

Couples and Singles' Savings After Retirement

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Abstract

We model the saving problem of retired couples and singles facing uncertain longevity and medical expenses in presence of means-tested social insurance. Households can save to self-insure against uncertain longevity and medical expenses, and to leave bequests. Individuals in a couple can be altruistic towards their spouse and other heirs and split bequests optimally. Single people can care about leaving bequests to children and others. Using AHEAD data, we first estimate the model and we then evaluate the relative importance of the various savings motives and the risk exposure of couples' vs. singles.

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1 Introduction

In the Assets and Health Dynamics of the Oldest Old (AHEAD), among American people aged 70 and older, about 50% of individuals are in a couple, while about 50% are single. Being in a couple during retirement allows the members of the couple to pool longevity and medical expense risk, but also exposes each household member to the loss of one's spouse and to the longevity and medical expenses risk of one's partner.

Much of the previous literature, including our own work, only study singles. In a previous paper (De Nardi, French, and Jones [20]) we have shown that after retirement medical expenses and government-provided insurance are important to explain the saving patterns of U.S. single retirees at all income levels, including those of the high permanent-income elderly who keep large amounts of assets until very late in life. These savings patterns are due to two important features of out-of-pocket medical expenses. First, out-of-pocket medical and nursing-home expenses can be large. Second, average medical expenditures rise very rapidly with age and permanent income. Medical expenses that rise with age provide the elderly with a strong incentive to save, and medical expenses that rise with permanent income encourage the rich to be more frugal. In other work, we showed that heterogeneous life expectancy is important to match the savings pattern of the retired elderly singles (De Nardi, French and Jones [19]).

This paper builds on these previous contributions by studying the determinants of household savings during retirement for both couples and singles in a framework that allows for the observed heterogeneity in life expectancy and medical expenses, and that explicitly models means-tested social insurance.

Our first goal is to match important facts about savings, medical expenses, and longevity, for both singles and couples. Our second goal is to estimate couples' bequest motives toward the surviving spouse and other heirs and compare it to the bequest motive of single people. Our third goal is to evaluate to what extent medical risk affects retired couples, compared to singles, and whether couples can better insure such risks due to intra-household risk-sharing and economies of scale. Our fourth goal is to evaluate the potentially different responses of the saving of couples, compared to that of singles, to changes in publicly-provided health insurance.

Using data from the AHEAD survey, we begin by documenting asset growth at each age for members of different cohorts, for both couples and

singles. We also estimate the data generating process for shocks faced by post-retirement households.

We then estimate the preference parameters of the model with the method of simulated moments. In particular, we compare the simulated asset profiles generated by the model to the empirical asset profiles generated by the data, and find the parameter vector that yields the closest match. In matching the model to the data, we take particular care to control for cohort effects. Moreover, by explicitly modeling demographic transitions, we account for mortality bias.

Our first-step estimates on mortality, health transition, and medical expenses allow us to uncover new evidence of how these risks compare for singles and couples. They indicate that singles live less long than people who are part of a couple, and that they are much more likely to end up in a nursing home. For that reason, singles also have higher medical spending, per person, than people who are part of a couple. We also find that assets drop sharply with the death of a spouse. By the time the second spouse dies, a large fraction of the wealth of the original couple has vanished, with the wealth falls at the time of death of each spouse explaining most of the decline. A large share of these drops is explained by the high medical expenses at the time of death.

Our second-step estimates will allow us to evaluate to what extent the risk sharing and economies of scale of a couple help insure against longevity and medical-expense risk and to what extent couples get a better or worse deal from publicly provided health insurance. They will also allow us to estimate bequest motives towards the surviving spouse, in contrast to bequests to children or others. Finally, it will enable us to study in what ways the responses of a couple will differ from the responses of a single person when, for example, public health insurance becomes more limited, or its quality worsens, or some of its means-testing criteria become tighter, or in relation to the different institutions across countries

The rest of the paper is organized as follows. In section 2 we describe the key previous papers on which our paper builds on. In section 3 we introduce our model and in section 4 we discuss our estimation procedure. In section 5 we describe some key features of the data and the estimated shock processes that households face. We discuss our results in section 6 and we and conclude in section 7.

2 Related Literature

Poterba et al. ([43]) show that relatively little dissaving occurs amongst retirees whose family composition does not change, but that assets fall significantly when households lose a spouse. We find similar results. Furthermore, as in French et al. ([26]), we document significant drops in assets when the last member of the household passes away.

Previous literature had shown that high income individuals live longer than low income individuals (see Attanasio and Emmerson [4] and Deaton and Paxson [17]). This means that high income households must save a larger share of their lifetime wealth if they are to smooth consumption over their retirement. Differential mortality rates thus provide a potential explanation for why high income households have higher savings rates than low income households. We extend the analysis along this dimension by explicitly modeling the interaction of life expectancy for individuals in couples and the differential life expectancy for couples and singles.

Even in presence of social insurance such as Medicare and Medicaid, households face potentially large out-of-pocket medical and nursing home expenses (see French and Jones [28, 27], Palumbo [41], Feenberg and Skinner [25], and Marshall et al. [36]). The risk of incurring such expenses might generate precautionary savings, over and above those accumulated against the risk of living a very long life ([35]).

Hubbard, Skinner and Zeldes [32] argue that means-tested social insurance programs such as Supplemental Security Income and Medicaid provide strong incentives for low income individuals not to save. De Nardi et al. [21] finds that these effects extend to singles in higher permanent income quintiles as well. In this paper, to allow for these important effects, we model means-tested social insurance explicitly for both singles and couples.

Bequest motives could be another reason why households, and especially those with high permanent income, retain high levels of assets at very old ages (Dynan, Skinner and Zeldes [23] and Americks et al. [2]). De Nardi [15] and Castañeda et al. [13] argue that bequest motives are necessary to explain why the observed distribution of wealth is more skewed and concentrated than the distribution of income (Quadrini et al. [18]). De Nardi et al [21] shows that bequest motives help fit both assets and Medicaid reciprocity profiles for singles. We allow for a richer structure of bequest motives, in that couples might want to leave resources to the surviving spouse, children and other heirs, while singles might want to leave bequests to children and

other heirs.

Previous quantitative papers on savings have used simpler models that omit one or more of these features. Hurd [33] estimates a structural model of bequest behavior in which the time of death is the only source of uncertainty. Palumbo [41] focuses on the effect of medical expenses and uncertain lifetimes, but omits bequests. Dynan, Skinner and Zeldes [23, 24] consider the interaction of mortality risk, medical expense risk and bequests, but use a stylized two-period model. Moreover, none of these papers model household survival dynamics, assuming instead that households (while “alive”) always have the same composition. In contrast, we explicitly model household survival dynamics: when the first household member dies, assets are optimally split among the surviving spouse and other heirs. Although Hurd [34] extends his earlier model to include household survival dynamics, he omits medical expense risk, and, in contrast to his earlier work, he does not estimate his model.

Our work also complements the one on retirement behaviour by couples by Blau and Gilleskie [8], Casanova [12], and Gallipoli and Turner [30].

Including couples and simultaneously considering bequest motives, social insurance, uncertain medical expenses, and uncertain life expectancy is important for at least two reasons. First, Dynan, Skinner and Zeldes [24] argue that explaining why the rich have high savings rates requires a model with precautionary motives, bequest motives and social insurance. Second, simultaneously considering multiple savings motives allows us to identify their relative strengths. This is essential for policy analysis. For example, the effects of estate taxes depend critically on whether rich elderly households save mainly for precautionary reasons, or mainly to leave bequests (see for example Gale and Perozek [29]).

3 The Model

Consider a retired household, composed either of a single person, or a couple, seeking to maximize its expected lifetime utility at household head age t , $t = t_r, t_r + 1, \dots, T + 1$, where t_r is the retirement age, while T is the maximum potential lifespan.

We use w to denote women and h to denote men. We use family structure, f_t , which equals to zero if single, and to one if couple, to distinguish between married and singles. Each person is either in good health ($hs^i = 0$), bad

health ($hs^i = 1$), or in a nursing home ($hs^i = 2$). For tractability, we assume a fixed age gap between the husband and the wife in a couple.

To be consistent with the data frequency, and to reduce computation time, our time period is two years long.

3.1 Preferences

Households maximize their utility by choosing savings, bequests and current and future consumption. The discount factor is given by β . Each period, the household's utility depends on its total consumption, c , and the health status of each member, hs^i , where $i \in \{h, w\}$.

The within-period utility function for singles is given by

$$u^i(c, hs^i) = (1 + \delta hs^i) \frac{c^{1-\nu}}{1-\nu}, \quad (1)$$

with $\nu \geq 0$. When $\delta = 0$, health status does not affect utility.

We assume that the preferences of couples can be represented by the following utility function:¹

$$u^c(c, hs^h, hs^w) = [1 + \delta(hs^h) + 1 + \delta(hs^w)] \frac{(c/\eta)^{1-\nu}}{1-\nu}, \quad (2)$$

where $1 < \eta \leq 2$ determines the extent to which couples enjoy economies of scale in the transformation of consumption goods to consumption services.

When a household member dies, the estate can be left to the surviving spouse (if there is one) or to other heirs, including the household's children. Estates are subject to estate taxes, but the exemption level during the time period in our sample is above the actual assets of the vast majority of the households in our sample. For this reason, we abstract from explicitly modeling estate taxation.

We indicate with b the part of the estate, that does *not* go to the surviving spouse, and assume that the deceased member of the household derives utility $\theta_j(b)$ from leaving that part of the estate to heirs other than the spouse. The subscript j indicates whether there is a surviving spouse or not, and whether

¹Mazzocco [37] shows that under full commitment, the behavior of a couple can be characterized by a unique utility function if the husband and wife share identical discount factors, identical beliefs and Harmonic Absolute Risk Aversion utility functions with identical curvature parameters.

one or two people have just died. In particular, $\theta_0(b)$ gives the utility from bequests for a single person with no surviving spouse, $\theta_1(b)$ gives the utility from bequests when there is a surviving spouse, and that $\theta_2(b)$ gives the utility from bequests when both spouses die at the same time. More specifically, the bequest function takes the form

$$\theta_j(b) = \phi_j \frac{(b + k_j)^{(1-\nu)}}{1 - \nu}, \quad (3)$$

where k_j determines the curvature of the bequest function. Our formulation can support several interpretations of the “bequest motive”: dynastic or “warm glow” altruism (as in Becker and Tomes [6] or Andreoni [3]); strategic motives (as in Bernheim, Schleifer and Summers [7] or Brown [9]); or some form of utility from wealth itself, as in (Carroll [11] and Hurd [33]).

3.2 Technology and Sources of Uncertainty

We assume that non-asset income at time t , y_t , is a deterministic function of the household’s permanent income, I , age, and family structure.

$$y_t(\cdot) = y(I, t, f_t). \quad (4)$$

There are several sources of uncertainty:

1) Health status uncertainty. The transition probabilities for the health status of a person depend on one’s current health status, permanent income, gender, age, and marital status. Hence the elements of the health status transition matrix for a person are given by

$$\pi_t(\cdot) = \Pr(hs_{t+1}|hs_t, I, g, t, f_t). \quad (5)$$

2) Survival uncertainty. Let $s_t(\cdot) = s(hs, I, g, t, f_t)$ denote the probability that an individual is alive at age $t + 1$, conditional on being alive at age t , having time- t health status hs , enjoying household permanent income I , being of a given gender and age, and being in a couple or not.

3) Medical expense uncertainty at the household level. We define m_t as the sum of all out-of-pocket medical expenses, including insurance premia, and medical expenses covered by the consumption floor. We assume that medical expenses depend upon the health status of each family member,

household permanent income, family structure, gender if single, age, and an idiosyncratic component, ψ_t :

$$\ln m_t = m(hs_t^h, hs_t^w, I, g, t, f_t) + \sigma(hs_t^h, hs_t^w, I, g, t, f_t) \times \psi_t. \quad (6)$$

Following Feenberg and Skinner [25] and French and Jones [27], we assume that ψ_t can be decomposed as

$$\psi_t = \zeta_t + \xi_t, \quad \xi_t \sim N(0, \sigma_\xi^2), \quad (7)$$

$$\zeta_t = \rho_m \zeta_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2), \quad (8)$$

where ξ_t and ϵ_t are serially and mutually independent. In practice, we discretize ξ and ζ , using quadrature methods described in Tauchen and Hussey [46].

The timing is the following: at the beginning of the period the health shock and the medical cost shocks are realized, income is received, and if the household qualifies, means-tested transfers are also received. Then the household consumes and saves. Finally the survival shock hits. Household members who die leave assets to their heirs. Bequests are included in the heirs' assets at the beginning of next period.

Let us denote assets at the beginning of the period with a_t . Assets have to satisfy a borrowing constraint $a_t \geq 0$. Let us indicate the constant and risk-free, rate of return with r , and total post-tax income with $y(r a_t + y_t(\cdot), \tau)$, with the vector τ describing the tax structure.

3.3 Government Transfers

Following Hubbard et al. [31, 32], we assume that the government provides means-tested transfers, $tr_t(\cdot)$, that bridge the gap between a minimum consumption floor and the household's resources, net of an asset disregard amount. Both the household consumption floor, $c_{min}(f_t)$, and the asset disregard amount, $a_d(f_t)$, are functions of one's family structure, f_t . We impose that if transfers are positive, $c_t = c_{min}(f_t)$.

3.4 Recursive Formulation

To save on state variables we follow Deaton [16] and redefine the problem in terms of cash-on-hand:

$$x_t = a_t + y(r a_t + y_t(\cdot), \tau) - m_t + tr_t(\cdot), \quad (9)$$

The law of motion for cash on hand next period is given by

$$x_{t+1} = x_t - c_t - I_{bt}b_t + y(r(x_t - c_t) + y_{t+1}(\cdot), \tau) - m_{t+1} + tr_t(\cdot) \quad (10)$$

where I_{bt} is an indicator function which turns to one if there is a spouse and the spouse dies during the current period, and b_t are the bequests that the deceased spouse leaves to other heirs (e.g., children). We do not include received bequests as a source of income, because very few households aged 65 or older receive them. When all household members die their assets are bequeathed to the remaining heirs.

Define available resources next period, before government transfers with

$$s_{t+1} = x_t - c_t - I_{bt}b_t + y(r(x_t - c_t) + y_{t+1}(\cdot), \tau) - m_{t+1}, \quad (11)$$

which is expressed as a function of cash on hand, rather than assets, which are no longer a state variable. Hence, consistently with the main Medicaid and SSI rules, we can express government transfers next period as a function of s_{t+1} as follows

$$tr_{t+1}(s_{t+1}) = \max\{0, c_{\min}(f_t) - [\max(0, s_{t+1} - a_d(f_t))]\}, \quad (12)$$

The law of motion for cash on hand next period can thus be rewritten as

$$x_{t+1} = s_{t+1} + tr_{t+1}(s_{t+1}). \quad (13)$$

To ensure that cash on hand is always non-negative, we require

$$c_t \leq x_t, \quad \forall t. \quad (14)$$

Using the definition of cash-on-hand, the value function for a single individual i , $i \in \{h, w\}$, can be written as

$$V_t^i(x_t, h s_t, I, \zeta_t) = \max_{c_t, x_{t+1}} \left\{ u^i(c_t, h s_t) + \beta s(h s, I, i, t, 0) E_t \left(V_{t+1}^i(x_{t+1}, h s_{t+1}, I, \zeta_{t+1}) \right) + \beta (1 - s(h s, I, i, t, 0)) \theta_0(x_t - c_t) \right\}, \quad (15)$$

subject to equations (4)-(8) and(11)-(14), with $f_t = 0$, since we are referring to singles.

The value function for couples can be written as

$$\begin{aligned}
V_t^c(x_t, hs_t^h, hs_t^w, I, \zeta_t) = & \max_{c_t, x_{t+1}, b^w, b^h} \left\{ u^c(c_t, hs_t^h, hs_t^w) \right. \\
& + \beta s(hs, I, w, t, 1) s(hs, I, h, t, 1) E_t(V_{t+1}^c(x_{t+1}, hs_{t+1}^h, hs_{t+1}^w, I, \zeta_{t+1})) \\
& + \beta s(hs, I, w, t, 1) (1 - s(hs, I, h, t, 1)) \left[\omega E_t(V_{t+1}^w(x_{t+1}^w, hs_{t+1}^w, I, \zeta_{t+1})) + \theta_1(b_t^h) \right] \\
& + \beta (1 - s(hs, I, t, w, 1)) s(hs, I, h, t, 1) \left[\omega E_t(V_{t+1}^h(x_{t+1}^h, hs_{t+1}^h, I, \zeta_{t+1})) + \theta_1(b_t^w) \right] + \\
& \left. \beta (1 - s(hs, I, w, t, 1)) (1 - s(hs, I, h, t, 1)) \theta_2(x_t - c_t) \right\}, \quad (16)
\end{aligned}$$

where η is the altruism parameter toward one's surviving spouse and the value function is subject to equations subject to equations (4)-(8) and (11)-(17), with $f_t = 1$ since we are referring to couples, and bequests are constrained by

$$b_t \leq x_t - c_t. \quad (17)$$

4 Estimation Procedure

We adopt a two-step strategy to estimate the model. In the first step, we estimate or calibrate those parameters that, given our assumptions, can be cleanly identified outside our model. In particular, we estimate health transitions, out-of-pocket medical expenses, and mortality rates from raw demographic data. We calibrate the household economies of scale parameter and the minimum consumption floor for singles and couples based on previous work.

In the second step, we estimate the rest of the model's parameters (discount factor, risk aversion, health preference shifter, and bequest parameters)

$$\Delta = (\beta, \nu, \delta, \omega, \phi_0, \phi_1, \phi_2, k_0, k_1, k_2)$$

with the method of simulated moments (MSM), taking as given the parameters that were estimated in the first step. In particular, we find the parameter values that allow simulated life-cycle decision profiles to "best match" (as measured by a GMM criterion function) the profiles from the data.

Because our underlying motivations are to explain why elderly individuals retain so many assets and to explain why individuals with high income save

at a higher rate, we match median assets by cohort, age, and permanent income. Because we wish to study differences in savings patterns of couples and singles, we match profiles for the singles and couples separately. Finally, to help identify bequest motives towards' one spouse, we also match the fraction of net worth left to the surviving spouse in a couple affected by death.

In particular, the moment conditions that comprise our estimator are given by

1. Median asset holdings by PI-cohort-year for the singles who are still alive when observed.
2. Median asset holdings by PI-cohort-year for the initially couples with both members currently alive.

When there is a death in a couple, the surviving spouse is included in the singles' profile of the appropriate age, cohort, and permanent income cell; in keeping with our assumption that all singles differ only in their state variables.

3. For estates of positive value, the median fraction of the estate left to the surviving spouse by PI and age. We do not condition this moment by cohort for two reasons. First, the sample size is too small, and second there does not seem to be a discernible cohort effect in this moment.
 *** Check and if necessary change [ebf: my hunch is that ???]

The cells are computed as follows.² Household i has family structure f_t ; which indexes households that are initially couples and those that either are initially singles (both men and women), or become singles through death of their spouse. We sort type- f households in cohort c by their permanent income levels, separating them into $Q = 5$ quintiles. Suppose that household i 's permanent income level falls in the q th permanent income interval of households in its cohort and family structure.

Let $a_{cft}(\Delta, \chi)$ be the model-predicted median observed asset level in calendar year t for a household that was in the q th permanent income interval of cohort c . Assuming that observed assets have a continuous density, at the

²As was done when constructing the figures from the HRS data, we drop cells with less than 10 observations from the moment conditions. Simulated agents are endowed with asset levels drawn from the 1996 data distribution, and thus we only match asset data 1998-2010.

“true” parameter vector (Δ_0, χ_0) exactly half of the households in group $cfqt$ will have asset levels of $a_{cfqt}(\Delta_0, \chi_0)$ or less. This leads to the following moment condition:

$$E(1\{\tilde{a}_{it} \leq a_{cfqt}(\Delta_0, \chi_0)\} - 1/2 | c, f, q, t, \text{household alive at } t) = 0, \quad (18)$$

for all c, f, q and t . In other words, for each permanent income-family structure-cohort grouping, the model and the data have the same median asset levels.

Similarly for the fraction of wealth left to the surviving spouse ... ****

The mechanics of our MSM approach are as follows. We compute life-cycle histories for a large number of artificial households. Each of these households is endowed with a value of the state vector $(t, f_t, x_t, I, hs_t^h, hs_t^w, \zeta_t)$ drawn from the data distribution for 1996, and each is assigned the entire health and mortality history realized by the household in the AHEAD data with the same initial conditions. This way we generate attrition in our simulations that mimics precisely the attrition relationships in the data (including the relationship between initial wealth and mortality).

We discretize the asset grid and, using value function iteration, we solve the model numerically. This yields a set of decision rules, which, in combination with the simulated endowments and shocks, allows us to simulate each individual’s net worth, medical expenditures, health, and mortality. We compute assets from the artificial histories in the same way as we compute them from the real data. We use these profiles to construct moment conditions, and evaluate the match using our GMM criterion. We search over the parameter space for the values that minimize the criterion. Appendix E contains a detailed description of our moment conditions, the weighting matrix in our GMM criterion function, and the asymptotic distribution of our parameter estimates.

When estimating the life-cycle profiles, and subsequently fitting the model to those profiles, we face two well-known problems. First, in a cross-section, older households were born in an earlier year than younger households and thus have different lifetime incomes. Because lifetime incomes of households in older cohorts will likely be lower than the lifetime incomes of younger cohorts, the asset levels of households in older cohorts will likely be lower also. Therefore, comparing older households born in earlier years to younger households in later years leads to understate asset growth. Second, households with lower income and wealth tend to die at younger ages than richer

households. Therefore, the average survivor in a cohort has higher lifetime income than the average deceased member of the cohort. As a result, “mortality bias” leads the econometrician to overstate the average lifetime income of members of a cohort. This bias is more severe at older ages, when a greater share of the cohort members are dead. Therefore, “mortality bias” leads to overstate asset growth.

We use panel data to overcome these first two problems. Because we are tracking the same households over time, we are obviously tracking members of the same cohort over time. Similarly, we do separate sets of simulations for each cohort, so that the (initial) wealth and income endowments behind the simulated profiles are consistent with the endowments behind the empirical profiles. As for the second problem, we explicitly simulate demographic transitions so that the simulated profiles incorporate mortality effects in the same way as the data, both for couples and singles.

5 Data

We use data from the Asset and Health Dynamics Among the Oldest Old (AHEAD) dataset. The AHEAD is a sample of non-institutionalized individuals, aged 70 or older in 1993. These individuals were interviewed in late 1993/early 1994, and again in 1996, 1998, 2000, 2002, 2004, 2006, 2008, and 2010. We do not use 1994 assets, nor medical expenses, due to underreporting (Rohwedder et al. [45]).

We only consider retired households to abstract from the retirement decisions and focus on the determinants of savings and consumption. Because we only allow for household composition changes through death, we drop households where an individual enters a household or an individual leaves the household for reasons other than death. Fortunately, attrition for reasons other than death is a minor concern in our data.

To keep the dynamic programming problem manageable, we assume a fixed difference in age between spouses, and we take the average age difference from our data. In our sample, husbands are on average 3 years older than their wives. To keep the data consistent with this assumption, we drop all households where the wife is more than 4 years older or 10 years younger than her husband.

We begin with 6047 households. After dropping 401 households who get married, divorced, were same sex couples, or who report making other

transitions not consistent with the model, 753 households who report earning at least \$3,000 in any period, 171 households with a large difference in the age of husband and wife, and 87 households with no information on the spouse in a household, we are left with 4,634 households, of whom 1,388 are couples and 3,246 are singles. This represents 24,274 household-year observations where at least one household member was alive.

We break the data into 5 cohorts. The first cohort consists of individuals that were ages 72-76 in 1996; the second cohort contains ages 77-81; the third ages 82-86; the fourth ages 87-91; and the final cohort, for sample size reasons, contains ages 92-102. Even with the longer age interval, the final cohort contains relatively few observations. In the interest of clarity, we exclude this cohort from our graphs, but we use all cells with at least 10 observations when estimating the model.

We use data for 8 different years; 1996, 1998, 2000, 2002, 2004, 2006, 2008, and 2010. We calculate summary statistics (e.g., medians), cohort-by-cohort, for surviving individuals in each calendar year—we use an unbalanced panel. We then construct life-cycle profiles by ordering the summary statistics by cohort and age at each year of observation. Moving from the left-hand-side to the right-hand-side of our graphs, we thus show data for four cohorts, with each cohort’s data starting out at the cohort’s average age in 1996.

Since we want to understand the role of income, we further stratify the data by post-retirement permanent income (PI). Hence, for each cohort our graphs usually display several horizontal lines showing, for example, median assets in each PI group in each calendar year. These lines also identify the moment conditions we use when estimating the model.

Our PI measure can be thought of as the level of income if there were two people in the household at age 70. We measure post-retirement PI as the individual’s average non-asset non-social insurance income over all periods during which he or she is observed. [ebf: I think we are using non asset non social insurance annuitized income, right?] Actual income is related to PI and is adjusted for age and household structure. Non-asset income includes the value of Social Security benefits, defined benefit pension benefits, veterans benefits and annuities. Since we model social insurance explicitly, we do not include SSI transfers. See appendix A for details. Because there is a roughly monotonic relationship between lifetime earnings and the income variables that we use, our measure of post-retirement PI is also a good measure of lifetime permanent income.

The AHEAD has information on the value of housing and real estate,

autos, liquid assets (which include money market accounts, savings accounts, T-bills, etc.), IRAs, Keoghs, stocks, the value of a farm or business, mutual funds, bonds, “other” assets and investment trusts less mortgages and other debts.

We do not include pension and Social Security wealth for four reasons. First, we wish to to maintain comparability with other studies (Hurd [33], Attanasio and Hoynes [5] for example). Second, because it is illegal to borrow against Social security wealth pension and difficult to borrow against most forms of pension wealth, Social Security and pension wealth are much more illiquid than other assets. Third, their tax treatment is different from other assets. Finally, differences in Social Security and pension are captured in our model by differences in the permanent income measure we use to predict annual income.

One important problem with our asset data is that the wealthy tend to underreport their wealth in virtually all household surveys (Davies and Shorrocks [14]). This will lead us to understate asset levels at all ages. However, Juster et al. (1999) show that the the wealth distribution of the AHEAD matches up well with aggregate values for all but the richest 1 % of households. Given that we match medians (conditional on permanent income), underreporting at the very top of the wealth distribution should not seriously affect our results.

6 Results

6.1 Health and Mortality

We estimate health transitions and mortality rates simultaneously by fitting the transitions observed in the HRS to a multinomial logit model. We allow the transition probabilities to depend on age, sex, current health status, marital status, permanent income, as well as polynomials and interactions of these variables.

Using the estimated transition probabilities, we simulate demographic histories, beginning at age 70, for different gender-PI-health combinations. Tables 1 and 2 show life expectancies for singles and couples at age 70, respectively. All tables use the appropriate distribution of people over state variables to compute the number which is object of interest. We find that rich people, women, married people, and healthy people live much longer

Income Percentile	Nursing Home	Males		Nursing Home	Females		All
		Bad Health	Good Health		Bad Health	Good Health	
10	2.86	6.72	8.45	3.79	11.13	13.21	11.46
30	2.86	7.30	9.36	3.78	11.78	14.03	12.20
50	2.86	7.99	10.34	3.78	12.48	14.89	13.01
70	2.88	8.83	11.37	3.78	13.33	15.76	13.85
90	2.90	9.76	12.44	3.85	14.14	16.54	14.73
By gender:							
Men							9.97
Women							13.72
By health status:							
Bad Health							11.11
Good Health							14.20

Table 1: Life expectancy in years for singles, conditional on reaching age 70.

than their poor, male, single, and sick counterparts.

Table 1 shows that a single male at the 10th permanent income percentile in a nursing home expects to live only 2.9 more years, while a single female at the 90th percentile in good health expects to live 16.5 more years. The far right column shows average life expectancy conditional on permanent income, averaging over both genders all health states. It shows that singles at the 10th percentile of the permanent income distribution live on average 11.5 years, whereas singles at the 90th percentile live on average 14.8 years.

Table 2 shows that a 70 year old male married to a 67 year old woman with the same health as himself, at the 10th permanent income percentile in a nursing home expects to live only 2.7 more years, while a 70 year old married female at the 90th percentile in good health (married to a 73 year old man in the same health state as herself) expects to live 18.6 more years.

Income Percentile	Nursing Home	<u>Males</u>		<u>Females</u>			All
		Bad Health	Good Health	Nursing Home	Bad Health	Good Health	
10	2.65	6.88	8.46	3.59	11.47	13.49	10.48
30	2.69	7.77	9.74	3.67	12.46	14.68	11.59
50	2.76	8.87	11.12	3.76	13.55	15.98	12.86
70	2.85	10.19	12.62	3.90	14.89	17.29	14.23
90	2.98	11.69	14.22	4.06	16.26	18.55	15.67
By gender:							
Men							12.24
Women							16.93
By health status:							
Bad Health							11.88
Good Health							15.60
Oldest Survivor							18.78
Prob Oldest Survivor is Woman							67.5%

Table 2: Life expectancy in years for couples, conditional on reaching age 70.

Averaging over both genders and all health states, married 70 year old people at the 10th percentile of the income distribution live on average 10.5 years, while those at the 90th percentile live 15.7 years. (Is this consistent with the age difference between men and women?) Singles at the 10th percentile live longer than members of couples at the same percentile. The reason for this is that singles at the 10th percentile of the income distribution are overwhelmingly women who live longer, whereas 50% of the members of couples are male. Conditional on gender, those in either good or bad health at the 10th percentile live longer if in a couple. Conditional only on gender, members of couples live almost 3 years longer than singles: single 70 year

Income Percentile	<u>Males</u>		<u>Females</u>		All
	Bad Health	Good Health	Bad Health	Good Health	
10	20.8	21.5	34.0	35.6	32.2
30	20.5	21.6	34.1	36.1	32.7
50	20.3	21.7	34.1	36.8	32.8
70	20.1	21.6	34.1	36.9	33.1
90	19.5	21.5	33.9	36.9	33.0
By gender:					
Men					21.0
Women					35.7
By health status:					
Bad Health					31.2
Good Health					33.2

Table 3: Probability of ever entering a nursing home, singles alive at age 70.

old women live on average 13.8 years versus 16.9 for married women. But a comparison of tables 1 and 2 reveals that, conditional on PI and health, the differences in longevity are much smaller. Married people live longer than singles, but much of the difference is explained by the fact that married people tend to have higher PI.

The bottom part of table 2 shows the number of years of remaining life of the oldest survivor in a household when the man is 70 and the woman is 67. On average the last survivor lives an additional 18.8 years. The woman is the oldest survivor 68% of the time.

The strongest predictor for nursing home entry is gender: men are less likely to end up in a nursing home than women. Although married people tend to live longer than singles, they are slightly less likely to end up in a nursing home.

Income Percentile	<u>Males</u>		<u>Females</u>		All
	Bad Health	Good Health	Bad Health	Good Health	
10	16.1	16.6	31.8	33.0	24.6
30	16.6	17.5	32.3	34.3	25.7
50	17.3	18.6	33.0	35.7	26.5
70	17.8	19.4	34.0	36.4	27.5
90	18.5	20.6	34.9	37.3	28.3
By gender:					
Men					19.0
Women					36.3
By health status:					
Bad Health					25.6
Good Health					28.4

Table 4: Probability of ever entering a nursing home, married alive at age 70.

Table 3 shows that single men and women face on average a 21% and 36% chance of being in a nursing home for an extended stay, respectively. Table ?? shows that married men and women face on average a 19% and 36% chance of being in a nursing home for an extended stay. Married people are much less likely to transition into a nursing home at any age (as also documented by Goda and Shoven?), but married people often become single as their partner dies and as they age. Furthermore, married people tend to live longer than singles, so they have more years of life to potentially enter a nursing home. Permanent income has only a small effect on ever being in a nursing home. Those with a high permanent income are less likely to be in a nursing home at each age, but they tend to live longer.

6.2 Income

We model income as a function of a third order polynomial in age, dummies for family structure, family structure interacted with an age trend, and a fifth order polynomial in permanent income. The estimates use a fixed effects estimation procedures, where the fixed effect is a transformation of initial permanent income. Hence the regression results can be interpreted as the effect of changing age or family structure for the same household: see appendix A for details of the procedure.

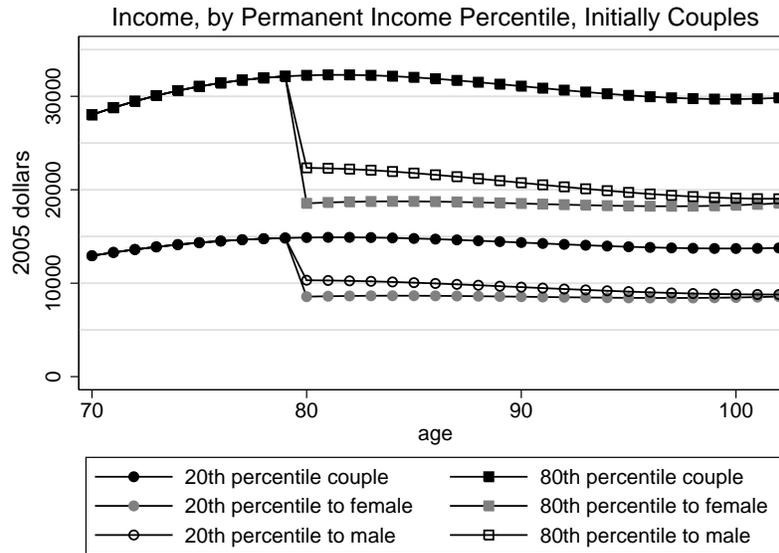


Figure 1: Income, conditional on permanent income and family structure. Figure assumes all households begin as couples, then potentially change to a single male or single female at age 80.

Figure 1 presents predicted income profiles for those at the 20th and 80th percentiles of the permanent income distribution. For each permanent income level, we display three scenarios, all commencing from the income of a couple. Under the first scenario, the household remains a couple until age 100. Under the second one, the man dies at age 80. Under the third one, the woman dies at age 80.

Figure 1 shows that average annual income ranges from about \$14,000 per year for couples in the 20th percentile of the PI distribution to over

\$30,000 for couples in the 80th percentile of the PI distribution. As a point of comparison, median wealth holdings for the two groups are XX and XX, respectively. Our estimates suggest that couples in which the male spouse dies at age 80 suffer a 40% decline in income, while couples in which the female spouse dies at 80 suffer a 30% decline in income. These income losses at the death of a spouse reflect the fact that although both Social Security and defined benefit pensions have spousal benefits, these replace only a fraction of the deceased spouse's income. Based on the Social Security rules, if only one member of the household worked when younger, Social Security benefits would fall by 33% with the death of one family member. If both members earned similar amounts, Social Security benefits would fall by 50% with the death of a spouse.

6.3 Medical Spending

One drawback of the AHEAD data is that it contains information only on out of pocket medical spending and not on the portion of medical spending covered by Medicaid. To be consistent with the model, in which the consumption floor is explicitly modeled, we need to include both Medicaid payments and out of pocket medical spending in our measure of out-of-pocket medical spending.

Fortunately, the Medicare Current Beneficiary Survey (MCBS) has extremely high quality information on Medicaid payments and out of pocket medical spending. It uses a mixture of both administrative and survey data. One drawback of the MCBS, however, is that although it has information on marital status and household income, but does not have information on the medical spending or health of the spouse. To exploit the best of both datasets, we use the procedures described in Appendix C to obtain our measured out-of-pocket medical expenditures, including those paid for by Medicaid.

We model the logarithm of medical spending as a fourth order polynomial in age, dummies for family structure, the health of each family member (including whether that family member just died), family structure interacted with an age trend, and a fifth order polynomial in permanent income. As with the income estimate, we use fixed effects estimation procedures, so the regression results can be interpreted as the effect of changing age or family structure for the same household.

Figure 2 displays our predicted medical expense profiles for those at the

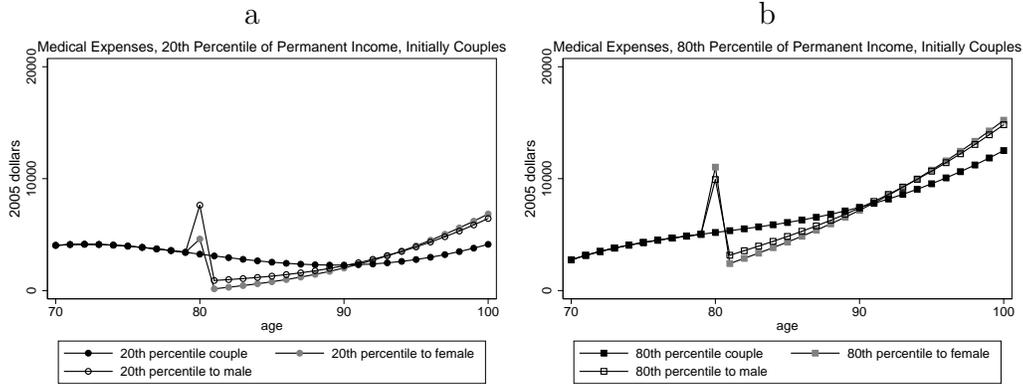


Figure 2: Mean out-of-pocket medical expenditures by age, conditional on permanent income and family structure. Figure assumes all households begin as couples, then potentially change to a single male or single female at age 80.

20th (left panel) and 80th (right panel) percentiles of the permanent income distribution for those in bad health. For each permanent income level, we present three scenarios, all of which start out with a couple. Under the first scenario, the household remains a couple until age 100. Under the second one, the man dies at age 80. Under the third one, the woman dies at age 80. The jump in medical spending shown at age 80 represents the elevated medical spending during the year of death of a family member and amounts to almost \$5,000 on average. Both average medical spending and the jump at time of death are larger for those with higher permanent income.

The figure shows that, before age 80, average annual medical spending hovers around \$4,000 per year for the couples in both income groups. For those at the 20th percentile of the PI distribution, medical expenses change little with age. However, for couples in the 80th percentile, medical expenses rise to well over \$10,000 per year by age 95.

In order to estimate the variability of medical spending, we take medical expense residuals (the difference between actual and predicted medical spending) and we regress the squared residuals on the same covariates we used for the regression for the logarithm of medical spending.

6.4 Life Cycle Asset Profiles

The following figures display median assets, conditional on birth cohort and income quintile, for couples and singles that are classified by permanent income quintile based on the permanent incomes of that same subpopulation (couples or singles). We choose to classify profiles in this fashion because couples are richer than singles and classifying a subpopulation according to the permanent income of the entire population would create cells that in some cases contain a large fraction of the population, while in other they generate a much smaller cell. The graphs based on the classification of permanent income by the whole population are in appendix B and display similar patterns.

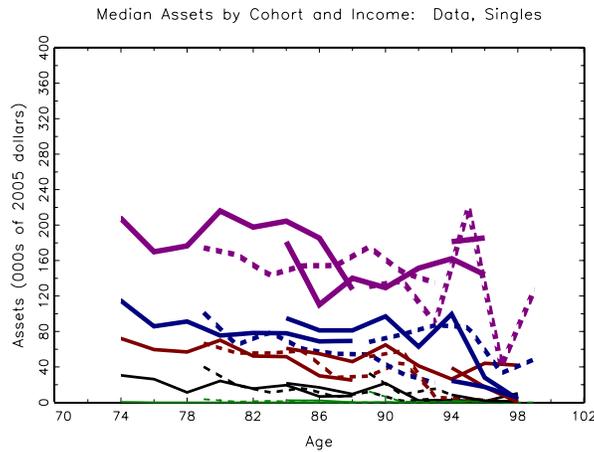


Figure 3: Median assets for singles, PI percentiles computed using singles only.

Figure 3 displays the asset profiles for the unbalanced panel of singles, classified according to the PI of the singles. Median assets are increasing in permanent income, with the 74-year-olds in the highest PI income of the singles holding about \$200,000 in median assets, while those at the lowest PI quintiles holding essentially no assets. Over time, those with the highest PI tend to hold onto significant wealth well into their nineties, those with the lower PIs never save much, while those in the middle PIs display quite some asset decumulation as they age.

Figure 4 reports median assets for the population of those who are ini-

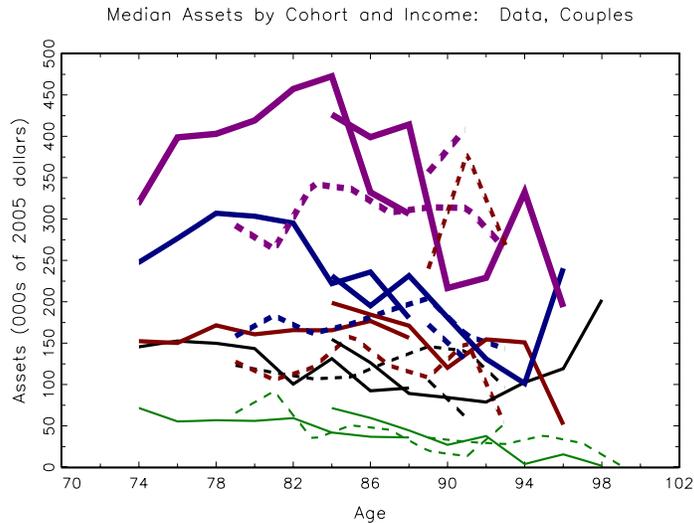


Figure 4: Median assets for initially couples, PI percentiles computed using couples only.

tially in a couple. The first thing to notice is that couples are richer than singles. The younger couples in the highest PI quintile for couples hold over \$340,000, compared to \$200,000 for singles, and even the couples in the lowest PI quintile bin hold over \$60,000 in the earlier years of their retirement, compared to zero for the singles. As for the singles, the couples in the highest PI quintile hold on to large amount of assets well into their nineties, while those in the lowest income PIs display more asset decumulation. Some of these couples lose one spouse during the period in which we observe them, in which case we report the assets of the surviving spouse, hence some of the decumulation that we observe in this graph is due to the death of one spouse and consequent disappearance of some of the couple’s assets.

Figure 4 Displays median assets for those who initially are in couples and remain in a couple during our sample period. That is, they do not experience the death of a partner. Interestingly, these graphs display flat to increasing asset profiles, going from low to high permanent income. This is consistent with the observation that much of the asset decumulation for a couple happens at death of one spouse.

To formalize and quantify this observation, we perform regression analysis in which (??? Eric describe). Figure 6 reports the assets of a couple starting

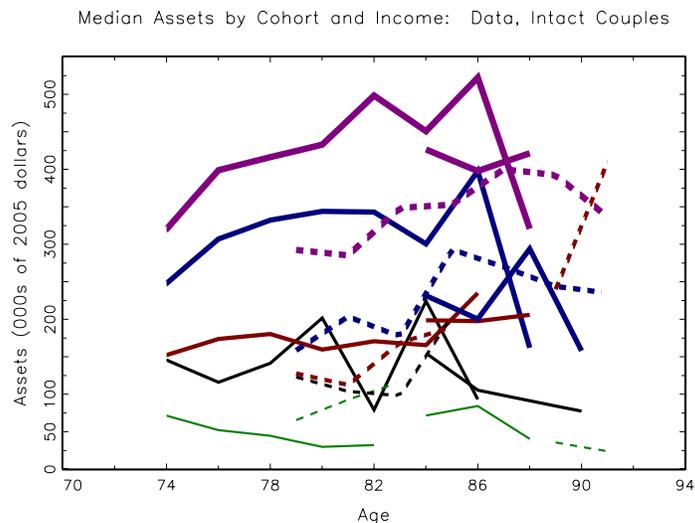


Figure 5: Median assets for intact couples, PI percentiles computed using couples only.

out, respectively, in the top PI income quintile, and in the bottom one, and display the assets under three scenarios. Under the first one, the couple remains intact until age 90. Under the second one, the male dies at age 80, while under the third one, the female dies at age 80. For both PI levels, assets stay roughly constant if both partners are alive. In contrast, assets display a significant drop at death of one of the spouses for the higher PI couples. Interestingly, assets experience a large drop when the male dies also for the lowest PI quintile, but not much of a drop when the female dies first. (??? How robust is this feature and what do we want to make of this? Do we think we have identification?)

7 Conclusions

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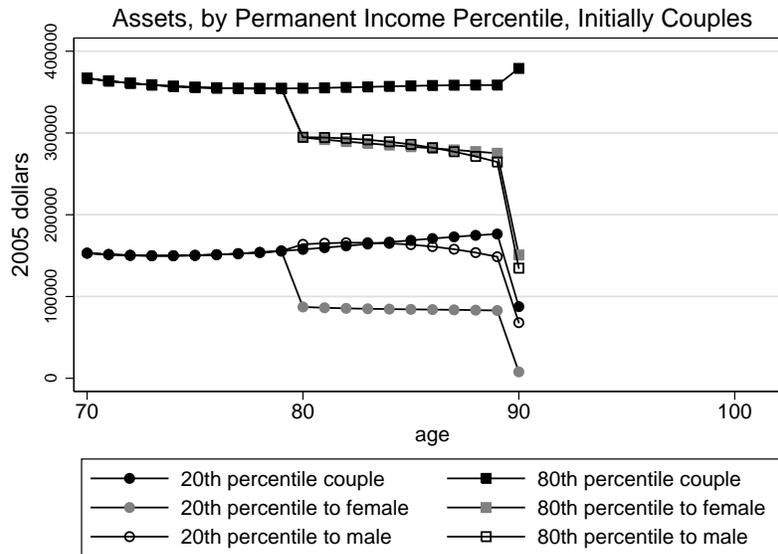


Figure 6: Net Worth, conditional on permanent income and family structure. Figure assumes all households begin as couples, then potentially change to a single male or single female at age 80, then everyone dies at age 90.

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Appendix A: Inferring Permanent Income

We assume that log income follows the process

$$\ln y_{it} = g(t, f_{it}) + h(I_i) + \omega_{it} \quad (19)$$

where $g(t, f_{it})$ is a flexible functional form of age t and family structure f_{it} (i.e., couple, single male, or single female) and ω_{it} represents measurement error. The variable I_i the household's percentile rank in the permanent income distribution. Since it is a summary measure of lifetime income at retirement, it should not change during retirement and is thus a fixed effect over our sample period. However, income could change as households age and potentially lose a family member. Our procedure to estimate equation (19) is to first estimate the fixed effects model

$$\ln y_{it} = g(t, f_{it}) + f_i + \omega_{it} \quad (20)$$

which allows us to obtain a consistent estimate of the function $g(t, f_{it})$. Next, note that as the number of time periods over which we can measure income and other variables for individual i (denoted T_i) becomes large,

$$plim_{T_i \rightarrow \infty} \frac{1}{T_i} \sum_{t=1}^{T_i} [\ln y_{it} - g(t, f_{it}) - \omega_{it}] = \frac{1}{T_i} \sum_{t=1}^{T_i} [\ln y_{it} - g(t, f_{it})] = h(I_i). \quad (21)$$

Thus we can calculate the percentile ranking of permanent income I_i for every household in our sample by taking the percentile ranking of $\frac{1}{T_i} \sum_{t=1}^{T_i} [\ln y_{it} - \hat{g}(t, f_{it})]$ where $\hat{g}(t, f_{it})$ is the estimated value of $g(t, f_{it})$ from equation (20). Finally, we need a mapping from the estimated index I_i back to a predicted level of income that can be used in the dynamic programming model. To do this we estimate the function

$$[\ln y_{it} - g(t, f_{it})] = h(I_i) + \omega_{it} \quad (22)$$

where $g(t, f_{it})$ is replaced with its estimated value $\hat{g}(t, f_{it})$ and the function $h(I_i)$ is a flexible functional form.

In practice we model $g(t, f_{it})$ as a third order polynomial in age, dummies for family structure, and family structure interacted with an age trend. We model $h(I_i)$ as a fifth order polynomial in permanent income.

Given that we have for every member of our sample t, f_{it} , and estimates of I_i and the functions $g(.,.), h(.,)$, we can calculate the predicted value of

$\ln \hat{y}_{it} = \hat{g}(t, f_{it}) + \hat{h}(\hat{I}_i)$. It is $\ln \hat{y}_{it}$ that we use when simulating the model for each household. A regression of $\ln y_{it}$ on $\ln \hat{y}_{it}$ yields a R^2 statistic of .74, suggesting that our predictions are accurate.

Appendix B: Asset Profiles Based on PI percentiles computed on whole population rather than on the sub-population of interest.

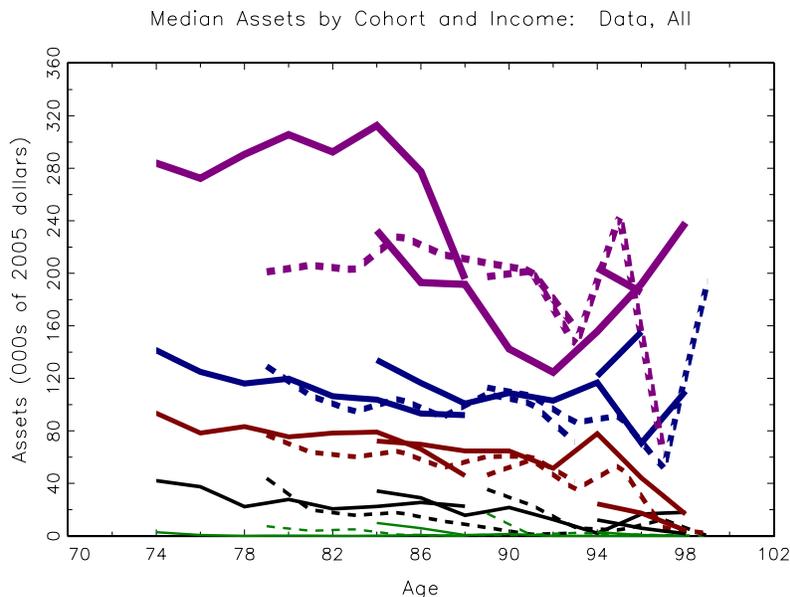


Figure 7: Median assets, whole sample.

Appendix C: Imputing Medicaid plus Out of Pocket Medical Expenses

Our goal is to measure the data generating process of the sum of Medicaid payments plus out of pocket expenses: this is the variable $\ln(m_{it})$ in equation (6) of the main text. If the household is drawing Medicaid benefits, then the household will spend less than $\ln(m_{it})$ on out of pocket medical spending (Medicaid picking up the remainder).

The AHEAD data contains information on out of pocket medical spending, but not on Medicaid payments. Fortunately, the Medicare Current Beneficiary Survey (MCBS) has extremely high quality information on Medicaid

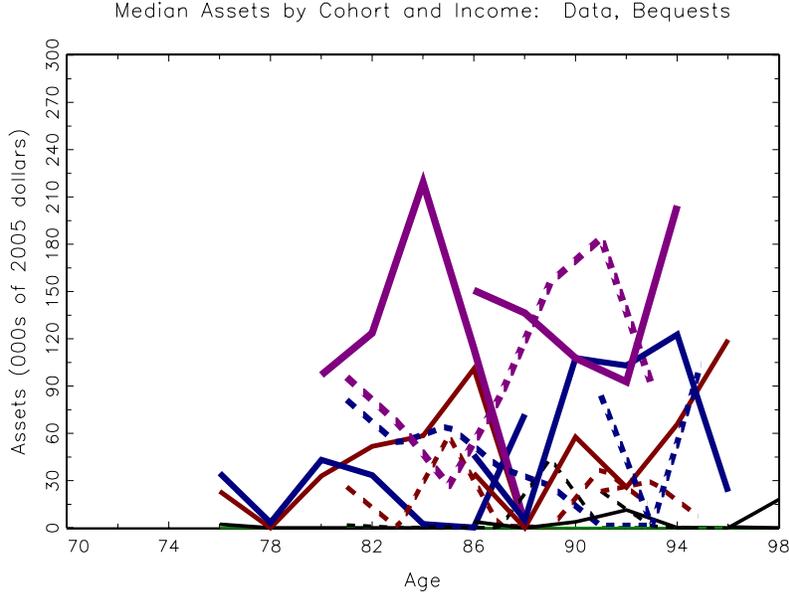


Figure 8: Bequests, whole sample.

payments plus out of pocket medical spending. One drawback of the MCBS, however, is that although it has information on marital status and household income, it does not have information on the medical spending or health of the spouse. Here we explain how to exploit the best of both datasets.

We use a two step estimation procedure. First, we estimate the process for household out of pocket medical spending using the AHEAD. Second, we estimate the process for Medicaid payments plus out of pocket medical spending as a function of out of pocket medical spending and other state variables using the MCBS. Because the MCBS lacks information on spouse's medical spending, and because most elderly Medicaid recipients are singles, we employ the second step estimation procedure only for singles. For couples we use only the first stage estimates estimated using the AHEAD data.

First Step Estimation Procedure: Define $\ln(oop_{it})$ as the log of out of pocket medical expenses and $X_{it} = (f_t, hs_{I,t}^h, hs_{I,t}^w, t, I)$. Using AHEAD data, we first use fixed effects estimation procedures to estimate

$$\ln(oop_{it}) = g(X_{it}) + \epsilon_{it}. \quad (23)$$

Next we construct the estimated residuals $\hat{\epsilon}_{it} = \ln(oop_{it}) - \hat{g}(X_{it})$ and estimate

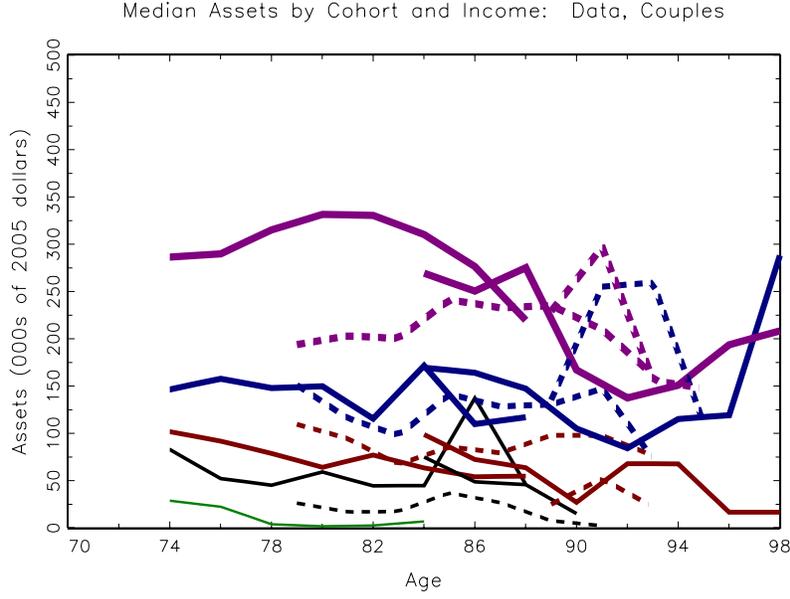


Figure 9: Median assets for couples, PI percentiles computed using whole population.

the regression (using OLS and no fixed effects):

$$\hat{\epsilon}_{it}^2 = h(X_{it}) + \zeta_{it}. \quad (24)$$

This allows us to recover the variance of ϵ_{it} conditional on X_{it} .

Second Step Estimation Procedure: Once we estimate equations (23) and (24) on the AHEAD data, we use the MCBS to estimate the link between $\ln(m_{it})$ and $\ln(oop_{it})$ for singles using OLS with no fixed effects:

$$\ln(m_{it}) = \alpha \ln(oop_{it}) + k(X_{it}) + u_{it}, \quad (25)$$

so we can construct $\hat{u}_{it} = \ln(m_{it}) - (\hat{\alpha} \ln(oop_{it}) + \hat{k}(X_{it}))$ and estimate the conditional variance of this:

$$\hat{u}_{it}^2 = \hat{l}(X_{it}) + \xi_{it} \quad (26)$$

Combining equations (23) and (25) yields

$$\ln(m_{it}) = \alpha g(X_{it}) + k(X_{it}) + (\alpha \epsilon_{it} + u_{it}). \quad (27)$$

Median Assets by Cohort and Income: Data, Intact Couples

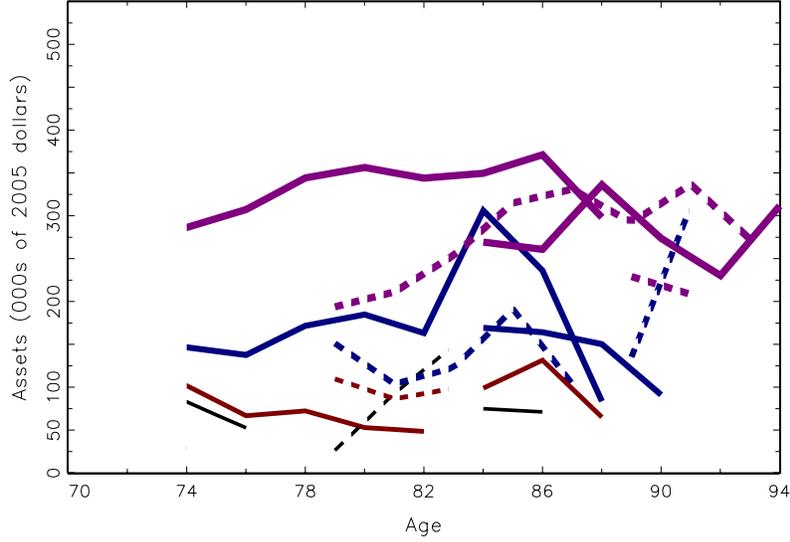


Figure 10: Median assets for intact couples, PI percentiles computed using whole population.

Recall that equation (6) of the main text is given by $\ln(m_{it}) = m(X_{it}) + \sigma(X_{it}) \times \psi_{it}$. Equation (27) then implies that $m(X_{it}) = \alpha g(X_{it}) + k(X_{it})$.

In order to infer the variance of medical expenditures conditional on X_{it} , note that from equations (6) and (27)

$$\sigma(X_{it}) \times \psi_{it} = \ln(m_{it}) - m(X_{it}) = (\alpha \epsilon_{it} + u_{it}). \quad (28)$$

so we can obtain the conditional variance by noting that $E[\psi_{it}^2] = 1$ and assuming that $E[\epsilon_{it} u_{it}] = 0$

$$[\sigma(X_{it})]^2 = E[(\alpha \epsilon_{it} + u_{it})^2 | X_{it}] = \alpha^2 E[\epsilon_{it}^2 | X_{it}] + E[u_{it}^2 | X_{it}] \quad (29)$$

where we estimate $E[\epsilon_{it}^2 | X_{it}] = h(X_{it})$ in equation (24) and $E[u_{it}^2 | X_{it}]$ in equation (26).

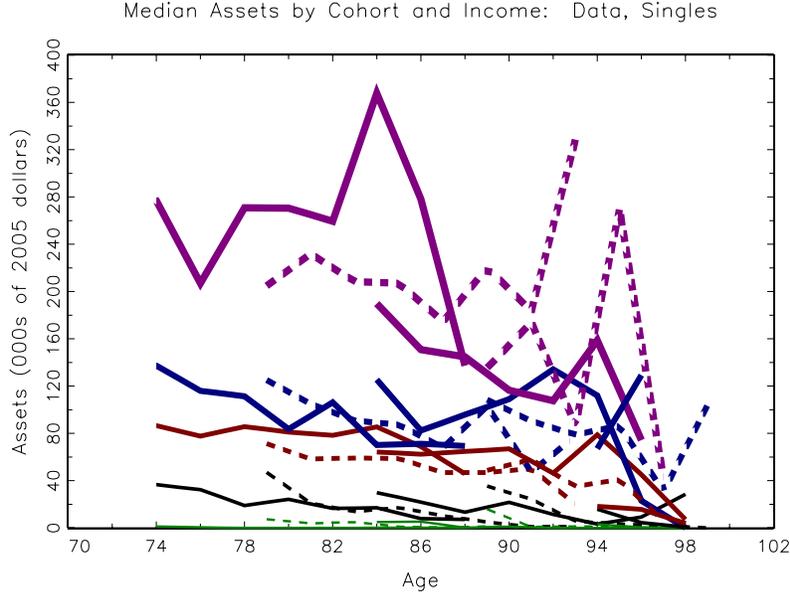


Figure 11: Median assets for singles, PI percentiles computed using whole population.

Appendix D: Outline of the computation of the value functions and optimal decision rules

We compute the value functions by backward induction. We start from the singles, find their time T value function and decision rules by maximizing equation (15) subject to the relevant constraints, and $V_{T+1}^i = \theta_0(x_t - c_t)$, $i = h, w$. This yields the value function V_T^i and the decision rules for time T . We then find the decision rules at time $T - 1$ by solving equation (15) with V_T^i . Continuing this backward induction yields decision rules for time $T - 2, T - 3, \dots, 1$.

We find the decisions for couples by maximizing equation (16), subject to the relevant constraints and the value function for the singles, and setting V_{T+1}^c to the appropriate bequest motive value. This yields the value function V_T^c and the decision rules for time T . We then find the decision rules at time $T - 1$ by solving equation (16) using $V_T^c, V_T^i, i = h, w$. Continuing this backward induction yields decision rules for time $T - 2, T - 3, \dots, 1$.

We discretize the persistent component and the transitory components of

the health shock, and interest rate into Markov Chain following Tauchen and Hussey (1991). We assume a finite number of permanent income categories. We take cash-on-hand to lay into a finite number of grid points.

Given each level of permanent income of the household, we solve for decision rules for each possible combination of cash-on-hand, income, health status, and persistent component of the health shock. We use linear interpolation within the grid and linear extrapolation outside of the grid to evaluate the value function at points that we do not directly compute.

For the singles, for simplicity of notation, here for the most part we drop any reference to the missing spousal variables everywhere, and just use similar notation to the one for couples (except for the number of arguments in the function). In the code, we have different names, say for example for the survival of single people and couples.

The value function for the singles, $i = h, w$ and $f_t = 0$ is given by:

$$V_t^i(x_t, h_{st}, I, \zeta_t) = \max_{c_t, x_{t+1}} \left\{ u^i(c_t, h_{st}) + \beta \left(1 - s(h_{st}, I, i, t, 0) \right) \theta_0(x_t - c_t) + \right. \\ \left. \beta s(h_{st}, I, i, t, 0) \left[\sum_{k=1}^{d_m} \sum_{l=1}^{d_\zeta} \sum_{n=1}^{d_\xi} Pr(h_{s_{t+1}} = h_{s_k} | h_{st}, I, g, t, 0) Pr(\zeta_{t+1} = \zeta_l | \zeta_t) Pr(\xi_{t+1} = \xi_n) \right. \right. \\ \left. \left. V_{t+1}^i \left(x_{t+1}(k, l, n), h_{s_{t+1}}(k), I, \zeta_{t+1}(l) \right) \right] \right\}.$$

Subject to:

$$z_{t+1} = x_t - c_t + y \left(r(x_t - c_t) + y_{t+1}(I, 0), \tau \right)$$

$$x_t > c_{\min}(i), \quad c_t \leq x_t, \quad \forall t,$$

$$\ln(m_{t+1}(k, l, n)) = m^i(h_{s_{t+1}}(k), I, t+1) + \sigma^i(h_{s_{t+1}}(k), I, t+1) \psi_{t+1}(l, n)$$

$$\psi_{t+1}(l, n) = \zeta_{t+1}(l) + \xi_{t+1}(n),$$

$$s_{t+1}(k, l, n) = z_{t+1} - m_{t+1}(k, l, n)$$

$$tr_{t+1}(k, l, n) = \max \left\{ 0, c_{\min}(f_t) - \max(0, s_{t+1} - a_d(i)) \right\}$$

$$x_{t+1}(k, l, n) = s_{t+1}(k, l, n) + tr_{t+1}(k, l, n)$$

$\forall x_t$ COH level, determine maximum consumption (and hence savings)

$\forall c_t \in (c_{\min}, x_t)$, compute $u^i(c, m)$ and $\theta_0(x_t - c_t)$.

$\forall (i, t, I)$ For each gender, age, PI, compute savings = z_{t+1}

$\forall h_{st}, \zeta_t$ For each health state and pers. medex shock TODAY

$\forall h_{s_{t+1}}(k), \zeta_{t+1}(l), \xi(n)$ tomorrow's shocks

$x_{t+1}(k, l, n)$ compute tomorrow's COH for each state

Interpolate and extrapolate $V_{t+1}^i(x_{t+1}(k, l, n), h_{s_{t+1}}(k), I, \zeta_{t+1}(l))$

Compute

$$\beta s(h_{st}, I, t) \sum_{k=1}^{d_m} \sum_{l=1}^{d_\zeta} \sum_{n=1}^{d_\xi} \Pi(k, l, n) V_{t+1}^i(x_{t+1}(k, l, n), h_{s_{t+1}}(k), I, \zeta_{t+1}(l))$$

$$\beta (1 - s(h_{st}, I, t)) \phi_0(x_t - c_t) + u^i(c_t, h_{st})$$

$W(c_t, x_t, I, h_{st}, \zeta_t)$ = sum of the two lines just above

$\max_{c_t} W(c_t, x_t, I, h_{st}, \zeta_t)$

$c_t^i(x_t, I, h_{st}, \zeta_t)$ = maximizer

$V_t^i(x_t, I, h_{st}, \zeta_t)$ = maximum

The value function for the couples is given by:

$$\begin{aligned}
V_t^c(x_t, hs_t^w, hs_t^h, I, \zeta_t) = & \max_{c_t, x_{t+1}, b_t^w, b_t^h} \left\{ u^c(c_t, hs_t^h, hs_t^w) + \right. \\
& + \beta \left(1 - s(hs^w, I, w, t, 1) \right) \left(1 - s(hs^h, I, h, t, 1) \right) \theta_2(x_t - c_t) + \\
& \beta s(hs^w, I, w, t, 1) (1 - s(hs^h, I, h, t, 1)) \left(\theta_1(b_t^h) + \omega \sum_{k=1}^{d_m} \sum_{l=1}^{d_\zeta} \sum_{n=1}^{d_\xi} Pr(\zeta_{t+1} = \zeta_l | \zeta_t) \right. \\
& \left. Pr(\xi_{t+1} = \xi_n) Pr(hs_{t+1}^w = hs_k | hs_t^w, I, w, t, 1) V_{t+1}^w(x_{t+1}^w(k, l, n), hs_{t+1}^w(k), I, \zeta_{t+1}(l)) \right) + \\
& \beta (1 - s(hs^w, I, w, t, 1)) s(hs^h, I, h, t, 1) \left(\theta_1(b_t^w) + \omega \sum_{k=1}^{d_m} \sum_{l=1}^{d_\zeta} \sum_{n=1}^{d_\xi} Pr(\zeta_{t+1} = \zeta_l | \zeta_t) \right. \\
& \left. Pr(\xi_{t+1} = \xi_n) Pr(hs_{t+1}^h = hs_{kh} | hs_t^h, I, h, t, 1) V_{t+1}^h(x_{t+1}^h(k, l, n), hs_{t+1}^h(k), I, \zeta_{t+1}(l)) \right) + \\
& \beta s(hs^w, I, w, t, 1) s(hs^h, I, h, t, 1) \\
& \left(\sum_{kh=1}^{d_m} \sum_{kw=1}^{d_m} \sum_{l=1}^{d_\zeta} \sum_{n=1}^{d_\xi} V_{t+1}^c(x_{t+1}(k_h, k_w, l, n), hs_{t+1}^w, hs_{t+1}^h, I, \zeta_{t+1}(l)) \right. \\
& \left. Pr(\zeta_{t+1} = \zeta_l | \zeta_t) Pr(\xi_{t+1} = \xi_n) Pr(hs_{t+1}^h = hs_{kh} | hs_t^h, I, h, t, 1) Pr(hs_{t+1}^w = hs_{kw} | hs_t^w, I, w, t, 1) \right) \left. \right\} \\
\end{aligned} \tag{30}$$

subject to

$$\begin{aligned}
x_t &> c_{min}(c), \quad c_t \leq x_t, \quad \forall t \\
0 &< b_t \leq x_t - c_t, \quad \forall t, \quad i = h, w, \\
y_{t+1}^c &= y(c, I, t + 1) \\
y_{t+1}^i &= y(i, I, t + 1) \\
z_{t+1}^c &= \{x_t - c_t + y(r(x_t - c_t) + y_{t+1}^c, \tau)\} \\
z_{t+1}^i &= \{x_t - c_t + y(r(x_t - c_t) + y_{t+1}^i, \tau)\}
\end{aligned}$$

$$\begin{aligned}
\ln m_{t+1}^c(k_h, k_w, l, n) &= m^c(hs_{t+1}^h(k_h), hs_{t+1}^w(k_w), t + 1, I) \\
&+ \sigma^c(hs_{t+1}^h(k_h), hs_{t+1}^w(k_w), I, t + 1) \psi_{t+1}(l, n)
\end{aligned}$$

$$\ln m_{t+1}^i(k, l, n) = m^i(i, hs_{t+1}^i(k), t+1, I) + \sigma^i(i, hs_{t+1}^i(k), I, t+1)\psi_{t+1}(l, n)$$

$$\psi_{t+1}(l, n) = \zeta_{t+1}(l) + \xi_{t+1}(n),$$

$$x_{t+1}^c(k_h, k_w, l, n) = \max\{z_{t+1}^c - m_{t+1}^c(k_h, k_w, l, n), c_{\min}(c)\}$$

$$x_{t+1}^i(j, k, l, n) = \max\{z_{t+1}^i - m_{t+1}^i(k, l, n) - b_t, c_{\min}(i)\}$$

Computation

$\forall(x_t)$

$\forall c_t \in (c_{\min}, x_t)$

$\theta_2(x_t - c_t)$

$\forall(b_t^i \in (0, x_t))$

$\theta_1(b_t)$

$\forall(m_t^h, m_t^w)$

$u(c_t, m_t^h, m_t^w)$

$\forall t, \zeta_t$

sumC: expected value next period in case both survive

sumH: expected value next period in case husband dies

sumW: expected value next period in case wife dies

bequestUCM: utility from bequests when both die

$W(c_t, x_t, I, m_t^h, m_t^w, \zeta_t) = u(c_t, m_t^h, m_t^w) + \beta s_{m,I,t}^w s_{m,I,t}^h \text{sumC}$

$+ \beta s_{m,I,t}^w (1 - s_{m,I,t}^h) \text{sumH}$

$+ \beta (1 - s_{m,I,t}^w) s_{m,I,t}^h \text{sumW} + \beta (1 - s_{m,I,t}^w) (1 - s_{m,I,t}^h) +$

$\beta (1 - s_{m,I,t}^w) (1 - s_{m,I,t}^h) \text{bequestUCM}$

$\max_{c_t} W(c_t, x_t, I, m_t^h, m_t^w, \zeta_t)$

$c_t^c(x_t, I, m_t^h, m_t^w, \zeta_t) = \text{maximizer}$

$V_t^c(x_t, I, m_t^h, m_t^w, \zeta_t) = \text{maximum}$

In the calculation of next period's expected value in the case in which one spouse dies (sumH and sumW), we compute the optimal decision rule for bequests, conditional on each consumption choice. In the end when we get the decision rule for consumption, we automatically get the optimal decision rule of bequest corresponding to each state $b_t^c(x_t, I, m_t^h, m_t^w, \zeta_t)$

Appendix E: Moment Conditions and the Asymptotic Distribution of Parameter Estimates

Our estimate, $\hat{\Delta}$, of the “true” preference vector Δ_0 is the value of Δ that minimizes the (weighted) distance between the estimated life cycle profiles for assets found in the data and the simulated profiles generated by the model. For each calendar year $t \in \{1, \dots, T\}$, we match median assets for 5 permanent income quintiles in 5 birth year cohorts, both for singles and couples, leading to a total of $50T$ moment conditions. Sorting households into quintiles also requires us to estimate the 1/5th, 2/5th, 3/5th and 4/5th permanent income quantiles for each birth year cohort and initial family type. This produces a total of 40 nuisance parameters, which we collect into the vector γ . Each of these parameters has its own moment condition.

The way in which we construct these moment conditions builds on the approach described in French and Jones [28]. Useful references include Buchinsky [10] and Powell [44]. Consider first the permanent income quantiles. Let $q \in \{1, 2, \dots, Q - 1\}$ index the quantiles. Assuming that the permanent income distribution is continuous, the π_q -th quantile of permanent income for initial family type f of cohort c , $g_{\pi_q}(c, f)$, is defined as

$$\Pr(I_i \leq g_{\pi_q}(c, f) | c, f) = \pi_q. \quad (31)$$

In other words, the fraction of households with less than g_{π_j} in permanent income is π_j . Using the indicator function, the definition of π_j -th conditional quantile can be rewritten as

$$E([1\{I_i \leq g_{\pi_q}(c, f)\} - \pi_q] \times 1\{c_i = c\} \times 1\{f_i = f\}) = 0, \quad (32)$$

for $c \in \{1, 2, \dots, C\}$, $f \in \{\text{single}, \text{couple}\}$, $q \in \{1, 2, \dots, Q - 1\}$.

The more important set of moment conditions involves the permanent income-conditional age-asset profiles. Suppose that household i 's permanent income level falls in the q th permanent income interval of households in its cohort, i.e.,

$$g_{\pi_{q-1}}(c, f) \leq I_i \leq g_{\pi_q}(c, f). \quad (33)$$

We assume that $\pi_0 = 0$ and $\pi_Q = 1$, so that $g_{\pi_0}(c, f) \equiv -\infty$ and $g_{\pi_Q}(c, f) \equiv \infty$. Let $a_{cfqt}(\Delta, \chi)$ be the model-predicted median observed asset level for group $cfqt$. Recall that the median is just the 1/2 quantile. Assuming that observed assets have a continuous conditional density, we arrive at the following moment condition:

$$E(1\{\tilde{a}_{it} \leq a_{cfqt}(\Delta_0, \chi_0)\} - 1/2 | c, f, q, t, \text{household observed at } t) = 0. \quad (34)$$

Equation (34) is merely equation (18) in the main text, adjusted to allow for “missing” as well as deceased households, as in French and Jones [27]. Using indicator function notation, we can convert this conditional moment equation into an unconditional one:

$$E\left(\left[1\{\tilde{a}_{it} \leq a_{cft}(\Delta_0, \chi_0)\} - 1/2\right] \times 1\{c_i = c\} \times 1\{f_i = f\} \times 1\{g_{\pi_{q-1}}(c, f) \leq I_i \leq g_{\pi_q}(c, f)\} \times 1\{\text{household observed at } t\} \mid t\right) = 0, \quad (35)$$

for $c \in \{1, 2, \dots, C\}$, $f \in \{\text{single, couple}\}$, $q \in \{1, 2, \dots, Q\}$, $t \in \{t_1, t_2, \dots, t_T\}$.

Suppose we have a data set of I independent households that are each observed at T separate calendar years. Let $\varphi(\Delta, \gamma; \chi_0)$ denote the $(50T+40)$ -element vector of moment conditions described immediately above, and let $\hat{\varphi}_I(\cdot)$ denote its sample analog. Letting $\widehat{\mathbf{W}}_I$ denote a $(50T+40) \times (50T+40)$ weighting matrix, the MSM estimator $(\hat{\Delta}', \hat{\gamma}')'$ is given by

$$\arg \min_{\{\Delta, \gamma\}} \frac{I}{1 + \tau} \hat{\varphi}_I(\Delta, \gamma; \chi_0)' \widehat{\mathbf{W}}_I \hat{\varphi}_I(\Delta, \gamma; \chi_0),$$

where τ is the ratio of the number of observations to the number of simulated observations.

In practice, we estimate χ_0 as well, using the approach described in the main text. Computational concerns, however, compel us to treat χ_0 as known in the analysis that follows. Under regularity conditions stated in Pakes and Pollard [40] and Duffie and Singleton [22], the MSM estimator $\hat{\theta}$ is both consistent and asymptotically normally distributed:

$$\sqrt{I} \left(\begin{pmatrix} \hat{\Delta} \\ \hat{\gamma} \end{pmatrix} - \begin{pmatrix} \Delta_0 \\ \gamma_0 \end{pmatrix} \right) \rightsquigarrow N(0, \mathbf{V}),$$

with the variance-covariance matrix \mathbf{V} given by

$$\mathbf{V} = (1 + \tau)(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}\mathbf{S}\mathbf{W}\mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1},$$

where: \mathbf{S} is the variance-covariance matrix of the data;

$$\mathbf{D} = \left[\begin{array}{cc} \frac{\partial \varphi(\Delta, \gamma; \chi_0)}{\partial \Delta'} \Big|_{\Delta=\Delta_0} & \frac{\partial \varphi(\Delta, \gamma; \chi_0)}{\partial \gamma'} \Big|_{\gamma=\gamma_0} \end{array} \right] \quad (36)$$

is the $(50T + 40) \times (9 + 40)$ gradient matrix of the population moment vector; and $\mathbf{W} = \text{plim}_{T \rightarrow \infty} \{\widehat{\mathbf{W}}_I\}$. Moreover, Newey [38] shows that if the model is properly specified,

$$\frac{I}{1 + \tau} \hat{\varphi}_I(\hat{\Delta}, \hat{\gamma}; \chi_0)' \mathbf{R}^{-1} \hat{\varphi}_I(\hat{\Delta}, \hat{\gamma}; \chi_0) \rightsquigarrow \chi_{50T-9}^2,$$

where \mathbf{R}^{-1} is the generalized inverse of

$$\begin{aligned} \mathbf{R} &= \mathbf{PSP}, \\ \mathbf{P} &= \mathbf{I} - \mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}. \end{aligned}$$

The asymptotically efficient weighting matrix arises when $\widehat{\mathbf{W}}_I$ converges to \mathbf{S}^{-1} , the inverse of the variance-covariance matrix of the data. When $\mathbf{W} = \mathbf{S}^{-1}$, \mathbf{V} simplifies to $(1 + \tau)(\mathbf{D}'\mathbf{S}^{-1}\mathbf{D})^{-1}$, and \mathbf{R} is replaced with \mathbf{S} . But even though the optimal weighting matrix is asymptotically efficient, it can be severely biased in small samples. (See, for example, Altonji and Segal [1].) We thus use a “diagonal” weighting matrix, as suggested by Pischke [42]. The diagonal weighting scheme uses the inverse of the matrix that is the same as \mathbf{S} along the diagonal and has zeros off the diagonal of the matrix.

We estimate \mathbf{D} , \mathbf{S} and \mathbf{W} with their sample analogs. For example, our estimate of \mathbf{S} is the $(50T + 40) \times (50T + 40)$ estimated variance-covariance matrix of the sample data. When estimating preferences, we use sample statistics, so that $a_{cfqt}(\Delta, \chi)$ is replaced with the sample median for group $cfqt$. When computing the chi-square statistic and the standard errors, we use model predictions, so that the sample medians for group $cfqt$ is replaced with its simulated counterpart, $a_{cfqt}(\hat{\Delta}, \hat{\chi})$.

One complication in estimating the gradient matrix \mathbf{D} is that the functions inside the moment condition $\varphi(\Delta, \gamma; \chi)$ are non-differentiable at certain data points; see equations (32) and (35). This means that we cannot consistently estimate \mathbf{D} as the numerical derivative of $\hat{\varphi}_I(\cdot)$. Our asymptotic results therefore do not follow from the standard GMM approach, but rather the approach for non-smooth functions described in Pakes and Pollard [40], Newey and McFadden [39] (section 7) and Powell [44].

In finding \mathbf{D} , it proves useful to partition $\varphi(\cdot)$ into the $50T$ -element vector $\varphi_{\Delta}(\cdot)$, corresponding to the moment conditions described by equation (35) and the 40 – *element* vector $\varphi_{\gamma}(\cdot)$, corresponding to the moment conditions described by equation (32). Using this notation, we can rewrite equation

(36) as

$$\mathbf{D} = \begin{bmatrix} \frac{\partial \varphi_{\Delta}(\Delta_0, \gamma_0; \chi_0)}{\partial \Delta'} & \frac{\partial \varphi_{\Delta}(\Delta_0, \gamma_0; \chi_0)}{\partial \gamma'} \\ \frac{\partial \varphi_{\gamma}(\Delta_0, \gamma_0; \chi_0)}{\partial \Delta'} & \frac{\partial \varphi_{\gamma}(\Delta_0, \gamma_0; \chi_0)}{\partial \gamma'} \end{bmatrix}, \quad (37)$$

and proceed element-by-element.

It immediately follows from equation (32) that

$$\frac{\partial \varphi_{\gamma}(\Delta_0, \gamma_0; \chi_0)}{\partial \Delta'} = \mathbf{0}. \quad (38)$$

To find $\frac{\partial \varphi_{\gamma}(\Delta_0, \gamma_0; \chi_0)}{\partial \gamma'}$, we rewrite equation (32) as

$$\Pr(c_i = c \ \& \ f_i = f) \times [F(g_{\pi_q}(c, f)|c, f) - \pi_j] = 0, \quad (39)$$

where $F(g_{\pi_q}(c, f)|c, f)$ is the c.d.f. of permanent income for family type- f members of cohort c evaluated at the π_j -th quantile. Differentiating this equation shows that $\frac{\partial \varphi_{\gamma}(\Delta_0, \gamma_0; \chi_0)}{\partial \gamma'}$ is a diagonal matrix whose diagonal elements are given by

$$\Pr(c_i = c \ \& \ f_i = f) \times f(g_{\pi_q}(c, f)|c, f). \quad (40)$$

In practice we find $f(g_{\pi_q}(c, f)|c, f)$, the conditional p.d.f. of permanent income evaluated at the π_j -th quantile, with a kernel density estimator.

To find $\frac{\partial \varphi_{\Delta}(\Delta_0, \gamma_0; \chi_0)}{\partial \Delta'}$ and $\frac{\partial \varphi_{\Delta}(\Delta_0, \gamma_0; \chi_0)}{\partial \gamma'}$, it is helpful to rewrite equation (35) as

$$\Pr(c_i = c \ \& \ f_i = f \ \& \ \text{household observed at } t) \times \int_{g_{\pi_{q-1}}(c, f)}^{g_{\pi_q}(c, f)} \left[\int_{-\infty}^{a_{cfqt}(\Delta_0, \chi_0)} f(\tilde{a}_{it}|c, f, I_i, t) d\tilde{a}_{it} - \frac{1}{2} \right] f(I_i|c, f) dI_i = 0, \quad (41)$$

It follows that the rows of $\frac{\partial \varphi_{\Delta}(\Delta_0, \gamma_0; \chi_0)}{\partial \Delta'}$ are given by

$$\Pr(c_i = c \ \& \ f_i = f \ \& \ g_{\pi_{q-1}}(c, f) \leq I_i \leq g_{\pi_q}(c, f) \ \& \ \text{household observed at } t) \times f(a_{cfqt}|c, f, g_{\pi_{q-1}}(c, f) \leq I_i \leq g_{\pi_q}(c, f), t) \times \frac{\partial a_{cfqt}(\Delta_0, \gamma_0; \chi_0)}{\partial \Delta'}. \quad (42)$$

Proceeding similarly, it can be shown that each row of $\frac{\partial \varphi_{\Delta}(\Delta_0, \gamma_0; \chi_0)}{\partial \gamma'}$ has the following two non-zero elements:

$$\Pr(c_i = c \ \& \ f_i = f \ \& \ \text{household observed at } t) \times \begin{pmatrix} -f(g_{\pi_{q-1}}(c, f)|c, f) [F(a_{cfqt}|c, f, g_{\pi_{q-1}}(c, f), t) - 1/2] \\ f(g_{\pi_q}(c, f)|c, f) [F(a_{cfqt}|c, f, g_{\pi_q}(c, f), t) - 1/2] \end{pmatrix}', \quad (43)$$

with

$$f(g_{\pi_0}(c, f)|c, f) [F(a_{cft}|c, f, g_{\pi_0}(c, f), t) - 1/2] \equiv 0, \quad (44)$$

$$f(g_{\pi_Q}(c, f)|c, f) [F(a_{cft}|c, f, g_{\pi_Q}(c, f), t) - 1/2] \equiv 0. \quad (45)$$

In practice, we find $F(a_{cft}|c, f, I, t)$, the conditional c.d.f. of assets evaluated at the median a_{cft} by finding kernel estimates of the mean regression of $1\{a_{it} \leq a_{cft}\}$ on I (holding c, f and, t fixed).