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IDENTIFYING THE EFFECT OF PERSUASION*

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Abstract. We set up an econometric model of persuasion and study identification of key parameters under various scenarios of data availability. We find that a commonly used measure of persuasion does not estimate the persuasion rate of any population in general. We provide formal identification results, recommend several new parameters to estimate, and discuss their interpretation. We revisit two strands of the empirical literature on persuasion to show that the persuasive effect is highly heterogeneous and studies based on binary instruments provide limited information about the average persuasion rate in a heterogeneous population.

Key Words: Communication, Media, Persuasion, Partial Identification, Treatment Effects

JEL Classification Codes: C21, D72, L82

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1. Introduction

How effectively one can accomplish persuasion has been of interest to ancient Greek philosophers in the Lyceum of Athens,[1] to early–modern English preachers in St Paul's Cathedral,[2] and to contemporary American news producers at Fox News in New York City.[3] Recently economists have been endeavoring to build theoretical models of persuasion (e.g. Kamenica and Gentzkow, 2011; Che, Dessein, and Kartik, 2013; Gentzkow and Kamenica, 2017; Bergemann and Morris, 2017; Prat, 2017) and to quantify empirically to what extent persuasive effort affects the behavior of consumers, voters, donors, and investors (see DellaVigna and Gentzkow, 2010, for a survey of the recent literature).

In this paper, we set up an econometric model of persuasion, point out the key parameters of interest, and study their identification under various scenarios of data availability. Since we have observational data in mind, it is important that we allow for endogeneity, i.e. the possibility that an agent's decision on an exposure to persuasive information is correlated with her potential actions. To convey the idea, we use DellaVigna and Kaplan (2007, DK hereafter) as a running example. DK study the effect of an exposure to Fox News on the probability of voting for a Republican presidential candidate. Here, the persuasive information of interest is the viewership of the Fox News channel, where an agent's decision about whether to watch Fox News or not may be correlated with her political orientation. To address this issue, we assume that the econometrician has an instrumental variable at his disposal: in DK's study, they rely on the premise that Fox News availability via local cable in 2000 seems random after controlling for a set of covariates. Later we will discuss this example again within the potential outcome framework.

Before introducing the key parameter in the paper, we recall the persuasion rate used in DK, DellaVigna and Gentzkow (2010) and many others: for a binary outcome, it is defined by

$$f = \frac{y_T - y_C}{e_T - e_C} \cdot \frac{1}{1 - y_0},$$

where y_j is the share of group j adopting the behavior of interest (e.g. voting for a Republican candidate), e_j is the share of group j exposed to persuasion, $j \in \{T, C\}$, and T and C denote treatment and control groups (e.g. having Fox News available via local cable or not), respectively. Here, y_0 is

[1] See Rapp (2010) for three technical means of persuasion in Aristotle's Rhetoric.

[2] See Kirby (2008) for historic details of the public persuasion at Paul's Cross, the open-air pulpit in St Paul's Cathedral in the 16th century.

[3] DellaVigna and Kaplan (2007) and Martin and Yurukoglu (2017) measure the persuasive effects of slanted news using data on Fox News.

the share of the population that would take the action of interest without an exposure to persuasion. If y_0 is unobserved, DK propose using y_C in its place as an approximation. The quantity f is intended to make it easier to compare persuasive effects across different studies. Since DK first introduced the concept of the persuasion rate, it has been used and modified by many authors: e.g. Enikolopov, Petrova, and Zhuravskaya (2011); Gentzkow, Shapiro, and Sinkinson (2011); DellaVigna, Enikolopov, Mironova, Petrova, and Zhuravskaya (2014); Martin and Yurukoglu (2017); Bassi and Rasul (2017). In their survey, DellaVigna and Gentzkow (2010) use the persuasion rate as the key summary statistic to compare persuasive impacts across different studies.

In order to make our discussion more formal, we use the potential outcome framework. Let T_i denote the binary indicator that equals 1 if individual i is exposed to persuasive information such as Fox News. Let $Y_i(t)$ be a binary indicator, which shows agent i's action when T_i is exogenously set to $t \in \{0,1\}$: in section 2 we model $Y_i(t)$ as an expected–utility–maximizing action with or without an exposure to persuasion. For example, $Y_i(1)$ equals 1 if individual i votes for a Republican candidate when she watched Fox News. The econometrician never observes both $Y_i(0)$ and $Y_i(1)$ but can only observe either of the two, i.e. $Y_i = T_i Y_i(1) + (1 - T_i) Y_i(0)$. Then, the fraction of individuals among the entire population who changed their behavior *because of* their exposure to persuasive information can be denoted by

$$\theta = \mathbb{P}\{Y_i(1) = 1 | Y_i(0) = 0\}. \tag{1}$$

Here, the conditioning event $Y_i(0) = 0$ describes the counterfactual instance that individual i would not vote for a Republican candidate if she was excluded from the viewership of Fox News. If $Y_i(0) = 1$, then there is no room for persuasion for individual i. In section 5.1, we point out that θ generally differs from the probability limit of f: heterogeneity in the effect of persuasion is an important reason for the discrepancy.

It is a challenging task to identify θ since (i) we never observe $Y_i(1)$ and $Y_i(0)$ jointly, (ii) T_i may not be observed at all, and (iii) T_i can be highly endogenous. As we mentioned earlier, DK use entries of the Fox News cable channel to local markets as a natural experiment, which helps to address issue (iii). Like DK, a large body of the empirical literature on measuring the effect of persuasion makes use of data from natural or field experiments (e.g. DellaVigna and Gentzkow, 2010), where "intent–to–treat" is randomized by design. In our identification analysis, we presume that by the design of an empirical study, there exists an instrumental variable Z_i that is independent of unobservables but affects T_i. Note, however, that even in an experimental setup, where Z_i is initially randomized, it is rare that the agent's *actual* exposure T_i to the treatment is randomly determined. Individuals can choose to watch Fox News, whereas the Fox News channel may be randomly available in the local cable package. Also, we note that "intent–to–treat" is easier to observe than the "actual" treatment.

We build on the econometrics literature on partial identification (e.g. Manski, 2003, 2007; Tamer, 2010) and the literature on program evaluation (see e.g. Heckman and Vytlacil, 2007; Imbens and Wooldridge, 2009, for surveys of the literature). Our work is the first paper that formulates quantification of the effect of persuasion within the framework of causal inference and provides a formal identification analysis. We are explicit about the possibility that an actual exposure to persuasion can be endogenous and its effect can be heterogeneous. At this level of generality, the commonly used quantity f, or its approximation, is different from θ. As a matter of fact, the approximation of f proposed by DK may not represent a well–defined probability in a heterogeneous population. For example, the approximation of f can even be larger than 1, which is undesirable since f measures the rate of persuasion. In this regard, we build on DK but add important clarifications to the literature. Further, we establish the sharp identified bounds of θ under various data scenarios, which we will discuss below.

In deriving the sharp identified bounds of θ, we consider three different data scenarios: i.e. the outcome and the treatment are jointly observed, they are separately observed, or the treatment is not observed at all.[4] The first case is most ideal, the third case is least informative, and the second case is motivated by the data structure in DK. In addition to θ, we introduce the local and marginal persuasion rates, say θ_{local} and θ_{mte}, and we investigate their identification as well. The former is defined as the persuasive effect for the subpopulation of compliers (e.g. Imbens and Angrist, 1994) and the latter is the persuasive effect defined at a particular value of the unobserved random variable governing selection assignment as in e.g. Heckman and Vytlacil (2005).

The main findings of this paper are as follows. First, the persuasion rate θ is partially identified and its sharp lower bound is the same across the three data scenarios: the sharp lower bound depends only on the joint distribution of (Y_i, Z_i). In terms of DK's notation used above, the sharp lower bound of θ is shown to be $(y_T - y_C)/(1 - y_C)$, which is often computed as a lower bound of f when y_0 is approximated by y_C and $e_T - e_C$ is unknown. Therefore, our identification results show that the bound $(y_T - y_C)/(1 - y_C)$ is not only sharp but also robust to the presence of endogeneity

[4] The case that the outcome and the treatment are separately observed belongs to an identification problem called the "ecological inference" problem. For instance, Cross and Manski (2002) and Manski (2017) discuss bounding a "long regression" by using information from a "short regression." Their substantive concerns are distinct from ours.

as well as heterogeneity in the persuasion effect. Further, knowing e_T and e_C does *not* improve the lower bound at all, which we find surprising. Second, the sharp upper bound of θ depends on the data scenarios, with more favorable data scenarios yielding tighter bounds: i.e. knowing e_T and e_C improves the upper bound of θ in general. Third, the local persuasion rate θ_{local} is point identified when the outcome, the actual exposure to persuasion, and the intent–to–treat are jointly observed. Otherwise, it is only partially identified. Finally, the marginal persuasion rate θ_{mte} can be point identified if Z_i is continuously distributed.

We illustrate the usefulness of our identification results by applying them to two strands of the empirical literature on persuasion, i.e. the effects of media on voting and door–to–door fund raising. When we revisit DK using their original data, we find that the identification region for the average persuasion rate θ is between 1% and 99% and that the lower bound for the local average persuasion rate θ_{local} is either 12% or 37%, depending on the specification of the fixed effects. These results suggest that the persuasive effect of Fox News is fairly large for compliers, i.e. those who would watch Fox News if and only if it is randomly available, but that DK's data are uninformative about the general population. Overall, our empirical results show that heterogeneity in the persuasion effect is an important issue and randomizing the "intent–to–treat" does not render identification of θ in general.

The remainder of the paper is organized as follows. In section 2, we present a simple economic model to motivate our setup. Focusing on a binary treatment and a binary outcome, we formulate a model of persuasion within the framework of expected utility maximization. This formulation naturally leads to a potential outcome setup with a certain monotonicity restriction. In section 3 we discuss identification results for θ, and in section 4 we provide corresponding results for θ_{local} and θ_{mte}. In section 5.1, we clarify the difference between f and θ as well as the relationship between f and θ_{local}. Section 5.2 summarizes our recommendations about what to estimate and how to interpret. Specifically, we discuss in detail which parameters should be reported among the identified ones in each data scenario and how they should be interpreted. In section 6, we revisit the empirical literature on the effects of news media on voting, where we apply our identification results to three published articles. In section 7, we look at the literature on door-to-door fund raising and illustrate the usefulness of our results by applying them to two published papers. In section 8, we give concluding remarks. The appendix contains the proofs.

2. A Binary Choice Model Under Uncertainty

We consider a binary choice problem under binary states. The states are unknown to the agent at the time of the decision and the agent relies on her subjective belief about them to make a decision. We assume that there is an informational treatment such as watching a particular news channel, which potentially affects the agent's belief about the states and hence her choice as well. Our interest is in the econometric analysis of the informational treatment effect, i.e. the effect of persuasion. For instance, a news media is often claimed to be biased and it tries to convince voters to choose a candidate from a particular party: e.g. Fox News promoting Republican candidates or The Washington Post supporting Democratic ones. Our framework below provides a formal way of analyzing the persuasive effect when treatment is potentially endogenous.[5]

Suppose that there are two possible states, denoted by $S \in \mathscr{S} = \{\text{High}, \text{Low}\}$. Let $T_i \in \{0, 1\}$ indicate individual i's status of the informational treatment. Further, let $q_i(t)$ describe individual i's subjective belief about the state when T_i is set to $t \in \{0, 1\}$: i.e. $q_i(t) = \mathbb{P}(S = \text{High} | T_i = t, I_i)$, where I_i denotes all other information available to individual i. Table 1 describes the utility individual i receives from each choice conditional on the state. The payoffs matrix in table 1 is from Bergemann and Morris (2017, see matrix (3)).

TABLE 1. Utility by choice and state

	$S = \text{Low}$	$S = \text{High}$
Vote (1)	-1	$U_i \geq 0$
Not vote (0)	0	0

The utility from option 0 is normalized to be 0 for each state. Since the expected utility is all that matters for the decision, the utility from option 1 when the state is "low" is normalized to be -1: the sign restrictions are to make the choice nontrivial. The utility term U_i is not observed by the econometrician.

Suppose that individual i maximizes her expected utility. Then, individual i chooses option 1 if and only if her expected utility, $-(1 - q_i) + q_i U_i$, is positive with her belief q_i about the state. Therefore, when the informational treatment is set to be $t \in \{0, 1\}$, the potential outcome $Y_i(t)$ can

[5] As a different example, imagine consumers facing a decision problem about whether to purchase a durable good or not when the quality of the good, which can be "high" or "low", is uncertain. Each consumer has her own belief about the quality of the good, which may depend on whether she has read an advertisement brochure or not. Reading the brochure is an informational treatment of interest.

be written as follows:

$$Y_i(t) = \mathbb{1}\big[-\{1 - q_i(t)\} + q_i(t)U_i \geq 0\big], \tag{2}$$

where $\mathbb{1}[\cdot]$ is the usual indicator function. We now make the following assumptions.

Assumption A. *U_i has a conditional density $f\{\cdot|q_i(0), q_i(1)\}$ such that $f\{u|q_i(0), q_i(1)\} > 0$ for all $u \in [0, \infty)$ with probability one.*

Assumption B. *$q_i(0) \leq q_i(1)$ with probability one.*

Assumption A says that U_i is continuously distributed given $q_i(0), q_i(1)$. However, it does not rule out the possibility that U_i and $q_i(t)$ are dependent on each other. Assumption B simply means that the informational treatment may shift an agent's belief *only in one direction*.

As mentioned before, the outcome variable Y_i observed by the econometrician is given by

$$Y_i = T_i Y_i(1) + (1 - T_i)Y_i(0), \tag{3}$$

where T_i may not always be observed. For example, in the context of marketing (see footnote 5), it is easily observable if a consumer has received an advertisement brochure, but it is rarely observed whether the consumer has actually read it or not. We will analyze a few different scenarios regarding how much information associated with T_i is available to the econometrician.

In addition to the outcome Y_i, we assume that the econometrician observes a binary "*intent–to–treat*" variable, which will be denoted by Z_i.[6] Throughout the paper we assume that Z_i is randomly assigned and T_i has a simple threshold structure, i.e.

$$T_i = \mathbb{1}\{V_i \leq e(Z_i)\}. \tag{4}$$

We summarize this in the following assumption.

Assumption C. *T_i has the threshold structure in equation (4), where V_i is uniformly distributed, and $0 \leq e(0) < e(1) \leq 1$. Further, Z_i is independent of $\big(q_i(t), U_i, V_i\big)$ for $t = 0, 1$.*

Therefore, the function e is the propensity score, or more descriptively in our context, it can be referred to as the *exposure rate*.

Recall that we are interested in the rate of persuasion: $\theta = \mathbb{P}\{Y_i(1) = 1|Y_i(0) = 0\}$. Assumption B has an important implication for us, which we state as a lemma.

[6] In section 4, we consider the case that Z_i is continuously distributed.

Lemma 1. *Under assumption A, assumption B is equivalent to $Y_i(0) \leq Y_i(1)$ with probability one. Therefore,*

$$\theta = \frac{\mathbb{P}\{Y_i(1) = 1\} - \mathbb{P}\{Y_i(0) = 1\}}{1 - \mathbb{P}\{Y_i(0) = 1\}}. \tag{5}$$

Therefore, identification of θ can be achieved by identifying two counterfactual probabilities $\mathbb{P}\{Y_i(1) = 1\}$ and $\mathbb{P}\{Y_i(0) = 1\}$: i.e. we do not need to know the joint distribution of $Y_i(0)$ and $Y_i(1)$.[7] In fact, since $Y_i(t)$ is binary, θ is the average treatment effect (ATE) divided by $\mathbb{P}\{Y_i(0) = 0\}$.

In addition to (Y_i, T_i, Z_i), one may observe covariates X_i. In many applications, it would be important to control for covariates. Throughout the paper, we implicitly assume that all assumptions and results are conditional on the value of covariates. We suppress it from the notation, unless it is necessary.

3. Identification of θ

As we mentioned earlier, T_i can be difficult to observe directly. The most favorable situation is probably the one in which there is no difference between the actual treatment and the intent–to–treat. We will refer to this case as the *sharp persuasion design*. However, in social sciences, the sharp design is rather an exceptional scenario. More realistically, the actual treatment tends to differ from the intent–to–treat, which will be referred to as the *fuzzy persuasion design*.

The most challenging scenario in the fuzzy design is the one where we do not have any information other than the joint distribution of (Y_i, Z_i). The most ideal situation is the one where (Y_i, T_i, Z_i) is jointly observed. However, these two extremes are not the only possibilities. For example, the researcher may have two different data sources from which $\mathbb{P}(Y_i = 1|Z_i = z)$ and $e(z) = \mathbb{P}(T_i = 1|Z_i = z)$ are separately revealed. In fact, DK used town–level election data to estimate $\mathbb{P}(Y_i = 1|Z_i = z)$ and microlevel audience data to infer $e(z) = \mathbb{P}(T_i = 1|Z_i = z)$. In our analysis, we consider these three different scenarios in the fuzzy design regarding data availability.[8]

It turns out that point identification of the persuasion rate θ is generally not available with binary Z_i, even if the full joint distribution of (Y_i, T_i, Z_i) is observable. Indeed, among the cases we have studied in the paper, the sharp persuasion design is the only case where θ is point identified, and in all fuzzy design cases, θ is only partially identified up to an interval with the same lower bound

[7] If Y_i were non-binary, identification of θ would be much harder. In that case, it would be necessary to know about the joint distribution of $Y_i(1)$ and $Y_i(0)$.

[8] In our analysis we assume T_i is correctly measured if it is observed. See Nguimkeu, Denteh, and Tchernis (2016) and Calvi, Lewbel, and Tommasi (2017) for the issues of mismeasured treatment. Their subject matters are distinct from ours.

θ_L, which is defined by

$$\theta_L = \frac{\mathbb{P}(Y_i = 1|Z_i = 1) - \mathbb{P}(Y_i = 1|Z_i = 0)}{1 - \mathbb{P}(Y_i = 1|Z_i = 0)}. \tag{6}$$

The parameter θ_L is identified with the joint distribution of (Y_i, Z_i) only.

3.1. **The Sharp Persuasion Design.** We first consider the simplest scenario, i.e. the one where everybody *complies* with the intent–to–treat. This case arises if and only if $e(1) - e(0) = 1$, in which case the entire population only consists of compliers.

Assumption D. *We have $e(1) - e(0) = 1$, i.e. $T_i = Z_i$ with probability one.*

Under assumption D, there is essentially no difference between T_i and Z_i, and therefore it is a standard exercise to show the identification of the distribution of the potential outcomes.

Theorem 1. *Suppose that assumptions A to D hold. Then, for $z = 0, 1$, we have $\mathbb{P}\{Y_i(z) = 1\} = \mathbb{P}(Y_i = 1|Z_i = z)$. In particular, $\theta = \theta_L$.*

Under assumption D there is no difference between the actual treatment assignment and the intent–to–treat. Therefore, T_i is essentially observed and it is randomly assigned. In this case, the identification analysis is the same as that of the average treatment effect under unconfoundedness. In fact, the potential outcomes are binary and they have a similar structure to the treatment assignment T_i, from which more intuition for theorem 1 can be obtained.

To see this, note that, by lemma 1, there are three subpopulations:

$$\begin{cases} \text{Never–voters: } Y_i(0) = 0, Y_i(1) = 0, \\ \text{Responders: } Y_i(0) = 0, Y_i(1) = 1, \\ \text{Always–voters: } Y_i(0) = 1, Y_i(1) = 1. \end{cases}$$

In the context of voting as in DK, "Always–voters" can be referred to as "Republicans," and "non–Republicans" consists of "never–Republicans" and "those who respond to Fox News." Then, what the observation of (Y_i, Z_i) can reveal about which group individual i belongs to can be summarized as follows.

Table 2 provides an intuitive illustration about the identification of θ by θ_L: θ_L captures the portion of "responders" among the "non–always–voters." However, note that we cannot identify which individual is a "persuaded one" just as the group of compliers is generally unidentified in the population.

TABLE 2. Identification of the Persuasion Rate

	$Y_i = 0$	$Y_i = 1$
$Z_i = 0$	Never–voter or Responder	Always–voter
$Z_i = 1$	Never–voter	Always–voter or Responder

3.2. **The Fuzzy Persuasion Design.** Assumption D assumes away the existence of never–takers and always–takers. In this subsection we discuss how far we can go without assumption D. It will be shown that θ is only partially identified even if data reveal the full joint distribution of (Y_i, T_i, Z_i). If Y_i and T_i are not jointly observed or T_i is not observed at all, then the sharp identified interval of θ becomes wider. However, it turns out that the sharp lower bound stays the same in all three cases we consider, and it coincides with θ_L. Throughout this section, we assume that the joint distribution of (Y_i, Z_i) is identified directly from data.

3.2.1. *Identification with the Joint Distribution of* (Y_i, T_i, Z_i). We start with the case where the richest dataset is available.

Assumption E. *The joint distribution of* (Y_i, T_i, Z_i) *is known, where both* $e(0)$ *and* $1 - e(1)$ *are bounded away from zero.*

Assumption E does not generally deliver point identification of θ. Before we discuss partial identification results under assumption E, we first explain what challenges we face here with table 3.

TABLE 3. Lack of Identification of θ

		$Y_i = 0$	$Y_i = 1$
$Z_i = 0$	$T_i = 0$ (Never–taker or Complier)	Never–voter or Responder	Always–voter
	$T_i = 1$ (Always–taker)	Never–voter	Always–voter or Responder
$Z_i = 1$	$T_i = 0$ (Never–taker)	Never–voter or Responder	Always–voter
	$T_i = 1$ (Always–taker or Complier)	Never–voter	Always–voter or Responder

The event $(Z_i, T_i) = (0, 0)$ represents a different subpopulation from what $(Z_i, T_i) = (0, 1)$ does. Therefore, comparing $\mathbb{P}(Y_i = 1 | Z_i = 0, T_i = 0)$ with $\mathbb{P}(Y_i = 1 | Z_i = 0, T_i = 1)$ does not lead to anything meaningful. Indeed, the only subpopulation we can learn about from $Z_i = 0$ and $Z_i = 1$ in common is the one of "compliers", just as the Wald statistic estimates only the local average treatment effect (LATE), not the ATE. Although θ is not generally point–identified, we can derive its sharp identified bounds under assumption E.

Theorem 2. *Suppose that assumptions A to C and E are satisfied. Then, the sharp identified interval of* θ *is given by* $[\theta_L, \theta_U]$*, where* θ_L *is given in equation (6) and*

$$\theta_U = \frac{\mathbb{P}(Y_i = 1, T_i = 1 | Z_i = 1) - \mathbb{P}(Y_i = 1, T_i = 0 | Z_i = 0) + 1 - e(1)}{1 - \mathbb{P}(Y_i = 1, T_i = 0 | Z_i = 0)}.$$

The bounds in theorem 2 shrink to a singleton as $\big(e(0), e(1)\big)$ approaches $(0, 1)$, which is not surprising given the result in theorem 1. Also, it is worth noting that the lower bound θ_L only depends on the distribution of (Y_i, Z_i): observing T_i along with (Y_i, Z_i) helps only for the upper bound. If $e(1)$ is too small, then the upper bound will not be very informative: θ_U converges to 1 as $e(1)$ approaches 0: i.e. if nobody reads a brochure, then we do not learn much about how "persuading" the brochure is. However, even if $e(1)$ approaches 1, the upper bound does not necessarily shrink to the lower bound. In other words, even if everybody is always exposed to an advertisement, we do not necessarily pin down the persuasion rate of the advertisement.

3.2.2. *Identification with the Knowledge of the Exposure Rates.* We now consider a situation in which T_i is not observed but the researcher has knowledge about the exposure rates $e(0)$ and $e(1)$. For instance, one may have two different data sources for (Y_i, Z_i) and (T_i, Z_i) as in DK.

In this section we treat $e(0)$ and $e(1)$ as "parameters," and we obtain the sharp identified bounds of θ under the assumption that $e(0)$ and $e(1)$ are known. If the researcher's prior knowledge about $\{e(0), e(1)\}$ is only probabilistic, then $e(0)$ and $e(1)$ in the bounds can be averaged out.

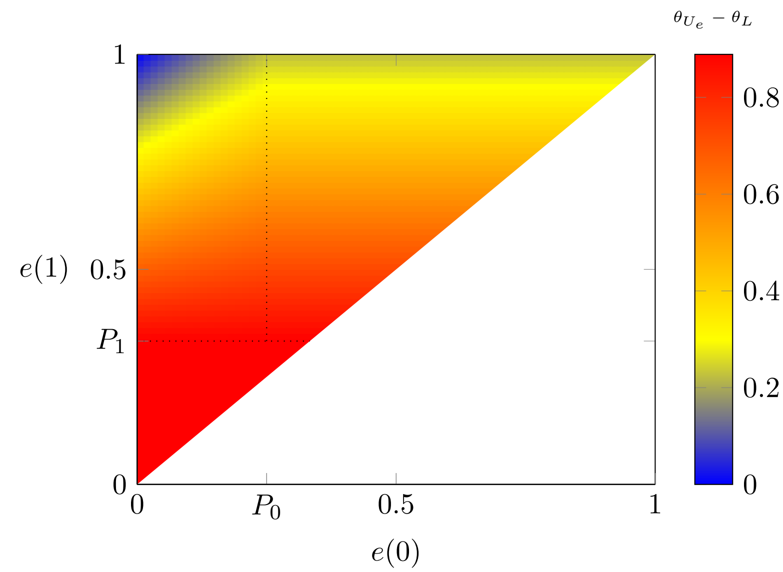
Assumption F. *T_i is not observed but the exposure rates $\{e(0), e(1)\}$ are known.*

Theorem 3. *Suppose that assumptions A to C and F are satisfied. Then, the sharp identified interval of* θ *is given by* $[\theta_L,\ \theta_{U_e}]$*, where* θ_L *is given in equation (6) and*

$$\theta_{U_e} = \frac{\min\{1, \mathbb{P}(Y_i = 1 | Z_i = 1) + 1 - e(1)\} - \max\{0,\ \mathbb{P}(Y_i = 1 | Z_i = 0) - e(0)\}}{1 - \max\{0,\ \mathbb{P}(Y_i = 1 | Z_i = 0) - e(0)\}}. \tag{7}$$

Therefore, the upper bound in this case is nontrivial if and only if $e(1) > \mathbb{P}(Y_i = 1 | Z_i = 1)$: it is the relative size of the take–up rate $e(1)$, i.e. the probability of reading a brochure when it is mailed, that determines how much we can hope to learn about the persuasion rate. Figure 1 illustrates how the identified set of θ changes for different values of $e(z)$ when $\mathbb{P}(Y_i = 1 | Z_i = z)$ is given. The difference between $\theta_{U_e} - \theta_L$ gets smaller as $e(1)$ approaches 1 for each value of $e(0)$.

FIGURE 1. An Example of the Length of the Identified Interval of θ



Note: We set $P_0 = \mathbb{P}(Y_i = 1|Z_i = 0) = 1/4$ and $P_1 = \mathbb{P}(Y_i = 1|Z_i = 1) = 1/3$. Therefore, $\theta_L = 1/9$ but θ_{U_e} varies as $e(0)$ and $e(1)$ vary.

3.2.3. *Identification with No Information Associated with T_i.* Figure 1 readily shows that the upper bound of θ becomes trivial if $e(0), e(1)$ are totally unknown. For the sake of completeness we state this in a separate theorem.

Assumption G. *No information associated with T_i is available: i.e. the distribution of (Y_i, Z_i) is all that is known.*

Theorem 4. *Suppose that assumptions A to C and G are satisfied. Then, the sharp bound of θ is given by $[\theta_L, 1]$, where θ_L is given in equation (6).*

4. THE LOCAL AND MARGINAL PERSUASION RATES

In this section we consider two alternative parameters, i.e. the local and marginal persuasion rates, defined by

$$\theta_{\text{local}} = \mathbb{P}\{Y_i(1) = 1|Y_i(0) = 0, e(0) < V_i \leq e(1)\} \tag{8}$$

and

$$\theta_{\text{mte}}(v) = \mathbb{P}\{Y_i(1) = 1|Y_i(0) = 0, V_i = v\} \tag{9}$$

for $0 < v < 1$. Here, θ_{local} is the persuasion rate for the subpopulation characterized by $e(0) < V_i \leq e(1)$, i.e. the *compliers* (e.g. Imbens and Angrist, 1994), whereas $\theta_{\text{mte}}(v)$ is for the subpopulation such that $V_i = v$ (e.g. Heckman and Vytlacil, 2005).

To analyze these two parameters, we replace assumption A with the following condition.

Assumption H. *U_i has a conditional density $f\{\cdot|q_i(0), q_i(1), V_i\}$ such that $f\{u|q_i(0), q_i(1), V_i\} > 0$ for all $u \in [0, \infty)$ with probability one.*

We first focus on θ_{local}. By the same reasoning as lemma 1, we have

$$\theta_{\text{local}} = \frac{\mathbb{E}\{Y_i(1) - Y_i(0) \mid e(0) < V_i \leq e(1)\}}{\mathbb{P}(Y_i(0) = 0 \mid e(0) < V_i \leq e(1)\}}, \tag{10}$$

where the numerator is the local average treatment effect (LATE), which has received a great deal of attention in the econometrics literature (see, e.g., Deaton, 2010; Heckman, 2010; Imbens, 2010, for a recent debate).

Just like LATE, it is contentious whether or not θ_{local} should be the parameter of interest since the compliers are concerned with an unidentified subgroup of the population. We take a practical view that the identification results on θ_{local} can complement the results obtained in section 3. The following theorem shows the identification of θ_{local} under the different scenarios of data availability.

Theorem 5. *Suppose that assumptions B, C and H are satisfied.*

(i) Under assumption E, θ_{local} is point identified by $\theta_{local} = \theta^*$, where

$$\theta^* = \frac{\mathbb{P}(Y_i = 1|Z_i = 1) - \mathbb{P}(Y_i = 1|Z_i = 0)}{\mathbb{P}(Y_i = 0, T_i = 0|Z_i = 0) - \mathbb{P}(Y_i = 0, T_i = 0|Z_i = 1)}.$$

(ii) Under assumption F, the sharp identified interval of θ_{local} is given by $[\theta_L^*, 1]$, where

$$\theta_L^* = \frac{\mathbb{P}(Y_i = 1|Z_i = 1) - \mathbb{P}(Y_i = 1|Z_i = 0)}{\min\{1 - \mathbb{P}(Y_i = 1|Z_i = 0),\ e(1) - e(0)\}}.$$

(iii) Under assumption G, the sharp identified interval of θ_{local} coincides with that of θ, i.e. $\left[\theta_L,\ 1\right]$.

It is well known that the numerator of equation (10) is identified by the Wald statistic:

$$\mathbb{E}\{Y_i(1) - Y_i(0) \mid e(0) < V_i \leq e(1)\} = \frac{\mathbb{P}(Y_i = 1|Z_i = 1) - \mathbb{P}(Y_i = 1|Z_i = 0)}{e(1) - e(0)}.$$

In addition, we need to identify the denominator of equation (10). It is important to notice that the identification of the LATE requires the distribution of (T_i, Z_i) and that of (Y_i, Z_i) separately, but not the joint distribution of (Y_i, T_i, Z_i). Unlike the LATE, the point identification in part *(i)* of theorem 5 demands the knowledge of the joint distribution of (Y_i, T_i, Z_i): the denominator of equation (10) requires that we know the marginal distribution of $Y_i(0)$ for the compliers (see e.g. Imbens and Rubin, 1997). Part *(ii)* of theorem 5 shows that this requirement is not only sufficient but also necessary to achieve the point identification of θ_{local}. The local persuasion rate θ_{local} represents the average persuasive effect for a population that is different from the entire population. Given this caveat, it is interesting to note that in part *(ii)*, the upper bound of θ_{local} is always trivial in contrast to θ, but the lower bound of θ_{local} can never be worse than that of θ. Therefore, in principle, the length of the identified interval of θ can be smaller than that of θ_{local}. If T_i is not observed at all, then there is no advantage in focusing on the compliers. Part *(iii)* of theorem 5 confirms the intuition that the bound for θ_{local} is identical to θ if the distribution of (Y_i, Z_i) is the only piece of information available. This corresponds to the uninteresting case for θ_{local} though since we have no information on compliers.

We now move to $\theta_{\text{mte}}(v)$. If Y_i and T_i are jointly observed along with a continuous instrument Z_i, then $\theta_{\text{mte}}(v)$ can be point identified as in Heckman and Vytlacil (2005); Carneiro, Heckman, and Vytlacil (2011). Examples of continuous instruments can be found in the literature on the media effects on voting. For instance, Enikolopov, Petrova, and Zhuravskaya (2011) and DellaVigna, Enikolopov, Mironova, Petrova, and Zhuravskaya (2014) use the signal strength of NTV and Serbian radio as instruments, respectively: in both of the papers, (Y_i, T_i, Z_i) are jointly observed.

The following assumption describes the situation in which we discuss the identification of $\theta_{\text{mte}}(v)$. Using the standard results in the literature (e.g. Heckman and Vytlacil, 2005), it is then straightforward to obtain the identification of $\theta_{\text{mte}}(v)$, which we state in the subsequent theorem.

Assumption I. *(i) The joint distribution of (Y_i, T_i, Z_i) is known.*

(ii) T_i has the threshold structure in equation (4), where V_i is uniformly distributed, and Z_i is independent of $\big(q_i(t), U_i, V_i\big)$ for $t = 0, 1$.

(iii) The distribution of $e(Z_i)$ is absolutely continuous with respect to Lebesgue measure, where v is in the interior of the support of $e(Z_i)$.

Theorem 6. *Suppose that assumptions B, H and I are satisfied. Then $\theta_{mte}(v)$ is point identified by*

$$\theta_{mte}(v) = \frac{\partial \mathbb{P}\{Y_i = 1 | e(Z_i) = e\} / \partial e|_{e=v}}{1 - \partial \mathbb{P}\{Y_i = 1, T_i = 0 | e(Z_i) = e\} / \partial e|_{e=v}}, \tag{11}$$

provided that $\mathbb{P}\{Y_i = 1 | e(Z_i) = e\}$ and $\mathbb{P}\{Y_i = 1, T_i = 0 | e(Z_i) = e\}$ are continuously differentiable with respect to e.

Theorem 6 does not consider the other two scenarios of data availability. We do this for two reasons. One is that continuous instruments are rare in the context of persuasion and we are not aware of any applications, where continuous instruments are available while the outcome and treatment are not jointly observed. Second, attempts to bound $\theta_{\text{mte}}(v)$ in the other data scenario will involve bounding a derivative, for which we need more than the bounds of the level. For example, if we only know $e(Z_i)$ and the joint distribution of (Y_i, Z_i), then the denominator of equation (11) is not identified. Further, bounding $\mathbb{P}\{Y_i = 1, T_i = 0 | e(Z_i) = e\}$ does not generally lead to useful bounds of its derivative without additional assumptions. Therefore, we do not pursue this in the current paper.

If the support of the exposure rate $e(Z_i)$ is equal to the unit interval $[0, 1]$, then theorem 6 shows the identification of $\theta_{\text{mte}}(v)$ for all v in the unit interval. Then, we can use $\theta_{\text{mte}}(v)$ to construct different policy oriented quantities as in Heckman and Vytlacil (2005) and Carneiro, Heckman, and Vytlacil (2011). For instance, the persuasion rate of the entire population θ will be given by

$$\theta = \int_0^1 \theta_{\text{mte}}(v) dv,$$

because V_i is uniformly distributed on the unit interval.

5. Discussions

5.1. **Measuring Persuasive Effects in the Literature.**

We now discuss the relationship between our parameters of persuasive effects and the ones that were used in the literature. For this purpose we focus on the binary instrument case.

The population version of DK's proposal f is

$$\theta_{DK} = \frac{\mathbb{P}(Y_i = 1 | Z_i = 1) - \mathbb{P}(Y_i = 1 | Z_i = 0)}{e(1) - e(0)} \frac{1}{1 - \mathbb{P}\{Y_i(0) = 1\}}, \tag{12}$$

which is often approximated by

$$\tilde{\theta}_{DK} = \frac{\mathbb{P}(Y_i = 1 | Z_i = 1) - \mathbb{P}(Y_i = 1 | Z_i = 0)}{e(1) - e(0)} \frac{1}{1 - \mathbb{P}(Y_i = 1 | Z_i = 0)}. \tag{13}$$

Note here that $\tilde{\theta}_{DK}$ does not require any knowledge about the joint distribution of (Y_i, T_i) given Z_i.

We now discuss the relationship between $\theta, \theta_{\text{local}}$, and θ_{DK}. By equation (27) in the proof of theorem 5, we have

$$\mathbb{P}\{Y_i(0) = 0\}\theta_{DK} = \mathbb{P}\{Y_i(0) = 0 | e(0) < V_i \leq e(1)\}\theta_{\text{local}} = \mathbb{E}\{Y_i(1) - Y_i(0) | e(0) < V_i \leq e(1)\}$$

under assumptions B, C and H. Further, recall from lemma 1 that

$$\mathbb{P}\{Y_i(0) = 0\}\theta = \mathbb{E}\{Y_i(1) - Y_i(0)\}.$$

Therefore, $\theta, \theta_{\text{local}}$, and θ_{DK} are different parameters in general. For example, θ_{DK} rescales the LATE with an unconditional probability, and hence it does not render a well–defined conditional probability in general.

There are some special cases where the three parameters coincide. For example, $\theta = \theta_{DK}$ holds if and only if the ATE equals the LATE. This happens, for example, if at least one of the following three conditions hold:

(i) the entire population consists of compliers, i.e. $e(0) = 0$ and $e(1) = 1$, as in the sharp persuasion design;

(ii) $Y_i(1) - Y_i(0)$ is a constant;

(iii) V_i is independent of $\big(q_i(t), U_i\big)$ for $t = 0, 1$, in which case the potential outcome $Y_i(t)$ is independent of T_i conditional on Z_i.

Condition (ii) corresponds to the situation with no heterogeneity in the treatment effect. This is probably the least interesting condition because there are only two unrealistic possibilities for this: either $Y_i(1) - Y_i(0) = 1$ (everyone is persuaded) or $Y_i(1) - Y_i(0) = 0$ (no one has room for persuasion). Under condition (i), there is no difference between the intent–to–treat and the actual treatment, in which case randomizing the intent–to–treat is sufficient to identify θ. Condition (iii) is often referred to as the condition of *unconfoundedness or selection on observables* in econometrics. Since $\mathbb{P}\{Y_i(0) = 1\} = \mathbb{P}\{Y_i(0) = 1 \mid e(0) < V_i \leq e(1)\}$ under conditions (i) or (iii), we have $\theta = \theta_{\text{local}} = \theta_{DK}$ under either of the two conditions.

Unlike the three parameters, $\tilde{\theta}_{DK}$ generally does not measure the effect of persuasion even under condition (iii). However, as DK correctly pointed out, it is an approximation of $\theta_{DK}(= \theta = \theta_{\text{local}})$ when $e(0)$ is close to zero or $\theta = 0$.[9] $\tilde{\theta}_{DK}$ has some interesting features though: observing the two marginals of (Y_i, Z_i) and (T_i, Z_i) is sufficient for its identification, and it has a simple lower bound θ_L that can be identified without observing T_i at all. Indeed, DellaVigna and Gentzkow (2010) extensively reports $\tilde{\theta}_{DK}$ or its lower bound θ_L, depending on whether T_i is observed or not. However, $\tilde{\theta}_{DK}$ should be interpreted with caution: θ_L is *always* a meaningful estimand but $\tilde{\theta}_{DK}$ is not. When information about $e(0)$ and $e(1)$ is available, it seems a better practice to report θ_L together with θ_L^* than to estimate $\tilde{\theta}_{DK}$.

[9]Under condition (iii), we have $\mathbb{P}(Y_i = 1|Z_i = z) = \mathbb{P}(Y_i = 1, T_i = 1|Z_i = z) + \mathbb{P}(Y_i = 1, T_i = 0|Z_i = z) = \mathbb{P}\{Y_i(0) = 1\} + \big[\mathbb{P}\{Y_i(1) = 1\} - \mathbb{P}\{Y_i(0) = 1\}\big]e(z)$.

It is worth pointing out that θ_L is not only a lower bound of $\tilde{\theta}_{DK}$ but also the sharp lower bound of θ in a much more general sense. Specifically, neither condition (iii) nor approximation by $\tilde{\theta}_{DK}$ is needed, and therefore the bound is robust to both the presence of endogeneity in the treatment assignment and poor approximation of θ_{DK} by $\tilde{\theta}_{DK}$.

Finally, as an aside we point out that without condition (iii), θ_{DK} does not measure the persuasion rate of any subpopulation correctly: the first factor on the right–hand side of equation (12) focuses on a subpopulation of "compliers," while the second factor is not conditioned on the complier group.

5.2. **Main Takeaways from Our Results.** Focusing on the binary instrument case, we have proposed five estimands, namely $\theta_L, \theta_L^*, \theta^*, \theta_U$, and θ_{U_e}. Which of those parameters can be estimated depends on whether and how T_i is observed. Below is the summary of the proposed estimands and their interpretation in each of the three data scenarios. Note that "no endogeneity" here means that either there is no difference between the intent–to–treat and the actual treatment, or the potential outcomes are independent of the actual treatment given the intent–to–treat.

(1) If (Y_i, T_i, Z_i) are jointly observed, then estimate θ_L, θ_U, and θ^*.
   - (a) In general, $[\theta_L,\ \theta_U]$ is the sharp identified interval of the persuasion rate for the population, and θ^* is the persuasion rate for the group of compliers.
   - (b) With no endogeneity, θ^* is the persuasion rate for the group of compliers as well as for the population. (θ_L is just a lower bound of it, which is not sharp.)

(2) If (Y_i, Z_i) and (T_i, Z_i) are separately observed, then estimate θ_L, θ_{U_e}, and θ_L^*.
   - (a) In general, $[\theta_L,\ \theta_{U_e}]$ is the sharp identified interval for the persuasion rate for the population, and $[\theta_L^*,\ 1]$ is the sharp identified interval for the persuasion rate for the group of compliers.
   - (b) With no endogeneity, $[\theta_L^*,\ 1]$ is the sharp identified interval for the persuasion rate for the group of compliers as well as for the entire population.

(3) If (Y_i, Z_i) is all that is observed, then estimate θ_L.
   - (a) With or without endogeneity, $[\theta_L,\ 1]$ is the sharp identified interval for the persuasion rate for the group of compliers as well as for the entire population.

Note that θ_L should *always* be estimated. Further, we recommend that θ_L^* or θ^* should also be reported together, depending on data availability, for two reasons. First, θ_L is always a lower bound of the persuasion rate for the population but it may not be sharp when there is no endogeneity.

Second, even if endogeneity is a potential concern, θ_L^* and θ^* provide information about the persuasion rate for the group of compliers. If the upper bound is also of interest, then θ_U or θ_{U_e} can be reported as well.

As we mentioned earlier, the actual treatment T_i may be difficult to observe. For example, it is more costly to obtain data on whether or not individuals have read The Washington Post than to observe if they received free subscriptions to it. The value of observing T_i depends on which parameter a researcher is interested in. Suppose that θ is the parameter of interest. If the informational treatment T_i is not observed, then θ_L is the only parameter that can be estimated. Moreover, it is the sharp lower bound even if we observe T_i. Therefore, if a researcher would like to learn about the persuasive effect for the entire population, the benefit of an attempt to observe T_i by e.g. conducting a follow–up survey is limited and can only come from tightening the upper bound. If the cost of collecting extra data on T_i is too high, then an alternative approach is to compute the upper bound θ_{U_e} as a function of $\{e(1), e(0)\}$ so that readers can rely on their own prior on the exposure rate. However, the value of observing T_i can be high if θ_{local} is the parameter of interest. It is point-identified if (Y_i, T_i, Z_i) are jointly observed, and its lower bound is improved even if the exposure rates $\{e(1), e(0)\}$ are only known. In a nutshell, our identification analysis shows that the value of observing T_i depends crucially on which population is of interest to a researcher.

6. The Effects of Media on Voting

In this section, we revisit the recent empirical literature on the effects of media on voting and apply our identification results.

6.1. **Reading a Newspaper: Gerber, Karlan, and Bergan (2009) Revisited.**

Gerber, Karlan, and Bergan (2009, GKB hereafter) reported findings from a field experiment to measure the effect of political news. In GKB, there were three treatments, or more precisely three statuses in the intention to treat: a control group, an offer of free subscription to The Washington Post, and one to The Washington Times. To illustrate the usefulness of our paper, we focus on The Washington Post and drop all observations from The Washington Times subscription. That is, $Z_i = 1$ if the i^{th} individual received free subscription to The Washington Post, and $Z_i = 0$ if not.

GKB focused on the intent–to–treat (ITT) analysis and have reported ITT estimates for various outcomes Y_i. When DellaVigna and Gentzkow (2010) computed persuasion rates for GKB, they considered that $T_i = 1$ if the i^{th} individual opted into the free subscription and $T_i = 0$ if he/she opted out of it.

In this section, for the purpose of illustrating our identification results, we consider a different treatment variable: $T_i = 1$ if the i^{th} individual reads the newspaper at least several times per week and $T_i = 0$ otherwise. Therefore, the relevant treatment we consider differs from GKB's ITT analysis, but it is whether individuals have *actually* read the newspaper or not, which is kept track of in a follow–up survey. The binary outcome we consider is as follows : $Y_i = 1$ if the i^{th} individual reported voting for the Democratic candidate in the 2005 gubernatorial election and $Y_i = 0$ if the subject did not vote for the Democratic candidate or did not vote at all. We use only a subsample of the GKB data with those who responded to the follow–up survey to use information on (Y_i, T_i, Z_i) jointly. After dropping observations for The Washington Times subscription and removing missing data, we summarize the GKB data in table 4.

TABLE 4. Summary statistics of the GKB data

The Washington Post ($Z_i = 1$)			
	Reads the newspaper		Total
Voted for Democrat	$T_i = 0$	$T_i = 1$	
$Y_i = 0$	94	93	187
$Y_i = 1$	31	68	99
Total	125	161	286
Control ($Z_i = 0$)			
	Reads the newspaper		Total
Voted for Democrat	$T_i = 0$	$T_i = 1$	
$Y_i = 0$	162	130	292
$Y_i = 1$	46	77	123
Total	208	207	415

We can now compute all of our bounds by hand using table 4. First of all, the ITT effect is estimated by

$$\hat{\mathbb{P}}(Y_i = 1 | Z_i = 1) - \hat{\mathbb{P}}(Y_i = 1 | Z_i = 0) = 0.0498.$$

Throughout this section, a hat refers to the sample estimate based on table 4. Although the joint distribution of (Y_i, T_i, Z_i) is observed in this example, we also considered using the two marginals of (Y_i, Z_i) and (T_i, Z_i) separately, to make a comparison. The estimates are summarized in table 5

First, we discuss the case where the full joint distribution of (Y_i, T_i, Z_i) is used. In this data scenario, the average effect of persuasion by reading The Washington Post is bounded between 7% and 63%. In contrast, the persuasion rate for the group of complies is point estimated by 81%. It is

TABLE 5. The Estimates of the Key Parameters

	(Y_i, T_i, Z_i)	(Y_i, T_i) and (T_i, Z_i)
θ	[0.0707, 0.6343]	[0.0707, 0.7832]
θ_{local}	0.8067	[0.7759, 1]

[a] The first row corresponds to $[\hat{\theta}_L,\ \hat{\theta}_U]$ and $[\hat{\theta}_L,\ \hat{\theta}_{U_e}]$, respectively. The second row shows $\hat{\theta}^*$ and $[\theta_L^*,\ 1]$, respectively.

interesting to note that the estimate of θ_{local} is so large that it is greater than the upper bound of θ. This suggests that individuals are highly heterogeneous in this example, indicating that $\tilde{\theta}_{DK}$ might not be a well–defined parameter here. Indeed, its estimate is

$$\tilde{\theta}_{DK} = \frac{\hat{\mathbb{P}}(Y_i = 1|Z_i = 1) - \hat{\mathbb{P}}(Y_i = 1|Z_i = 0)}{\hat{e}(1) - \hat{e}(0)} \frac{1}{1 - \hat{\mathbb{P}}(Y_i = 1|Z_i = 0)} = 1.1027,$$

which is greater than 1.

When the marginals of (Y_i, Z_i) and (T_i, Z_i) are used separately, the upper bound of θ increases from 63% to 78%. Further, θ_{local} is not point estimated anymore but we only know that it is bounded between 78% and 100%. This difference illustrates the loss of identification power if we do not observe the joint distribution of (Y_i, T_i, Z_i).

6.2. **The Effect of Fox News: DellaVigna and Kaplan (2007) Revisited.** In DK, the entry of Fox News in cable markets plays a role of an instrument conditional on a set of covariates. That is, Z_i is a binary variable that equals one if Fox News was part of local cable package in the town where the i^{th} individual was living in 2000. To apply our result to DK, let Y_i be the binary dependent variable that equals one if individual i voted for the Republican candidate in the 2000 presidential election. As DK argue in their paper, Fox News availability in 2000 is likely to be idiosyncratic, only after controlling for a set of covariates. We will be explicit about conditioning on covariates X_i to apply our identification results, and we write the lower bound as a function of the values of X_i: i.e.

$$\theta_L(x) = \frac{\mathbb{P}(Y_i = 1|Z_i = 1, X_i = x) - \mathbb{P}(Y_i = 1|Z_i = 0, X_i = x)}{1 - \mathbb{P}(Y_i = 1|Z_i = 0, X_i = x)}, \tag{14}$$

which is the sharp lower bound of $\mathbb{P}\{Y_i(1) = 1|Y_i(0) = 0, X_i = x\}$, the conditional persuasion rate. Then, to obtain the lower bound for the persuasion rate in the population, we integrate (14) with respect to the distribution F_X of X_i, so that

$$\theta_L = \int \theta_L(x) dF_X(x).$$

Note that X_i is first controlled for and is averaged out.

To estimate θ_L, we use DK's data[10] and adopt similar specifications as in DK. They estimated $\mathbb{P}(Y_i = 1|Z_i, X_i)$ using a town–level linear regression model, where the dependent variable is the Republican two–party vote share for the 2000 presidential election minus the same variable for the 1996 election. To be consistent with our econometric framework, we modify the dependent variable to be the votes cast for the Republican candidate in the 2000 presidential election divided by the population of age 18 and older. Recall that in our setup, $Y_i = 0$ if individual i did not voted for the Republican candidate. This event includes the case of voting for different candidates or that of not voting for any candidate at all. As the town–level covariates, we include the Republican vote share as a share of the voting–age population in the 1996 election, census controls for both 1990 and 2000, cable system controls, and US House district fixed effects (or county fixed effects). These specifications correspond to the main specifications of DK (see columns (4) and (5) of table IV in DK). In the regression, the town–level observations are weighted by the population of age 18 and older in 1996.

DK used two different data sources for (Y_i, Z_i) and (T_i, Z_i). Hence, we can look at the upper bound for θ and the lower bound for θ_{local} using these. Again, making use of the covariates explicitly, we rewrite (7) as

$$\theta_{U_e} = \int \theta_{U_e}(x) dF_X(x),$$

where

$$\theta_{U_e}(x) = \frac{\min\{1, \mathbb{P}(Y_i = 1|Z_i = 1, X_i = x) + 1 - e(1,x)\} - \max\{0,\ \mathbb{P}(Y_i = 1|Z_i = 0, X_i = x) - e(0,x)\}}{1 - \max\{0,\ \mathbb{P}(Y_i = 1|Z_i = 0, X_i = x) - e(0,x)\}},$$

and $e(z, x) = \mathbb{P}(T_i = 1|Z_i = z, X_i = x)$. We also re-write the bounds in part (ii) of Theorem 5 as

$$\int \frac{\mathbb{P}(Y_i = 1|Z_i = 1, X_i = x) - \mathbb{P}(Y_i = 1|Z_i = 0, X_i = x)}{\min\{1 - \mathbb{P}(Y_i = 1|Z_i = 0, X_i = x),\ e(1,x) - e(0,x)\}} dF_X(x) \le \theta_{\text{local}} \le 1. \tag{15}$$

DK estimated $e(z, x)$ using the microlevel Scarborough data on television audiences. We focus on "diary audience" measure in DK[11] and take the same specifications as in columns (2) and (3) of table VIII from DK.

[10] The data used in DK are available at http://eml.berkeley.edu/~sdellavi/index.html.

[11] The microlevel Scarborough data contain the "recall" measure regarding whether a respondent watched a given channel in the past seven days and the "diary" measure on whether a respondent watched a channel for at least one full half-an-hour block according to the seven-day diary.

TABLE 6. Persuasion Rates: Fox News Effects

	(1) U.S. House district fixed effects	(2) County fixed effects
θ	[0.005,0.991]	[0.011,0.992]
θ_{local}	[0.118,1]	[0.374,1]

Table 6 summarizes our empirical results.[12] Column (1) shows estimation results when U.S. House district fixed effects are controlled for and column (2) displays corresponding results for county fixed effects.

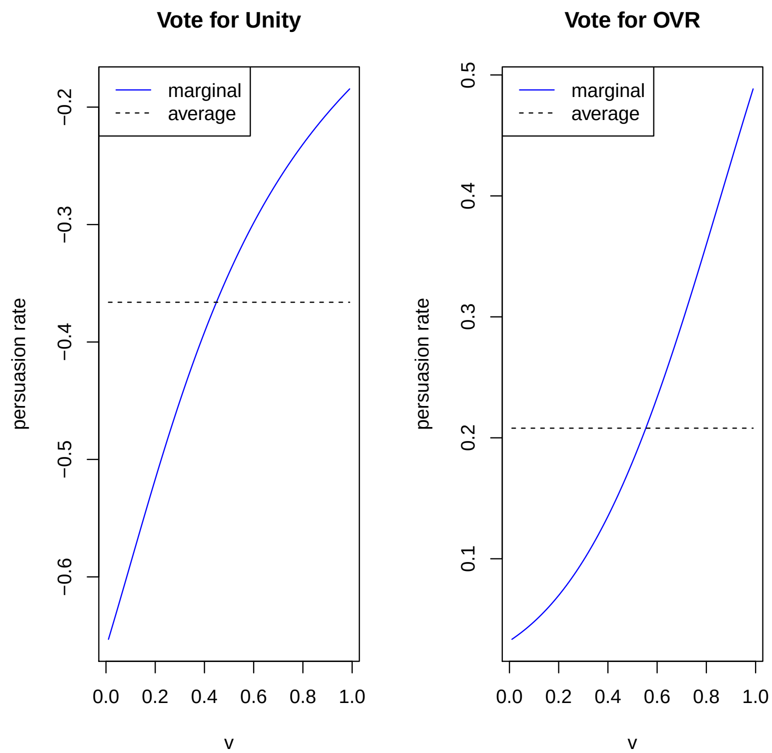
The bounds for θ are wide and uninformative. However, the lower bounds for θ_{local} are sizable and also comparable to the estimates of the persuasion rates reported in DK (0.11 and 0.28, respectively). In sum, we conclude that the persuasive effect of Fox News seems fairly large for the compliers, that is, those who would watch the Fox News channel if and only if it is randomly available.

6.3. **The NTV Effect: Enikolopov, Petrova, and Zhuravskaya (2011) Revisited.** As mentioned earlier, Enikolopov, Petrova, and Zhuravskaya (2011, EPZ hereafter) used a continuous instrument, i.e. the signal strength of NTV, to measure the persuasive effect of watching NTV (the anti-Putin TV station) on a parliamentary election in 1999. Further, in the individual–level survey data in EPZ, (Y_i, T_i, Z_i) are jointly observed. Therefore, in this subsection, we apply the identification result of the marginal persuasion rate to this example using the EPZ data.

To be consistent with our theoretical framework and other empirical examples, we let $Y_i = 1$ if an individual voted for the party of interest and $Y_i = 0$ otherwise, including the case of not voting at all. We look at two parties: the progovernment party "Unity" and the most popular opposition party OVR ("Fatherland–All Russsia"). During the 1999 election campaign, Unity was opposed by NTV, while OVR were supported by NTV. Thus, EPZ presumed a negative persuasion rate for voting for Unity but a positive persuasion rate for OVR. As in the previous section, it is necessary to condition on covariates. We take the baseline covariates as in columns (1) and (2) of table 6 and table 7 in EPZ. They include individual characteristics such as gender, age, marital status, and education, and subregional variables such as population size and average wage.

[12]To estimate the unconditional bounds reported in the table, the conditional ones are weighted by the number of respondents in a town for the Scarborough data. In addition, the predicted probabilities are truncated to be between 0 and 1.

FIGURE 2. Estimates of Marginal and Average Persuasion Rates



Notes: The left and right panels of the figure show estimates of the marginal and average persuasion rates for voting for Unity and OVR, respectively.

For the sake of simplicity, we estimate θ_{mte} parametrically. The population conditional probabilities, $e(z,x)$, $\mathbb{P}(Y_i = 1 | e(Z_i) = e, X_i = x)$ and $\mathbb{P}(Y_i = 1, T_i = 0 | e(Z_i) = e, X_i = x)$, are estimated by probit,[13] and the conditional estimates of equation (11) are averaged out with respect to covariates by sample survey weight.

Figure 2 presents the estimation results. In the left panel, $\theta_{\text{mte}}(v)$ and θ are plotted as a function of v, when the outcome variable is to vote for Unity. It can be seen that the marginal persuasive rate is about -60% at $v = 0.1$ but just -20% for $v = 0.9$. In view of equation (4), V_i can be interpreted as the unobserved cost of watching NTV. The estimation results suggest that the negative persuasive effect for Unity is much stronger for those whose unobserved cost of watching NTV is lower. In the right panel, corresponding results are shown for OVR. In this case, the positive persuasive effect is much weaker for those with lower values of v.

[13] The exposure rate $e(z,x)$ is first estimated and its predicted values are included linearly as a regressor to estimate the other two conditional probabilities.

A striking patten we can learn from figure 2 is that persuasive effects are highly heterogenous. This may partially answer the puzzle reported in EPZ. They found relatively modest positive persuasive effects for opposition parties but much stronger persuasive effects for Unity using aggregate voting outcomes, while the magnitudes are similar using individual survey data.[14] Our estimation results indicate that the marginal persuasive effects are highly heterogenous, thereby implying that different aggregate averages can be substantially different from each other. The average persuasive effect θ is plotted as a horizontal line in each panel of figure 2: it is -36.6% for Unity and 20.8% for OVR. In short, this application exemplifies the identification power of continuous instruments that can uncover the patterns of heterogeneity in persuasive effects.

7. Door-to-Door Fundraising

Landry, Lange, List, Price, and Rupp (2006) and DellaVigna, List, and Malmendier (2012) designed field experiments of door–to–door fund raising to examine various aspects of charity giving. In this section, we use their data to illustrate the usefulness of our identification results.

The common data structure in both papers is that for each type of experimental treatments, we observe (Y_i, T_i, Z_i):

- $Y_i = 1$ if a household made a contribution to door–to–door fund raising,
- $T_i = 1$ if a household answered the door and spoke to a solicitor,
- $Z_i = 1$ if a household was approached by a solicitor.

If $Z_i = 0$ (a household was not approached by a solicitor), then $T_i = 0$ and furthermore it is very likely that $Y_i = 0$. Hence, in this section, we assume that $\mathbb{P}(Y_i = 1, T_i = 0 | Z_i = 0) = \mathbb{P}(Y_i = 1 | Z_i = 0) = 0$. In addition, we assume that if $Y_i = 1$, it must be the case that $T_i = 1$. In other words, we assume that it is impossible to have both $Y_i = 1$ and $T_i = 0$ (making a contribution without answering the door). Thus, $\mathbb{P}(Y_i = 1, T_i = 1 | Z_i = 1) = \mathbb{P}(Y_i = 1 | Z_i = 1)$. These assumptions were also used in computation of the persuasion rates for donors in DellaVigna and Gentzkow (2010). Under these assumptions, we have the bound for θ as

$$\theta_L = \mathbb{P}(Y_i = 1 | Z_i = 1) \quad \text{and} \quad \theta_U = \mathbb{P}(Y_i = 1 | Z_i = 1) + 1 - e(1).$$

In addition,

$$\theta_{\text{local}} = \mathbb{P}(Y_i = 1 | Z_i = 1) / e(1);$$

θ_{local} is the same as the usual LATE.

[14] EPZ estimated the persuasion rate using a continuous version of DK. See equations (3) and (4) in EPZ for their formulae.

7.1. **Landry, Lange, List, Price, and Rupp (2006) Revisited.** In this study, there were four treatments: VCM (voluntary contributions mechanism), VCM with seed money, single-prize lottery, and multiple-prize lottery. Using Table II of Landry, Lange, List, Price, and Rupp (2006), we compute the persuasive effects by treatment and report results in table 7.

TABLE 7. Persuasive Effect by Treatment in Landry, Lange, List, Price, and Rupp (2006)

Treatment	$\mathbb{P}(Y_i = 1 \vert Z_i = 1)$	$e(1)$	θ_L	θ_U	θ_{local}
VCM	9.5%	37.6%	9.5%	71.9%	25.3%
VCM with seed money	5.2%	35.3%	5.2%	69.9%	14.8%
Single-prize lottery	17.1%	37.7%	17.1%	79.4%	45.5%
Multiple-prize lottery	12.6%	35.2%	12.6%	77.5%	35.9%
All	10.8%	36.3%	10.8%	74.5%	29.7%

Based on the lower bound and the LATE parameter, it seems that the single–prize lottery is the most effective fund raising tool, whereas the VCM with seed money is the least effective. However, the identification regions for θ of all four treatments overlap and there is no clear ranking based on those. This suggests that if one cares about the persuasive effect for the population, the evidence is inconclusive.

7.2. **DellaVigna, List, and Malmendier (2012) Revisited.** In their study of charity giving, DellaVigna, List, and Malmendier (2012, DLM hereafter) designed both fund raising and survey treatments to test for altruism and social pressure in charity giving. In this section, we focus only on three fund raising treatments: namely, the baseline treatment, the flyer treatment, and the opt-out treatment. The baseline treatment is the standard door–to–door funding raising campaign, the flyer treatment is with the flyer that provided information on fund raising the date before the solicitation, and the opt–out treatment is with the flyer that had an additional feature of a "Do Not Disturb" checkbox. There were two charities in each of the fund raising treatments: La Rabida Children's Hospital and the East Carolina Hazard Center.

DLM pointed out that treatments were randomized within a date–solicitor time block and estimated linear probability models with covariates: solicitor fixed effects, date–town fixed effects, hourly time block fixed effects, and area rating dummies. We use the same specification as in DLM, estimate $\mathbb{P}(Y_i = 1 \vert Z_i = 1, X_i = x)$ and $e(1, x)$, and then average out the conditional estimates as in

TABLE 8. Persuasive Effect by Treatment in DLM

Treatment	$\mathbb{P}(Y_i = 1 \mid Z_i = 1)$	$e(1)$	θ_L	θ_U	θ_{local}
La Rabida Children's Hospital					
Baseline	7.1%	40.5%	7.1%	66.6%	17.2%
Flyer	6.8%	36.4%	6.8%	70.4%	18.6%
Opt–Out	5.4%	30.4%	5.4%	74.9%	17.3%
East Carolina Hazard Center					
Baseline	4.7%	43.0%	4.7%	61.7%	10.9%
Flyer	5.1%	39.6%	5.1%	65.5%	13.0%
Opt–Out	3.0%	34.4%	3.0%	68.6%	8.6%

section 6.2.[15] The resulting estimates are reported in table 8, where we report the persuasive effect by treatment/charity.

The local persuasion rate is point identified and is higher for the in–state charity, La Rabida Children's Hospital. The estimates of θ_{local} are the highest for the flyer treatment in both charities. This does not mean that the flyer treatment is the most effective in fund raising for the general population. Note that the compliers of the baseline treatment are different from those of the flyer treatment. For example, it could be the case that households at the margin of giving might have decided to not answer the door after they noticed the flyer. Unlike θ_{local}, θ_L and θ_U are comparable across different treatments. However, as in the previous section, it is difficult to see whether there is a significant difference across treatments if we focus on the bounds for θ.[16]

8. CONCLUSIONS

We have set up a simple econometric model of persuasion, have introduced several parameters of interest, and have analyzed their identification. Our extensive empirical examples demonstrate that the persuasive effects are highly heterogenous in both settings of media and fund raising. We have focused on the case of binary treatments. In applications, treatments could be multivalued, for example watching Fox News, CNN or MSNBC. It would be fruitful to build on recent developments in multiple treatments (e.g. Heckman and Pinto, 2017; Lee and Salanié, 2015) to investigate identification of persuasive effects. It would be also interesting to estimate deep parameters in

[15] The data collected in DLM are available at `http://eml.berkeley.edu/~sdellavi/index.html`. As before, the predicted probabilities are truncated to be between 0 and 1, when they are averaged out.

[16] In addition to the fund raising treatments, DLM relied on survey treatments and structural estimates to draw conclusions in their paper.

an economic model of persuasion using a more structural approach in the setup of multiple treatments. These are topics of future research.

Appendix A. Proofs

A.1. **Proof of lemma 1.** If $q_i(0) \leq q_i(1)$, then $Y_i(0) = 1$ and $Y_i(1) = 0$ cannot happen. Now, conversely, suppose that $Y_i(0) \leq Y_i(1)$. If $\mathbb{P}\{q_i(1) < q_i(0)\} > 0$, then assumption A implies that $\mathbb{P}\{q_i(1) < 1/(1+U_i) < q_i(0)\} > 0$. This contradicts $\mathbb{P}\{Y_i(0) \leq Y_i(1)\} = 1$. The denominator on the right-hand side of equation (5) is nonzero since $\mathbb{P}\{Y_i(0) = 1\} < 1$ by assumption A. Finally, equation (5) follows from the fact that $Y_i(1) - Y_i(0) = \mathbb{1}\{Y_i(1) = 1, Y_i(0) = 0\}$ with probability one. □

A.2. **Proof of theorem 1.** By assumptions C and D,

$$\mathbb{P}\{Y_i(z) = 1\} = \mathbb{P}(Y_i(z) = 1|Z_i = z) = \mathbb{P}(Y_i = 1|T_i = z) = \mathbb{P}(Y_i = 1|Z_i = z). \tag{16}$$

So, the assertion follows from lemma 1 and the definition of θ_L. □

A.3. **Under assumption E.**

Lemma A.1. *$\mathbb{P}\{Y_i(1) = 1 \mid e(0) < V_i \leq e(1)\}$ is identified by*

$$\frac{\mathbb{P}(Y_i = 1, T_i = 1|Z_i = 1) - \mathbb{P}(Y_i = 1, T_i = 1|Z_i = 0)}{e(1) - e(0)}.$$

Similarly, $\mathbb{P}\{Y_i(0) = 1 \mid e(0) < V_i \leq e(1)\}$ is identified by

$$\frac{\mathbb{P}(Y_i = 1, T_i = 0|Z_i = 0) - \mathbb{P}(Y_i = 1, T_i = 0|Z_i = 1)}{e(1) - e(0)}.$$

Proof. The first assertion follows from

$$\mathbb{P}(Y_i = 1, T_i = 1|Z_i = z) = \mathbb{P}\{Y_i(1) = 1, V_i \leq e(z)\}. \tag{17}$$

The second statement is similar. □

Lemma A.2. *For $z = 0, 1$, $\mathbb{P}\{Y_i(1) = 1 \mid V_i \leq e(z)\}$ is identified by*

$$\mathbb{P}(Y_i = 1, T_i = 1|Z_i = z)/e(z).$$

Similarly, $\mathbb{P}\{Y_i(0) = 1 \mid V_i > e(z)\}$ is identified by

$$\mathbb{P}(Y_i = 1, T_i = 0|Z_i = z)/\{1 - e(z)\}.$$

Proof. The first assertion follows from

$$\mathbb{P}(Y_i = 1, T_i = 1 | Z_i = z) = \mathbb{P}\{Y_i(1) = 1, V_i \le e(z)\}. \tag{18}$$

The second assertion is similar. □

Lemma A.3. *The sharp identified interval of* $\mathbb{P}\{Y_i(1) = 1\}$ *is given by*

$$\big[\mathbb{P}(Y_i = 1 | Z_i = 1),\ \mathbb{P}(Y_i = 1, T_i = 1 | Z_i = 1) + 1 - e(1)\big].$$

Similarly, the sharp identified interval of $\mathbb{P}\{Y_i(0) = 1\}$ *is given by*

$$\big[\mathbb{P}(Y_i = 1, T_i = 0 | Z_i = 0),\ \mathbb{P}(Y_i = 1 | Z_i = 0)\big].$$

Proof. For the first assertion, note that

$$\begin{aligned} \mathbb{P}\{Y_i(1) = 1\} &= \mathbb{P}\{Y_i(1) = 1 \mid e(0) < V_i \le e(1)\}\{e(1) - e(0)\} \\ &\quad + \mathbb{P}\{Y_i(1) = 1 \mid V_i \le e(0)\}e(0) + \mathbb{P}\{Y_i(1) = 1 \mid V_i > e(1)\}\{1 - e(1)\}. \end{aligned} \tag{19}$$

By lemmas A.1 and A.2, the first two terms on the right–hand side of equation (19) are identified and their sum is equal to $\mathbb{P}(Y_i = 1, T_i = 1 | Z_i = 1)$. For the third term on the right–hand side of equation (19), note that

$$\begin{aligned} &\mathbb{P}\{Y_i(1) = 1 \mid V_i > e(1)\}\{1 - e(1)\} \\ &\qquad \ge \mathbb{P}\{Y_i(0) = 1 \mid V_i > e(1)\}\{1 - e(1)\} = \mathbb{P}(Y_i = 1, T_i = 0 | Z_i = 1), \end{aligned} \tag{20}$$

where $\mathbb{P}\{Y_i(1) = 1 \mid V_i > e(1)\} \le 1$. Therefore, the sharp bounds of the third term on the right–hand side of equation (19) is the interval between $\mathbb{P}(Y_i = 1, T_i = 0 | Z_i = 1)$ and $1 - e(1)$. Combining all these proves the first assertion. The second assertion is similar. □

Proof of Theorem 2: Let $a = \mathbb{P}\{Y_i(1) = 1\}$ and $b = \mathbb{P}\{Y_i(0) = 1\}$: so, $\theta = (a - b)/(1 - b)$. Let m_a, M_a be the lower and upper bounds of a provided in lemma A.3. Similarly, let m_b, M_b be the bounds of b given in lemma A.3. By lemma A.3 and the fact that the dependence between $Y_i(0)$ and $Y_i(1)$ is unrestricted except that $Y_i(0) \le Y_i(1)$, the identified bounds of θ can be obtained by

$$\max_{a,b} \text{ and } \min_{a,b} \frac{a - b}{1 - b} \quad \text{subject to} \quad a \in [m_a,\ M_a],\ b \in [m_b,\ M_b],\ a \ge b. \tag{21}$$

Here, note that $M_a \geq m_b$ because $e(0) < e(1)$ and $Y_i(0) \leq Y_i(1)$. Also, $m_a \geq M_b$ because $\theta \geq 0$. Therefore, the constraint $a \geq b$ is redundant. So, the minimum is $\theta_L = (m_a - M_b)/(1 - M_b) \geq 0$ and the maximum is $\theta_U = (M_a - m_b)/(1 - m_b)$: the monotonicity of the probability measure trivially shows that $\theta_U \leq 1$. Finally, sharpness follows from the intermediate value theorem because $(a - b)/(1 - b)$ varies continuously between θ_L and θ_U. □

A.4. **Under assumption F.**

Lemma A.4. *The sharp identified interval of* $\mathbb{P}\{Y_i(1) = 1\}$ *is given by*

$$\big[\mathbb{P}(Y_i = 1|Z_i = 1),\ \min\{1, \mathbb{P}(Y_i = 1|Z_i = 1) + 1 - e(1)\}\big] \tag{22}$$

Similarly, the sharp identified interval of $\mathbb{P}\{Y_i(0) = 1\}$ *is given by*

$$\big[\max\{0,\ \mathbb{P}(Y_i = 1|Z_i = 0) - e(0)\},\ \mathbb{P}(Y_i = 1|Z_i = 0)\big]. \tag{23}$$

Proof. First, we focus on $\mathbb{P}\{Y_i(1) = 1\}$. Let $C = \min\big[\mathbb{P}\{Y_i(0) = 1, V_i > e(1)\},\ \mathbb{P}\{Y_i(1) = 0, V_i \leq e(1)\}\big]$. Then, $C \geq 0$. Further,

$$C + \mathbb{P}(Y_i = 1, T_i = 1|Z_i = 1) = \min\{\mathbb{P}(Y_i = 1|Z_i = 1),\ e(1)\}. \tag{24}$$

Therefore, by lemma A.3,

$$\begin{aligned} \mathbb{P}(Y_i = 1|Z_i = 1) \leq \mathbb{P}\{Y_i(1) = 1\} &\leq \min\{\mathbb{P}(Y_i = 1|Z_i = 1) + 1 - e(1),\ 1\} - C \\ &\leq \min\{\mathbb{P}(Y_i = 1|Z_i = 1) + 1 - e(1),\ 1\}, \end{aligned}$$

where the last inequality follows from $C \geq 0$. For sharpness, we only need to show that C can take any value between 0 and $\min\{1 - e(1),\ 1 - \mathbb{P}(Y_i = 1|Z_i = 1)\}$. But this follows from the fact that

$$\begin{aligned} &0 \leq \mathbb{P}\{Y_i(0) = 1, V_i > e(1)\} \leq \mathbb{P}\{V_i > e(1)\} = 1 - e(1), \\ &0 \leq \mathbb{P}\{Y_i(1) = 0, V_i \leq e(1)\} = \mathbb{P}\{Y_i = 0, T_i = 1|Z_i = 1\} \leq \mathbb{P}\{Y_i = 0|Z_i = 1\}. \end{aligned}$$

For instance, if either $\mathbb{P}\{Y_i(0) = 1|V_i > e(1)\} = 0$ or $\mathbb{P}\{Y_i(1) = 0|V_i \leq e(1)\} = 0$, then $C = 0$, and if $\mathbb{P}\{Y_i(0) = 1|V_i > e(1)\} = 1$, then $C = \min\{1 - e(1),\ 1 - \mathbb{P}(Y_i = 1|Z_i = 1)\}$.[17]

The second assertion is similar. □

[17] Note that $\mathbb{P}(Y_i = 0|Z_i = 1) = \mathbb{P}\{Y_i(1) = 0,\ V_i \leq e(1)\} + \{1 - e(1)\}\big[1 - \mathbb{P}\{Y_i(0) = 1|V_i > e(1)\}\big]$.

Proof of theorem 3: Similarly to the proof of theorem 2, we need to consider

$$\max_{a,b} \text{ and } \min_{a,b} \frac{a-b}{1-b} \quad \text{subject to} \quad a \in [m_a, \tilde{M}_a],\ b \in [\tilde{m}_b, M_b],\ a \geq b, \tag{25}$$

where $m_a, \tilde{M}_a, \tilde{m}_b, M_b$ are given in lemma A.4. Follow the same reasoning as theorem 2. □

A.5. **Under assumption G. Proof of theorem 4:** Since theorem 3 uses more information but its lower bound only depends on the distribution of (Y_i, Z_i), it suffices to focus on the upper bound. From theorem 3, we can find the sharp upper bound in this case by

$$\max_{0<e(0)\leq e(1)<1} \theta_{U_e} = \frac{\min\{1, \mathbb{P}(Y_i = 1|Z_i = 1) + 1 - e(1)\} - \max\{0,\ \mathbb{P}(Y_i = 1|Z_i = 0) - e(0)\}}{1 - \max\{0,\ \mathbb{P}(Y_i = 1|Z_i = 0) - e(0)\}}. \tag{26}$$

Note that setting $e(0) = \mathbb{P}(Y_i = 1|Z_i = 0) \leq \mathbb{P}(Y_i = 1|Z_i = 1) = e(1)$ yields the maximum value 1. Sharpness follows from the fact that θ_{U_e} is continuous in $\big(e(0), e(1)\big)$. □

A.6. **For the Compliers. Proof of theorem 5:** For part *(i)*, note that

$$\mathbb{P}(Y_i = 1|Z_i = z) = \mathbb{P}\{Y_i(1) = 1, V_i \leq e(z)\} + \mathbb{P}\{Y_i(0) = 0, V_i > e(z)\},$$

from which it follows that

$$\theta_{\text{local}} = \frac{\mathbb{P}(Y_i = 1|Z_i = 1) - \mathbb{P}(Y_i = 1|Z_i = 0)}{\{e(1) - e(0)\}\mathbb{P}\{Y_i(0) = 0|e(0) < V_i \leq e(1)\}}. \tag{27}$$

Finally, note that the denominator on the right–hand side of equation (27) is equal to

$$\mathbb{P}\{Y_i(0) = 0, e(0) < V_i \leq e(1)\} = \mathbb{P}\{Y_i(0) = 0, e(0) < V_i\} - \mathbb{P}\{Y_i(0) = 0, e(1) < V_i\},$$

where $\mathbb{P}\{Y_i(0) = 0, e(z) < V_i\} = \mathbb{P}\{Y_i = 0, T_i = 0|Z_i = z\}$.

For part *(ii)*, we look for sharp bounds for $\mathbb{P}\{Y_i(0) = 0, e(0) < V_i \leq e(1)\}$ under assumption F. Using the fact that the sharp bounds of $\mathbb{P}(A \cap B \cap C)$ when $\mathbb{P}(A \cap B), \mathbb{P}(B \cap C)$, and $\mathbb{P}(C \cap A)$ are given are equal to the interval between 0 and $\min\{\mathbb{P}(A \cap B), \mathbb{P}(B \cap C), \mathbb{P}(C \cap A)\}$, we know that

$$\begin{aligned} 0 &\leq \mathbb{P}\{Y_i(0) = 0, e(0) < V_i \leq e(1)\} \\ &\leq \min\big[\mathbb{P}\{Y_i(0) = 0, V_i > e(0)\},\ e(1) - e(0),\ \mathbb{P}\{Y_i(0) = 0, V_i \leq e(1)\}\big], \end{aligned} \tag{28}$$

where it suffices to look for the sharp upper bound of the expression on the utmost left–hand side. First,

$$\mathbb{P}\{Y_i(0) = 0, V_i > e(0)\} = \mathbb{P}(Y_i = 0, T_i = 0|Z_i = 0) \leq \mathbb{P}(Y_i = 0|Z_i = 0),$$

where the inequality holds with equality when $\mathbb{P}(Y_i = 0, T_i = 1 | Z_i = 0) = 0$. Second, note that

$$\mathbb{P}\{Y_i(0) = 0, V_i \leq e(1)\} = \mathbb{P}\{Y_i(0) = 0, T_i = 1 | Z_i = 1\}$$

is totally unidentified. So, we conclude that the sharp upper bound of the term on the right–hand side of equation (28) is

$$\min\{\mathbb{P}(Y_i = 0 | Z_i = 0), e(1) - e(0)\}. \tag{29}$$

The bound in part *(iii)* corresponds to the case where $e(1) - e(0) = 1$. □

Proof of theorem 6: By the same reasoning as lemma 1, we have

$$\theta_{\text{mte}}(v) = \frac{\mathbb{E}\{Y_i(1) - Y_i(0) \,| V_i = v\}}{\mathbb{P}(Y_i(0) = 0 \mid V_i = v\}}. \tag{30}$$

Then as shown in Heckman and Vytlacil (2005),

$$\mathbb{E}\{Y_i(1) - Y_i(0) \,| V_i = v\} = \left. \frac{\partial \mathbb{P}\{Y_i = 1 | e(Z_i) = e\}}{\partial e} \right|_{e=v}.$$

Also, by the same argument as in Heckman and Vytlacil (2005),

$$\mathbb{E}\{Y_i(0) \,| V_i = v\} = -\left. \frac{\partial \mathbb{P}\{Y_i = 1, T_i = 0 | e(Z_i) = e\}}{\partial e} \right|_{e=v}.$$

The desired result follows immediately. □

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