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Thomas Crossley
Peter Levell
Stavros Poupakis

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Thomas F. Crossley

European University Institute, Institute for Fiscal Studies and ESCoE

Peter Levell

Institute for Fiscal Studies

Stavros Poupakis

University College London

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Abstract

Researchers are often interested in the relationship between two variables, with no single data set containing both. A common strategy is to use proxies for the dependent variable that are common to two surveys to impute the dependent variable into the data set containing the independent variable. We show that commonly employed regression or matching-based imputation procedures lead to inconsistent estimates. We offer an easily-implemented correction and correct asymptotic standard errors. We illustrate these with empirical examples using data from the US Consumer Expenditure Survey (CE) and the Panel Study of Income Dynamics (PSID).

Keywords: Imputation; Measurement error; Consumption.

JEL codes: C81, C13, E21

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1 Introduction

In empirical research we are often interested in the relationship between two variables, but no available data set contains both variables. For example, a key question in fiscal policy and macroeconomics is the effect of income or wealth (or changes in income or wealth) on consumption. Traditionally, consumption has been measured in dedicated household budget surveys which contain limited information on income or wealth. Income or wealth, and particularly changes in income and wealth, are measured in panel surveys with limited information on consumption.

A common strategy to overcome such problems is to use proxies for the dependent variable that are common to both surveys and impute that dependent variable into the data set containing the independent variable. In the first stage the dependent variable is regressed on the proxies in the donor data set. In the second stage, the coefficients, and possibly residuals, from the donor data set are combined with observations on the proxies in the main data set to generate an imputed value of the missing dependent variable in the main data set. Hereafter we refer to this as the **RP** procedure (for "regression prediction"). The addition of residuals to the regression prediction seeks to give the imputed variable a stochastic component and mimic the dispersion of the missing variable, and we refer to this as the **RP+** procedure. For example, in a well-known paper, Skinner (1987) proposed using the U.S Consumer Expenditure Survey (CE) and the **RP** procedure to impute a consumption measure into the Panel Study of Income Dynamics (PSID).[1] In this paper we consider the consequences of estimating a regression with an imputed dependent variable, and how those

[1] For panel data on consumption, an alternative approach is to invert the inter-temporal budget constraint and calculate spending as income minus saving where the latter is often approximated by changes in wealth. This was initially suggested by Ziliak (1998) for the PSID, but has more recently been adopted for administrative (tax) data on income and wealth (Browning et al., 2003). While attractive this procedure has several drawbacks. First it identifies only total household spending, and in many applications the distinctions between consumption spending, nondurable consumption and household investment spending are important. Second, in the case of our motivating example, this procedure results in income or wealth being on both the right and left-hand side of the equation so that any measurement error in income or wealth can cause quite serious problems (Browning et al., 2014). Baker et al. (2018) show that even with administrative data on income and wealth there can be significant measurement error in implied spending.

consequences depend on the imputation procedure adopted. We show that the **RP** procedure introduces a Berkson measurement error into the dependent variable, leading to inconsistent estimates of the regression coefficients of interest, as does the **RP+** procedure. Under mild assumptions, the asymptotic attenuation factor is equal to the population R^2 on the first stage regression of the variable to be imputed on the proxy or proxies. This leads us to suggest a "rescaled-regression-prediction" (hereafter **RRP**) procedure. We then show that, in the case a single proxy variable is used, the **RRP** procedure is numerically identical to a procedure developed by Blundell et al. (2004, 2008) (hereafter **BPP** after the authors), also for imputing consumption, in which the first stage involves, in contrast to **RP**, regressing the proxy on the variable to be imputed, and then inverting. Relative to **BPP** our proposed **RRP** approach has the advantage that it can more flexibly incorporate multiple proxies.

Both **RP** and **BPP** are currently used to impute dependent variables. However, the choice of imputation methods used in different papers appears to be ad hoc. **BPP** do not give explicit reasons for favouring their imputation method over others, and on occasion the same authors have switched from using **BPP** to **RP** in later papers. Examples of papers using versions of **RP** include Mulligan (1999); Browning et al. (2003); Meyer and Sullivan (2003); Attanasio and Pistaferri (2014); Charles et al. (2014); Arrondel et al. (2015); Fisher et al. (2016) and Kaplan et al. (2020). Examples of papers using **BPP** include Schulhofer-Wohl (2011); Guvenan and Smith (2014) and Attanasio et al. (2015). Some studies have observed that, empirically, **RP** imputation seems to lead to biased estimates in specific contexts. However, they neither offer an explanation nor realise that the problem is a general one. For example, Charles et al. (2014) note that the intergenerational elasticity of consumption spending is lower when **RP** is used to impute consumption to the PSID than when true consumption data is used. Palumbo (1999) also obtains lower estimates of risk aversion when using **RP** to impute consumption than when using a version of **BPP**. We account for these findings by formally setting out the nature of the biases associated with **RP**, by demonstrating that it specifically leads to an attenuation bias, and by offering a bias correction.

In the next section we lay out our basic framework, and derive the main results. We also relate our results to the prior literature, including Lusardi (1996), who combines CE consumption data with PSID income data using the 2-sample IV approach proposed by Klevmarken (1982) and Angrist and Krueger (1992). We clarify the relationship between that approach and the imputation procedures we study.

Section 3 takes up the question of inference. We show that the usual OLS standard errors from a regression of an imputed dependent variable, derived from the **RRP** or **BPP** procedures, are too small (a point that the literature also seems to have overlooked). We provide an estimator of the correct asymptotic standards errors of the regression coefficient of interest. Section 4 illustrates our main points with a Monte Carlo study, and Section 5 provides two empirical examples using the CE and PSID. Section 6 concludes.

2 Set-up And Main Results

Consider the following linear regression model

$$y = X\beta + \epsilon \tag{1}$$

where β is the $K \times 1$ parameter vector of interest. To make things concrete, the $n \times 1$ vector y could be consumption (or nondurable consumption), and the $n \times K$ matrix X would include income or wealth and other determinants of consumption. To keep the notation compact, variables have been de-meaned so there is no constant, but the addition of constants (and non-zero means) is not important for the analysis that follows.

We assume that for any random sample $i = 1, ..., n$ of $\{y_i, X_i'\}_{i=1}^n$ from the population the following hold:

A1 $E(X_i'X_i) = \Sigma_{XX}$ is finite and non-singular, and $E(X_i'\epsilon_i) = 0$

This means that given such a sample, an unbiased and consistent estimate of β can be obtained by OLS on Equation (1).

Suppose however that we have no such data on $\{y_i, X_i'\}$. In this case there are conditions under which we can consistently estimate β given a proxy for y, denoted Z, and samples where y and Z (but not X), and Z and X (but not y) are observed.

Let subscripts 1 and 2 denote whether variables correspond to sample 1 or sample 2; from here forward, the absence of a sample subscript indicates a population quantity. Using this notation, we would have a sample data on $\{y_{1i}, Z_{1i}'\}$ for $i = 1, ..., n_1$ and a second sample of data on $\{X_{2j}', Z_{2j}'\}$ $j = 1, ..., n_2$. Z_m is an $L \times n_m$ matrix of proxies ($l = 1, ..., L$) for y from sample m; if we have only a single proxy (a vector) we denote it by z_m. Similarly, when $K = 1$ we refer to X as x. In our consumption example z is often food spending. Food spending is captured in many general purpose surveys, and is thought to be well-measured.

To derive asymptotic results for different estimators of β that impute y using Z, we make the following additional assumptions:

A2 $\{y_{1i}, Z_{1i}'\}_{i=1}^{n_1}$ and $\{X_{2j}', Z_{2j}'\}_{j=1}^{n_2}$ are i.i.d random samples from the same population, with finite second moments and which are independent.

A3 $E(Z_{1i}'Z_{1i}) = \Sigma_{ZZ}$. Σ_{ZZ} is non-singular. $E\left(X_{2j}'X_{2j}\right) = \Sigma_{XX}$ (when $K = 1$, $E\left(x_{2j}^2\right) = \sigma_{xx} > 0$) and $E(y_{1i}^2) = E(y_{2j}^2) = \sigma_{yy} > 0$.

These assumptions guarantee the existence of linear projections of Z onto y and of y onto Z.[2]

$$Z_{1i} = y_{1i}\gamma + u_{1i} \tag{2}$$

where γ is $1 \times L$ and u_{1i} is $n_1 \times L$.

[2]In the derivations below we do not need to impose that $E\left(Z_{2j}'Z_{2j}\right) = \Sigma_{ZZ}$ or that $E\left(X_{1i}'X_{1i}\right) = \Sigma_{XX}$ as it is never ambiguous as to which samples are being used to calculate these objects. These equalities are however guaranteed by Assumptions A2 and A3.

$$y_{1i} = Z_{1i}\zeta + \xi_{1i} \tag{3}$$

The residuals u_{1i} and ξ_{1i} satisfy the conditions for equations (2) and (3) to be linear projections (i.e. $E(y'_{1i}u_{1i}) = 0$ and $E(Z'_{1i}\xi_{1i}) = 0$). However note that this is completely general in that we are not making any structural assumptions about the joint distributions of y_{1i} and Z'_{1i}; the orthogonality of y and Z variables with the error terms u and ξ arise by construction. In addition, no homoscedasticity assumptions are placed on u or ξ.

We also assume that:

A4 $\lim\limits_{n_2\to\infty} \frac{n_2}{n_1} = \alpha$ for some $\alpha > 0$.

This ensures that as n_1 tends to infinity, n_2 does as well.

The key assumptions that we make to allow consistent estimation of β are:

A5 $E\left(Z'_{1i}y_{1i}\right) = E\left(Z'_{2j}y_{2j}\right) = \Sigma_{Zy}$ which has at least one non-zero entry.

A6 $E\left(X'_{2j}u_{2j}\right) = 0$.

A5 ensures that the proxies Z_1 have information about y (that the slope of the linear projections in equations (2) and (3) are not zero.) Assumption A6 will be discussed further below.

Assumptions A1-A3 and A5 allow us to define the population R^2 from a regression of y on Z, $\phi_{y,Z} \equiv \Sigma_{yZ}\Sigma_{ZZ}^{-1}\Sigma_{Zy}/\left(\sigma_{\epsilon\epsilon} + \beta'\Sigma_{XX}\beta\right)$, and to guarantee that $0 < \phi_{y,Z} \leq 1$. Assumption A3 is necessary to ensure that this quantity is defined. Assumption A5 ensures that it is strictly positive and thus that its reciprocal is also defined.

To compute variances for different estimators allowing for general forms of heteroscedasticity, we make the following further assumptions:

A7 $E(Z_{1i}\xi_{1i}\xi'_{1i}Z'_{1i}) = \Omega_{Z\xi}$ which is finite and positive semi-definite.

A8 $E(X_{2j}\delta_{2j}\delta'_{2j}X'_{2j}) = \Omega_{X\delta}$ which is finite and positive semi-definite. δ_2 are residuals from a regression of $Z_2/\phi_{y,Z}$ on X_2.

A9 $\{y_{1i}, Z'_{1i}\}_{i=1}^{n_1}$ and $\{X'_{2j}, Z'_{2j}\}_{j=1}^{n_2}$ have finite fourth moments.

Finally, in what follows we also make use of the following definitions and notation:

D1 $E\left(X'_{2j}y_{2j}\right) = \Sigma_{Xy}$ and $E\left(X'_{2j}Z_{2j}\right) = \Sigma_{XZ}$. When $K = 1$, $E\left(x_{2j}y_{2j}\right) = \sigma_{xy}$.

D2 We define for instance $\Sigma'_{Zy} = \Sigma_{yZ}$.

D3 $E\left(u'_{1i}u_{1i}\right) = \Sigma_{uu}$ which under A2 is finite and positive semi-definite and $E\left(\epsilon_{2j}^2\right) = \sigma_{\epsilon\epsilon} \geq 0$. With a single proxy, $E\left(u_{1i}^2\right) = \sigma_{uu} \geq 0$.

D4 $R^2_{y_1,Z_1}$ is the sample analog of $\phi_{y,Z}$ (taken from sample 1).

D5 $E(y_{1i}) = \mu_y$

2.1 Alternative Imputation Strategies

Skinner (1987) suggested regressing y_1 on Z_1 in the CE and using the resulting coefficients to predict $\hat{y}_2$ in the PSID (and then regressing $\hat{y}_2$ on X_2), which we call the **RP** procedure. Note that with a single spending category as the proxy, the first stage linear projection here resembles an "inverse" Engel curve. Alternatively, Blundell et al. (2004, 2008), again using the CE and PSID, first regress z_1 (food spending) on y_1 then predict $\hat{y}_2 = z_2\frac{1}{\hat{\gamma}}$ (the **BPP** procedure). That is, they estimate an Engel curve and then invert it to predict consumption. A third alternative is to not impute consumption at the household level at all, but to recover the parameter of interest (β) from a combination of moments taken from the two surveys. This was first suggested (for a different application) by Arellano and Meghir (1992) (hereafter **AM**). Here, (again with a single proxy) one could regress z_1 on y_1 to get $\hat{\gamma}$, then regress z_2 on X_2 to get $\widehat{\beta\gamma}$, and take ratio of the two to estimate β.[3]

[3] Kaplan et al. (2020) use a imputation approach similar to the **RP** method. They regress county-level consumption spending on local house prices in the US. Since data on total nondurable consumption is not available at county level, they use county-level data on a subset of nondurable expenditures (grocery spending) from the Kilts-Nielsen Retail Scanner Dataset (KNRS) as their dependent variable, and then scale up their coefficients using household-level data on the relationship between grocery and total spending from the CE Survey. This is analogous to our set-up in a case where the regression of interest is $y_c = X_c\beta + \epsilon_c$

We first consider the **RP** procedure (with possibly multiple proxies). Regression of $\hat{y}_2^{RP}$ on X does not, in general, give a consistent estimate of β.

Proposition 1. *Given assumptions A1 - A5, Regression of $\hat{y}_2^{RP}$ on X yields inconsistent estimates of β unless (i) X is contained within the span of Z (ii) y is contained within the span of Z or (iii) $\Sigma_{Xy} = \Sigma_{XZ} = 0$.*

Proof.

$$
\begin{aligned}
plim\left(\hat{\beta}^{RP}\right) &= plim\left\{\left(\frac{X_2'X_2}{n_2}\right)^{-1}\frac{X_2'Z_2}{n_2}\left(\frac{Z_1'Z_1}{n_1}\right)^{-1}\frac{Z_1'y_1}{n_1}\right\} \\
&= plim\left\{\left(\frac{X_2'X_2}{n_2}\right)^{-1}\frac{X_2'Z_2}{n_2}\left(\frac{Z_1'Z_1}{n_1}\right)^{-1}\frac{Z_1'(Z_{1i}\zeta+\xi_{1i})}{n_1}\right\} \\
&= plim\left\{\left(\frac{X_2'X_2}{n_2}\right)^{-1}\frac{X_2'Z_2\zeta}{n_2}\right\} = plim\left\{\left(\frac{X_2'X_2}{n_2}\right)^{-1}\frac{X_2'(y_2-\xi_2)}{n_2}\right\} \\
&= \beta - plim\left\{\left(\frac{X_2'X_2}{n_2}\right)^{-1}\frac{X_2'\xi_2}{n_2}\right\} \\
&= \beta - plim\left\{\left(\frac{X_2'X_2}{n_2}\right)^{-1}\frac{(X_2'y_2 - X_2'Z_2(Z_2'Z_2)^{-1}Z_2'y_2)}{n_2}\right\}
\end{aligned}
$$

Given Assumption A5, the 2nd term will be zero if and only if (i) $\Sigma_{Zy} = \Sigma_{XZ} = 0$ or (ii) $\exists$ some finite, non-zero $L \times K$ matrix ϕ s.t $X = Z\phi$ or (iii) $\exists$ some finite, non-zero L -vector λ s.t $y = Z\lambda$. □

If (i) holds both parts of the bias term are zero but note that this would imply that $\beta = 0$ so that the estimator is consistent at only one point in the parameter space. If (ii) or (iii) hold, the two parts of the bias term are equal (and so cancel). Note though that (iii) implies that the first stage R^2 is one, and if (ii) holds there is no need for data combination. Thus the **RP** procedure only consistently estimates β in extreme cases.

(where the subscript c denotes county), the first stage regression is $y_h = \zeta z_h + \xi_h$ (where h denotes a household), and where the researcher takes the additional step of projecting z_h onto a set of county dummies to obtain z_c (and then proxying y_c using an estimate of ζz_c).

The source of the problem is that regression prediction results in a prediction, $\hat{y}_2^{RP}$, that differs from y_2 by a prediction error or Berkson measurement error, ξ_2 that is uncorrelated with z_2 but not uncorrelated with y_2 and, in general, not uncorrelated with X_2. As is well known, classical measurement in an independent variable causes bias in linear regression, but classical measurement errors in the dependent variable does not. This is because classical measurement errors in y are by assumption (and in contrast to Berkson errors) uncorrelated with y and X. It is also widely recognized that Berkson errors in an independent variable does not cause bias in a linear region (Berkson, 1950; Wansbeek and Meijer, 2000). What is less frequently recognized is that Berkson errors in a dependent variable do cause bias.

Figure 1 gives a geometric intuition for the problem. The solid lines represent the vectors y, z and X. The dashed lines illustrate orthogonal projections. The orthogonal projection of y onto X (which would be obtained by regression with complete data) is labelled $X\beta$. The **RP** procedure first projects the y onto z, giving $\hat{y} = z\gamma$, and then projects this vector onto X giving $X\beta^{RP}$. Note that $X\beta^{RP} \neq X\beta$.

[FIGURE 1 HERE]

The same problem arises with the **RP+** procedure. The true value of (unobserved) y_{2j} can be decomposed into its projection onto Z and an orthogonal error

$$y_{2j} = \hat{y}_j + \hat{\xi}_{2j} \tag{4}$$

Consider then drawing a random residual from the first stage regression (ω_{1j}) to create a stochastic imputation

$$\hat{\hat{y}}_{2j} = \hat{y}_j + \hat{\omega}_{1j} = y_{2j} - \hat{\xi}_{2j} + \hat{\xi}_{1j} \tag{5}$$

Then $\hat{\hat{y}}_2$ differs from y_2 by the error $\hat{\xi}_1 - \hat{\xi}_2$ which is by construction orthogonal to Z_1, but not y_2 or X_2.[4]

[4]Note that, because it is randomly drawn from a separate random sample, $\hat{\omega}_1$ is orthogonal to y_2. The problem with the composite error $\hat{\omega}_1 - \hat{\omega}_2$ lies in the prediction error $\hat{\omega}_2$.

The proof of Proposition 1 notes that, given Assumptions A1-A5, $E(X_2'\xi_2) = 0$ can hold only in extreme cases. The same is not true of the alternative projection error, u_2, associated with (2). The next proposition shows that the further assumption A6 ($E(X_2'u_2) = 0$) implies a bias in $\hat{\beta}^{RP}$ that takes a simple form.

Proposition 2. *Given assumptions A1 - A6,*

$$plim\left(\hat{\beta}^{RP}\right) = \beta\phi_{y,Z} \tag{6}$$

Proof. See Appendix. □

Thus, with A6, $\hat{\beta}^{RP}$ is attenuated and the degree of attenuation depends on the first stage population R^2 ($\phi_{y,Z}$).[5] It is important to note that we are working with de-meaned versions of the variables: More generally, $R^2_{y_1,Z_1}$ is the *centered* sample R^2, $\phi_{y,Z}$ is the *centered* population R^2 and the result holds without demeaning the data. In our motivating example, R^2s for food Engel curves are typically between 50 and 70%, implying inflation factors of between 1.4 and 2 (or attenuation of between 30 and 50%).

In A6, note that u_2 is not observed so this condition is not empirically verifiable. It states that X should not affect Z independently of Y.

This condition has analogies with the exclusion restriction imposed in instrumental variable procedures (discussed further below). However, in many settings it is likely that suitable proxies will be easier to identify than suitable instruments. This is because we have already assumed that X is uncorrelated with other determinants of y (ϵ) (A1), and by construction u is uncorrelated with y. That is, if X is indeed exogenous, the imputation error u would have to be correlated with X despite being uncorrelated with y.

Moreover, the reason it may often be possible to find proxies for which $E(X_{2i}'u_{2i}) = 0$, whereas $E(X_{2i}'xi_{2i}) = 0$ can only be satisfied in very special cases follows from the fact that

[5] A6 and A1 are in fact sufficient but not necessary for the proof of Proposition 2. If $E(X_{2j}'\epsilon_j) \neq 0$ and $E(X_{2j}'u_{2j}) \neq 0$ but $E(X_{2j}'(\epsilon_{2j}\gamma + u_{2j})) = 0$ we would still be able to obtain consistent estimates of $\beta\gamma$ by regressing Z on X. This is all that is needed for the proof of Proposition 2 below.

y and ξ are correlated by construction while y and u are by construction uncorrelated.

As the attenuation in the **RP** procedure is an estimable quantity, the bias can be corrected. One can rescale $\hat{y}^{RP}$ by the estimated first stage (centered) $R^2_{y_1,Z_1}$, or, equivalently, rescale $\hat{\beta}^{RP}$ by the estimated first stage (centered) $R^2_{y_1,Z_1}$. We refer to this procedure as "Re-scaled Regression Prediction" (**RRP**), with the rescaled impute of y_2 denoted $\hat{y}_2^{RRP}$ and the resulting estimate of β denoted $\hat{\beta}^{RRP}$. The consistency of $\hat{\beta}^{RRP}$, is a consequence of Proposition 2.

Corollary.

$$plim\left(\hat{\beta}^{RRP}\right) = plim\left(\frac{\hat{\beta}^{RP}}{R^2_{y_1,Z_1}}\right) = \beta.$$

Finally, consider the **BPP** and **AM** procedures, with resulting estimates $\hat{\beta}^{BPP}$ and $\hat{\beta}^{AM}$.

Proposition 3. *If and only if there is a single proxy z (a vector) $\hat{\beta}^{RRP}$, $\hat{\beta}^{BPP}$ and $\hat{\beta}^{AM}$ are numerically identical.*

Proof. See Appendix. □

Consistency of $\hat{\beta}^{AM}$ follows either directly from the Slutsky theorem or by numerical equivalence to $\hat{\beta}^{RRP}$ and $\hat{\beta}^{BPP}$.

An attraction of the **RRP** procedure over **BPP** and **AM** is that extends naturally to multiple proxies. Applied researchers may also be more comfortable with a two-step regression based approach.

It is useful also to think about other moments, as these imputation procedures have been used to study dispersion as well as regression coefficients. For example, Blundell et al. (2008), Attanasio and Pistaferri (2014) and Fisher et al. (2016) study consumption inequality. There are a number of reasons why one might wish to impute y from some other sample to calculate means, variances and covariances rather than calculating them directly in the initial sample. For example, one might want to calculate variances among subsets of the population that cannot be defined in the first sample but can be defined in the second. In addition, we might be interested in the growth of y among particular individuals along with variances and covariances for these growth rates from panel data, but may only directly observe y in cross-sectional data. For instance Blundell et al. (2008) calculate variances of growth rates in total consumption expenditures for households in the PSID, and covariances of these growth rates with the growth in incomes, where total consumption is imputed to the PSID from the cross-sectional CE.

We continue with the case of a single proxy to allow comparison of **BPP** to **RP** and **RRP**, and consider the case of a single x variable for ease of exposition (though the results extend naturally to a vector X). **AM** recovers β directly, and does not generate unit level estimates of y. The imputes $\hat{y}^{RP}$ and $\hat{y}^{RRP}$ are numerically different,

$$\hat{y}^{RP} = z_2(z_1'z_1)^{-1}z_1'y_1, \tag{7}$$

$$\hat{y}^{RRP} = z_2(z_1'z_1)^{-1}z_1'y_1/R^2_{y_1,z_1} \tag{8}$$

Algebra analogous to the proof of Proposition 3 shows that $\hat{y}^{RRP}$ and $\hat{y}^{BPP}$ are numerically identical for the case when all variables have been de-meaned. They will differ by an additive constant in the event a non-zero intercept shift is present in equation (2).

Denote sample moments based on $\hat{y}^{RP}$ by s^{RP}_{yy} and s^{RP}_{yx}; and analogously for $\hat{y}^{RRP}$ and $\hat{y}^{BPP}$,

$$s^{RP}_{yy} = \frac{1}{n_2}\hat{y}^{RP\prime}\hat{y}^{RP} = \frac{1}{n_2}z_1'y_1(z_1'z_1)^{-1}z_2'z_2(z_1'z_1)^{-1}z_1'y_1, \tag{9}$$

$$plim\left(s^{RP}_{yy}\right) = \frac{(\gamma\sigma_{yy})^2}{\gamma\sigma_{yy}+\sigma_{uu}} = \sigma_{yy}\times\phi_{y,z}, \tag{10}$$

where again $\phi_{y,Z}$ is the population R^2 from the first stage regression. The sample variance of $\hat{y}^{RP}$ underestimates the population variance of y. A similar calculation gives:

$$plim\left(s^{RP}_{yx}\right) = \sigma_{yx}\times\phi_{y,z}. \tag{11}$$

Note that with a scalar x the OLS estimate of β is just s_{yx}^{RP}/s_{yy}^{RP} and this gives an additional intuition for the inconsistency of $\hat{\beta}^{RP}$ as an estimator of β: s_{yx}^{RP} is not a consistent estimator of σ_{yx}. Moreover, adding a residual to $\hat{y}^{RP}$, (the **RP+** procedure) does not correct this.

For the rescaled impute $\hat{y}^{RRP}$, it follows from Equations (10) and (11) and the definition of $\hat{y}^{RRP}$ that

$$plim\left(s_{yy}^{RRP}\right) = \sigma_{yy}/\phi_{y,z} \tag{12}$$

and

$$plim\left(s_{yx}^{RRP}\right) = \sigma_{yx}. \tag{13}$$

Continuing with the bivariate regression intuition, the **RRP** procedure is consistent for β because it is consistent for σ_{yx}.

Finally, simple algebra establishes that

$$s_{yy}^{RRP} = s_{yy}^{BPP} \tag{14}$$

and

$$s_{yx}^{RRP} = s_{yx}^{BPP}, \tag{15}$$

This follows from the numerical equivalence of the de-meaned values of $\hat{y}^{RRP}$ and $\hat{y}^{BPP}$. Thus $plim\ s_{yy}^{BPP} = plim\ s_{yy}^{RRP} > \sigma_{yy} > plim\ s_{yy}^{RP}$. Turning again to our motivation consumption example, Attanasio and Pistaferri (2014) show that trends in s_{yy}^{BPP} and s_{yy} (where y is observed) are similar, but that there is a level difference. The similarity in trends suggests that the first stage $R^2_{y_1,Z_1}$ is roughly constant across years in their data. We confirm this in our empirical example below.

For completeness we can also consider means. Had we not de-meaned the data, then it is straightforward to show that the sample of $\hat{y}^{RP}$ gives an consistent (and unbiased) estimate of the population mean of y. However, if the **RPP** procedure is implemented by rescaling $\hat{y}^{RP}$ (rather than rescaling β^{RP}), it then immediately follows that the mean of this rescaled prediction of y is not a consistent estimator of the mean of y. One implication is that a Statistical Agency aiming to add an imputed $\hat{y}$ to a data release could not add a single variable that would be appropriate both for use as a regressand and for estimating quantities that depend on the first moment of y (poverty rates, for example).

Table 1: Summary of Imputation Methods (Consistency)

	μ_y	σ_{yy}	β
Regression Prediction (**RP**)	✓	×	×
Regression Prediction + $\hat{e}$ (**RP+**)	✓	✓	×
Rescaled Regression Prediction (**RRP**)	×	×	✓
Blundell et al., 2004; 2008 (**BPP**)	✓	×	✓
Arellano and Meghir, 1992 (**AM**)	-	-	✓

Notes: a ✓indicates that the procedure given by the row leads to a consistent estimate of the population parameter given by the column (μ_y, σ_{yy} or β). A × indicates that the procedure leads to an inconsistent estimate of the relevant parameter, and a dash indicates that the procedure does not provide an estimate via the analogous sample moment. The table assumes that the **RPP** procedure is implemented by rescaling $\hat{y}^{RP}$ (rather than rescaling β^{RP}).

Table 1 summarizes these consistency results. For the case of a single proxy any of the **RRP**, **BPP** and **AM** procedures give a consistent estimate of a regression coefficient β, but for estimating unconditional moments, imputations from **RP**, **BPP** and especially **RP+** are preferable.

2.2 *Matching and Hot-deck Imputation*

However, the problem we highlight with regression prediction extends to several related imputation procedures. In particular, Lillard et al. (1986) and David et al. (1986) note that commonly employed hot-deck imputation procedures can be interpreted as regression prediction plus an added residual. Such procedures draw a matched observation, y_{1i}, of the missing variable y_{2j}, from a cell in the donor data set. Cells are defined by categorical variables derived from Z. This replacement of the missing y_{2j} with y_{1i} from an donor observation matched on Z can be viewed as a prediction using the coefficients from a saturated first stage regression on those categorical indicators for Z, plus a residual from the first stage regression, and so is an example of the **RP+** procedure, and our results apply.

2.3 Practicalities

The analysis above is trivially extended to handle additional covariates. If additional covariates W are added to both the first stage regression and regression of interest, then the results above hold by straightforward application of the Frisch-Waugh-Lovell theorem (y, X and Z can be "residualized" and then the results apply directly to the residualized variables). There are two points to note: First, the additional covariates W must be added to both the first stage regression and regression of interest. Second, if covariates are added, then the relevant first stage R^2 is the *partial* R^2 associated with Z.

Often a researcher will want to estimate a panel version of Equation (1): $\Delta y = \Delta X\beta + \Delta\epsilon$ where $\Delta y = y^1 - y^0$ and superscripts denote time (and similarly for X and ϵ). As before β is the main object of interest and could be estimated consistently by OLS if we had complete data (that is, $E(\Delta X_i \times \Delta\epsilon_i) = 0$). Suppose we have no data from which to compute $\frac{1}{n}\sum \Delta y_i \times \Delta X_i$, but do have have some data on $\{y_1^1, Z_1\}$, $\{y_2^0, Z_2\}$, and a third sample with $\{\Delta X_3, Z_3\}$. In our running example, one often wants to estimate the effect of income or wealth changes on consumption and the available data would be a repeated cross-sectional household budget survey combined with a panel survey on income or wealth. Then y_3 can be imputed year by year. It is straightforward extension of the results above to show that $\hat{\beta}^{RRP}$ is consistent in this case, and with one proxy $\hat{\beta}^{BPP}$ remains numerically identical to $\hat{\beta}^{RRP}$.

2.4 Related Literature

In this paper we study the use of proxies to predict a dependent variable.[6] Regression prediction of a dependent variable induces a prediction or Berkson measurement error. Berkson measurement errors in a dependent variable cause bias in a linear regression, and this seems to be much less noted than innocuous cases of Berkson measurement error in an independent

[6] Wooldridge (2002) contains an excellent overview of the use of proxies for independent variables and Lubotsky and Wittenberg (2006) and Bollinger and Minier (2015) are recent papers on the optimal use of multiple proxies for an independent variable.

variable, or classical measurement error in a dependent variable.[7] Two exceptions are Hyslop and Imbens (2001) and Hoderlein and Winter (2010). Hyslop and Imbens (2001) show attenuation bias in a regression of $\hat{y}$ on X where $\hat{y}$ is an optimal linear prediction generated by a survey respondent (not the econometrician). Relative to the imputation problem we study, key differences include the fact that it is the survey respondent doing the prediction and the assumption that the respondent's information set includes Z, β and $E(X)$. They also assume (in our notation) that $Z = y + u$; $(\gamma = 1)$. Hoderlein and Winter (2010) study a similar problem, but in a nonparametric setting. Again, in their model it is the survey respondent, rather than the econometrician, doing the predicting.[8]

Dumont et al. (2005) study corrected standard errors in a regression with a "generated regressand". Their work is motivated by the two-stage procedure for mandated-wage regression proposed by Feenstra and Hanson (1999). In this paper, domestic prices are first regressed on some structural determinants (trade and technology variables). The estimated contributions of these variables to price changes are then in turn regressed on factor shares to identify the changes in factor prices 'mandated' by changes in product prices.

In this context the first stage is

$$z_i = Y_i \gamma + u_i \tag{16}$$

and the second stage is not (1) but rather

$$Y_i^k \gamma^k = X_i \beta^k + \epsilon_i^k \tag{17}$$

where the k superscript denotes the kth element of a vector. Here $Y_i^k \gamma^k$ is not observed and so is replaced by the first stage estimate $Y_i^k \hat{\gamma}^k$. Of course the vector $\hat{\gamma}$ differs from γ by an estimation error $(Y'Y)^{-1}Y'\hat{u}$, but, given the set-up, the stochastic element $\hat{u}$ is orthogonal

[7] Berkson measurement error in an independent variable is also a problem in nonlinear models. See for example Blundell et al. (2019).

[8] They illustrate their results using self-reported data on consumption expenditure.

to Y, and so also X, and thus causes problems for inference but not bias. Although the motivation and second-stage regressand are different, this procedure essentially regresses z on Y, analogously to the **BPP** procedure, rather than y on Z as in the **RP** procedure, so the Berkson measurement error problem does not arise.

Finally, it is also useful to contrast the imputation procedures studied in this paper with the 2-sample IV (2SIV) and 2-sample 2SLS approaches (Klevmarken (1982), Angrist and Krueger (1992), Inoue and Solon (2010) and Pacini and Windmeijer (2016)) applied to the combination of CE consumption data and PSID income data by Lusardi (1996).

Starting from (1), suppose we consider the linear projection of X_2 on Z_2:

$$X_{2j} = Z_{2j}\theta + \nu_{2j} \tag{18}$$

Note that $E(Z'_{2j}\nu_{2j}) = 0$ by definition. Then replace assumptions with A5 and A6 with parallel assumptions:

A11 $E\left(Z'_{1i}X_{1i}\right)$ and $E\left(Z'_{2j}X_{2j}\right)$ both have rank K.

A12 $E\left(Z'_{1i}\epsilon_{1i}\right) = 0$.

Then the 2-sample-2SLS estimator:

$$\hat{\beta}^{2S2SLS} = (\hat{X}_1{}'\hat{X}_1)^{-1}\hat{X}_1{}'y_1 \tag{19}$$

where $\hat{X}_1 = Z_1(Z'_2Z_2)^{-1}Z'_2X_2$ is consistent for β.

This approach is typically taken where Z is a grouping variable or variables (e.g., birth cohort, occupation, birth cohort $\times$ education). The key assumption is that Z is linearly independent of y given X, which is the polar opposite to the assumption that Z is a proxy or proxies: a useful proxy must have information about y over and above the information in X). To see this, considering the combination of equations (1) and (2):

$$Z_{1i} = X_{1i}\beta\gamma + \epsilon_{1i}\gamma + u_{1i} \tag{20}$$

Given A5, then A12 can only hold in a knife-edge case.[9]

With 2-sample 2SLS, we use Z to impute X (and as the resulting prediction or Berkson error is in an independent variable, this two-stage procedure does not cause inconsistency).[10] An additional virtue of this procedure is that inherent measurement error in y poses no additional difficulties as long as that measurement error is uncorrelated with Z. However, it is important to note that, as the key assumption that supports the use of Z as an instrument in general contradicts the assumption required to use Z as a proxy (and vice-versa), a variable may be a plausible instrument or a plausible proxy, or neither; but not both.[11]

3 Inference and Precision

3.1 Asymptotic Standard Errors - One Proxy, Homoscedastic Case

If we strengthened the assumptions listed in Section 2 to include homoscedasticity ($E\left(\epsilon_i^2|X_i\right) = \psi_{\epsilon_i\epsilon_i} > 0$) and conditional independence of the error term ($E(\epsilon_i|X_i) = 0$), then direct estimation of (1) on complete data would result in an asymptotic variance for $\hat{\beta}$ of $\left(\Sigma_{XX}\right)^{-1}\psi_{\epsilon\epsilon}$. When we impute $\hat{y}$ from one data set to another, there are two losses of precision resulting from (i) imputation and (ii) the combination of two different samples of the underlying population. Moreover, applying the usual OLS standard error formula the regression of $\hat{y}$ on X results in standard errors that are too small. We use the one-proxy (and single X variable), and homoscedastic case to illustrate these points, and then give a correct formula for the

[9] Given equations (1) and (2), $E(Z_{1i}'\epsilon_{1i}) = E\left((X_{1i}\beta\gamma + \gamma\epsilon_{1i} + u_{1i})'\epsilon_{1i}\right) = 0$. Given A1 this implies $E\left((\gamma\epsilon_{1i} + u_{1i})'\epsilon_{1i}\right) = 0$ which can only be true if $E\left((\gamma\epsilon_{1i}'\epsilon_{1i}\right) = -E(u_{1i}'\epsilon_{1i})$ (note also that A5 implies that $\gamma \neq 0$).

[10] Inoue and Solon (2010) show that 2SIV is not in general efficient because it does not take account of fact that Z_1 and Z_2 will be different in finite samples. They suggest the 2-Sample Two-Stage Least Squares procedure is therefore preferred.

[11] A similar point is made with respect to proxy and IV approaches to an "omitted variable" (a missing independent variable) in Wooldridge (2002).

asymptotic standard errors with possibly multiple proxies and heteroscedastic errors.

With a single proxy, $\hat{\beta}^{AM}$, $\hat{\beta}^{RRP}$ and $\hat{\beta}^{BPP}$ are numerically identical, so we derive the asymptotic variance from the **AM** approach. The first stage (2) and reduced form (20) give two moments

$$E\left(y_{1i}'(z_{1i} - \gamma y_{1i})\right) = E\left(y_{1i}' u_{1i}\right) = 0,$$

$$E\left(x_{2j}'(z_{2j} - \gamma x_{2j}\beta)\right) = E\left(x_{2j}'(\gamma \epsilon_{2j} + u_{2j})\right) = 0$$

which identify the parameters γ and β.

It is informative to first consider implementing $\hat{\beta}^{AM}$ (or equivalently $\hat{\beta}^{BPP}$ or $\hat{\beta}^{RRP}$) on a single sample, containing all of y, z, x (of course, a researcher would have no reason to do this, but it delivers a useful intuition). In this one-sample case, given the further assumptions $E\left(u_i^2|y_i\right) = \psi_{uu}$ and $E(u_i|y_i) = 0$, the asymptotic variance-covariance matrix of the moments is

$$F = \begin{bmatrix} \psi_{uu}\sigma_{yy} & \psi_{uu}\sigma_{XX}\beta \\ \psi_{uu}\sigma_{XX}\beta & \left(\gamma^2\psi_{\epsilon\epsilon} + \psi_{uu}\right)\sigma_{XX} \end{bmatrix} \tag{21}$$

where the off-diagonal terms are not zero because the moments come from the same random sample. The asymptotic variance covariance matrix of (β, γ) is $\left(G'F^{-1}G\right)^{-1}$ where G is the gradient of the moments with respect the parameters. The asymptotic variance of $\hat{\gamma}$ is of course $\sigma_{yy}^{-1}\psi_{uu}$. The asymptotic variance of $\hat{\beta}$ is

$$Asymp\ Var(\hat{\beta}) = \frac{\left(\sigma_{XX}\right)^{-1}\psi_{\epsilon\epsilon}}{\phi_{y,Z}}. \tag{22}$$

Thus the loss of asymptotic precision due to imputation (relative to the direct estimation of (1)), is inversely related to the first stage population R^2 $(\phi_{y,Z})$. Note the similarity of this precision loss to the precision loss in the case of linear IV estimation (relative to OLS), which is related to a first stage R^2 in the same way(Shea, 1997).

Turning now to the realistic two-sample case, the asymptotic variance-covariance matrix of the moments becomes

$$F = \begin{bmatrix} \alpha\psi_{uu}\sigma_{yy} & 0 \\ 0 & \left(\gamma^2\psi_{\epsilon\epsilon} + \psi_{uu}\right)\sigma_{XX} \end{bmatrix}$$

where note that the off-diagonal terms are now zero because the moments come from independent random samples. The asymptotic variance covariance matrix of (β, γ) is again $(G'F^{-1}G)^{-1}$ where G is the gradient of the moments with respect the parameters. The asymptotic variance of $\hat{\gamma}$ is the same as before (though now multiplied by the term α). $\alpha\left(\sigma_{yy}\right)^{-1}\psi_{uu}$. The asymptotic variance of $\hat{\beta}$ is

$$\begin{aligned} Asymp\ Var(\hat{\beta}) &= \left(\sigma_{XX}\right)^{-1}\left(\psi_{\epsilon\epsilon} + \gamma^{-2}\psi_{uu}\right) + \alpha\sigma_{yy}^{-1}\beta^2\gamma^{-2}\psi_{uu} \\ &= \left(\sigma_{XX}\right)^{-1}\psi_{\epsilon\epsilon} + \gamma^{-2}\left(\sigma_{XX}\right)^{-1}\psi_{uu} + \alpha\beta^2\gamma^{-2}\sigma_{yy}^{-1}\psi_{uu}. \end{aligned}$$

This can be written as

$$Asymp\ Var(\hat{\beta}) = \frac{\left(\sigma_{XX}\right)^{-1}\psi_{\epsilon\epsilon}}{\phi_{y,Z}} + (1+\alpha)\beta^2\left(\frac{1-\phi_{y,Z}}{\phi_{y,Z}}\right) \tag{23}$$

The second term inside the brackets represents the loss of asymptotic precision, due to the use of two different samples. Precision is greater in (22) because the covariances between moments in equation (21) have a stabilising influence on the estimates $\hat{\beta}$. These covariance terms are zero in the two sample case.

Finally, the usual OLS standard errors from a regression of an imputed dependent variable (derived from the **RRP** or **BPP** procedures) are incorrect, but can easily be corrected. The

OLS standard errors (as produced by standard software packages) are

$$\hat{V}^{OLS}(\hat{\beta}^{BPP}) = (x_2 x_2)^{-1}\left(\hat{y}_2 - x_2\hat{\beta}\right)'\left(\hat{y}_2 - x_2\hat{\beta}\right) = (x_2 x_2)^{-1}\left(\hat{y}_2'\hat{y}_2 - \hat{y}_2' x_2 (x_2' x_2) x_2' \hat{y}_2\right)$$
$$= (x_2' x_2)^{-1}\left[y_1' y_1 (z_1' y_1)^{-1} z_2' z_2 (z_1' y_1)^{-1} y_1' y_1 - y_1' y_1 (z_1' y_1)^{-1} z_2' x_2 (x_2' x_2) x_2' z_2 (z_1' y_1)^{-1} y_1' y_1\right].$$

With some algebra, it is straightforward to show that

$$plim\left(\hat{V}^{OLS}(\hat{\beta})\right) = \frac{(\sigma_{XX})^{-1}\psi_{\epsilon\epsilon}}{\phi_{y,Z}} + \beta^2\left(\frac{1-\phi_{y,Z}}{\phi_{y,Z}}\right)$$
$$= Asym\, Var(\hat{\beta}) - \alpha\beta^2\left(\frac{1-\phi_{y,Z}}{\phi_{y,Z}}\right). \tag{24}$$

So, the usual OLS standard errors are too small, by $\alpha\beta^2\left(\frac{1-\phi_{y,Z}}{\phi_{y,Z}}\right)$. Given assumption A9, the OLS standard errors can be corrected using available consistent estimates of α, β and $\phi_{y,Z}$, $\frac{n_2}{n_1}$, $\hat{\beta}$ and $R^2_{y_1,Z_1}$.

3.2 Asymptotic Standard Errors - General Case

If there is more than one proxy $\hat{\beta}^{RRP} \neq \hat{\beta}^{AM}$. Here we derive the asymptotic variance of $\hat{\beta}^{RRP}$ and relate it to uncorrected, 'naive' estimates of the standard errors one would obtain from the second stage **RRP** regression (of $\hat{y}$ on X). Our formula allows for possibly heteroscedastic errors and can, for example, straightforwardly be extended to provide cluster-robust standard errors.[12]

Proposition 4. *Given assumptions A1 - A8, $\hat{\beta}^{RRP}$ has asymptotic variance*

$$Asymp\, Var(\hat{\beta}^{RRP}) = \Sigma_{XX}^{-1}\left[\Omega_{X\delta} + \alpha\frac{\Sigma_{XZ}}{\phi_{y,Z}}\Sigma_{ZZ}^{-1}\Omega_{Z\xi}\Sigma_{ZZ}^{-1}\frac{\Sigma_{ZX}}{\phi_{y,Z}}\right]\Sigma_{XX}^{-1}$$

Proof. See Appendix. □

[12] Our approach closely follows that of Pacini and Windmeijer (2016) who provide robust standard errors for 2 sample 2SLS.

Given the further assumption A9, this formula can be estimated using sample analogs of Σ_{XX}, $\phi_{y,z}$, Σ_{XZ}, α, $\Omega_{Z\xi}$ and $\Omega_{X\delta}$. It can also be written as

$$Asymp\,Var(\hat{\beta}^{RRP}) = V_{OLS}(\hat{\beta}) + \alpha\Sigma_{XX}^{-1}\frac{\Sigma_{XZ}}{\phi_{y,Z}}V_{OLS}(\hat{\gamma})\frac{\Sigma_{XZ}}{\phi_{y,Z}}\Sigma_{XX}^{-1}$$

which can be used to adjust robust estimates for the variances of coefficients in the first and second stage regressions that are provided by Stata and other software packages. A Stata package that implements the **RRP** procedure and provides the correct standard errors is available from the authors at https://github.com/spoupakis/rrp. The results in Proposition 4 can straightforwardly be extended to situations where we impute the dependent variable into panel data and where we use instrumental variables for X.

4 Monte Carlo Experiments

To demonstrate the points made above in finite samples we first present a small Monte Carlo study. The baseline data generating process is as follows. There is a single regressor $x \sim N(0, 2)$. The dependent variable of interest is $y = 1 + \beta \times x + \epsilon$ with $\sigma_{\epsilon\epsilon} = 1$. The parameter of interest is $\beta = 1$. We cannot regress y_2 on x_2 directly, because information on these quantities is collected in separate surveys (We only observe y_1 and x_2, so that we cannot calculate the empirical covariance, $\frac{1}{n_1}\sum y_1 \times x_1$ or $\frac{1}{n_2}\sum y_2 \times x_2$). However, both surveys contain a potential proxies for y. We begin with the case of a single proxy, z, which we generate as follows,

$$z_1 = 1 + 0.5 \times y_1 + u_1 \qquad \text{and} \qquad z_2 = 1 + 0.5 \times y_2 + u_2$$

with $u_1, u_2 \sim N(0, \sigma_{uu})$. We consider the case where $\sigma_{uu} = 1$ and a first stage R^2 of 0.56.

We simulate this population multiple times, each time drawing two data sets (y_1, z_1) and (x_2, z_2), and implementing the **RP**, **RP+**, **RRP**, **BPP** and **AM** procedures. Sample size is 500 for both samples (so that $\frac{n_2}{n_1} = 1$) and we perform 10,000 replications.

The results are presented in Table 2. The first column shows the case of complete data (OLS on a data set with both y and x); the remaining columns display results for different imputation procedures. The first row gives the mean over 10,000 replications of the estimate of β. With complete data OLS is unbiased for β. The **RP** and **RP+** procedures are systematically biased and the mean attenuation factor is equal to the population first stage R^2 of 0.56. The **RRP**, **BPP** and **AM** procedures (which are numerically identical here) are approximately unbiased for β.

Table 2: Monte Carlo Experiment: One proxy

	FULL	RP	RP+	RRP	BPP	AM
Mean of $\hat{\beta}$	1.000	0.556	0.555	1.002	1.002	1.002
Std. Dev. of $\hat{\beta}$	0.022	0.036	0.049	0.065	0.065	0.065
Mean of SE($\hat{\beta}$)	0.022	0.028	0.043	0.050	0.050	
Mean of Corrected SE($\hat{\beta}$)				0.064		
Mean of $\frac{1}{n}\sum \hat{y}_i$	1.000	1.000	0.999	1.805	1.000	
Mean of $\frac{1}{n-1}\sum(\hat{y}_i - \bar{\hat{y}})^2$	4.999	2.784	5.000	9.048	9.048	

Note: Results based on 10,000 replications, $n_1 = n_2 = 500$, $\beta = 1$, $E(y_i) = 1$, $\sigma_{yy} = 5$.

Rows two through four show the standard deviation of $\hat{\beta}$ across replications along with the mean of the OLS standard error across replications and (in the case of the **RRP** procedure) the mean of the corrected standard error. When regressing an imputed y on x, the usual OLS variance formula leads to a standard error that is too small, but the corrected standard error correctly captures the variation of $\hat{\beta}$ in repeated sampling.

Finally, rows five and six consider estimating the first two unconditional moments of y. The mean of imputed $\hat{y}$ from the **RP** procedure is unbiased for the population mean of y but of course the variance of $\hat{y}$ from this procedure is not unbiased for the population variance. Adding a stochastic residual from the first stage (the **RP+** procedure) corrects this. Because the **RRP** and **BPP** procedures amount to an upward rescaling of $\hat{y}$, the variances of the resulting imputations are quite biased estimates of the population variance of y. However, in the case of **BPP**, there is no bias in the mean.

Table 3 illustrates the case when there are two proxies available, A and B. We generate these as:

$$z_{a,1} = 1 + \gamma_A \times y_1 + u_{A,1} \qquad \text{and} \qquad z_{a,2} = 1 + \gamma_A \times y_2 + u_{A,2}$$
$$z_{b,1} = 1 + \gamma_B \times y_1 + u_{B,1} \qquad \qquad z_{b,2} = 1 + \gamma_B \times y_2 + u_{B,2}$$

where $\gamma_A = 0.4$, $\gamma_B = 0.3$ and $u_A, u_B \sim MVN(0, \Sigma_{uu})$ with covariance $\Sigma_{uu} = \begin{pmatrix} 1 & -0.5 \\ -0.5 & 1 \end{pmatrix}$.

Table 3: Monte Carlo Experiment: Two proxies

	FULL	RP	RP+	RRP	AM
Mean of $\hat{\beta}$	1.000	0.712	0.712	1.000	1.001
Std. Dev. of $\hat{\beta}$	0.022	0.034	0.044	0.048	0.048
Mean of SE($\hat{\beta}$)	0.022	0.028	0.039	0.039	
Mean of Corrected SE($\hat{\beta}$)				0.048	

Note: Based on 10,000 replications, $n_1 = n_2 = 500$, $\beta = 1$, $E(y_i) = 1$, $\sigma_{yy} = 5$.

The key points are that the **RRP** and **AM** procedures remain approximately unbiased for β, and that the additional proxy improves precision.

The simulation study of two proxies is repeated for different values for the variance of the error term of the proxy B equation, with $\Sigma_{uu}[2, 2]$ equal to 1, 2 and 4. Table 4 reports the standard deviation of $\hat{\beta}$. This illustrates that in finite samples, the **RRP** procedure can be more efficient than **AM**.

Table 4: Monte Carlo Experiment: Two proxies, varying $\sigma_{u,B}$

Mean of $\hat{\beta}$	FULL	RP	RP+	RRP	AM
For $\sigma_{u,A} = 1$ and $\sigma_{u,B} = 1$	0.022	0.034	0.044	0.048	0.048
For $\sigma_{u,A} = 1$ and $\sigma_{u,B} = 2$	0.022	0.036	0.048	0.060	0.066
For $\sigma_{u,A} = 1$ and $\sigma_{u,B} = 4$	0.022	0.036	0.050	0.067	0.089

Note: Based on 10,000 replications, $n_1 = n_2 = 500$, $\beta = 1$, $E(y_i) = 1$, $\sigma_{yy} = 5$.

Finally, Table 5 considers a hot-deck imputation. Here z_1 and z_2 are partitioned into bins, and the missing y_2 is imputed by drawing a y_1 from the relevant z-bin. As noted above, this is formally equivalent to **RP+**. The results are in column 2 (titled "Hot-deck"). As expected, estimates of β are significantly attenuated, with bias equal to the first stage

R^2. Estimates of the unconditional mean and variance are unbiased. In column 3 we rescale the donated y_1 by the first stage R^2 (from a regression on bin indicator variables), and refer to this a "rescaled hot-deck" (RHD). As expected, the result is very limited empirical bias in estimates of β but significant bias in estimates of the unconditional mean and variance.

Table 5: Monte Carlo Experiment: Hot-deck Imputation

	FULL	Hot-deck (HD)	RHD
Mean of $\hat{\beta}$	1.000	0.532	0.986
Std. Dev. of $\hat{\beta}$	0.022	0.049	0.088
Mean of $\frac{1}{n}\sum \hat{y}_i$	1.000	1.001	1.858
Mean of $\frac{1}{n-1}\sum(\hat{y}_i - \bar{\hat{y}})^2$	4.999	4.990	17.218

Note: Based on 10,000 replications, $n_1 = n_2 = 500$, $\beta = 1$, $E(y) = 1$, $V(y) = 5$. The imputation is based on 1 proxy, partitioned into 10 bins.

5 Empirical Illustrations

In this section we illustrate the our results with two empirical examples using the PSID (Panel Study of Income Dynamics, 2019) and the CE Interview Survey.

5.1 Housing Wealth Effects

We begin with an exercise similar to that of Skinner (1989) (making use of the imputation procedure set out in Skinner (1987)). This is to estimate the elasticity of consumption spending with respect to changes in housing wealth by regressing nondurable consumption spending on demographics, lags and leads of total family income and house values. We do this using the 2005-2013 waves of the PSID when a more-or-less complete measure of nondurable expenditures is available. Following the approach taken by Skinner (1989) for an earlier period when spending data was only available for a subset of goods, we also impute nondurable consumption spending from the CE Survey into the PSID.[13] This allows us to

[13]Prior to 1999 the PSID only included food and utility spending, which was then broadened to include health expenditures, gasoline, car maintenance, transportation, child care and education. In 2005, additional categories for clothing and entertainment were added.

compare results from different imputation procedures with the complete data case (using the PSID's own consumption measures). In this respect our exercise is similar to that used in Attanasio and Pistaferri (2014) who assess the accuracy of the imputed consumption measures they use in the early years of the PSID with those available in the PSID in later years.

Our measure of nondurable consumption is the sum of spending on food at home, food away from home, utilities (including gas and electricity), gasoline, car insurance, car repairs, clothing, vacations and entertainment. For proxies we use the log sum of total food spending (whether at home or away from home), log utility spending and the number of cars owned by the household (up to a maximum of two). Our demographics controls are the size of the household, age, age squared, the log earnings of the household head (set to zero for those with zero earnings), and a dummy for having zero earnings. We annualise consumption measures and then take logs in both surveys.

Our sample selection choices in the PSID are chosen to mirror those used in Skinner (1989). In particular we take a sample of homeowners, who are observed in all waves from 2005-2013, who do not move, are not observed with zero incomes and who are not observed renting over the sample period. To prevent our results being driven by extreme values, we also exclude those with incomes or house values in the top and bottom 1% of the PSID sample.

In the CE Interview Survey we take a sample of homeowners. The CE Interview Survey aims to interview households over a four quarters, asking retrospective consumption questions over the previous three months in each interview. We take only those individuals who were observed in all four interviews, and whose final interview was held a year coinciding with the biennial PSID survey waves from 2005-2013. We then average spending over each of the previous four quarters they were observed and keep only one observation per household. By averaging over multiple waves we reduce measure error in consumption and get consumption values which are more in the spirit of the questions households are asked in the PSID (households in the PSID are asked about their spending over the previous year, or 'usual' spending in an average week or month). We run our imputation regressions separately in each year, which would for example allow for the fact that changes in relative prices might change the relationship between food and total spending from one period to the next.[14]

Table 6 shows the results from our first stage imputation regressions. We note that the relationships between the proxy variables and total nondurable consumption and the fit of the imputation regressions remain very stable across the different survey years. Column (6) shows results pooling across all years (2005, 2007, 2009, 2011 and 2013).

Table 6: Imputing nondurable consumption spending using CES

	(1) 2005	(2) 2007	(3) 2009	(4) 2011	(5) 2013	(6) All years
log Food	0.562***	0.545***	0.541***	0.549***	0.555***	0.549***
	(0.012)	(0.008)	(0.008)	(0.008)	(0.008)	(0.004)
log Utilities	0.377***	0.382***	0.410***	0.384***	0.389***	0.389***
	(0.015)	(0.010)	(0.011)	(0.010)	(0.011)	(0.005)
Cars	0.035***	0.036***	0.027***	0.042***	0.031***	0.034***
	(0.005)	(0.004)	(0.003)	(0.004)	(0.004)	(0.002)
Partial R^2	0.728	0.755	0.751	0.761	0.753	0.751
N	1,590	2,896	2,759	2,668	2,470	12,383

Note: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Standard errors in parentheses. Additional controls for age, age squared, family size, log of head's earnings (set to zero if earnings are zero), a dummy for head's earnings being greater than zero, and (in the pooled regression) year dummies. "Cars" refers to the number of cars capped at a maximum of two. The partial R^2 reported here is obtained by regressing our dependent variables on our proxies after partialling out the effects of other covariates in an inital regression.

Table 7 shows the results from regressions of consumption spending on income variables and house values in the PSID. The first column shows results using the consumption measure

[14]Our approach differs from the approach used in Skinner (1989) in two key respects. First, Skinner (1989) imputes the absolute level of consumption using the absolute levels of food and utilty spending before taking logs of the imputed values in the PSID, while we use the log of nondurable consumption, food and utility spending throughout. To avoid the need to throw out observations who do not report spending on food away from home, we combine food at home and food away from into a food spending variable. Second, we use a measure of nondurable consumption that is narrower than that used in Skinner (who takes the sum of all spending, less mortgage interest, furniture and automobiles and including imputed spending on owner-occupied housing). This allows us to compare the results we obtain without imputed spending measures with those in the PSID.

available in the PSID. This is the complete data case. The second column shows results using the **RP** procedure employed by Skinner, and the third column shows results using the **RRP** approach we set out above.

Table 7: Empirical Example: Log nondurable consumption on house values

	(1) PSID	(2) CE (RP)	(3) CE (RRP)
log Income_{t-3}	0.047**	0.036*	0.048*
	(0.017)	(0.016)	(0.021)
log Income_{t-2}	0.064***	0.043**	0.057**
	(0.016)	(0.014)	(0.018)
log Income_{t-1}	0.040**	0.024	0.032
	(0.015)	(0.014)	(0.019)
log Income_{t}	0.109***	0.080***	0.107***
	(0.022)	(0.020)	(0.026)
log Income_{t+1}	0.105***	0.074***	0.099***
	(0.016)	(0.015)	(0.020)
log House value	0.114***	0.083***	0.111***
	(0.016)	(0.015)	(0.020)
N	5,406	5,406	5,406

Note: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Standard errors in parentheses. Standard errors are clustered at the individual level. Additional controls for age, age squared, family size, log of head's earnings (set to zero if earnings are zero), a dummy for head's earnings being greater than zero, and year dummies. Column (1) shows results using the measures of nondurable consumption contained in the PSID as the dependent variable. Column (2) uses the unscaled regression prediction (RP) procedure to impute consumption spending into the PSID from the CE survey. Column (3) shows results when nondurable consumption is imputed to the PSID from the CE using the re-scaled regression prediction (RRP) procedure.

The complete data results from the PSID suggests that each 10% increase in house values is associated with a 1.14% increase in consumption spending. When we impute consumption using the **RP** procedure, we underestimate the effects of housing wealth on consumption (with the estimated effect falling to 0.83%). Using the **RRP** procedure, we obtain a value of 1.11% which is very similar to that obtained using the complete data in the PSID. This illustrates the theoretical results of Section 2. There we argued that the exclusion restriction

A6 was very weak. These results suggest that it holds in these data, and moreover, that our demographic covariates adequately control for any sample differences between the PSID and the CE Interview Survey.

5.2 Consumption Inequality

As a second exercise we examine the evolution of consumption inequality using actual and imputed nondurable consumption measures. This is in the spirit of the longer-run analysis of consumption and inequality carried out in Attanasio and Pistaferri (2014).

To do this we impute consumption measures for all households in the PSID (this time including non-homeowners) and plot the standard deviation over time for imputed consumption from the **RP** procedure and from the **RRP** procedure. We then compare this with the standard deviation of nondurable consumption spending as measured in the PSID. To prevent this measure being unduly influenced by extreme values, we also trim the top and bottom 1% of consumption spending in the PSID. The results are shown in Figure 2.

[FIGURE 2 HERE]

The standard deviation of consumption spending shows similar trends in all three series. The fact that imputed and observed consumption move in similar ways over time is consistent with the findings of Attanasio and Pistaferri (2014) who use the latter as a check for the former in their analysis. The link between movements in the **RP** and **RRP** imputed measures reflects the stability of the first stage R^2 over time.

We also note that the **RP** measure tends to understate the *level* of consumption inequality, while the re-scaled (**RRP**) procedure tends to overstate it. This was shown analytically in Section 2. This example reinforces the point made in Table 1 that while the **RRP** procedure does not lead to biased estimates of regression coefficients, it does lead to biased estimates of the unconditional population mean and variance. When we apply the correction implied in equation (12) to the RRP estimate of the standard deviation we obtain roughly the correct standard deviation. Once again, this suggests that the key assumption (A6) is appropriate in this application.

6 Summary and Conclusion

Although using regression prediction to impute the dependent variable in a regression model induces measurement errors "on the left", it is not necessarily innocuous. We have shown that the resulting Berkson errors in the dependent variable result in inconsistent estimates of the regression slope. This procedure has been much used to impute consumption to data sets with income or wealth, following a suggestion by Skinner (1987). This inconsistency can be overcome by rescaling by the first stage (imputation) R^2 (the **RRP** procedure) or by employing reverse regression in the first stage (the **BPP** procedure). Even then, we have shown that the usual OLS standard errors are not correct, but they can be corrected with estimable quantities.

Our results have use beyond the applications we demonstrate. For example, suppose a researcher has data with which to estimate a regression, but the suspects that the dependent variables is measured with error. The researcher also has a validation sample including the same dependent variable and a 'gold-standard' measure of the dependent variable (for example from administrative data). Then our analysis points to how to use this validation sample to estimate the regression model of interest (treating the original measure of the dependent variable as the proxy, z, and the 'gold-standard' measure as y.)

Our analysis demonstrates that the preferred method of imputation may depend on the intended application. This poses a challenge to data providers who may wish to include imputed variables in a standardized data set for multiple users.

Imputation of a dependent variable from a complimentary data set is a potentially useful part of the applied econometrician's toolkit, but it must be done with care.

Figures

Figure 1: RP Imputation Procedure as Projections

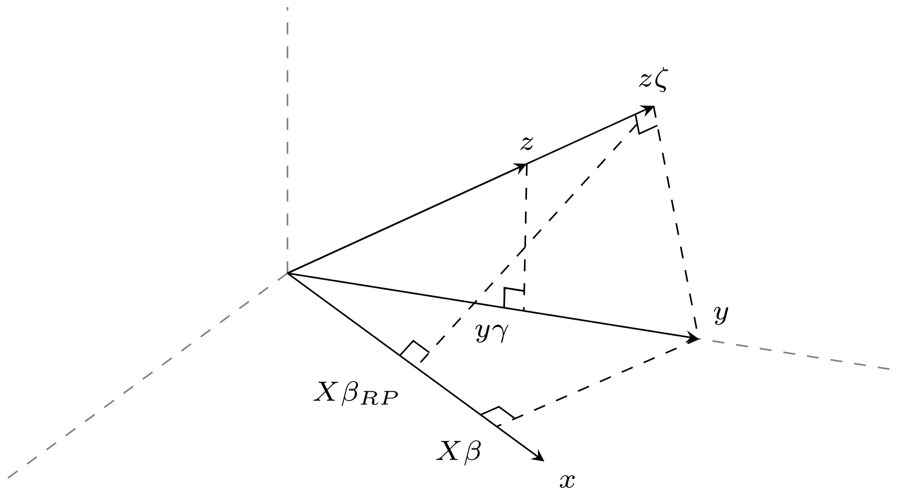
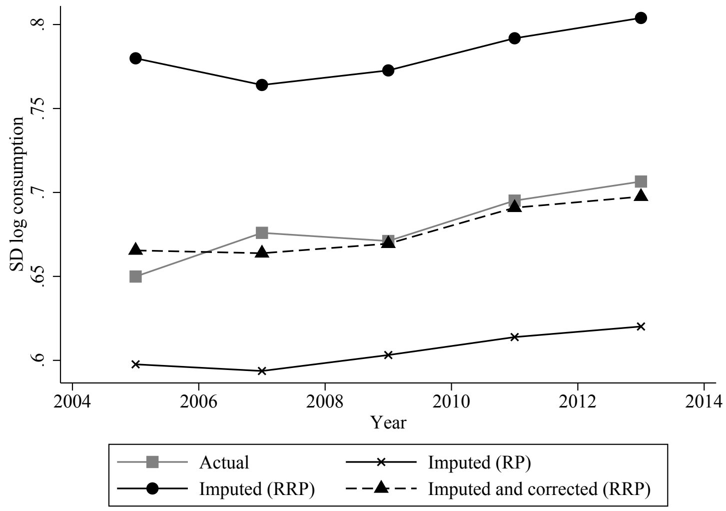


Figure 2: Standard deviation of log consumption



Note: Authors' calculations using the PSID. Lines show the standard deviation of log nondurable spending in the PSID ("Actual"), the standard deviation of imputed log consumption using regression prediction ("Imputed (RP)"), the standard deviation of imputed log consumption using re-scaled regression prediction ("Imputed (RRP)"), and the standard deviation of log consumption using re-scaled regression prediction corrected using the relationship in equation (12) ("Imputed and corrected (RRP)").

Appendices

A Proofs of propositions

Proof of Proposition 2

Proof.

$$
\begin{aligned}
plim\left(\hat{\beta}^{RP}\right) &= plim\left\{\left(\frac{X_2'X_2}{n_2}\right)^{-1}\frac{X_2'Z_2}{n_2}\left(\frac{Z_1'Z_1}{n_1}\right)^{-1}\frac{Z_1'y_1}{n_1}\right\}\\
&= plim\left\{\left(\frac{X_2'X_2}{n_2}\right)^{-1}\frac{X_2'Z_2}{n_2}\left(\frac{Z_1'Z_1}{n_1}\right)^{-1}\frac{Z_1'y_1}{n_1}\frac{1}{R^2_{y_1,Z_1}}R^2_{y_1,Z_1}\right\}\\
&= plim\left\{\left(\frac{X_2'X_2}{n_2}\right)^{-1}\frac{X_2'Z_2}{n_2}\left(\frac{Z_1'Z_1}{n_1}\right)^{-1}\frac{Z_1'y_1}{n_1}\left[\frac{y_1'Z_1}{n_1}\left(\frac{Z_1'Z_1}{n_1}\right)^{-1}\frac{Z_1'y_1}{n_1}\right]^{-1}\frac{y_1'y_1}{n_1}R^2_{y_1,Z_1}\right\}\\
&= \left[\beta\gamma + plim\left\{\left(\frac{X_2'X_2}{n_2}\right)^{-1}\frac{X_2'u_2}{n_2}\right\}\right]\Sigma_{ZZ}^{-1}\gamma'\sigma_{yy}\left(\sigma_{yy}\gamma\Sigma_{ZZ}^{-1}\gamma'\sigma_{yy}\right)^{-1}\sigma_{yy}\phi_{y,Z}\\
&= \beta\phi_{y,Z} + \Sigma_{ZZ}^{-1}\gamma'\sigma_{yy}\left(\sigma_{yy}\gamma\Sigma_{ZZ}^{-1}\gamma'\sigma_{yy}\right)^{-1}\sigma_{yy}\phi_{y,Z}\Sigma_{XX}^{-1}\times plim\left\{\frac{X_2'u_2}{n_2}\right\}\\
&= \beta\phi_{y,Z}
\end{aligned}
$$

□

Proof of Proposition 3

Proof. We have

$$\hat{\beta}^{RRP} = (X_2'X_2)^{-1}(X_2'z_2)(z_1'z_1)^{-1}(z_1'y_1)\left[(y_1'z_1)(z_1'z_1)^{-1}(z_1'y_1)\right]^{-1} y_1'y_1$$
$$= (X_2'X_2)^{-1}X_2'z_2(y_1'z_1)^{-1}y_1'y_1 = \hat{\beta}^{BPP}. \quad (25)$$

Thus, under the assumptions listed in Proposition 2, $\hat{\beta}^{BPP}$ is also consistent.

The **AM** procedure takes the ratio of $\widehat{\beta\gamma} = (X_2'X_2)^{-1}X_2'z_2$ and $\hat{\gamma} = (y_1'y_1)^{-1}y_1'z_1$, to give $\hat{\beta}^{AM} = \widehat{\beta\gamma}/\hat{\gamma}$.

$$\hat{\beta}^{AM} = \widehat{\beta\gamma}/\hat{\gamma} = (X_2'X_2)^{-1}X_2'z_2\left[(y_1'y_1)^{-1}y_1'z_1\right]^{-1}$$
$$= (X_2'X_2)^{-1}X_2'z_2(y_1'z_1)^{-1}y_1'y_1 = \hat{\beta}^{RRP} = \hat{\beta}^{BPP}. \quad (26)$$

□

Proof of Proposition 4

Proof. Consider the second stage regression

$$\frac{Z_{2j}\zeta}{\phi_{y,z}} = X_{2j}\beta + \delta_{2j} \quad (27)$$

This implies that

$$\frac{Z_{2j}\hat{\zeta}}{R^2_{y_1,z_1}} = X_{2j}\beta + \delta_{2j} + Z_{2j}\left(\frac{\hat{\zeta}}{R^2_{y_1,z_1}} - \frac{\zeta}{\phi_{y,z}}\right) \quad (28)$$

Our estimate of β is

$$\hat{\beta}^{RRP} = (X_2'X_2)^{-1}X_2'\frac{Z_2\hat{\zeta}}{R^2_{y_1,z_1}} \tag{29}$$

Combined with (28) this gives

$$\hat{\beta} - \beta = (X_2'X_2)^{-1}X_2'\delta_2 + (X_2'X_2)^{-1}X_2'Z_2\left(\frac{\hat{\zeta}}{R^2_{y_1,z_1}} - \frac{\zeta}{\phi_{y,z}}\right) \tag{30}$$

Given our assumptions, this implies that

$$\sqrt{n_2}(\hat{\beta} - \beta) \rightarrow_d N(0, Asymp\,Var(\hat{\beta}^{RRP})) \tag{31}$$

where

$$Asymp\,Var(\hat{\beta}^{RRP}) = \Sigma_{XX}^{-1}\left[\Omega_{X\delta} + \alpha\frac{\Sigma_{XZ}}{\phi_{y,Z}}\Sigma_{ZZ}^{-1}\Omega_{Z\xi}\Sigma_{ZZ}^{-1}\frac{\Sigma_{ZX}}{\phi_{y,Z}}\right]\Sigma_{XX}^{-1} \tag{32}$$

□

References

Angrist, J. D. and Krueger, A. B. (1992). The effect of age at school entry on educational attainment: An application of instrumental variables with moments from two samples. *Journal of the American Statistical Association*, 87(418):328–336.

Arellano, M. and Meghir, C. (1992). Female labour supply and on-the-job search: An empirical model estimated using complementary data sets. *The Review of Economic Studies*, 59(3):537–559.

Arrondel, L., Lamarche, P., and Savignac, F. (2015). Wealth effects on consumption across the wealth distribution: Empirical evidence. Technical Report 1817, ECB Working Paper Series.

Attanasio, O., Hurst, E., and Pistaferri, L. (2015). The evolution of income, consumption, and leisure inequality in the United States, 1980–2010. In Carroll, C. D., Crossley, T. F., and Sabelhaus, J., editors, *Improving the Measurement of Consumer Expenditures*, pages 100–140. National Bureau of Economic Research, University of Chicago Press.

Attanasio, O. and Pistaferri, L. (2014). Consumption inequality over the last half century: Some evidence using the new PSID consumption measure. *American Economic Review: Papers & Proceedings*, 104(5):122–126.

Baker, S. R., Kueng, L., Meyer, S., and Pagel, M. (2018). Measurement error in imputed consumption. Working Paper 25078, National Bureau of Economic Research.

Berkson, J. (1950). Are there two regressions? *Journal of the American Statistical Association*, 45(250):164–180.

Blundell, R., Horowitz, J., and Parey, M. (2019). Estimation of a nonseparable heterogenous demand function with shape restrictions and berkson errors. Technical report.

Blundell, R., Pistaferri, L., and Preston, I. (2004). Imputing consumption in the PSID using food demand estimates from the CEX. IFS Working Paper WP04/27, The Institute for Fiscal Studies.

Blundell, R., Pistaferri, L., and Preston, I. (2008). Consumption inequality and partial insurance. *American Economic Review*, pages 1887–1921.

Bollinger, C. R. and Minier, J. (2015). On the robustness of coefficient estimates to the inclusion of proxy variables. *Journal of Econometric Methods*, 4(1):101–122.

Browning, M., Crossley, T. F., and Weber, G. (2003). Asking consumption questions in general purpose surveys. *Economic Journal*, 113(491):F540–F567.

Browning, M., Crossley, T. F., and Winter, J. (2014). The measurement of household consumption expenditures. *Annual Review of Economics*, 6(1):475–501.

Charles, K. K., Danziger, S., Li, G., and Schoeni, R. (2014). The Intergenerational Correlation of Consumption Expenditures. *American Economic Review: Papers & Proceedings*, 104(5):136–140.

David, M., Little, R. J. A., Samuhel, M. E., and Triest, R. K. (1986). Alternative methods for CPS income imputation. *Journal of the American Statistical Association*, 81(393):29–41.

Dumont, M., Rayp, G., Thas, O., and Willemé, P. (2005). Correcting standard errors in two-stage estimation procedures with generated regressands. *Oxford Bulletin of Economics and Statistics*, 67(3):421–433.

Feenstra, R. C. and Hanson, G. H. (1999). The Impact of Outsourcing and High-technology Capital on Wages: Estimates for the United States, 1979-1990. *The Quarterly Journal of Economics*, 114(3):907–940.

Fisher, J., Johnson, D., Latner, J. P., Smeeding, T., and Thompson, J. (2016). Inequality and mobility using income, consumption, and wealth for the same individuals. *RSF: The Russell Sage Foundation Journal of the Social Sciences*, 2(6):44–58.

Guvenan, F. and Smith, A. A. (2014). Inferring Labor Income Risk and Partial Insurance From Economic Choices. *Econometrica*, 82(6):2085–2129.

Hoderlein, S. and Winter, J. (2010). Structural measurement errors in nonseparable models. *Journal of Econometrics*, 157(2):432–440.

Hyslop, D. R. and Imbens, G. W. (2001). Bias from classical and other forms of measurement error. *Journal of Business & Economic Statistics*, 19(4):475–481.

Inoue, A. and Solon, G. (2010). Two-sample instrumental variables estimators. *The Review of Economics and Statistics*, 92(3):557–561.

Kaplan, G., Mitman, K., and Violante, G. L. (2020). Non-durable consumption and housing net worth in the great recession: Evidence from easily accessible data. *Journal of Public Economics*, Forthcoming.

Klevmarken, A. (1982). Missing variables and Two-Stage Least-Squares estimation from more than one data set. Working Paper Series 62, Research Institute of Industrial Economics.

Lillard, L., Smith, J. P., and Welch, F. (1986). What do we really know about wages: The importance of non-reporting and census imputation. *Journal of Political Economy*, 94(3):489–506.

Lubotsky, D. and Wittenberg, M. (2006). Interpretation of regressions with multiple proxies. *The Review of Economics and Statistics*, 88(3):549–562.

Lusardi, A. (1996). Permanent income, current income, and consumption: Evidence from two panel data sets. *Journal of Business & Economic Statistics*, 14(1):81–90.

Meyer, B. and Sullivan, J. X. (2003). Measuring the Well-Being of the Poor Using Income and Consumption. *Journal of Human Resources*, 38(4):1180–1220.

Mulligan, C. (1999). Galton versus the Human Capital Approach to Inheritance. *Journal of Political Economy*, 107(S6):184–224.

Pacini, D. and Windmeijer, F. (2016). Robust inference for the Two-Sample 2SLS estimator. *Economics Letters*, 146(C):50–54.

Palumbo, M. (1999). Uncertain Medical Expenses and Precautionary Saving Near the End of the Life Cycle. *The Review of Economic Studies*, 66(2):395–421.

Panel Study of Income Dynamics (2019). Public Use Dataset. Produced and distributed by the Survey Research Center, Institute for Social Research, University of Michigan, Ann Arbor, MI.

Schulhofer-Wohl, S. (2011). Heterogeneity and Tests of Risk Sharing. *Journal of Political Economy*, 119(5):925–958.

Shea, J. (1997). Instrument relevance in multivariate linear models: A simple measure. *The Review of Economics and Statistics*, 79(2):348–352.

Skinner, J. (1987). A superior measure of consumption from the Panel Study of Income Dynamics. *Economics Letters*, 23(2):213–216.

Skinner, J. (1989). Housing wealth and aggregate saving. *Regional Science and Urban Economics*, 19(2):305–324.

Wansbeek, T. and Meijer, E. (2000). *Measurement Error and Latent Variables in Econometrics*. Elsevier, Amsterdam.

Wooldridge, J. M. (2002). *Econometric Analysis of Cross Section and Panel Data*. MIT Press, Cambridge, MA.

Ziliak, J. P. (1998). Does the choice of consumption measure matter? An application to the permanent-income hypothesis. *Journal of Monetary Economics*, 41(1):201–216.