<td>Univ ((\delta_3))</td>
<td>Stat ((\delta_1))</td>
<td>HS ((\delta_2))</td>
</tr>
<tr>
<td>age ((\delta_1))</td>
<td>0.475 (.05)</td>
<td>0.606 (.04)</td>
<td>0.923 (.07)</td>
<td>-0.232 (.08)</td>
<td>0.144 (.07)</td>
</tr>
<tr>
<td>age sq ((\delta_2))</td>
<td>-0.252 (.04)</td>
<td>-0.302 (.03)</td>
<td>-0.524 (.06)</td>
<td>0.125 (.07)</td>
<td>-0.172 (.06)</td>
</tr>
<tr>
<td>age cubic ((\delta_3))</td>
<td>0.042 (.01)</td>
<td>0.050 (.01)</td>
<td>0.094 (.01)</td>
<td>-0.017 (.02)</td>
<td>0.052 (.02)</td>
</tr>
<tr>
<td>((\rho))</td>
<td>0.502</td>
<td>0.594</td>
<td>0.416</td>
<td>0.811</td>
<td>0.820</td>
</tr>
<tr>
<td>((\sigma_\xi^2))</td>
<td>0.024</td>
<td>0.012</td>
<td>0.026</td>
<td>0.030</td>
<td>0.035</td>
</tr>
<tr>
<td>Var ME ((\sigma_\epsilon^2))</td>
<td>0.004</td>
<td>0.007</td>
<td>0.001</td>
<td>0.012</td>
<td>0.005</td>
</tr>
<tr>
<td>N</td>
<td>9,116</td>
<td>11,990</td>
<td>4,291</td>
<td>8,432</td>
<td>7,469</td>
</tr>
</tbody>
</table>
Earnings process

**Table**: Earnings levels by gender and education

<table>
<thead>
<tr>
<th></th>
<th>Stat</th>
<th>HS</th>
<th>Univ</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Men</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ability type 1</td>
<td>2.61</td>
<td>2.74</td>
<td>2.74</td>
</tr>
<tr>
<td></td>
<td>(.01)</td>
<td>(.01)</td>
<td>(.03)</td>
</tr>
<tr>
<td>ability type 2</td>
<td>3.13</td>
<td>3.25</td>
<td>3.24</td>
</tr>
<tr>
<td></td>
<td>(.00)</td>
<td>(.00)</td>
<td>(.00)</td>
</tr>
<tr>
<td><strong>Women</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ability type 1</td>
<td>1.95</td>
<td>2.00</td>
<td>2.82</td>
</tr>
<tr>
<td></td>
<td>(.03)</td>
<td>(.04)</td>
<td>(.04)</td>
</tr>
<tr>
<td>ability type 2</td>
<td>2.95</td>
<td>2.95</td>
<td>3.10</td>
</tr>
<tr>
<td></td>
<td>(.01)</td>
<td>(.00)</td>
<td>(.02)</td>
</tr>
</tbody>
</table>
Ability and Education: Singles

Table: Proportion of ability type 1 among singles by gender and education

<table>
<thead>
<tr>
<th></th>
<th>secondary</th>
<th>high school</th>
<th>university</th>
</tr>
</thead>
<tbody>
<tr>
<td>men</td>
<td>0.753</td>
<td>0.705</td>
<td>0.505</td>
</tr>
<tr>
<td></td>
<td>(.03)</td>
<td>(.02)</td>
<td>(.03)</td>
</tr>
<tr>
<td>women</td>
<td>0.439</td>
<td>0.171</td>
<td>0.109</td>
</tr>
<tr>
<td></td>
<td>(.04)</td>
<td>(.06)</td>
<td>(.08)</td>
</tr>
</tbody>
</table>
Preferences

- Preferences for work are parameterized as a cubic polynomial in age.
- This differs by education and gender.
- It also includes an unobserved preference component for preferences for work and a transitory preference shock.
- These random preferences are unknown to the individuals at the time of marriage.
The Economic Surplus from marriage

- Given the estimates we can compute the economic Surplus.
- We classify it on the basis increasing human capital
- Positive assortative matching

Table: Economic surplus from marriage by human capital of both spouses

<table>
<thead>
<tr>
<th></th>
<th>Low HC</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1, 1)</td>
<td>(2, 1)</td>
<td>(1, 2)</td>
<td>(2, 2)</td>
<td>(3, 1)</td>
<td>(3, 2)</td>
</tr>
<tr>
<td>Men H:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low H:</td>
<td>(1, 1)</td>
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<td></td>
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<td>2:</td>
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<td>3:</td>
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<td>4:</td>
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<tr>
<td>5:</td>
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<td></td>
</tr>
<tr>
<td>High H:</td>
<td>(3, 2)</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

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Marriage Market
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Intrahousehold allocations - The Pareto weights

- The Probability of man with HC $H_m$ marrying a woman of HC $H$ is

$$\Pr(H \mid H_m) = \Pr\left(\bar{U}(H_m, H) + \beta_m^H \geq \bar{U}(H_m, H') + \beta_m^{H'}\right)$$

$$= \Pr\left(\beta_m^H - \beta_m^{H'} \geq \bar{U}(H_m, H') - \bar{U}(H_m, H)\right)$$

- Include the single option
- Now recall that

$$\bar{U}(H_m, H) = \exp\left(\frac{1 - \delta}{1 - \delta T} V_m(H_m, H, \mu)\right)$$

where $V_m(H_m, H, \mu) = \Upsilon(H_m, H) + \exp\left(\frac{1 - \delta}{1 - \delta} \ln(\mu(H_m, H))\right)$
Intrahousehold allocations - The Pareto weights

- Assuming an extreme value distribution we can recover $\mu$ from $Pr(H|H_m)$
- Since we do not observe human capital we do not directly observe $Pr(H|H_m)$
- However because individuals sort on HC and not separately on ability and education we know that

$$Pr(H | H_i) = Pr(S, \theta, | S_i, \theta_i)$$

- This has been estimated
Intrahousehold allocations - The Pareto weights

**Table: Women’s Pareto weights by spouses’ human capital**

<table>
<thead>
<tr>
<th>Men ((s, \theta))</th>
<th>(\text{LowH (1,1)})</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(\text{High H (3,2)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low H: ((1,1))</td>
<td>0.19</td>
<td>0.27</td>
<td>0.45</td>
<td>0.67</td>
<td>1.62</td>
<td>2.60</td>
</tr>
<tr>
<td>2: ((2,1))</td>
<td>0.13</td>
<td>0.19</td>
<td>0.31</td>
<td>0.45</td>
<td>1.81</td>
<td>1.49</td>
</tr>
<tr>
<td>3: ((3,1))</td>
<td>0.10</td>
<td>0.16</td>
<td>0.29</td>
<td>0.44</td>
<td>1.39</td>
<td>2.57</td>
</tr>
<tr>
<td>4: ((1,2))</td>
<td>0.09</td>
<td>0.14</td>
<td>0.24</td>
<td>0.37</td>
<td>2.04</td>
<td>1.95</td>
</tr>
<tr>
<td>5: ((2,2))</td>
<td>0.05</td>
<td>0.08</td>
<td>0.15</td>
<td>0.22</td>
<td>0.64</td>
<td>1.31</td>
</tr>
<tr>
<td>HighH: ((3,2))</td>
<td>0.05</td>
<td>0.08</td>
<td>0.15</td>
<td>0.15</td>
<td>0.63</td>
<td>1.04</td>
</tr>
</tbody>
</table>
Intrahousehold allocations - The Pareto weights

- These are estimated Pareto weights based on observed behavior.
- The model is over identified since one can use females or males to estimate the weights - in principle the model is testable.
- For the purposes of counterfactual simulations one needs to solve the model to find a new equilibrium in the marriage market.
- This involves computing the new surplus function. Then finding the set of stable matches given education. Then iterating on the education choice.
The final element of estimation is the education choice.

This is an important part of the story because it defines the endogenous supply of human capital.

Policies that change educational choices have the potential of affecting the marriage market and intrahousehold allocations.
Concluding Remarks

- This a rich equilibrium framework for considering life-cycle decisions in a unified way
- It allows us to analyze the long term effects of policies in a systematic way
- It also provides an empirical framework for understanding marital patterns
- On the agenda (shorter and longer run):
  - Counterfactual policy simulations
  - Divorce
  - Imperfectly transferable utility
  - Limited Commitment in equilibrium