An improved bootstrap test of density ratio ordering

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Abstract

Two probability distributions with common support on the real line are said to exhibit density ratio ordering when they admit a nonincreasing density ratio. This property implies, but is not implied by, first order stochastic dominance. Existing statistical tests of the null hypothesis of density ratio ordering are known to be conservative, with limiting rejection rates below nominal size at all but one point in the null: the so-called least favorable case, at which the two distributions are equal. In this paper we show how a bootstrap procedure can be used to shrink the critical values used in existing procedures such that the limiting rejection rate is increased to nominal size at all points on what is, in a sense, the boundary of the null. This improves power against relevant local alternatives. Our procedure is based on preliminary estimation of a contact set, the form of which is obtained from a novel representation of the Hadamard directional derivative of the least concave majorant operator. Numerical simulations indicate that the improvements to power can be large in modestly sized samples.