A lattice test for additive separability

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A Lattice Test for Additive Separability

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Abstract: We derive necessary and sufficient conditions for a finite data set of price and demand observations to be consistent with an additively separable preference. We do so without imposing concavity on any of the subutility functions or convexity of the budget set *a priori*, thereby generalizing earlier results. Our simple and intuitive *lattice test* easily accommodates departures from rationality, or errors, which subsequently facilitates a rich empirical analysis. We apply our econometric techniques to the food consumption of a panel of British households. The primary empirical finding is that additive separability has considerable success in explaining the data.

Keywords: additive separability, consumer demand, lattice test, revealed preference

JEL Classification Numbers: C14, C60, D11, D12

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1. Introduction

The separability of a consumer's preference—across goods, time periods, states of the world, and so on—is a frequently held assumption in economics. Separability is useful because it restricts the attention of both the agent and the econometrician to a single commodity or commodity group, and this simplification pays dividends in numerous theoretical and empirical settings. One might therefore want to test for the separability of a consumer's preference, i.e., to characterize the empirical content of separability in the form of its implied restrictions on observable data, which are typically prices and demands. The question of separability and its empirical implications has therefore spawned a long and rich literature.[1]

Suppose that we have access to a finite data set containing price and demand observations. In this paper, we derive restrictions on this data set which are necessary and sufficient for it to have arisen from the maximization of a separable preference of a particular form known as *additivity*, which is a type of *strong* separability. We do so in the nonparametric revealed preference tradition of Afriat (1967), Diewert (1973), and Varian (1982), i.e., we do not assume anything about the specific functional shape of the preference, only the primitive essentials. Furthermore, unlike previous revealed preference work on additive separability (see Varian (1983), Diewert and Parkan (1985), and Fleissig and Whitney (2007)), we do not require convexity of the preference or of the budget set *a priori*. Instead we extend the *lattice approach* (see Polisson, Quah, and Renou (2017)) to achieve *domain reduction*; crucially, the lattice approach does not appeal to the techniques of convex optimization, which allows for an easy extension to accommodate departures from full economic rationality, or errors, in the form of cost inefficiencies (see, e.g., Afriat (1972, 1973) and Varian (1990)).

We implement our procedures on a panel of British consumers known as the Kantar Worldpanel, previously used to study nutrition in Dubois, Griffith, and Nevo (2014), among some other papers.[2] The data set is a representative sample of British households observed

[1] For surveys on functional structure and separability, see Blackorby, Primont, and Russell (1978) and Deaton and Muellbauer (1980); for seminal references on separability, see Leontief (1947), Strotz (1957, 1959), Frisch (1959), Gorman (1959, 1968), Debreu (1960), Houthakker (1960), Pearce (1961), and Goldman and Uzawa (1964); for nonparametric tests of separability in the spirit of those which are developed and implemented in this paper, see Varian (1983), Diewert and Parkan (1985), Swofford and Whitney (1987, 1988), Fleissig and Whitney (2003, 2007, 2008), Echenique (2014), Quah (2014), and Cherchye *et al.* (2015).

[2] See Griffith and O'Connell (2009) and Leicester and Oldfield (2009) for a description of the data set,

between 2005 and 2012. We focus our empirical analysis on 4 aggregate food groups—fruit, vegetables, red meat, and poultry/fish—and we explore the relationships *within* two subgroups: {fruit, vegetables} and {red meat, poultry/fish}. The primary empirical finding is that additive separability has considerable success in explaining consumption choices within *both* subgroups; this success is more pronounced within the subgroup {fruit, vegetables} than {red meat, poultry/fish}, suggesting greater substitutability/complementarity within the latter subgroup, or equivalently, greater demand independence within the former.

This paper is a contribution to the econometric investigation of the empirical content of separability, and in particular, additive separability. It has long been thought that additivity is extremely empirically restrictive, and that while often a convenient simplifying assumption to make, it is unsuitable for applied empirical work due to the restrictions it imposes *a priori* on substitution terms, and hence own- and cross-price elasticities of demand.[3] However, in this paper we argue that additive separability, in the absence of auxiliary parametric assumptions and in the presence of finite data, may not be as restrictive as it might at first appear. The remainder of the paper is organized as follows. Section 2 defines separability, and in particular additive separability, and underscores its central role in simplifying the economic analysis of consumer behavior. Section 3 outlines the revealed preference approach as a means of characterizing the empirical content of additive separability. Section 4 describes the empirical implementation and results. Section 5 concludes.

2. Separability

Suppose that a consumer has a rational preference ordering $\succcurlyeq$ over a consumption set, which we assume to be $\mathbb{R}^{\ell}_{+}$. Further suppose that these commodities can be divided into two commodity groups X and Y. The consumption set is then given by $X \times Y$, with typical element (x, y). It is well known that preferences are separable on X if $(x, y) \succcurlyeq (x', y)$ for some $y \in Y$ implies $(x, y') \succcurlyeq (x', y')$ for all $y' \in Y$, i.e., the weak preference of x to x' is

as well as a discussion of the costs and benefits of using household scanner data (rather than conventional household surveys) to study nutrition.

[3] Deaton (1974) concludes "the main argument of this paper is that *the assumption of additive preferences is almost certain to be invalid in practice and the use of demand models based on such an assumption will lead to severe distortion of measurement.* [emphasis Deaton's] So that if the price to be paid for the theoretical consistency of demand models is the necessity of assuming additive preferences, then the price is too high". See also Blundell (1988), who systematically examines the empirical demand literature since Stone (1954).

independent of the choice from Y. It is also known that if $\succcurlyeq$ has a utility representation, then preferences are separable on X if and only if there exist some functions $v : X \to \mathbb{R}$ and $f : \mathbb{R} \times Y \to \mathbb{R}$, where f is increasing in its first argument, such that $u(x, y) = f(v(x), y)$. Assuming differentiability, notice that the marginal rates of substitution between any two commodities in X are independent of the choice from Y. We could of course separate the preference even further, and the strongest form of separability typically invoked is *additive* across all commodities, i.e., if there are ℓ commodities, then $u(x_1, x_2, \ldots, x_\ell) = \sum_{i=1}^{\ell} v_i(x_i)$, where $v_i : \mathbb{R}_+ \to \mathbb{R}$ for all $i = 1, 2, \ldots, \ell$.[4] Again assuming differentiability, it is easy to see that the marginal rates of substitution between any two commodities are independent of the amounts of any other commodities. The literature on functional separability and its role in the economics of consumer behavior is comprehensively summarized by Blackorby, Primont, and Russell (1978) and Deaton and Muellbauer (1980).

The separability of a consumer's preference is often an assumption made out of necessity, either explicitly or implicitly. *Any* empirical work in consumer demand and industrial organization necessarily focuses on a subset of products, the justification for which typically involves invoking some type of separability argument. It is therefore well known that substitution patterns across groups are restricted *a priori*, e.g., across goods (consumption/leisure), time periods (today/tomorrow), and states of the world (good/bad). In fact, it is not uncommon to assume even *stronger* forms of separability, e.g., quasilinear utility (e.g., in consumer demand, industrial organization, and mechanism design), exponential discounting, and expected utility. Many of the canonical models in economics appeal to separability as a simplifying device, and it is therefore unsurprising that we might want to test for the separability of a consumer's preference.

3. Revealed Preference

In this paper, we suppose that we have access to a finite set of price and demand observations taken from an individual consumer. Our revealed preference analysis is in the spirit of Afriat (1967), Diewert (1973), and Varian (1982), i.e., we adopt a completely nonparametric

[4] Interestingly, Debreu (1960) shows that a necessary and sufficient condition for additive separability is the weak separability of every subset of commodities, so our revealed preference characterization will apply to this utility representation as well.

approach rather than assuming that preferences take a specific functional form, which can then be applied at the level of the individual consumer.[5] This approach is in contrast to conventional econometric approaches, which typically adopt functional forms and restrict observed and unobserved heterogeneity *a priori*.[6]

3.1 Afriat's Theorem

Let $\{(p^t, x^t)\}_{t=1}^T$ denote a finite set of observations drawn on an individual consumer. At every observation $t = 1, 2, \ldots, T$, an econometrician has access to the consumption bundle $x^t = (x_1^t, x_2^t, \ldots, x_\ell^t) \in \mathbb{R}_+^\ell$ and the corresponding price vector $p^t = (p_1^t, p_2^t, \ldots, p_\ell^t) \in \mathbb{R}_{++}^\ell$.

Definition 1. *For any consumption bundles x^t, x^s, the bundle x^t is* directly revealed preferred *to the bundle x^s ($x^t \succcurlyeq_0^* x^s$) whenever $p^t \cdot x^s \leqslant p^t \cdot x^t$.*

Definition 2. *For any consumption bundles x^t, x^s, the bundle x^t is* directly revealed strictly preferred *to the bundle x^s ($x^t \succ_0^* x^s$) whenever $p^t \cdot x^s < p^t \cdot x^t$.*

Definition 3. *For any consumption bundles x^t, x^s, the bundle x^t is* revealed preferred *to the bundle x^s ($x^t \succcurlyeq^* x^s$) whenever $p^t \cdot x^i \leqslant p^t \cdot x^t$, $p^i \cdot x^j \leqslant p^i \cdot x^i$, $\ldots$, $p^k \cdot x^l \leqslant p^k \cdot x^k$, $p^l \cdot x^s \leqslant p^l \cdot x^l$.*

In words, we have established that x^t is directly revealed preferred to x^s if x^s was affordable when x^t was purchased, and that x^t is directly revealed strictly preferred to x^s if, not only was x^s affordable when x^t was purchased, but also it cost *strictly* less. The transitive closure of the direct revealed preference relation then implies that x^t can be revealed preferred to x^s either directly or through a chain. These definitions lead naturally towards what has become a central axiom in the revealed preference literature.

Definition 4. *The data set $\{(p^t, x^t)\}_{t=1}^T$ obeys the Generalized Axiom of Revealed Preference (GARP) so long as $x^t \succcurlyeq^* x^s \implies x^s \not\succ_0^* x^t$.*

In words, GARP asserts that whenever x^t is revealed preferred to x^s, either directly or through a chain, it cannot be the case that x^s is directly revealed strictly preferred to x^t.

[5] See Chambers and Echenique (2016) and Crawford and De Rock (2014) for summaries of the theoretical and empirical revealed preference literatures, respectively.

[6] See Deaton and Muellbauer (1980) for a textbook treatment of the empirical demand literature.

An equivalent statement is that whenever

$$x^t \succcurlyeq_0^* x^i,\ x^i \succcurlyeq_0^* x^j,\ \ldots\ ,\ x^k \succcurlyeq_0^* x^l,\ x^l \succcurlyeq_0^* x^s,\ x^s \succcurlyeq_0^* x^t,$$

none of the weak relations ($\succcurlyeq_0^*$) can be replaced with a strict relation ($\succ_0^*$). Therefore, GARP is essentially an intuitive no-cycling condition on the data, and it is straightforward to show that such a condition is a necessary consequence of maximizing a well-behaved utility function. We now consider the problem formally.

Definition 5. *The data set* $\{(p^t, x^t)\}_{t=1}^T$ *is* rationalizable *if there exists a utility function* $u : \mathbb{R}_+^\ell \to \mathbb{R}$*, such that, at every observation* $t = 1, 2, \ldots, T$*,*

$$u(x^t) \geqslant u(x) \ \textit{ for any } \ x \in B^t = \{x \in \mathbb{R}_+^\ell : p^t \cdot x \leqslant p^t \cdot x^t\}.$$

A data set is therefore said to be rationalizable if there exists a utility function, such that, at every observation, the consumption bundle that was chosen delivers weakly greater utility than any other consumption bundle that was affordable. This definition is both natural and standard, and we can relate this rationalizability concept to GARP via Afriat's Theorem.

Theorem 1. *The following statements are equivalent:*

1. *The data set* $\{(p^t, x^t)\}_{t=1}^T$ *is rationalizable by a nonsatiated utility function* $u : \mathbb{R}_+^\ell \to \mathbb{R}$.
2. *The data set* $\{(p^t, x^t)\}_{t=1}^T$ *obeys GARP.*
3. *For the data set* $\{(p^t, x^t)\}_{t=1}^T$*, at every observation* $t = 1, 2, \ldots, T$*, there exist numbers* $u^t \in \mathbb{R}$ *and* $\lambda^t \in \mathbb{R}_{++}$*, such that*

$$u^{t'} \leqslant u^t + \lambda^t p^t \cdot (x^{t'} - x^t) \ \textit{ for all } \ t, t' = 1, 2, \ldots, T.$$

4. *The data set* $\{(p^t, x^t)\}_{t=1}^T$ *is rationalizable by a utility function* $u : \mathbb{R}_+^\ell \to \mathbb{R}$*, which is increasing, concave, and continuous.*

Proof. See Afriat (1967), Diewert (1973), and Varian (1982). □

Some brief remarks on Afriat's Theorem are in order. First, the theorem characterizes the entirety of the empirical content of utility maximization in a finite data setting, i.e., since the observable restrictions on a given data set are both necessary and sufficient for utility maximization, these restrictions exhaust the empirical implications of such a procedure. Second, the theorem provides two equivalent empirical tests for utility maximization in a finite data setting: one involves checking GARP, and the other involves finding a feasible solution to a set of linear inequalities, which have come to be known as the Afriat inequalities. Both procedures are computationally efficient. Third, in a finite data setting, if a data set is rationalizable by a nonsatiated utility function, it is also rationalizable by a utility function that is increasing, concave, and continuous. An equivalent statement is that monotonicity, concavity, and continuity of the utility function are, over and above nonsatiation, without loss of generality in a classical finite data setting, i.e., they are untestable. Since these properties are without loss of generality, they can be assumed without cost in this setting.[7]

3.2 Additive Separability

Testing for separability is typically thought to be a 'hard' problem, if not in concept then in computation and application. Nonparametric tests of *weak* separability date back to Varian (1983) and Diewert and Parkan (1985), in which each of the subutility functions is assumed to be concave and the budget set is linear, i.e., the characterization is not completely general. This approach gives rise to a nonlinear (bilinear) test, which is known to be computationally hard, although some practical advances have been made, e.g., see Swofford and Whitney (1987, 1988), Fleissig and Whitney (2003, 2008), and Cherchye *et al.* (2015). More recently, Quah (2014) establishes a testing procedure for weak separability that does not require convexity of either the preference or budget set, and which is in principle implementable on finite data sets that are not too large, while Echenique (2014) argues that separability is, in general, a computationally hard problem.

Contrary to these somewhat negative results on the nonparametric testing of weak separability, we argue that *additive* separability in its strongest but most common form is exactly and efficiently testable. Varian (1983) and Diewert and Parkan (1985) were again the first to establish necessary and sufficient conditions for additively separable rationalizability. The

[7] These stronger assumptions give rise straightforwardly to a set of necessary and sufficient conditions on the data; however, with additional structure on the preference and/or on the budget set, these assumptions are clearly no longer without cost, i.e., they have observable implications, which will soon be made clear.

Varian (1983) characterization partitions the ℓ goods into two groups, but the result can easily be generalized to some number of groups less than or equal to ℓ.[8] What is critical is that each of the subutility functions is *concave* and the budget set is *convex*, which allows for a first-order condition approach (appealing to the techniques of convex optimization) that produces a straightforward linear test. Varian (1983) and Diewert and Parkan (1985) therefore establish a *joint* test for additive separability *and* global concavity of each of the subutility functions, where the maximization of this preference is subject to a classical linear budget constraint. Notice that concavity of the subutility functions implies diminishing marginal utilities of consumption in every commodity, which is akin to assuming a smoothing of consumption across commodities (e.g., consumer products, time periods, and states of the world). Nothing about additive separability as it is defined requires concavity of the subutility functions or any such smoothing, and so a violation of the Varian (1983) and Diewert and Parkan (1985) tests is not necessarily a violation of additive separability *as such*.

In this paper, we develop and implement a test of additive separability that does not require concavity of the subutility functions *a priori*, nor does it require a convex budget set. In this sense, we provide a more general test than Varian (1983) and Diewert and Parkan (1985). Our results hold when *all* goods are additively separable, i.e., when there is a subutility function associated with each good defined over the consumption space for that good. In this sense, we are less general than Varian (1983) and Diewert and Parkan (1985), although this stronger notion of additive separability is common in many applied theoretical and empirical settings. Since we do not require convexity of either the preference or budget set, the procedure can be extended to account for departures from rationality, or errors, in the form of cost inefficiencies (see, e.g., Afriat (1972, 1973), Varian (1990), Halevy, Persitz, and Zrill (2016), and Polisson, Quah, and Renou (2017)); as will soon be made clear, the notion of Afriat (1972, 1973) cost inefficiencies requires a rationalization over a (modified) nonconvex budget set, even if the budget itself is convex. As a result, what emerges is a simple, intuitive, and easily implementable nonparametric test of additive separability in its most common form, which allows us to reconsider the question of the empirical plausibility of additivity in the absence of any confounding functional form assumptions.

[8] See Appendix A.1 for a version of the characterization in Varian (1983) and Diewert and Parkan (1985).

Before developing our revealed preference procedure, it is worth exploring what constitutes a violation of additive separability. Consider the example depicted in Figure 1. In this simple example with two goods and three observations, the consumer purchased $x^1 = (4, 1)$ from B^1, $x^2 = (2, 4)$ from B^2, and $x^3 = (5, 3)$ from B^3.[9] Since none of the budget lines intersect, it is immediate that these observations obey GARP and are therefore rationalizable as having arisen from the maximization of a nonsatiated utility function. They are *not*, however, rationalizable by additive separability, which is straightforward to demonstrate.

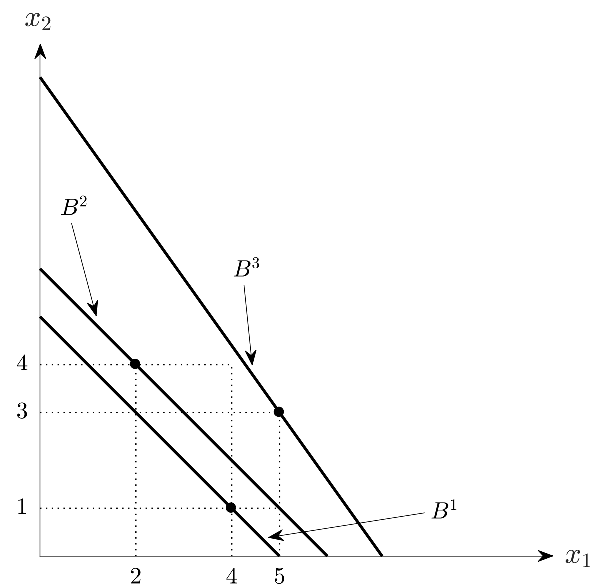


Figure 1: Violation of Additive Separability

At the first observation, it must be the case that $v_1(4) + v_2(1) \geqslant v_1(2) + v_2(3)$, since $(2, 3)$ was available when $(4, 1)$ was purchased; at the second observation, it must be the case that $v_1(2) + v_2(4) \geqslant v_1(5) + v_2(1)$ since $(5, 1)$ was available when $(2, 4)$ was purchased; together these inequalities imply that $v_1(4) + v_2(4) \geqslant v_1(5) + v_2(3)$, which contradicts the third observation since $(4, 4)$ costs *strictly* less than $(5, 3)$ when $(5, 3)$ was chosen. What this simple example illustrates is that an additive structure may give rise to further revealed preference relations through a cancellation of terms,[10] and this is precisely what we hope to

[9] The prices have been suppressed since we do not require them for this example.

[10] In fact, Debreu (1960) shows that with only two goods, as in our example, if preferences are representable by a utility function, then they can be represented by an additively separable utility function if and only if a *double cancellation* condition holds.

systematically capture in our revealed preference approach.

The procedure we develop makes use of a revealed preference concept known as the *lattice method*, which was first introduced by Polisson, Quah, and Renou (2017) in order to test a broad class of models of decision making under risk and under uncertainty. The essence of the approach is to achieve *domain reduction*, i.e., to convert (without loss) an *infinite* set of revealed preference relations on the consumption space into a *finite* set of revealed preference relations on the lattice. The latter, crucially, can be checked. In Polisson, Quah, and Renou (2017), the lattice method allows for revealed preference tests of many different models of decision making under risk and under uncertainty, and without requiring concavity of the Bernoulli utility function *a priori*. In these settings, many utility representations take a separable form, and so the problem in Polisson, Quah, and Renou (2017) has a functional structure similar to additive separability as described in this paper.

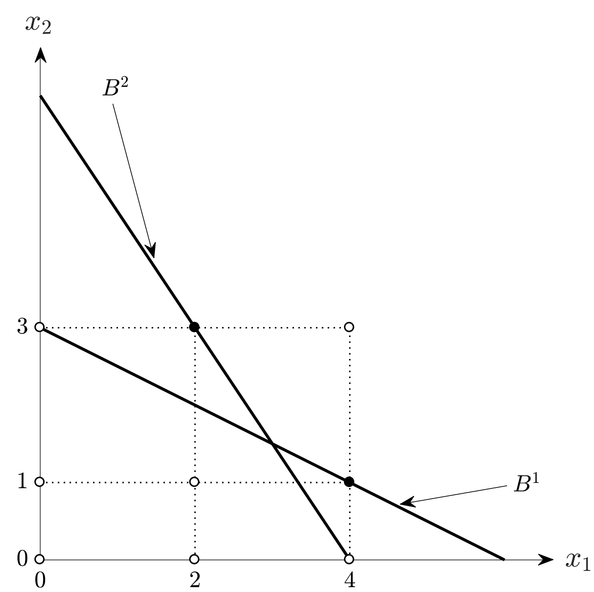


Figure 2: Rationalizability by Additive Separability

Before formalizing the main argument, we present another simple example in order to develop intuition. Suppose that we have observed a consumer choosing the bundle $x^1 = (4, 1)$ at the prices $p^1 = (2, 4)$, followed by the bundle $x^2 = (2, 3)$ at the prices $p^2 = (3, 2)$. This situation is depicted in Figure 2. The data obey GARP since $(4, 1)$ was purchased when $(2, 3)$ was unavailable, and vice versa, and therefore the data are rationalizable by utility maximization. Now we would like to know whether these data are also rationalizable by additive separability, i.e., if there exist increasing and continuous subutility functions $v_1 : \mathbb{R}_+ \to \mathbb{R}$ and $v_2 : \mathbb{R}_+ \to \mathbb{R}$, such that

$$v_1(4) + v_2(1) \geqslant v_1(x_1) + v_2(x_2) \text{ for any } (x_1, x_2) \in B^1,$$

$$v_1(2) + v_2(3) \geqslant v_1(x_1) + v_2(x_2) \text{ for any } (x_1, x_2) \in B^2,$$

where $B^1 = \{(x_1, x_2) \in \mathbb{R}^2_+ : 2x_1 + 4x_2 \leqslant 12\}$ and $B^2 = \{(x_1, x_2) \in \mathbb{R}^2_+ : 3x_1 + 2x_2 \leqslant 12\}$. Since we do not require that the subutility functions v_1 and v_2 are concave as in Varian (1983) and Diewert and Parkan (1985), we cannot adopt a first-order condition approach when deriving restrictions on this data set which are necessary and sufficient for rationalizability by additive separability. Instead, we appeal to the lattice method.

Notice that for the first good, we have observed the consumption levels 2 and 4, and for the second good, the consumption levels 1 and 3. At a minimum, the consumption set for the first good must contain the elements in $\mathcal{X}_1 = \{0, 2, 4\}$, i.e., any observed consumption levels plus zero, and similarly, the consumption set for the second good must contain the elements in $\mathcal{X}_2 = \{0, 1, 3\}$. Consequently, the consumption set must contain, again at a minimum, any elements in the finite lattice $\mathcal{L} = \mathcal{X}_1 \times \mathcal{X}_2$. It is certainly clear that as a *necessary* condition for rationalizability by additive separability, there must exist sets of real numbers $\{\bar{v}_1(0), \bar{v}_1(2), \bar{v}_1(4)\}$ and $\{\bar{v}_2(0), \bar{v}_2(1), \bar{v}_2(3)\}$, with $\bar{v}_1(0) < \bar{v}_1(2) < \bar{v}_1(4)$ and $\bar{v}_2(0) < \bar{v}_2(1) < \bar{v}_2(3)$,[11] such that

$$\bar{v}_1(4) + \bar{v}_2(1) \geqslant \bar{v}_1(x_1) + \bar{v}_2(x_2) \text{ for any } (x_1, x_2) \in B^1 \cap \mathcal{L},$$

$$\bar{v}_1(2) + \bar{v}_2(3) \geqslant \bar{v}_1(x_1) + \bar{v}_2(x_2) \text{ for any } (x_1, x_2) \in B^2 \cap \mathcal{L},$$

with the inequalities strict whenever (x_1, x_2) is in the interior of the budget set.[12] This simplifies to finding sets of real numbers satisfying $\bar{v}_1(4) + \bar{v}_2(1) \geqslant \bar{v}_1(0) + \bar{v}_2(3)$ and $\bar{v}_1(2) + \bar{v}_2(3) \geqslant \bar{v}_1(4) + \bar{v}_2(0)$.[13] It is clear that by letting $\bar{v}_1(0) = 0$, $\bar{v}_1(2) = 2$, $\bar{v}_1(4) = 4$, $\bar{v}_2(0) = 0$, $\bar{v}_2(1) = 1$, and $\bar{v}_2(3) = 3$, we meet this requirement. If we were unable to find such numbers, then we would reject that these data could have arisen from the maximization of an additively

[11] This is due to monotonicity, i.e., the subutility functions $v_1 : \mathbb{R}_+ \to \mathbb{R}$ and $v_2 : \mathbb{R}_+ \to \mathbb{R}$ are increasing.
[12] This is again due to monotonicity.
[13] The simplification arises since elements of the lattice which are weakly *dominated* by the bundles chosen satisfy the above conditions trivially as a result of monotonicity.

separable preference. What we claim in this paper is that the above requirements are also *sufficient* for rationalizability by additive separability.

We now address the problem formally. Below is the general notion of rationalizability by additive separability to which we appeal throughout the remainder of the paper.

Definition 6. *The data set $\{(p^t, x^t)\}_{t=1}^T$ is rationalizable by additive separability if there exist a collection of subutility functions $\{v_i\}_{i=1}^{\ell}$, with $v_i : \mathbb{R}_+ \to \mathbb{R}$ increasing and continuous for all $i = 1, 2, \ldots, \ell$, such that, at every observation $t = 1, 2, \ldots, T$,*

$$\sum_{i=1}^{\ell} v_i(x_i^t) \geqslant \sum_{i=1}^{\ell} v_i(x_i) \text{ for any } x \in B^t = \{x \in \mathbb{R}_+^{\ell} : p^t \cdot x \leqslant p^t \cdot x^t\}.$$

Notice that *all* goods are additively separable in this formulation, i.e., the marginal rates of substitution between any two goods are independent of the consumption of any other goods. This strong form of separability is very common but thought to be highly restrictive.

Theorem 2. *The following statements are equivalent:*

(1) The data set $\{(p^t, x^t)\}_{t=1}^T$ is rationalizable by additive separability.

(2) There exist numbers $\bar{v}_i(x_i) \in \mathbb{R}$ for each $x_i \in \mathcal{X}_i = \{x \in \mathbb{R}_+ : x = x_i^t \text{ for some } t\} \cup \{0\}$ for all $i = 1, 2, \ldots, \ell$, with $\bar{v}_i(x_i) > \bar{v}_i(y_i)$ whenever $x_i > y_i$ for any x_i, $y_i \in \mathcal{X}_i$, such that, at every observation $t = 1, 2, \ldots, T$,

$$\sum_{i=1}^{\ell} \bar{v}_i(x_i^t) \geqslant \sum_{i=1}^{\ell} \bar{v}_i(x_i) \text{ for any } x \in B^t \cap \mathcal{L}, \tag{A1}$$

$$\sum_{i=1}^{\ell} \bar{v}_i(x_i^t) > \sum_{i=1}^{\ell} \bar{v}_i(x_i) \text{ for any } x \in (B^t / \partial B^t) \cap \mathcal{L}, \tag{A2}$$

where $\mathcal{L} = \prod_{i=1}^{\ell} \mathcal{X}_i$ denotes a finite lattiace, and where $\partial B^t = \{x \in \mathbb{R}_+^{\ell} : p^t \cdot x = p^t \cdot x^t\}$ denotes the boundary of B^t.[14]

Proof. The proof is given in the Appendix. □

The necessity of statement (2) is a straightforward generalization of the principle outlined in the earlier example. The intuition for the sufficiency of statement (2) is that conditional

[14] Notice, therefore, that the interior of B^t is denoted by $B^t / \partial B^t = \{x \in \mathbb{R}_+^{\ell} : p^t \cdot x < p^t \cdot x^t\}$.

on being able to find subutility functions restricted to the finite lattice, a set of increasing and continuous subutility functions can *extend* these mappings to the entire space, i.e., we can always 'connect the dots'. A natural choice for each extension is a step function, which is neither increasing nor continuous; the sufficiency argument shows that we can construct piecewise linear extensions that are arbitrarily close to these steps.

While the lattice approach developed in this paper relates in concept to Polisson, Quah, and Renou (2017), it differs on two important accounts: (1) here we construct some arbitrary number ℓ subutility functions, one for *every* consumption good, each of which is required to be increasing and continuous, rather that a *single* Bernoulli utility function over state-contingent consumption; and (2) our method of proof involves constructing subutility functions which are *piecewise linear*, and this can be done explicitly, whereas Polisson, Quah, and Renou (2017) must allow for sufficient flexibility in the curvature of the Bernoulli utility function in order to accommodate a broad class of models of decision making under risk and uncertainty, and as a consequence, the proof by induction is significantly more complicated.

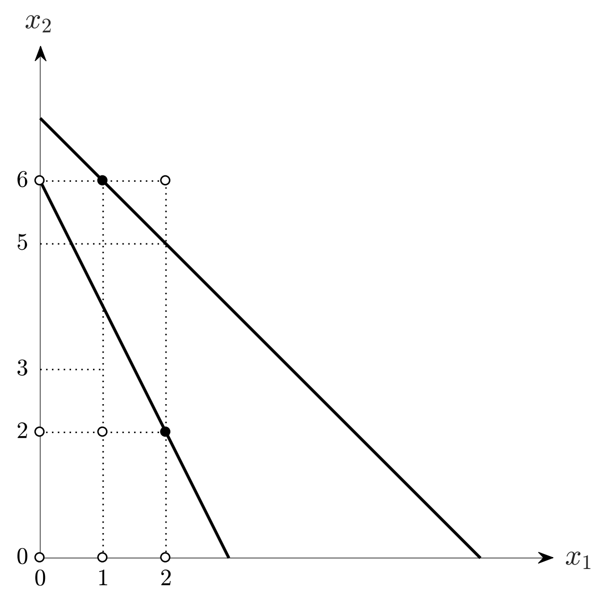


Figure 3: Violation of Concave Additive Separability

Returning to the example, we conclude that these data are, in fact, rationalizable by additive separability. Furthermore, they are also rationalizable by *concave* additive separability (see Varian (1983) and Diewert and Parkan (1985)), i.e., the consumer's choices are consistent with a smoothing across goods. Suppose instead, as shown in Figure 3, that we had observed the consumer choosing the bundle $x^1 = (2, 2)$ at prices $p^1 = (4, 2)$, followed by the bundle $x^2 = (1, 6)$ at prices $p^2 = (2, 2)$. The data clearly obey GARP, and it is easy to show using the procedure described in this paper that they are also rationalizable by additive separability. These data are *not*, however, rationalizable by concave additive separability. Since $(1, 3)$ costs strictly less than $(2, 2)$ when $(2, 2)$ was chosen, it must be the case that $v_1(2) + v_2(2) > v_1(1) + v_2(3)$. By the concavity of subutility function v_2, it must also be the case that $v_2(6) - v_2(3) \leqslant v_2(5) - v_2(2)$. Together these inequalities imply that $v_1(2) + v_2(5) > v_1(1) + v_2(6)$, which of course contradicts the optimality of $(1, 6)$. Intuitively, since income increases and good 1 becomes relatively cheaper between observations 1 and 2, a consumer who is smoothing consumption would never buy strictly less of good 1.

3.4 Departures from Rationality

So far the tests we have developed are for *exact* rationalizability by additive separability. When a data set is not exactly rationalizable, it is imperative to establish some notion of the 'distance' from full rationalizability, or what we might call *approximate* rationalizability. The predominant convention in the revealed preference literature is to capture these departures from rationality, or errors, in the form of cost inefficiencies. The stringency of a revealed preference test can always be reduced by requiring a rationalization over the budget set $B^t(e) = \{x^t\} \cup \{x \in \mathbb{R}^\ell_+ : p^t \cdot x \leqslant e\, p^t \cdot x^t\}$ for some $e \in [0, 1]$ for all $t = 1, 2, \ldots, T$. In other words, a consumer maximizes subject to a degree of cost efficiency e. The largest value of e which rationalizes a given data set is known as the *critical cost efficiency index* (CCEI) for that data set. The basic idea is that the degree to which a consumer is cost inefficient is a reflection of his departure from full economic rationality (see, e.g., Afriat (1972, 1973), Varian (1990), Halevy, Persitz, and Zrill (2016), and Polisson, Quah, and Renou (2017)). Notice, crucially, that the budget set $B^t(e)$ is *not* convex.[15] Since the lattice approach does not require this, it can be applied straightforwardly to check for rationalizations on nonconvex budget sets, and as in Polisson, Quah, and Renou (2017), this is one of its important features, particularly with regard to empirical work.[16]

[15] See Forges and Minelli (2009) for revealed preference tests on nonconvex budget sets.

[16] Let $\mathcal{C}^t = B^t(e) = \{x \in \mathbb{R}^\ell_+ : x \leqslant x^t\} \cup \{x \in \mathbb{R}^\ell_+ : p^t \cdot x \leqslant e\, p^t \cdot x^t\}$ for some $e \in [0, 1]$ for all $t = 1, 2, \ldots, T$, and apply Lemma 1 in Appendix A.2. Notice that $\{x \in \mathbb{R}^\ell_+ : x \leqslant x^t\}$ has replaced $\{x^t\}$ in the definition of $B^t(e)$; this is due to the fact that any rationalization must be monotone.

4. Implementation

4.1 Data

We apply our revealed preference procedures to a representative sample of British consumers from the Kantar Worldpanel, which is a rolling panel of British households that record (via a barcode scanner) all food purchases which enter the home. Data between 2005 and 2012 are aggregated to the household-year-month level, which is a convention in the literature on the economics of nutrition, driven by practical aggregation considerations and also by the need to treat a household's food purchases as non-durable and non-strorable across observations.[17] We subsequently drop any household-year-months over which a given household has not made a purchase for 7 or more consecutive days in order to exclude any infrequencies. We then restrict our sample to a subsample of households that have been observed for at least 50 year-months, and for each of these households, we randomly select 50 of these year-months. The final sample therefore contains 4,027 households, each of which has been observed on 50 occasions.

We focus our empirical analysis on 4 aggregate food groups: fruit, vegetables, red meat, and poultry/fish. It is standard in observational consumer panels to have access to *expenditure* data, i.e., the econometrician observes any quantities which have been purchased and their corresponding prices. For commodities which have not been purchased, both of these amounts are of course equal to zero, and the corresponding prices are effectively 'missing'.[18] Price indices (e.g., unit prices equal to expenditures divided by quantities purchased) are then constructed for those commodities which have been purchased, and imputed for those which have not been purchased. In this paper, we construct unit prices for each aggregate food group at the household-year-month level, and any missing prices (which are due to zero purchases) are then imputed by taking an arithmetic average of year-month unit prices across any households for which the corresponding unit prices have been observed. Across 4,027 households and 50 observations on every household, only 3.7 percent of any unit prices are unobserved or unobservable due to a zero purchase, and so it would appear that the imputation is only minimally intrusive in our nutritional application.

[17] The models that we are testing are assumed to be intertemporally separable.

[18] See Crawford and Polisson (2016) for a treatment of demand analysis when prices are partially observed.

	Budget Shares	Unit Prices
Fruit	8.01 (6.05)	1.55 (0.87)
Vegetables	9.54 (5.25)	1.43 (0.75)
Red Meat	13.79 (7.63)	5.03 (2.03)
Poultry/Fish	6.84 (5.25)	5.16 (2.54)

Table 1: Market Summary Statistics

Some simple market summary statistics are reported in Table 1. Each cell contains a mean and standard deviation (in brackets). Taken altogether, fruit, vegetables, red meat, and poultry/fish account for roughly 38 percent of a household's monthly spending on food on average. The largest share of the budget goes towards red meat, followed by vegetables, then fruit, and finally poultry/fish. Furthermore, red meat and poultry/fish are more expensive on average than fruit and vegetables. While these average descriptive statistics are meant to provide a backdrop for the empirical analysis which follows, we should also note that there is substantial heterogeneity in these data.

4.2 Rationalizability

We assume that food aggregates within the subgroup {fruit, vegetables} are weakly separable from those within the subgroup {red meat, poultry/fish}, i.e., preferences over fruit and vegetables are independent of the consumption of anything other than fruit and vegetables, and similarly for red meat and poultry/fish. These assumptions are standard simplifications in applied empirical work which allow us to focus our attention *within* each subgroup, i.e., on the interaction *between* fruit and vegetables and *between* red meat and poultry/fish. The assumption is of course not beyond question since one can imagine complementarities existing *across* subgroups. However, a decision-making procedure compatible with this assumption is multi-stage budgeting, where the household first allocates an amount of expenditure to each commodity group, and then chooses the consumption levels within each of these groups, i.e., the household first decides how much money to spend on fruit and vegetables, and on red meat and poultry/fish, and then decides how much of each to purchase.[19]

The assumptions stated above allow us to focus our analysis within each subgroup, i.e., we are able to rest for rationalizability by utility maximization and additive separability within the subgroups {fruit, vegetables} and {red meat, poultry/fish}. The first natural step in the empirical analysis is to perform a series of *exact* tests. Only 14 out of 4,027 households exhibit choice behavior that is consistent with the maximization of a stable preference over fruit and vegetables, and among these, just 7 are consistent with additive separability. Rates of rationalizability are similarly low for red meat and poultry/fish, with only 23 households rationalizable by utility maximization, and 21 by additive separability.

These rates are exceptionally low, which is unsurprising given the sharpness of the exact tests, which are applied to data sets containing 50 observations each. In order to proceed empirically in a meaningful direction, it is necessary to take into account the distances or departures from rationality, or in the language of econometrics, to allow for error terms. As described in Section 3.4, following Halevy, Persitz, and Zrill (2016) and Polisson, Quah, and Renou (2017), among others, we extend the notion of Afriat's (1972, 1973) cost inefficiencies to the additively separable case, which allows us to calculate a critical cost efficiency index (CCEI) for both utility maximization and additive separability.[20]

	Utility Maximization		Additive Separability	
	CCEI	95% CI	CCEI	95% CI
Fruit, Vegetables	0.9143 (0.0013)	[0.9117, 0.9169]	0.9084 (0.0013)	[0.9058, 0.9110]
Red Meat, Poultry/Fish	0.8765 (0.0017)	[0.8732, 0.8798]	0.8701 (0.0017)	[0.8668, 0.8733]

Table 2: Rationalizability Results

Some basic rationalizability results are displayed in Table 2. Each CCEI cell contains a mean and standard error (in brackets), from which we then construct 95 percent confidence intervals.[21] For fruit and vegetables, the mean CCEI associated with utility maximization is

[19] This procedure would, however, also require homotheticity within groups. See Gorman (1959).

[20] Of course, the conventional econometric strategy is to introduce additive error terms to consumption choices according to a minimum distance criterion. See Varian (1990) and Halevy, Persitz, and Zrill (2016) for a comparison of the revealed preference and distance based approaches.

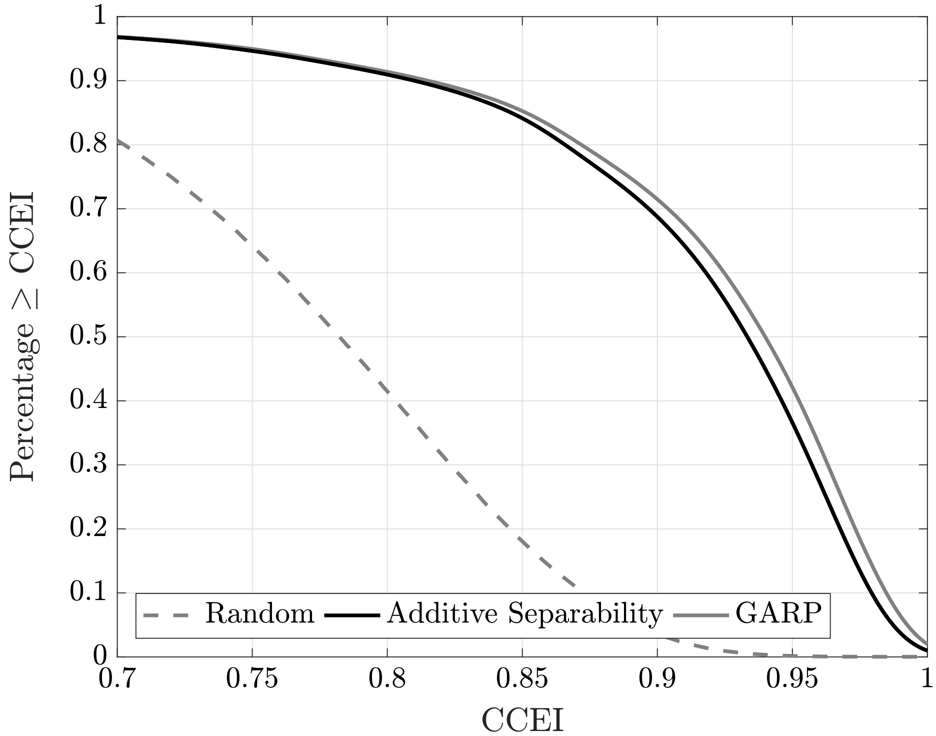
[21] With a sample size of 4,027 households, the standard errors are inevitably going to be very small, and

about 0.9143, which means that the average household's monthly budget needs to be reduced by about 9 percent in order for the data to be rationalizable by utility maximization. In other words, the average household is wasting about 9 percent of its monthly budget on fruit and vegetables by departing from rationality in the form of utility maximization. A household's CCEI for additive separability is *necessarily* going to be lower than for utility maximization, which is due to the fact that additive separability is a more restrictive nested model. As shown in Table 2, the mean CCEI for additive separability is 0.9084 for fruit and vegetables, which is only moderately lower than 0.9143 in a meaningful economic sense. The pattern is similar for red meat and poultry/fish, where the mean CCEI for utility maximization is 0.8765, and 0.8701 for additive separability.

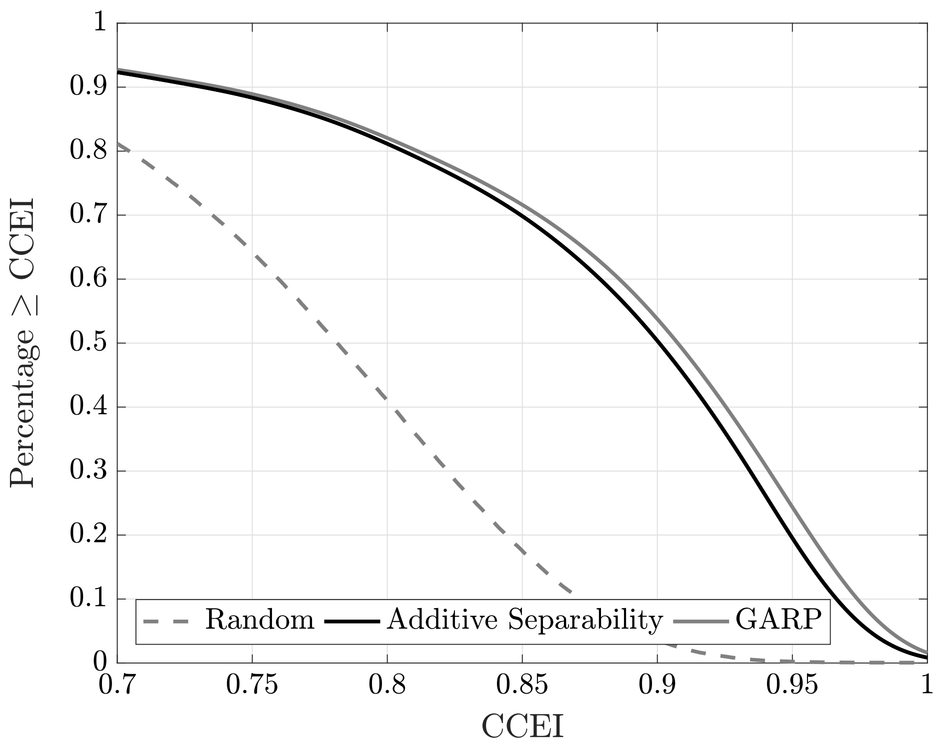
In order to examine the heterogeneity in CCEIs, we plot their distributions in Figure 4. For fruit and vegetables, about 72 percent of households have a CCEI above 0.9 for utility maximization, and about 91 percent have a CCEI above 0.8. In the additively separable case, these figures are 69 and 91 percent at the 0.9 and 0.8 efficiency levels, respectively. What emerges at first glance is that the data are largely rationalizable by *both* utility maximization *and* additive separability. In fact, the differences between the two models appear to be only very slight; one might argue as before that these differences are not economically meaningful. A similar story can be told for red meat and poultry/fish, where about 55 percent have a CCEI above 0.9 for utility maximization, and about 82 percent have a CCEI above 0.8. These figures are 51 percent and 81 percent at the 0.9 and 0.8 efficiency levels, respectively, in the additively separable case. What also emerges is that rates of rationalizability for both utility maximization and additive separability are lower within the subgroup {red meat, poultry/fish} than {fruit, vegetables} at all efficiency levels.

While at first glance these results lend some support to both utility maximization and additive separability as plausible modes of explanation, it is useful to make a comparison to an alternative behavioral hypothesis. Following the convention in the empirical revealed preference literature, we adopt as our alternative the 'irrational consumer' of Becker (1962), who is naively choosing randomly uniformly from the frontiers of his budget sets. In our

furthermore, any finite sample distributional properties are going to converge to large sample distributional properties. In Table 2, 95 percent confidence intervals are constructed using the standard normal distribution. Simple bootstrapping, i.e., resampling with replacement 10^6 times, produces identical results.



(a) Fruit, Vegetables



(b) Red Meat, Poultry/Fish

Figure 4: CCEI Distributions

procedure, we simulate data at the household level, first within the subgroup {fruit, vegetables} and then within the subgroup {red meat, poultry/fish}.[22] The dashed gray lines in Figures 4a and 4b correspond to the CCEI distributions for utility maximization among the simulated households. Since the CCEI distributions for additive separability among the simulated households would be even lower than for utility maximization, the visual comparisons in Figure 4 are conservative. The differences between our panel of British households and their simulated 'irrational' counterparts are noticeably pronounced, again suggesting that both utility maximization and additive separability provide considerable explanation.

While these basic rationalizability results are indeed suggestive, in the remaining three subsections we formalize our empirical claims, also in a statistical sense. To do so, we decompose the analysis into three parts, first focusing on pass rates, then on restrictive and predictive power, and finally on a measure of predictive success which aggregates the two.

4.3 Pass Rates

It is natural to more deeply examine the performances of these models in terms of their pass rates at different efficiency thresholds. For example, fixing the efficiency level at 0.95, we can ask how many households are rationalizable by utility maximization or additive separability, allowing for 5 percent cost inefficiency. Table 3 displays pass rates and exact 95% confidence intervals for utility maximization and additive separability at the 0.85, 0.9, and 0.95 efficiency thresholds;[23] panel (1) corresponds to utility maximization, panel (2) to additive separability, and panel (3) to additive separability *conditional on* utility maximization. Notice that with so many households, all of these pass rates are very precisely estimated.

As shown in panels (1) and (2) of Table 3, our most basic empirical finding is that utility maximization explains more of the data than additive separability,[24] but only modestly.

[22] For example, within the subgroup {fruit, vegetables}, we generate a random sample of 100,000 simulated or synthetic households, each of whom is choosing randomly uniformly from 50 budget lines. For each synthetic household, a set of 50 budgets is selected at random (and with equal probability) from among the 4,027 sets that have been observed in our sample (assuming that the sample is random and representative of the population). We conduct a similar exercise within the subgroup {red meat, poultry/fish}.

[23] At a given efficiency threshold, a household either passes or fails the test for a given model; the underlying random variable is therefore Bernoulli distributed, and the sample pass rates are then binomially distributed. Confidence intervals are *exact* in the sense that they are obtained from the binomial distribution using the procedure first proposed by Clopper and Pearson (1934).

[24] Recall that since the models are nested, it must be the case that utility maximization cannot do worse

(1)	Utility Maximization		
	$e = 0.85$	$e = 0.90$	$e = 0.95$
Fruit, Vegetables	0.85 [0.84, 0.87]	0.72 [0.70, 0.73]	0.43 [0.41, 0.45]
Red Meat, Poultry/Fish	0.72 [0.70, 0.73]	0.55 [0.53, 0.56]	0.24 [0.23, 0.26]
(2)	Additive Separability		
	$e = 0.85$	$e = 0.90$	$e = 0.95$
Fruit, Vegetables	0.84 [0.83, 0.85]	0.69 [0.68, 0.70]	0.37 [0.36, 0.39]
Red Meat, Poultry/Fish	0.70 [0.69, 0.72]	0.51 [0.50, 0.53]	0.19 [0.18, 0.20]
(3)	Additive Separability | Utility Maximization		
	$e = 0.85$	$e = 0.90$	$e = 0.95$
Fruit, Vegetables	0.99 [0.98, 0.99]	0.96 [0.95, 0.97]	0.87 [0.85, 0.88]
Red Meat, Poultry/Fish	0.98 [0.97, 0.98]	0.94 [0.93, 0.95]	0.77 [0.75, 0.80]

Table 3: Pass Rates and 95% Confidence Intervals

For example, for fruit and vegetables, and at an efficiency threshold of 0.85, about 85% of households are rationalizable by utility maximization, and 84% by additive separability; furthermore, this difference is statistically significant at a 0.05 significance level.[25] This empirical finding, that utility maximization explains statistically more of the data than additive separability, but only modestly, holds across both subgroups and all efficiency levels.

A second empirical finding, which is shown in panels (1) and (2) of Table 3, is that rates of rationalizability for both utility maximization *and* additive separability within the subgroup {fruit, vegetables} are noticeably higher than their counterparts within the subgroup {red meat, poultry/fish}, a statement which holds across efficiency thresholds, and which is also statistically robust.[26] It is of course natural to examine pass rates for additive separability *conditional* on already being rationalizable by utility maximization; these are displayed in

than additive separability in terms of rationalizing the data.

[25] If we observe a even single household that is rationalizable by utility maximization but not by additive separability (which is nested within utility maximization), then the true pass rates cannot be equal.

[26] By statistically robust, we mean that a difference in pass rates (which are non-nested but realized within a single sample) is statistically different from zero at a 0.05 significance level.

panel (3) of Table 3. Within both subgroups, and across efficiency levels, these conditional pass rates are very high; furthermore, they are modestly higher across efficiency thresholds within the subgroup {fruit, vegetables}, and these differences are statistically significant.

4.4 Power

Rationalizability, be it exact or approximate (in terms of cost inefficiencies), only measures whether a given data set is consistent with utility maximization, additive separability, or additive separability given utility maximization. These models, and their tests, can be more or less observationally stringent, depending on both the theoretical structure in play and the particular data which have been observed. Bronars (1987) proposed to measure the *power* of a model/test as the probability that a consumer who is choosing randomly uniformly from his budget frontiers will *fail* the test for a given model. Since Bronars (1987), it has become standard in applied empirical work in the revealed preference literature to assess power in this way. In practice, we numerically approximate the power by simulating data; we generate 100,000 synthetic data sets (as described in Section 4.2), and calculate the fraction of these data sets which fail the tests for utility maximization and additive separability at specified efficiency thresholds.[27] To numerically approximate the *conditional power* of additive separability, we first fix an efficiency threshold, say 0.9, and generate 100,000 synthetic data sets (again as described in Section 4.2), which also obey GARP, i.e., which are rationalizable by utility maximization in the first place.[28]

Table 4 displays the Bronars (1987) power measures for utility maximization and additive separability at the 0.85, 0.9, and 0.95 efficiency thresholds; panel (1) again corresponds to utility maximization, panel (2) to additive separability, and panel (3) to additive separability conditional on utility maximization. As shown in panels (1) and (2) of Table 4, the unconditional power for both utility maximization and additive separability is very high, and this holds within both subgroups and across efficiency thresholds. For example, for fruit and

[27] It is clear from Hoeffding's (1963) inequality (or by plotting the simulation results against the number of simulations) that 100,000 random draws are sufficiently many to guarantee convergence.

[28] The conditional power procedure is slightly more complicated. For each synthetic household, a set of 50 budgets has again been selected at random from among the sample of 4,027; however, to generate random choices from these 50 budgets which obey GARP, we do so *sequentially*. See Polisson, Quah, and Renou (2017) for the details of such a procedure.

(1)	Utility Maximization		
	$e = 0.85$	$e = 0.90$	$e = 0.95$
Fruit, Vegetables	0.82	0.96	1.00
Red Meat, Poultry/Fish	0.82	0.96	1.00
(2)	Additive Separability		
	$e = 0.85$	$e = 0.90$	$e = 0.95$
Fruit, Vegetables	0.85	0.98	1.00
Red Meat, Poultry/Fish	0.86	0.98	1.00
(3)	Additive Separability | Utility Maximization		
	$e = 0.85$	$e = 0.90$	$e = 0.95$
Fruit, Vegetables	0.43	0.76	0.98
Red Meat, Poultry/Fish	0.45	0.79	0.99

Table 4: Bronars Power

vegetables, and at an efficiency threshold of 0.85, about 18% of the simulated households are rationalizable by utility maximization, and 15% by additive separability; naturally, the power measures are greater still at higher efficiency thresholds. Furthermore, the differences in power across the two subgroups appear only to be very slight, i.e., their unconditional power is nearly identical at every efficiency threshold.

The *conditional power* of additive separability is shown in panel (3) of Table 4. Within both subgroups, and across efficiency levels, these conditional measures are much lower than in the unconditional case. For example, for fruit and vegetables, and at an efficiency threshold of 0.85, about 57% of the simulated households which are already rationalizable by utility maximization are subsequently rationalizeable by additive separability; as in the unconditional case, the conditional power measures are greater at higher efficiency thresholds, with nearly perfect power at the 0.95 efficiency threshold. Once again, the differences in conditional power across the two subgroups appear only to be very slight.

Given that the unconditional power of both utility maximization and additive separability is very high, and also that these power measures (both unconditional and conditional) exhibit very little variation across subgroups, many of the conclusions that were drawn from the investigation of basic pass rates in Section 4.3 would at first glance appear to be robust. We formalize this notion in the remaining subsection.

4.5 Adjusted Pass Rates

While the empirical results (in terms of pass rates and power) presented so far suggest that both utility maximization and additive separability have meaningful explanatory purchase in these data, we require a more formal procedure in order to make a robust claim. To this end, we appeal to an *index of predictive success* first proposed by Selten (1991). The Selten index aggregates two components: (1) the relative frequency of correct predictions, i.e., the 'hit rate' which is equal to the pass rate, and (2) the relative size of the set of predicted outcomes, i.e., the 'imprecision' which is equal to one minus the power. Selten (1991) shows that a simple difference between the two obeys several desirable axiomatic properties,[29] and subsequently proves that this index cannot be improved upon by any other measure of predictive success which takes into account the same information.[30] Building upon Bronars (1987), Beatty and Crawford (2011) introduced the Selten index into the revealed preference literature, and it has since been adopted widely in empirical work. The essence of the approach is to mitigate the success of model in making correct predictions with the (im)precision of those predictions, i.e., to *adjust* pass rates in light of power/precision. As such, we refer to the Selten index of predictive success as an *adjusted pass rate*.

Table 5 displays the adjusted pass rates, or Selten indices of predictive success, and 95% confidence intervals (obtained from a bootstrap) for utility maximization and additive separability across efficiency thresholds;[31] panels (1)–(3) correspond to utility maximization and additive separability (conditional on utility maximization) as in Tables 3 and 4. The first observation is that all models, both unconditionally and conditionally, and across efficiency thresholds, have positive predictive success which is statistically different from zero.[32] In

[29] The Selten index satisfies three axioms: (1) monotonicity, i.e., passing a perfectly stringent test is better than failing a perfectly lenient test; (2) equivalence, i.e., passing a perfectly lenient test is no more or less informative than failing a perfectly stringent test; and (3) aggregability, i.e., the measure can be aggregated across members of a heterogeneous sample, which is of course important in empirical work.

[30] To be precise, any other measure of predictive success which obeys the same set of axioms must be an affine transformation of this simple difference.

[31] Notice that the adjusted pass rates are equal to the pass rates minus the imprecision, where the latter is equal to one minus the power (see Section 4.4). Since power has been obtained numerically, we appeal to a simple bootstrap, i.e., a resampling (10^6 samples) of adjusted pass rates with replacement, in order to obtain standard errors and confidence intervals.

[32] To make this statement, we use bootstrapped standard errors in order to perform a t-test of the null hypothesis that predictive success is equal to zero against a two-sided alternative at a 0.05 significance level.

(1)	Utility Maximization		
	$e = 0.85$	$e = 0.90$	$e = 0.95$
Fruit, Vegetables	0.67 $[0.66, 0.69]$	0.68 $[0.67, 0.70]$	0.43 $[0.41, 0.44]$
Red Meat, Poultry/Fish	0.54 $[0.53, 0.56]$	0.51 $[0.50, 0.53]$	0.24 $[0.23, 0.26]$
(2)	Additive Separability		
	$e = 0.85$	$e = 0.90$	$e = 0.95$
Fruit, Vegetables	0.70 $[0.68, 0.71]$	0.67 $[0.65, 0.68]$	0.37 $[0.36, 0.39]$
Red Meat, Poultry/Fish	0.57 $[0.55, 0.58]$	0.49 $[0.48, 0.51]$	0.19 $[0.18, 0.20]$
(3)	Additive Separability | Utility Maximization		
	$e = 0.85$	$e = 0.90$	$e = 0.95$
Fruit, Vegetables	0.41 $[0.41, 0.42]$	0.72 $[0.72, 0.73]$	0.85 $[0.83, 0.86]$
Red Meat, Poultry/Fish	0.43 $[0.42, 0.43]$	0.73 $[0.72, 0.74]$	0.76 $[0.73, 0.79]$

Table 5: Adjusted Pass Rates and 95% Confidence Intervals

other words, the ability of these models to explain the data remains high, even after making adjustments for power/precision.

From Section 4.3, our most basic empirical finding was that utility maximization explains more of the data than additive separability, but only modestly, and that these differences are statistically significant; such a claim can be moderated further after adjusting for power/precision. The adjusted gaps (shown in panels (1) and (2) of Table 5) are narrower than the unadjusted gaps (shown in panels (1) and (2) of Table 3), and in some cases (at the 0.85 efficiency threshold) the direction is reversed, and all differences are statistically significant at a 0.05 significance level. Our primary empirical finding from Section 4.3 was that the rates of rationalizability for both utility maximization *and* additive separability are substantively (and statistically) higher within the subgroup {fruit, vegetables} than {red meat, poultry/fish}; this finding holds (statistically) even after adjusting for power/precision (see again panels (1) and (2) of Table 5). Lastly, the adjusted conditional pass rates (shown in panel (3) of Table 5) are noticeably lower than the unadjusted pass rates (shown in panel

(3) of Table 3), although they remain (statistically) greater than zero. Furthermore, the adjusted pass rates are substantially higher within the subgroup {fruit, vegetables} than {red meat, poultry/fish} at the 0.95 efficiency threshold (suggesting greater substitutability/complementarity within the latter subgroup), and only modestly lower at the 0.85 efficiency threshold; these differences are statistically significant at a 0.05 significance level.[33]

5. Conclusions

In this paper, we develop and implement a nonparametric lattice test for additive separability. While this preference structure is commonly used to simplify the analysis of consumer choice, it has long been thought that it imposes unreasonable restrictions on observable data, i.e., "the price [of additivity] is too high". The lattice approach, which is purely nonparametric and applied to finite data, suggests that perhaps the empirical price of additivity is not as high as it might first appear. Future work could extend the approach developed in this paper to new and different empirical settings with little difficulty.

Appendix A

A.1 Concave Additive Separability

Suppose that ℓ goods are divided into two groups, and that there are ℓ_1 goods in group 1 and ℓ_2 goods in group 2, so that $\ell_1 + \ell_2 = \ell$. Now the data set $\{(p^t, x^t)\}_{t=1}^T$ can be partitioned into two subsets $\{(q^t, y^t)\}_{t=1}^T$ and $\{(r^t, z^t)\}_{t=1}^T$, which correspond to groups 1 and 2.[34]

Definition 7. *The data set $\{(p^t, x^t)\}_{t=1}^T$, which can be partitioned into the two subsets $\{(q^t, y^t)\}_{t=1}^T$ and $\{(r^t, z^t)\}_{t=1}^T$, is rationalizable by concave additive separability if there exist increasing, concave, and continuous utility functions $v : \mathbb{R}_+^{\ell_1} \to \mathbb{R}$ and $w : \mathbb{R}_+^{\ell_2} \to \mathbb{R}$, such that, at every observation $t = 1, 2, \ldots, T$,*

$$v(y^t) + w(z^t) \geqslant v(y) + w(z) \ \text{ for any } \ x = (y, z) \in \{x \in \mathbb{R}_+^\ell : p^t \cdot x \leqslant p^t \cdot x^t\}.$$

Theorem 3. *The following statements are equivalent:*

[33] There is no statistical difference between the two subgroups at the 0.90 efficiency threshold.

[34] Notice that $y^t \in \mathbb{R}_+^{\ell_1}$, $z^t \in \mathbb{R}_+^{\ell_2}$, $q^t \in \mathbb{R}_{++}^{\ell_1}$, and $r^t \in \mathbb{R}_{++}^{\ell_2}$, so that $x^t = (y^t, z^t)$ and $p^t = (q^t, r^t)$ for all $t = 1, 2, \ldots, T$.

1. *The data set* $\{(p^t, x^t)\}_{t=1}^T$*, which can be partitioned into the two subsets* $\{(q^t, y^t)\}_{t=1}^T$ *and* $\{(r^t, z^t)\}_{t=1}^T$*, is rationalizable by concave additive separability.*

2. *For the data set* $\{(p^t, x^t)\}_{t=1}^T$*, which can be partitioned into the two subsets* $\{(q^t, y^t)\}_{t=1}^T$ *and* $\{(r^t, z^t)\}_{t=1}^T$*, at every observation* $t = 1, 2, \ldots, T$*, there exist numbers* v^t*,* $w^t \in \mathbb{R}$ *and* $\lambda^t \in \mathbb{R}_{++}$*, such that*

$$v^{t'} \leqslant v^t + \lambda^t q^t \cdot (y^{t'} - y^t) \text{ for all } t, t' = 1, 2, \ldots, T, \tag{V1}$$

$$w^{t'} \leqslant w^t + \lambda^t r^t \cdot (z^{t'} - z^t) \text{ for all } t, t' = 1, 2, \ldots, T. \tag{V2}$$

Proof of Theorem 3. (1) $\implies$ (2): If the data set $\{(p^t, x^t)\}_{t=1}^T$, which can be partitioned into the two subsets $\{(q^t, y^t)\}_{t=1}^T$ and $\{(r^t, z^t)\}_{t=1}^T$, is rationalizable by concave additive separability, then at every observation $t = 1, 2, \ldots, T$, there exists a number $\lambda^t \in \mathbb{R}_{++}$, such that $\lambda^t q^t \in \partial v(y^t)$ and $\lambda^t r^t \in \partial w(z^t)$ for all $t = 1, 2, \ldots, T$, where $\partial v(y^t)$ and $\partial w(z^t)$ denote the super-differentials of v and w evaluated at y^t and z^t. Notice that the elements of $\partial v(y^t)$ and $\partial w(z^t)$ are positive since v and w are increasing. Given the concavity of v and w,

$$v(y^{t'}) \leqslant v(y^t) + \partial v(y^t) \cdot (y^{t'} - y^t) \text{ for all } t, t' = 1, 2, \ldots, T,$$

$$w(z^{t'}) \leqslant w(z^t) + \partial w(z^t) \cdot (z^{t'} - z^t) \text{ for all } t, t' = 1, 2, \ldots, T.$$

Together with the first-order conditions, the above inequalities imply that at every observation $t = 1, 2, \ldots, T$, there exists a number $\lambda^t \in \mathbb{R}_{++}$, such that

$$v(y^{t'}) \leqslant v(y^t) + \lambda^t q^t \cdot (y^{t'} - y^t) \text{ for all } t, t' = 1, 2, \ldots, T,$$

$$w(z^{t'}) \leqslant w(z^t) + \lambda^t r^t \cdot (z^{t'} - z^t) \text{ for all } t, t' = 1, 2, \ldots, T.$$

Letting $v^t = v(y^t)$ and $w^t = w(z^t)$ for all $t = 1, 2, \ldots, T$, implies that at every observation $t = 1, 2, \ldots, T$, there exist numbers v^t, $w^t \in \mathbb{R}$ and $\lambda^t \in \mathbb{R}_{++}$, such that $(V1)$ and $(V2)$ hold.

(2) $\implies$ (1): Given the data set $\{(p^t, x^t)\}_{t=1}^T$, which can be partitioned into the two subsets $\{(q^t, y^t)\}_{t=1}^T$ and $\{(r^t, z^t)\}_{t=1}^T$, if, at every observation $t = 1, 2, \ldots, T$, there exist numbers v^t, $w^t \in \mathbb{R}$ and $\lambda^t \in \mathbb{R}_{++}$, such that $(V1)$ and $(V2)$ hold, then for any $y \in \mathbb{R}_+^{\ell_1}$ and $z \in \mathbb{R}_+^{\ell_2}$, define the functions $v : \mathbb{R}_+^{\ell_1} \to \mathbb{R}$ and $w : \mathbb{R}_+^{\ell_2} \to \mathbb{R}$ according to

$$v(y) = \min_t \{v^t + \lambda^t q^t \cdot (y - y^t)\},$$

$$w(z) = \min_t \{w^t + \lambda^t r^t \cdot (z - z^t)\}.$$

Notice that v and w are increasing, concave, and continuous. By the definitions of v and w,

$$v(y^t) = v^m + \lambda^m q^m \cdot (y^t - y^m) \leqslant v^t + \lambda^t q^t \cdot (y^t - y^t) = v^t \text{ for all } t = 1, 2, \ldots, T,$$

$$w(z^t) = w^n + \lambda^n r^n \cdot (z^t - z^n) \leqslant w^t + \lambda^t r^t \cdot (z^t - z^t) = w^t \text{ for all } t = 1, 2, \ldots, T,$$

for some m and n, which hold with equality in order not to violate $(V1)$ and $(V2)$. Therefore, $v(y^t) = v^t$ and $w(z^t) = w^t$ for all $t = 1, 2, \ldots, T$. At any observation $t = 1, 2, \ldots, T$, choose some $x = (y, z) \in \{x \in \mathbb{R}^\ell_+ : p^t \cdot x \leqslant p^t \cdot x^t\}$. Again by the definitions of v and w,

$$v(y) \leqslant v^t + \lambda^t q^t \cdot (y - y^t),$$

$$w(z) \leqslant w^t + \lambda^t r^t \cdot (z - z^t).$$

Summing the above inequalities implies that

$$\begin{aligned} v(y) + w(z) &\leqslant v^t + w^t + \lambda^t q^t \cdot (y - y^t) + \lambda^t r^t \cdot (z - z^t) \\ &= v^t + w^t + \lambda^t p^t \cdot (x - x^t) \\ &\leqslant v^t + w^t \\ &= v(y^t) + w(z^t). \end{aligned}$$

□

A.2 Proof of Theorem 2

The proof of Theorem 2 makes use of the following lemma.

LEMMA 1. *Let $\{(x^t, \mathcal{C}^t)\}_{t=1}^T$ denote a finite set of observations drawn on an individual consumer, where $x^t \in \mathbb{R}^\ell_+$ corresponds to the consumption bundle chosen from the constraint set $\mathcal{C}^t \subset \mathbb{R}^\ell_+$ at every observation $t = 1, 2, \ldots, T$. Further suppose that each $\mathcal{C}^t$ is compact and downward closed, i.e., for any $x \in \mathcal{C}^t$ and $y \in \mathbb{R}^\ell_+$, if $y < x$, then $y \in \mathcal{C}^t$.*[35] *Let $\partial\mathcal{C}^t$ denote the upper boundary of $\mathcal{C}^t$, i.e., $x \in \partial\mathcal{C}^t$ if there is no $y \in \mathcal{C}^t$ such that $y > x$, and assume that $x^t \in \partial\mathcal{C}^t$ for all $t = 1, 2, \ldots, T$. If there is a finite collection of sets $\{\mathcal{X}_i\}_{i=1}^\ell$,*

[35] For any x, $y \in \mathbb{R}^\ell_+$, we say that $x \geqslant y$ if $x_i \geqslant y_i$ for all $i = 1, 2, \ldots, \ell$, that $x > y$ if $x \geqslant y$ and $x \neq y$, and that $x \gg y$ if $x_i > y_i$ for all $i = 1, 2, \ldots, \ell$.

where each finite set $\mathcal{X}_i = \{x \in \mathbb{R}_+ : x = x_i^t \text{ for some } t\} \cup \{0\}$ *for all* $i = 1, 2, \ldots, \ell$, *a finite lattice* $\mathcal{L} = \prod_{i=1}^{\ell} \mathcal{X}_i$, *and a finite collection of functions* $\{\bar{v}_i\}_{i=1}^{\ell}$, *where each* $\bar{v}_i : \mathcal{X}_i \to \mathbb{R}$ *is increasing for all* $i = 1, 2, \ldots, \ell$, *such that, at every observation* $t = 1, 2, \ldots, T$,

$$\sum_{i=1}^{\ell} \bar{v}_i(x_i^t) \geqslant \sum_{i=1}^{\ell} \bar{v}_i(x_i) \text{ for all } x \in \mathcal{C}^t \cap \mathcal{L},$$

$$\sum_{i=1}^{\ell} \bar{v}_i(x_i^t) > \sum_{i=1}^{\ell} \bar{v}_i(x_i) \text{ for all } x \in (\mathcal{C}^t / \partial \mathcal{C}^t) \cap \mathcal{L},$$

then there is a finite collection of functions $\{v_i\}_{i=1}^{\ell}$, *where each* $v_i : \mathbb{R}_+ \to \mathbb{R}$ *is increasing and continuous for all* $i = 1, 2, \ldots, \ell$, *such that, at every observation* $t = 1, 2, \ldots, T$,

$$\sum_{i=1}^{\ell} v_i(x_i^t) \geqslant \sum_{i=1}^{\ell} v_i(x_i) \text{ for all } x \in \mathcal{C}^t.$$

Proof. For each $i = 1, 2, \ldots, \ell$, order the finite set $\mathcal{X}_i$ so that $\mathcal{X}_i = \{z_i^0, z_i^1, z_i^2, \ldots, z_i^{K_i-1}, z_i^{K_i}\}$, where $z_i^0 = 0 < z_i^1 < z_i^2 < \cdots < z_i^{K_i-1} < z_i^{K_i}$. For each $i = 1, 2, \ldots, \ell$, define the function $v_i : \mathbb{R}_+ \to \mathbb{R}$ according to

$$v_i(z) = \begin{cases} \bar{v}_i(z_i^0) + \left(\dfrac{\epsilon_i^1}{\Delta z_i^1}\right)(z - z_i^0) & \text{for } z \in [z_i^0, z_i^1 - \epsilon_i^1] \\ \bar{v}_i(z_i^1) + \left(\dfrac{\Delta \bar{v}_i^1}{\epsilon_i^1}\right)(z - z_i^1) & \text{for } z \in [z_i^1 - \epsilon_i^1, z_i^1] \\ \bar{v}_i(z_i^1) + \left(\dfrac{\epsilon_i^2}{\Delta z_i^2}\right)(z - z_i^1) & \text{for } z \in [z_i^1, z_i^2 - \epsilon_i^2] \\ \bar{v}_i(z_i^2) + \left(\dfrac{\Delta \bar{v}_i^2}{\epsilon_i^2}\right)(z - z_i^2) & \text{for } z \in [z_i^2 - \epsilon_i^2, z_i^2] \\ \cdots & \text{for } \cdots \\ \bar{v}_i(z_i^{K_i-1}) + \left(\dfrac{\epsilon_i^{K_i}}{\Delta z_i^{K_i}}\right)(z - z_i^{K_i-1}) & \text{for } z \in [z_i^{K_i-1}, z_i^{K_i} - \epsilon_i^{K_i}] \\ \bar{v}_i(z_i^{K_i}) + \left(\dfrac{\Delta \bar{v}_i^{K_i}}{\epsilon_i^{K_i}}\right)(z - z_i^{K_i}) & \text{for } z \in [z_i^{K_i} - \epsilon_i^{K_i}, z_i^{K_i}] \\ \bar{v}_i(z_i^{K_i}) + \epsilon_i^{K_i+1}(z - z_i^{K_i}) & \text{for } z \in [z_i^{K_i}, +\infty) \end{cases},$$

for some suitable vector $\epsilon_i = (\epsilon_i^1, \epsilon_i^2, \ldots, \epsilon_i^{K_i-1}, \epsilon_i^{K_i}, \epsilon_i^{K_i+1}) \gg 0$, where $\Delta z_i^k = (z_i^k - z_i^{k-1}) - \epsilon_i^k$ and $\Delta \bar{v}_i^k = (\bar{v}_i(z_i^k) - \bar{v}_i(z_i^{k-1})) - \epsilon_i^k$, so that each $v_i : \mathbb{R}_+ \to \mathbb{R}$ is increasing and continuous, and that, at every observation $t = 1, 2, \ldots, T$,

$$\sum_{i=1}^{\ell} v_i(x_i^t) \geqslant \sum_{i=1}^{\ell} v_i(x_i) \ \text{ for all } \ x \in \mathcal{C}^t.$$

Suppose, to the contrary, that no choice of $\{\epsilon_i\}_{i=1}^{\ell}$ satisfies the above requirements, i.e., for every choice of $\{\epsilon_i\}_{i=1}^{\ell}$, at some observation t,

$$\sum_{i=1}^{\ell} v_i(x_i^t) < \sum_{i=1}^{\ell} v_i(x_i) \ \text{ for some } \ x \in \mathcal{C}^t.$$

Therefore, it must be the case that at some observation t,

$$\begin{aligned}\sum_{i=1}^{\ell} \bar{v}_i(x_i^t) &= \sum_{i=1}^{\ell} v_i(x_i^t) \\ &< \sum_{i=1}^{\ell} v_i(x_i) \\ &= \sum_{i=1}^{\ell} \bar{v}_i(y_i) + \delta(\{\epsilon_i\}_{i=1}^{\ell}),\end{aligned}$$

for some $x \in \mathcal{C}^t$ and some $y \in \mathcal{C}^t \cap \mathcal{L}$ or $y \in (\mathcal{C}^t / \partial\mathcal{C}^t) \cap \mathcal{L}$, with $y \leqslant x$; since $\{\epsilon_i\}_{i=1}^{\ell}$ can be chosen in order to make $\delta(\{\epsilon_i\}_{i=1}^{\ell}) > 0$ sufficiently small, this can only be true when

$$\sum_{i=1}^{\ell} \bar{v}_i(x_i^t) < \sum_{i=1}^{\ell} \bar{v}_i(y_i) \ \text{ for some } \ y \in \mathcal{C}^t \cap \mathcal{L},$$

in the case where $y \in \mathcal{C}^t \cap \mathcal{L}$, or when

$$\sum_{i=1}^{\ell} \bar{v}_i(x_i^t) = \sum_{i=1}^{\ell} \bar{v}_i(y_i) \ \text{ for some } \ y \in (\mathcal{C}^t / \partial\mathcal{C}^t) \cap \mathcal{L},$$

in the case where $y \in (\mathcal{C}^t / \partial\mathcal{C}^t) \cap \mathcal{L}$. And so we have a contradiction. □

Proof of Theorem 2. (1) $\Longrightarrow$ (2): If the data set $\{(p^t, x^t)\}_{t=1}^{T}$ is rationalizable by additive separability, then for each $i = 1, 2, \ldots, \ell$, let $\bar{v}_i(x_i^t) = v_i(x_i^t)$ for all $t = 1, 2, \ldots, T$.

(2) $\Longrightarrow$ (1): Given the data set $\{(p^t, x^t)\}_{t=1}^{T}$, if there exist numbers $\bar{v}_i(x_i) \in \mathbb{R}$ for each $x_i \in \mathcal{X}_i = \{x \in \mathbb{R}_+ : x = x_i^t \ \text{ for some } \ t\} \cup \{0\}$ for all $i = 1, 2, \ldots, \ell$, with $\bar{v}_i(x_i) > \bar{v}_i(y_i)$ whenever $x_i > y_i$ for any x_i, $y_i \in \mathcal{X}_i$, such that $(A1)$ and $(A2)$ hold at every observation $t = 1, 2, \ldots, T$, then let $\mathcal{C}^t = B^t$ for all $t = 1, 2, \ldots, T$, and apply Lemma 1. □

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