NOISY SHARE PRICES AND THE *Q* MODEL OF INVESTMENT

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Abstract

We consider to what extent the empirical failings of the Q model of investment can be attributed to the use of share prices to measure average q. We show that the usual empirical formulation may fail to identify the Q model when stock market valuations deviate from the present value of expected net distributions to shareholders in ways that are consistent with weak and semi-strong forms of the Efficient Markets Hypothesis. We show that the structural parameters of the Q model can still be identified in this case using a direct estimate of the firm's fundamental value, and implement this using data on securities analysts' earnings forecasts for a large sample of publicly traded US firms. Our empirical results suggest that stock market valuations deviate significantly from fundamental values. Controlling for this, we find no evidence that the Q model of investment is seriously misspecified.

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Summary

The Q model of investment relates investment to average q. Abstracting from debt, other assets and taxes, average q is the ratio of the maximised value of the firm to the replacement cost of its capital. Hayashi (1982) established conditions under which this average q ratio should be a sufficient statistic for company investment rates. That is, given average q, no other variables should help to explain investment rates.

Traditionally the value of the firm has been measured using its stock market valuation. The resulting measures of 'Tobin's q' have been found to be only weakly related to investment, and the sufficient statistic prediction is soundly rejected. Notably, financial variables like cash flow help to explain investment in addition to Tobin's q, which may be suggestive of financing constraints.

We consider to what extent these empirical findings can be attributed to the use of share prices to measure average q. We analyse the measurement error in Tobin's q that occurs if stock market valuations deviate from the present value of expected net distributions to shareholders. We show that such 'noise' in share prices can be catastrophic for identification of the underlying investment model when these deviations are both persistent and correlated with the true value of the firm. This kind of deviation between stock market and fundamental values is quite consistent with the Efficient Markets Hypothesis and can occur, for example, in rational bubble or noise trader models of asset pricing. Summers (1986) has shown that such deviations would not be detected by standard tests of the Efficient Markets Hypothesis.

We also show that the Q model of investment can still be estimated consistently in this case by using a direct estimate of the firm's fundamental value in place of its stock market valuation when constructing the average q ratio. We implement this approach for a large sample of publicly traded US firms, using estimates of firms' fundamental values obtained by discounting securities analysts' forecasts of their future earnings. We find that the Q model performs dramatically better when this alternative measure of average q is used. In particular, we find that our alternative measure of average q dominates not only Tobin's q based on firms' stock market valuations, but also cash flow and sales variables. This is consistent with the Q model of investment, with firms taking investment decisions to maximise their fundamental values. One implication is that stock market values do deviate significantly from fundamental values. Given expectations of future earnings, share price fluctuations seem to be a sideshow for investment. A further implication is that financing constraints do not appear to be significant for this sample of US companies.

> "Perhaps no single empirical issue is of more fundamental importance to both the fields of financial economics and macroeconomics than the question of whether or not stock prices are a well-informed and rational assessment of the value of future earnings available to stockholders"
>
> Fischer and Merton (1984), p.94

1 Introduction

Hayashi (1982) showed that marginal q is a sufficient statistic for investment in a value-maximizing model of investment behavior with strictly convex adjustment costs, and established conditions under which marginal q and average q are equal.[1] Marginal q depends on the unobserved shadow value of an additional unit of installed capital. In the simplest case, average q is the ratio of the value of the firm to the replacement cost of its installed capital, and, in principle, can be measured. The key requirement for equality of average and marginal q is linear homogeneity of the net revenue function. The standard empirical formulation uses the stock market valuation as a measure of the value of the firm. These results thus provide a rigorous link between the firm's optimal investment decisions and expectations of future profitability, summarized by the observable stock market valuation.

This model of investment has been subjected to extensive empirical testing, using both micro and macro data in several countries. Almost invariably, the model has been rejected. The empirical relationship between investment and conventional measures of average q is weak; when the structural investment equation is estimated the marginal adjustment cost parameter is found to be implausibly high; and perhaps most importantly, the prediction that average q is a sufficient statistic for investment is generally rejected.[2] Abel and Blanchard (1986) find similar results for a version of the model that relaxes the equality between average and marginal q, and eschews the use of share price data.[3] Hayashi and Inoue (1991) find similar results for a version of the model which allows for multiple capital inputs. Standard econometric procedures to allow for measurement error in average q have yielded similar findings,[4] although Cummins,

[1] See also Abel (1980), Mussa (1977) and Lucas and Prescott (1971).

[2] See, for example, Fazzari, Hubbard and Petersen (1988), Hayashi and Inoue (1991) and Blundell, Bond, Devereux and Schiantarelli (1992).

[3] See Gilchrist and Himmelberg (1995) for an application of this approach using micro data.

[4] See, for example, Hayashi and Inoue (1991) and Blundell *et al.* (1992).

Hassett and Hubbard (1994, 1995) report more reasonable estimates of the structural parameters in periods around tax reforms when variation in average q is dominated by exogenous changes in its tax components. More recently, Cummins, Hassett, and Oliner (1999) have found a strong relationship between investment rates and an alternative measure of q, obtained by capitalising analysts' forecasts of the firm's future earnings, rather than relying on the firm's stock market valuation. Erickson and Whited (2000) have also reported more favorable results using an estimator that allows for persistent measurement error in the conventional measure of average q. These results are consistent with the idea that share prices may provide a noisy measure of the firm's true value, resulting in a severe measurement error problem with the standard implementation of this investment model. Nevertheless the large literature on the effects of capital market imperfections[5] and the growing literature that emphasizes non-convex adjustment costs and the option value of waiting to invest[6] suggest the consensus view is that the underlying investment model is seriously misspecified.

In this paper we analyze the measurement error problem with conventional measures of average q that occurs if stock market valuations deviate from the present value of expected future net distributions to shareholders. Equality between the stock market valuation and this fundamental value is not implied by the weaker concepts of stock market efficiency that are commonly tested in the finance literature.[7] Persistent deviations between stock market and fundamental values can occur, for example, in rational bubble or noise trader models of asset prices, both of which are consistent with weak market efficiency.[8] Moreover, such deviations may themselves be correlated with new information about the true value of the firm. We show that in this case the parameters of the adjustment cost function may not be identified, in the sense that there may be no valid moment conditions available for the investment equation that is usually estimated.

In this case, we also show that the investment equation can be identified by constructing an alternative measure of average or marginal q that does not depend on share price data. We consider an alternative measure of average q, along the lines

[5] See, for example, the surveys in Schiantarelli (1996) and Hubbard (1998).
[6] See, for example, Caballero (1998) and Dixit and Pindyck (1994).
[7] See, for example, Summers (1986) and the discussion in Campbell, Lo and MacKinlay (1997).
[8] See, respectively, Blanchard and Watson (1982) and Froot and Obstfeld (1991), and Campbell and Kyle (1993).

proposed by Cummins, Hassett, and Oliner (1999), based on the discounted value of securities analysts' earnings forecasts for individual firms, and we discuss how this approach differs from that considered in Abel and Blanchard (1986). Our approach identifies the adjustment cost parameters in the structural investment equation if the measurement error in our estimate of the firm's fundamental value is independent of the observed variables used as instruments, and we show how this condition can be tested. This also allows a rigorous test of the null hypothesis that the firm's investment is chosen to maximize the fundamental value of the firm, and is not influenced by the deviation between this fundamental value and the current stock market valuation.[9]

Our empirical results, using panel data for US companies that are publicly traded and followed by at least one analyst, are striking. Using the conventional measure of average q, based on stock market valuations, we replicate the usual empirical findings. Using alternative instrument sets has little effect on these results, and tests of overidentifying restrictions appear to reject the model. However we obtain quite different results when we use the alternative measure of average q based on analysts' earnings forecasts: the same tests of overidentifying restrictions do not reject the model in this case; we find more reasonable estimates of the marginal adjustment cost parameter; and using this measure we do not reject the prediction that average q is a sufficient statistic for investment. We further obtain similar empirical findings using a semi-log approximation to the standard investment equation, which we show to be robust to the presence of a particular form of measurement error in stock market valuations that is consistent with weak market efficiency.

Our results challenge the view that the Q model of investment, based on the assumptions of fundamental value-maximization, strictly convex adjustment costs and a linear homogeneous net revenue function, is seriously misspecified, at least for company-level data. They suggest that stock market valuations deviate significantly from well-informed assessments of the value of future earnings available to stockholders, although these share price anomalies are not a significant influence on company investment.

[9] See Mørck, Shleifer and Vishny (1990), Blanchard, Rhee and Summers (1993) and Galeotti and Schiantarelli (1994) for previous studies of this issue.

The remainder of the paper is organized as follows. Section 2.1 sets out the basic investment model we analyze. Section 2.2 considers estimation of this model in the presence of measurement error when stock market and fundamental values differ, and shows that the structural parameters may not be identified in one important case. Section 2.3 shows how identification can be achieved in this case by using an alternative estimate of average q based on analysts' earnings forecasts. We relate our approach to that suggested by Abel and Blanchard (1986), and discuss how it can be used to test the hypothesis that firms choose investment to maximize their fundamental value in cases where stock market and fundamental values deviate systematically. Section 2.4 considers a semi-log approximation to the Q model, and shows how this is robust to one particularly interesting form of measurement error. Section 2.5 presents the specification of the empirical investment equations. Section 3 describes our data. Section 4 presents our empirical results, and section 5 concludes. The Data Appendix describes in detail the construction of the different measures of average q.

2 The Q model

2.1 Basic model

The model we consider is standard in the investment literature. The objective of the firm when choosing investment at time t is to maximize the present value of the stream of current and expected future net distributions to its existing shareholders. Assuming, for simplicity, no taxes and no debt finance, the net distribution to shareholders (i.e., dividends paid minus the value of new shares issued) coincides with the net revenue generated by the firm in each period. Thus the firm's objective is to maximize:[10]

$$V_t = E_t\left[\sum_{s=0}^{\infty} \beta_{t+s}\Pi_{t+s}\right], \tag{1}$$

where Π_{t+s} denotes net revenue generated in period $t+s$; β_{t+s} is the discount factor used in period t to discount expected revenue in period $t+s$, with $\beta_t = 1$; and $E_t[.]$ denotes an expectation conditioned on information available in period t.

[10] The firm index i is suppressed except when needed for clarification.

We specify the net revenue function to have the form

$$\Pi_t\left(K_t, L_t, I_t\right) = p_t F(K_t, L_t) - w_t L_t - p_t^K\left[I_t + G(I_t, K_t)\right], \tag{2}$$

where K_t is the stock of capital in period t, L_t denotes a vector of variable inputs used in period t, I_t is gross investment in period t, p_t is the price of the firm's output, w_t is a vector of prices/wage rates for the variable inputs, and p_t^K is the price of capital goods in period t. $F(K_t, L_t)$ is the production function and $G(I_t, K_t)$ is the adjustment cost function. Our timing assumption is that current investment is immediately productive, and the stock of capital evolves according to

$$K_{t+s} = (1-\delta)K_{t+s-1} + I_{t+s}, \tag{3}$$

where δ is the rate of depreciation. We also assume that current prices and the realizations of current technology shocks are known to the firm when choosing current investment. The expected value in equation (1) is taken over the distribution of future prices and technology shocks. Other timing conventions are certainly possible, but would not affect the substance of our analysis in the following sections.

The firm chooses investment to maximize V_t subject to the capital accumulation constraint in equation (3). The first order conditions for this problem give

$$-\left(\frac{\partial \Pi_t}{\partial I_t}\right) = \lambda_t, \tag{4}$$

and

$$\lambda_t = E_t\left[\sum_{s=0}^{\infty} \beta_{t+s}(1-\delta)^s\left(\frac{\partial \Pi_{t+s}}{\partial K_{t+s}}\right)\right], \tag{5}$$

where λ_t is the shadow value of an additional unit of installed capital in period t.

Given equation (2) and price-taking behavior, the first order condition (4) can be rearranged as

$$\left(\frac{\partial G_t}{\partial I_t}\right) = (q_t - 1), \tag{6}$$

where $q_t = \lambda_t / p_t^K$ is marginal q, or the ratio of the shadow value of an additional unit of capital to its purchase cost. In the absence of adjustment costs, investment is chosen such that marginal q is unity, and in the presence of strictly convex adjustment costs investment is an increasing function of marginal q.

The average q model requires that $\Pi_t\,(K_t, L_t, I_t)$ is homogeneous of degree one in (K_t, L_t, I_t), sufficient conditions for which are that both the production function and the adjustment cost function exhibit constant returns to scale, and the firm is a price-taker in all markets. Given this linear homogeneity, Hayashi (1982) proved the equality of marginal q and average q, which with our timing convention yields

$$q_t = \frac{V_t}{p_t^K(1-\delta)K_{t-1}}. \tag{7}$$

Average q is the ratio of the value of a firm entering period t with a capital stock of $(1-\delta)K_{t-1}$ inherited from the past, to the replacement cost value of that capital in period t. Notice that the numerator of average q in (7) is the present value of current and expected future net distributions to shareholders, as in equation (1). As noted in the introduction, the firm's stock market valuation need not coincide with this fundamental value, even if stock markets satisfy weak and semi-strong forms of the Efficient Markets Hypothesis, as defined by Fama (1970).

Further assuming that adjustment costs have the symmetric, quadratic form

$$G(I_t, K_t) = \frac{b}{2}\left[\left(\frac{I_t}{K_t}\right) - c - e_t\right]^2 K_t, \tag{8}$$

then gives the convenient linear model

$$\begin{aligned} \left(\frac{I_t}{K_t}\right) &= c + \frac{1}{b}\left(\frac{V_t}{p_t^K(1-\delta)K_{t-1}} - 1\right) + e_t, \\ &= c + \frac{1}{b}Q_t + e_t \end{aligned} \tag{9}$$

in which the error term e_t is an adjustment cost shock, observed by the firm but not by the econometrician, which may be serially correlated.[11]

[11] It is well known that the Q model can be extended to allow for debt finance and the presence of taxes. See, for example, Summers (1981) and Hayashi (1982, 1985). We incorporate the standard adjustments for debt finance and tax in the empirical measures of Q_t used in section 4. These are detailed in the Data Appendix.

2.2 Stock market valuations and identification of the Q model

Under the assumption that the firm's stock market valuation (V_t^E) equals its fundamental value (V_t), the empirical investment equation (9) can be estimated consistently by using the equity valuation to measure the numerator of average q. We relax this assumption to allow for the possibility that $V_t^E \neq V_t$, and consider the implications of the resulting measurement error in average q for the identification and estimation of the Q investment model in equation (9).

We first write

$$V_t^E = V_t + m_t, \tag{10}$$

where m_t denotes the error or 'noise' in stock market values as a measure of V_t. The usual measure of Q_t that uses the firm's equity valuation then has the form

$$\begin{aligned} Q_t^E &= \frac{V_t + m_t}{p_t^K(1-\delta)K_{t-1}} - 1 \\ &= Q_t + \frac{m_t}{p_t^K(1-\delta)K_{t-1}} \\ &= Q_t + \mu_t. \end{aligned} \tag{11}$$

Substituting Q_t^E for Q_t in the investment model (9) then gives

$$\left(\frac{I_t}{K_t}\right) = c + \frac{1}{b}Q_t^E + \left(e_t - \frac{\mu_t}{b}\right). \tag{12}$$

It is useful to distinguish among three forms that the measurement error m_t in stock market valuations may take. We discuss what each form implies for the properties of the measurement error μ_t that enters the investment equation (12), and consequently for the identification and consistent estimation of the structural parameters.

For convenience we introduce the notation $\kappa_t = p_t^K(1-\delta)K_{t-1}$, so that $\mu_t = \frac{m_t}{\kappa_t}$, and assume throughout that $\kappa_t > 0$.

Proposition 1 *If m_t is mean zero, serially uncorrelated and independent of κ_s and V_s $\forall s,t$, then μ_t is serially uncorrelated and uncorrelated with Q_s^E for $s \neq t$.*

Proof: $Cov(\mu_t, \mu_s) = E(\mu_t\mu_s) = E(m_t m_s)E\left(\frac{1}{\kappa_t\kappa_s}\right) = 0$ for $s \neq t$.

$Cov(\mu_t, Q_s^E) = E(\mu_t Q_s^E) = E\left[\left(\frac{m_t}{\kappa_t}\right)\left(\frac{V_s}{\kappa_s} + \frac{m_s}{\kappa_s} - 1\right)\right]$

$= E(m_t)E\left(\frac{V_s}{\kappa_t \kappa_s}\right) + E(m_t m_s)E\left(\frac{1}{\kappa_t \kappa_s}\right) - E(m_t)E\left(\frac{1}{\kappa_t}\right) = 0$ for $s \neq t$.■

This is the case of pure random noise in stock market valuations. In this case, the measurement error in Q_t^E is also serially uncorrelated, and lagged values of Q_s^E itself are admissible as instrumental variables for Q_t^E in (12).[12] This result easily extends to the case where m_t is $MA(k)$, provided that it remains independent of κ_s and V_s $\forall s,t$. In that case, μ_t is also $MA(k)$, and Q_{t-k-1}^E and longer lags are available as instruments. Thus it would be possible to obtain consistent parameter estimates provided the time dimension of the panel exceeds k. However, previous research suggests that allowing for this type of measurement error in the Q model does not have a major impact on the empirical results.[13]

Several models of share price bubbles and noise trading predict highly persistent deviations of equity valuations from fundamental values. We thus consider the case of unrestricted serial correlation in m_t. Proposition 2 below shows that identification may still be possible, provided this persistent measurement error is independent of some observed instruments.

Proposition 2 *If m_t is mean zero and independent of κ_s and z_s $\forall s,t$, then μ_t is uncorrelated with z_s $\forall s,t$.*

Proof: $Cov(\mu_t, z_s) = E(\mu_t z_s) = E(m_t)E\left(\frac{z_s}{\kappa_t}\right) = 0$.■

This case rules out lagged values of Q_s^E as instruments for Q_t^E, but if the measurement error is independent of the true value of the firm V_s and its capital stock κ_s, then it may still be possible to obtain consistent estimates of the investment equation using lagged values of variables like sales, profits or investment itself as instruments.[14] The key to identification in this case would be the exclusion of lagged values of Q_s^E itself from the instrument set.

Propositions 1 and 2 provide sufficient conditions on the measurement error m_t in stock market valuations that permit the adjustment cost parameters c and $\frac{1}{b}$ to be identified from the standard empirical model (12). More generally these parameters

[12] This assumes that the adjustment cost shocks (e_t) are also serially uncorrelated. We relax this assumption later.

[13] See, for example, Hayashi and Inoue (1991) and Blundell *et al.* (1992).

[14] Current values of these variables will not be valid instruments if they are correlated with e_t. Erickson and Whited (2000) have recently proposed a different approach to identification of the Q model when average q is measured with persistent error, which also relies on independence between the measurement error and the true value of average q.

may not be identified using this conventional approach. The most problematic case occurs when m_t is both persistent and correlated with the true value of the firm and the resulting measurement error μ_t in (12) inherits these properties. In this case, very restrictive additional conditions are required to identify the model.

To illustrate this, we consider a simple example in which the fundamental value of the firm evolves as a random walk

$$V_t = V_{t-1} + \omega_t \tag{13}$$

with $\omega_t \sim iidN(0, \sigma_\omega^2)$ and

$$\begin{aligned} m_t &= \mu_t \kappa_t \\ \mu_t &= \theta_0 u_t + \theta_1 u_{t-1} + ... + \theta_k u_{t-k} \end{aligned} \tag{14}$$

with $u_t \sim iidN(0, \sigma_u^2)$ and $\theta_s \neq 0$ for $s = 0, 1, \ldots, k$. The measurement error introduced into the investment equation (12) is thus $MA(k)$, and we assume here that k exceeds the time dimension of the panel available. Let

$$u_t = \widetilde{u}_t + \phi \omega_t \tag{15}$$

with $E(\widetilde{u}_t \omega_t) = 0$ and $\phi \neq 0$, and consider a candidate instrumental variable z_t

$$z_t = \widetilde{z}_t + \psi \omega_t \tag{16}$$

with $E(\widetilde{z}_t \omega_t) = 0$ and $\psi \neq 0$. For simplicity, let $E(\widetilde{u}_t \widetilde{z}_s) = 0$ for all t, s, so that the only correlation between u_t and z_t comes through their dependence on ω_t. Then

$$\begin{aligned} E(\mu_t z_{t-s}) &= \theta_s E(u_{t-s} z_{t-s}) = \theta_s \phi \psi \sigma_\omega^2 \\ &\neq 0 \text{ for } s = 0, 1, ..., k. \end{aligned} \tag{17}$$

Consequently no observed lags of z_{t-s} are valid instruments for Q_t^E in (12), and if all candidate instruments depend on the innovation ω_t in the firm's fundamental value in a similar way then the parameters of the investment model (12) are not identified.

In this example the deviation between stock market and fundamental values is both persistent and correlated with the true value of the firm. That these are not sufficient conditions for identification to fail can be seen by considering the case in which ω_t is the sum of two orthogonal components

$$\omega_t = \omega_{1t} + \omega_{2t} \tag{18}$$

and in which

$$\begin{aligned} u_t &= \tilde{u}_t + \phi\omega_{1t} \\ z_t &= \tilde{z}_t + \psi\omega_{2t}. \end{aligned} \tag{19}$$

Again μ_t is persistent and correlated with V_t, but now

$$E(\mu_t z_{t-s}) = \theta_s E(u_{t-s} z_{t-s}) = \theta_s \phi\psi E(\omega_{1t-s}\omega_{2t-s}) = 0 \tag{20}$$

and lagged values of z_{t-s} are available as instruments for Q_t^E in (12). Nevertheless we can conclude that there is a potentially severe identification problem with the usual implementation of the Q investment model if there are persistent deviations between stock market and fundamental values, and if these deviations are themselves dependent on new information about the fundamental value of the firm. It is worth emphasizing that this form of the measurement error is consistent with both rational bubbles and noise trader models.[15]

Below we consider two approaches to identifying the underlying investment model that may still be useful in this case. First, we briefly relate our discussion to one particularly interesting form of measurement error which is consistent with much of the evidence on weak and semi-strong forms of the Efficient Markets Hypothesis. This is

[15] See, for example, Blanchard and Watson (1982), Froot and Obstfeld (1991) and Campbell and Kyle (1993).

the case where

$$\begin{aligned} V_t^E &= V_t e^{\upsilon_t} \\ \upsilon_t &= \upsilon_{t-1} + \xi_t \\ E_{t-1}(\xi_t) &= 0 \end{aligned} \tag{21}$$

so that

$$\ln V_t^E = \ln V_t + \upsilon_t \tag{22}$$

and

$$\Delta \ln V_t^E = \Delta \ln V_t + \xi_t. \tag{23}$$

This type of deviation between equity valuations and fundamental values thus leaves $\Delta \ln V_t^E$ unforecastable if the true $\Delta \ln V_t$ is unforecastable using past information. More generally, Summers (1986) has shown that if $\upsilon_t = \alpha \upsilon_{t-1} + \xi_t$ then for α sufficiently close to one, standard tests of weak form and semi-strong form stock market efficiency would have little or no power to reject the Efficient Markets Hypothesis.

This form of measurement error implies

$$V_t^E = V_t + (e^{\upsilon_t} - 1)\, V_t \tag{24}$$

so that

$$m_t = (e^{\upsilon_t} - 1)\, V_t. \tag{25}$$

Hence this rules out the requirements for Proposition 1 above, since both υ_t and V_t are serially correlated. It also rules out the requirements for Proposition 2 above, through the dependence between κ_t and V_t. However it may still be possible to obtain valid instruments for Q_t^E in (12) in the special case where υ_t is independent of the firm's fundamental value V_s, as shown in Proposition 3 below.[16]

[16]This discussion relates to the usual linear specification of the investment equation, as in (12). Section 2.4 below considers an alternative approach suggested by this particular form of measurement error.

Proposition 3 *Let $m_t = \zeta_t V_t$. Then if ζ_t is mean zero and independent of V_s, κ_s and z_s $\forall s, t$, then μ_t is uncorrelated with z_s $\forall s, t$.*

Proof: $Cov(\mu_t, z_s) = E(\mu_t z_s) = E\left(\frac{\zeta_t V_t z_s}{\kappa_t}\right) = E(\zeta_t)E\left(\frac{V_t z_s}{\kappa_t}\right) = 0.$■

Setting $\zeta_t = (e^{\upsilon_t} - 1)$ yields the result. However identification may again fail if correlation between υ_t and V_t results in a measurement error μ_t that is highly persistent and correlated with the fundamental value of the firm.

2.3 Identification using analysts' earnings forecasts

To identify the parameters of the underlying investment model (9) in cases where all lagged variables correlated with V_s are not valid instruments for Q_t^E, we require a measure of Q_t that does not use the firm's stock market valuation, and thus does not introduce the same measurement error component into the empirical specification. We consider using securities analysts' earnings forecasts to construct a direct estimate of the firm's fundamental value, along the lines suggested by Cummins, Hassett and Oliner (1999). This estimate will also be measured with error. Identification will depend on whether this new measurement error is orthogonal to available instruments.

Let $E_t^c\left[\Pi_{t+s}^a\right]$ denote the analysts' consensus forecast of earnings in period $t+s$, based on information available to analysts in period t; and let $\widetilde{\beta}_{t+s}$ denote an assumed discount factor between periods t and $t+s$. We then construct an estimate of the firm's fundamental value as

$$\widehat{V}_t = E_t^c\left[\Pi_t^a + \widetilde{\beta}_{t+1}\Pi_{t+1}^a + ... + \widetilde{\beta}_{t+s}\Pi_{t+s}^a\right]. \tag{26}$$

We then use this estimate in place of the firm's stock market valuation to obtain an alternative estimate of average q, and hence

$$\widehat{Q}_t = \frac{\widehat{V}_t}{p_t^K(1-\delta)K_{t-1}} - 1. \tag{27}$$

Letting $\nu_t = \widehat{Q}_t - Q_t$ denote the measurement error in this estimate of Q_t, the resulting econometric model is

$$\left(\frac{I_t}{K_t}\right) = c + \frac{1}{b}\widehat{Q}_t + \left(e_t - \frac{\nu_t}{b}\right). \tag{28}$$

The sources of measurement error here include truncating the series after a finite number of periods,[17] using an incorrect discount rate, differences in the information sets available to analysts and the firm, and the fact that analysts forecast net profits rather than net distributions to shareholders. Nevertheless this approach is potentially interesting since v_t does *not* depend on the deviation between stock market valuations and the firm's fundamental value, which, as we have shown above, may result in non-identification of the conventional investment model (12). Identification of the structural parameters in (28) clearly depends on whether v_t is uncorrelated with suitably lagged values of observable instruments, such as sales, profits or investment. We regard this as an empirical question that can be investigated using tests of overidentifying restrictions.

2.3.1 Relation to Abel and Blanchard (1986)

To implement the approach outlined above, we construct an estimate of

$$\text{average } q_t = \frac{E_t\left[\sum_{s=0}^{\infty} \beta_{t+s} \Pi_{t+s}\right]}{p_t^K (1-\delta) K_{t-1}}. \tag{29}$$

Abel and Blanchard (1986) proposed instead to construct an econometric estimate, based on forecasts from vector autoregressions, of

$$\text{marginal } q_t = \left(\frac{1}{p_t^K}\right) E_t \left[\sum_{s=0}^{\infty} \beta_{t+s} (1-\delta)^s \left(\frac{\partial \Pi_{t+s}}{\partial K_{t+s}}\right)\right]. \tag{30}$$

Our approach relies on the assumption that Π_t is homogeneous of degree one in (K_t, L_t, I_t), but avoids the need to specify a functional form for the marginal revenue product of capital. The practical appeal is that we can use published profit forecasts based on the information set available to professional securities analysts, which is likely to be richer than that available to the econometrician specifying the auxiliary forecasting model needed to implement the Abel-Blanchard approach.

When implementing their procedure, Abel and Blanchard (1986) and subsequent researchers (see, for example, Gilchrist and Himmelberg, 1995) assumed that Π_t is homogeneous of degree one in K_t *alone*. This is strictly inconsistent with the structure

[17] Or using an incorrect terminal value correction.

of the Q model outlined in section 2.1, and likely to result in biased estimates of the adjustment cost parameter. Given the assumption that Π_t is homogeneous of degree one in (K_t, L_t, I_t), we have[18]

$$\Pi_t = \left(\frac{\partial \Pi_t}{\partial K_t}\right) K_t + \left(\frac{\partial \Pi_t}{\partial I_t}\right) I_t \tag{31}$$

or

$$\frac{\partial \Pi_t}{\partial K_t} = \frac{\Pi_t}{K_t} - \left(\frac{\partial \Pi_t}{\partial I_t}\right)\left(\frac{I_t}{K_t}\right). \tag{32}$$

Thus the approximation $\left(\frac{\partial \Pi_t}{\partial K_t}\right) \approx \left(\frac{\Pi_t}{K_t}\right)$ omits terms in the rate of investment (and, for the adjustment cost function in equation (8), also terms in the square of the rate of investment) that will result in omitted variable bias.[19] Moreover, given the structure of adjustment costs assumed in equation (8) that forms the basis for a linear relationship between the investment rate and Q, it is difficult to see how net revenue Π_t could be homogeneous of degree one in K_t alone.

2.3.2 Testing the null hypothesis of fundamental value maximization

Conditional on the maintained structure of the Q investment model, our analysis allows us to test whether stock market valuations deviate from fundamental values and to characterize some properties of such deviations. If there is no deviation or the deviation takes the form of pure random noise, as in Proposition 1, then we should find that lagged values of Q_s^E are valid instruments in model (12). If there is a serially correlated deviation that evolves independently of the firm's fundamental value, as in Proposition 2 or 3, then we should find that lagged values of Q_s^E are not valid instruments but lagged values of other variables, such as sales, profits or investment, may be. If there is a deviation that is both persistent and correlated with the firm's fundamental value, then we may be unable to find any valid instruments for the conventional specification (12), although we may still be able to identify the parameters of the underlying investment

[18] Since $\left(\frac{\partial \Pi_t}{\partial L_t}\right) = 0$ for the variable inputs.

[19] Abel and Blanchard (1986) themselves noted this point in their footnote 5, and did not claim to be identifying the structural parameters.

model by constructing an alternative measure of Q which is purged of this error in stock market valuations.

This analysis is conditioned on the assumption that firms choose a path for investment which maximizes their fundamental value in (1). In the case where stock market valuations deviate systematically from fundamental values, this may not be the only objective that shareholders would want to pursue. Theoretical analyses have not produced a consensus on the appropriate objective for firms' investment decisions in an economic environment in which share prices deviate from fundamentals *and* the deviation may be manipulated by actions of the firm.[20] Should we find evidence that equity valuations do deviate systematically from fundamental values, it would also be useful to test the maintained hypothesis that investment decisions are chosen to maximize the firm's fundamental value.

Consider the extended investment model obtained by adding the conventional measure of Q based on equity valuations to the model derived in (9)

$$\left(\frac{I}{K}\right)_t = \alpha + \beta Q_t + \gamma Q_t^E + e_t. \tag{33}$$

Under the null hypothesis that *only* the fundamental value matters for the investment decision, we should find $\beta = 1/b$ and $\gamma = 0$. To implement this test, we consider using our estimate $\widehat{Q}_t$ as a measure of Q_t, with measurement error ν_t; and using the conventional variable as an accurate measure of Q_t^E. This gives the extended empirical model

$$\left(\frac{I}{K}\right)_t = \alpha + \beta \widehat{Q}_t + \gamma Q_t^E + (e_t - \beta \nu_t)\,. \tag{34}$$

Consistent estimation again requires instruments that are orthogonal to both e_t and ν_t. To the extent that we find such instruments when considering the basic investment equation (28), then we can also test the null hypothesis that investment is chosen to maximize only the firm's fundamental value.

[20] See, for example, the differing views expressed by Fischer and Merton (1984) and Stein (1996).

2.4 A semi-log approximation to the Q model

Our main approach to identification in the presence of systematic deviations between equity valuations and fundamental values relies on the availability of suitable instrumental variables that are orthogonal to the measurement error in our estimate of Q constructed using analysts' earnings forecasts. Should such instruments not be available, or as a further check on the results of overidentification tests, it would be useful to develop an alternative approach that yields approximately consistent estimates of the structural parameters. This will also be useful in contexts where data on analysts' earnings forecasts are not available.

To do this, we take seriously the multiplicative structure of the measurement error in share prices outlined in (21) above. This has the interesting implication that the first-difference of $\ln V_t^E$ is measured with an error that is orthogonal to past information. To exploit this property, we therefore consider a semi-log approximation to the Q model.

In the absence of adjustment costs, equation (6) shows that the firm chooses a path for investment that equates marginal q to unity. Thus for suitably low levels of adjustment costs, we would expect $(q-1)$ to be small. Using the approximation $x \approx \ln(1+x)$ for small x, we obtain $(q-1) \approx \ln q$. Hence at low levels of adjustment costs, an accurate approximation to the investment model (9) can be obtained as[21]

$$\left(\frac{I_t}{K_t}\right) = c + \frac{1}{b}\ln q_t + e_t. \tag{35}$$

Now invoking equality between average and marginal q, measuring average $q_t^E = V_t^E/\kappa_t$ in the usual way, maintaining the multiplicative measurement error structure in (21) and taking first-differences of the resulting empirical model,[22] we obtain

$$\Delta\left(\frac{I_t}{K_t}\right) = \frac{1}{b}\Delta \ln q_t^E + \left(\Delta e_t - \frac{\xi_t}{b}\right). \tag{36}$$

[21] Alternatively, we could obtain the adjustment cost function which yields the semi-log model (35) as the exact representation of the first order condition for investment (4). Whilst we have done this, we prefer to maintain the type of adjustment cost specification that is standard in this literature, and thus regard (35) as a potentially interesting approximation to this standard structural model.

[22] Estimating in first-differences is common practice in panel data applications of the Q investment model, allowing for possible correlation between Q_t and an unobserved firm-specific component of the adjustment cost parameter c, or for a firm-specific measurement error component. See, for example, Hayashi and Inoue (1991) and Blundell *et al.* (1992).

Given that the measurement error ξ_t in this case is an innovation, suitably lagged values of $\ln q_s^E$ itself as well as other variables should be valid instruments in this semi-log specification.[23]

One objection to this approach would be that adjustment costs are too high for this to provide an accurate approximation. Whilst most of the existing evidence based on linear specifications such as (12) has suggested that adjustment costs are incredibly high, it is also possible that conventional estimates of the parameter $1/b$ are seriously biased as a result of measurement error in the conventional measure of average q constructed using stock market valuations. It can also be observed that the usual implementation of the Abel-Blanchard approach, discussed in section 2.3.1, also provides a reasonable approximation to the structural investment equation in the case where adjustment costs are sufficiently low. We simply note that the semi-log specification outlined here is considerably easier to implement.

2.5 Empirical specification

Following Blundell *et al.* (1992), our empirical specification also allows for the adjustment cost shock (e_{it}) for firm i in period t to have the first-order autoregressive structure

$$e_{it} = \rho e_{i,t-1} + \varepsilon_{it}, \tag{37}$$

where ε_{it} can further be allowed to have firm-specific and time-specific components.[24] Allowing for this form of serial correlation in equation (12) gives the dynamic specification

$$\begin{aligned} \left(\frac{I_{it}}{K_{it}}\right) &= c(1-\rho) + \frac{1}{b}Q_{it}^E - \frac{\rho}{b}Q_{i,t-1}^E + \rho\left(\frac{I_{i,t-1}}{K_{i,t-1}}\right) \\ &\quad + \left[\varepsilon_{it} - \frac{1}{b}\left(\mu_{it} - \rho\mu_{i,t-1}\right)\right] \end{aligned} \tag{38}$$

[23]Notice that if we were to replace $\Delta \ln q_t^E$ in (36) by $\Delta \ln \hat{q}_t$, there is no reason to suppose that the resulting measurement error would be an innovation.

[24]Controlling for firm-specific and time-specific effects also allows for firm-specific and time-specific components of any measurement error in the measures of Q_{it}.

and a similar dynamic specification based on the model defined by equation (28), where $\hat{Q}$ replaces Q^E. We allow for time effects by including year dummies in the estimated specifications. Estimation allows for unobserved firm-specific effects by using first-differenced GMM estimators with instruments dated $t-3$ and earlier. This is implemented using DPD98 for GAUSS.[25] We confirmed that our sets of instruments have significant predictive power for both measures of Q and for the lagged dependent variable in first-differences, and that similar results are obtained if we normalize the equation on the measure of Q rather than on the investment rate. The common factor restriction is tested and imposed in the results reported below, using the minimum distance procedure described in Blundell *et al.* (1992).

3 Data

The Compustat dataset is an unbalanced panel of firms from the industrial, full coverage, and research files. The files contain data up to and including financial year 1999. The variables we use are defined as follows. The replacement value of the capital stock is calculated using the standard perpetual inventory method with the initial observation set equal to the book value of the firm's first reported net stock of property, plant, and equipment (data item 8) and an industry-level rate of economic depreciation constructed from Hulten and Wykoff (1981). Gross investment is defined as the direct measure of capital expenditures in Compustat (data item 30). Cash flow is the sum of net income (data item 18) and depreciation (data item 14). Sales is defined as the sum of sales (data item 12) and (when available) the change in finished goods inventory (data item 78). Cash flow and sales are divided by the current period replacement value of the capital stock and are denoted CF/K and S/K, respectively. The implicit price deflator (IPD) for total investment for the firm's three-digit SIC code is used to deflate the investment and cash flow variables and in the perpetual inventory calculation of the replacement value of the firm's capital stock. The three-digit IPD for gross output is used to deflate sales. The investment price deflators are from the NBER/Census database (http://www.nber.org/nberces) and the output price deflators are from the BEA (www.bea.doc.gov/dn2.htm). The construction of Q^E and $\hat{Q}$, which incorporate

[25] See Arellano and Bond (1991, 1998).

standard adjustments for the presence of corporate taxes, debt finance and current assets, is discussed in detail in the Data Appendix.

We employ data on expected earnings from I/B/E/S International Inc., a private company that has been collecting earnings forecasts from securities analysts since 1971. To be included in the I/B/E/S database, a company must be actively followed by at least one securities analyst, who agrees to provide I/B/E/S with timely earnings estimates. According to I/B/E/S, an analyst actively follows a company if he or she produces research reports on the company, speaks to company management, and issues regular earnings forecasts. These criteria ensure that I/B/E/S data come from well-informed sources. The I/B/E/S earnings forecasts refer to net income from continuing operations as defined by the consensus of securities analysts following the firm. Typically, this consensus measure removes from earnings a wider range of non-recurring charges than the "extraordinary items" reported on firms' financial statements.

For each company in the database, I/B/E/S asks analysts to provide forecasts of earnings per share over the next four quarters and each of the next five years. We focus on the annual forecasts to match the frequency of our Compustat data. To conform with the timing of the stock market valuation we use to construct Q^E, we use analysts' forecasts issued at the beginning of the financial year. In practice, few analysts provide annual forecasts beyond two years ahead. I/B/E/S also obtains a separate forecast of the average annual growth of the firm's net income over the next five years — the so-called "long-term growth forecast". I/B/E/S started collecting long-term growth forecasts in 1982 which means our sample extends from financial year 1982 to 1999.

We abstract from any heterogeneity in analyst expectations for a given firm-year by using the mean across analysts for each earnings measure (which I/B/E/S terms the "consensus" estimate). We multiply the one-year-ahead and two-year-ahead forecasts of earnings per share by the number of shares outstanding to yield forecasts of future earnings levels. The construction of $\hat{V}$ from these forecast earnings levels and the long-term growth forecasts is discussed in detail in the Data Appendix.

The sample we use for estimation includes all firms with at least five consecutive years of complete Compustat and I/B/E/S data. We determine whether the firm satisfies the five-year requirement after deleting observations that fail to meet a standard set of criteria for data quality. We deleted values of q^E that were non-positive or greater than the upper-decile of the empirical distribution. We also deleted values of the change in q^E that were greater than the ninety-fifth percentile or less than the fifth percentile. We applied the same rules to $\hat{q}$. Finally, we also deleted extreme outliers — never amounting to more than a percentile — in the level of, and changes in, CF/K and S/K. These types of rules are common in the literature and we employ them to maintain comparability to previous studies. The programs used to generate the data are available from www.econ.nyu.edu/user/cummins. Descriptive statistics for the variables we use in the empirical analysis are reported in the Data Appendix.

4 Empirical results

Table 1 contains the GMM estimates of the first-differenced investment equations corresponding to equations (12) and (28), using Q^E and $\hat{Q}$ respectively as measures of Q. Columns (1) – (3) present estimates of the conventional specification, in which stock market values are used to construct a tax-adjusted measure of average q. Columns (4) – (6) present estimates of the alternative specification in which the measurement of average q is identical in all respects except that the stock market valuation is replaced by our measure of the present value of earnings forecast by securities analysts.

Column (1) is presented mainly for the purpose of comparison. In this specification the instruments used are current and lagged values of Q^E itself, as well as lagged values of the dependent variable (I/K) and a cash flow variable (CF/K) dated $t-3$ and $t-4$, and year dummies. Regardless of any measurement error, the derivation of the investment model suggests that Q_t^E should be an invalid instrument, since the shadow value of capital (λ_t) is correlated with the current shock to adjustment costs (e_t) through the marginal revenue product of capital that appears in (5). For equations in first-differences this implies that Q_{t-1}^E should also be an invalid instrument. We find that the validity of these instruments in this specification is indeed strongly rejected by the Sargan test of overidentifying restrictions.

Column (2) reports a specification which could yield consistent estimates if the measurement error introduced by using Q_t^E as a measure of Q_t is serially uncorrelated. As we showed in Proposition 1, this would be the case if the difference between V_t^E and V_t is pure random noise. Given that our estimator allows for autoregressive adjustment cost shocks and unobserved firm-specific effects, this implies that only values of Q^E dated $t-3$ and earlier could be valid instruments in the first-differenced versions of (38). Accordingly we only use lagged values of endogenous variables dated $t-3$ and $t-4$ as instruments in this specification. Nevertheless we again find that the validity of these instruments is strongly rejected. If the underlying Q investment model is correct, this implies either that the adjustment cost shocks are more persistent than our $AR(1)$ formulation, or that the measurement error is not pure random noise.

Column (3) explores the latter possibility by omitting all lagged values of Q^E from the instrument set. As we showed in Proposition 2, lagged values of the investment rate and the cash flow variable may be valid instruments even if the difference between V_t^E and V_t is serially correlated, provided this difference evolves independently of the true value of the firm V, its capital stock κ, and these variables. However the Sargan test continues to reject the validity of these instruments in this empirical specification, and this was the case for all sets of instruments we experimented with.[26] This suggests that the underlying investment model is misspecified, the adjustment cost shocks are more persistent than our empirical specification allows, or identification of the model is undermined by the measurement error introduced by using stock market valuations to measure average q.

To discriminate among these possibilities, columns (4) – (6) report a parallel set of results when $\hat{V}$ is used in place of V^E. Not surprisingly, current values and recent lags of the corresponding $\hat{Q}$ measure are again rejected as valid instrumental variables, as predicted by the underlying investment model. However, the results in columns (5) and (6) are more interesting. The instruments used in column (5) require the measurement error in $\hat{Q}_t$ to be serially uncorrelated, whilst those used in column (6) only require this measurement error to be orthogonal to lagged values of the investment rate and the cash flow variable.[27] The former assumption is rejected by the Sargan test of overidentifying restrictions at the 10 percent level,[28] but the latter assumption is not rejected

[26]Independence between the measurement error in average q^E and the ratio of cash flow to capital, assumed in the approach of Erickson and Whited (2000), is rejected in our dataset which contains a much larger sample of firms and years.

[27]More precisely, as our empirical specification allows for firm-specific, time-specific and $AR(1)$ components in the error term, it is only the residual component of the measurement error that is required to satisfy these orthogonality conditions.

[28]A Difference Sargan test, comparing the specification in column (5) to that in column (6), rejects the validity of the lagged $\hat{Q}_{t-s}$ instruments additionally used in column (5) at the 2.5 percent level.

— although the previous columns have demonstrated that this test certainly has power to reject invalid moment conditions in this sample. The estimated coefficient on $\hat{Q}$ is also much larger than those found on Q^E, or those that have typically been reported in previous studies. We find evidence of an autoregressive component in the error term, consistent with either serially correlated adjustment cost shocks as in (37), or with an autoregressive measurement error component in $\hat{Q}_t$. Both of these possibilities are consistent with the underlying Q investment model. Our results therefore suggest that the use of stock market valuations to measure average q in previous studies can account for their failure to identify the structure of this model.

We can use the estimated coefficients on Q^E and $\hat{Q}$ to calculate the implied elasticities of the investment rate with respect to the average q ratio. The elasticities implied by using $\hat{Q}$ are four times as large as those from using Q^E. In contrast to many previous studies, these estimates indicate that investment spending is quite sensitive to fundamentals; the preferred estimates in column (6) imply that the elasticity is unity. When $\hat{Q}$ is used, the estimates also imply that the marginal adjustment costs for an additional \$1 of investment are substantially less than \$1, evaluated at either the means or medians of the sample variables.

In the final column of Table 1 we test the null hypothesis of fundamental value maximization set out in section 2.3.2. In that section, we showed how we can construct a test of the null hypothesis that only fundamentals matter for investment decisions by including both $\hat{Q}$ and Q^E in the empirical model (34). In particular, if firms choose investment to maximize their fundamental value, then the estimated coefficient on $\hat{Q}$ should equal $1/b$ while the estimated coefficient on Q^E should equal zero. We find that the estimated coefficient on $\hat{Q}$ in column (7) is very close to that in column (6), when Q^E is not included in the specification. In addition, the estimated coefficient on Q^E in column (7) is very small and not significantly different from zero. Conditional on the maintained structure of the Q model, our analysis thus indicates that firms choose investment to maximize their fundamental value.

In Table 2 we present some additional tests of the Q investment model. In columns (1), (4) and (7) we introduce the lagged ratio of cash flow to capital as an additional explanatory variable in the investment equations. In all other respects, the estimation method and data are identical to those used to generate columns (3), (6) and (7) in

Table 1 using the same instrument set. In this framework, the coefficient on cash flow measures its influence after controlling for expected future returns, and it should be zero if there are no binding financial constraints and the Q model is otherwise correctly specified.

The coefficient on Q^E in column (1) is little affected but, as many studies have found, the coefficient on cash flow is positive and statistically significant. However, in column (4) when we use $\hat{Q}$ as our measure of Q we find that investment is insensitive to cash flow.[29] Moreover, there is no evidence that the model is misspecified based on the diagnostic tests. Similarly when we include cash flow and both measures of Q in column (7), we find that neither Q^E nor cash flow is statistically significant conditional on $\hat{Q}$. These results are consistent with fundamental value maximization in the absence of binding financing constraints for this sample of firms.

In columns (2), (5) and (8) we consider the lagged ratio of sales to capital as an additional explanatory variable. Either imperfectly competitive product markets or decreasing returns to scale introduce a wedge between average and marginal q that can be related to expectations of future sales.[30] The significance of the lagged sales term in column (2) clearly indicates some form of misspecification of the standard Q investment model when average q is measured using stock market valuations. However, this sales variable is found to be insignificant in columns (5) and (8), when we condition on $\hat{Q}$.[31] Again there is no evidence of misspecification for the basic Q investment model when analysts' earnings forecasts are used to construct the alternative measure of average q.

In columns (3), (6) and (9) of Table 2 we introduce non-linear terms in the measures of Q. As before, the estimation method and data are identical to those used to generate the results in Table 1 using the same instrument set. In this framework, the coefficient on the squared term measures the extent to which investment responds non-linearly to fundamentals. The coefficient on the squared term should be zero if adjustment costs

[29]The current cash flow variable $(CF/K)_t$ was also insignificant conditional on $\hat{Q}_t$.
[30]See, for example, Hayashi (1982) and Schiantarelli and Georgoutos (1990).
[31]Similarly, we found that the current ratio of sales to capital, as well as current or lagged growth rates of real sales, were insignificant explanatory variables conditional on $\hat{Q}_t$.

take the symmetric, quadratic form of equation (8). However, significant non-linearities are consistent with a model of non-convex adjustment costs.[32]

The coefficient on Q^E in column (3) is higher than the comparable estimate in column (3) of Table 1. The coefficient on the square of Q^E is negative and statistically significant, indicating that the investment rate is concave in Q^E. In particular, the implied elasticity is much higher at the sample means than those in Table 1 but tails off, becoming negative at values of Q^E greater than 10. This could suggest that adjustment costs do not take the strictly convex form of equation (8). However there is good reason to suspect that this conclusion may be premature, given the findings we have already presented. The relevant question is whether this non-linearity is a primitive feature of the structural model, as emphasized by Abel and Eberly (1996), or whether this can also be attributed to measurement error.[33] The results in columns (6) and (9), where we perform the analogous experiment using $\hat{Q}$, support the latter interpretation. In these cases, we find no evidence of non-linearities or model misspecification, suggesting that measurement error in share prices, rather than non-convex adjustment costs, is responsible for the finding of non-linearity when using Q^E.

In Table 3 we consider the semi-log approximation to the Q model presented in section 2.4. Instead of relying on an alternative measure of fundamentals, the semi-log specification attempts to identify the Q model using only the information in Q^E. The basic idea is to rely as much as possible on the structure of the model to develop an estimator. If we take seriously the multiplicative structure of the measurement error in share prices outlined in equation (13), then we can use a semi-log approximation to obtain valid orthogonality restrictions provided that adjustment costs are not too high.

In Table 3 we report the results of this exercise using as instruments period $t-3$ and $t-4$ values of $\ln q^E$ and the investment rate, and the same sample as we used in Tables 1 and 2. The estimated coefficients are close to those obtained using our preferred specification in column (6) of Table 1. Despite the approximation required to obtain the semi-log model, this specification is not formally rejected by the Sargan test of overidentifying restrictions. We did find that lagged cash flow variables were not

[32] See Abel and Eberly (1996), Eberly (1997) and Barnett and Sakellaris (1998).

[33] More complex measurement error than the classical Gaussian form can distort the shape as well as the slope of a relationship. See, for example, Chesher (1991). Here it seems likely that higher values of Q^E could be subject to greater measurement error.

valid instruments in this specification of the model, and, for some instrument sets, the lagged ratio of cash flow to capital was statistically significant when introduced as an additional explanatory variable. This is not particularly surprising, and could indicate either that the approximation error introduced here is correlated with lagged cash flow, or that the measurement error in $\ln V_t^E$ is not an exact random walk.[34] Nevertheless, these alternative estimates increase our confidence that the Q model of investment can be identified successfully by using securities analysts' earnings forecasts to construct an estimate of average q.

5 Conclusions

In this paper we have analyzed the implications of 'noise' in share prices, or deviations between stock market values and the present value of expected net distributions to shareholders, for estimation of the Q model of investment. We show that the usual empirical implementation, with average q measured using stock market valuations, may not identify the structural parameters of the underlying investment model in one potentially interesting case when the deviation between stock market and fundamental values is both highly persistent and itself correlated with the true value of the firm. This case is consistent with the behavior of share prices in both rational bubble and noise trader models, and with both weak and semi-strong forms of the Efficient Markets Hypothesis. In this case, we show that the Q model may nevertheless be identified either by constructing an alternative measure of average q that does not use share price data, or by considering a semi-log approximation that will be accurate at low levels of adjustment costs.

Our empirical results suggest that this form of measurement error is very relevant, and can account for all the standard empirical failings of the conventional Q model. Constructing an alternative estimate of average q based on the present value of securities analysts' earnings forecasts for individual firms, we find evidence consistent with the theoretical prediction that average q is a sufficient statistic for investment rates. In particular, we find no information in sales and cash flow variables conditional on our measure of average q; and no evidence of non-linearities in the relationship

[34] See Campbell *et al.* (1997) for a discussion of evidence of long run mean reversion in share prices.

between investment rates and our measure of average q. We also find there is no additional information in the conventional measure of average q, which is consistent with the hypothesis that investment is chosen to maximize the firm's fundamental value, notwithstanding the presence of share price anomalies.

These results have important implications for both asset pricing and investment models. Conditional on the maintained structure of the Q model, which we do not reject, we strongly reject the hypothesis that stock market valuations coincide with the present value of expected net distributions to shareholders. Our findings indicate significant and highly persistent deviations between stock market and fundamental values, which do not evolve independently of new information about the true value of the firm. Our results also suggest that previous evidence of significant cash flow and sales effects or non-linearities in the investment-q relation can be attributed to the difficulty of controlling for measurement error of this form. At least for US company data, we find no evidence that the basic Q model of investment is seriously misspecified.

A Data Appendix

A.1 Construction of Tobin's Q

Incorporating the usual adjustments for debt, taxes, and current assets, Q is defined as

$$Q_{it} = \frac{V_{it} + B_{it} - A_{it} - C_{it}}{(1 - \Gamma_{it})\kappa_{it}} - 1,$$

where V is the unobservable present discounted value of expected net distributions to existing shareholders; B is the book value of outstanding debt; A is the present value of the depreciation allowances on investment made before period t; C is current assets; κ is the replacement value of the firm's inherited capital stock ($p_t^K(1-\delta)K_{i,t-1}$); and Γ is the present value of the tax benefit for each unit of current investment spending. For example, with an investment tax credit at rate k, Γ is:

$$\Gamma_{it} = k_{it} + \sum_{s=t}^{\infty}(1 + r_s + \pi_s^e)^{t-s}\tau_s DEP_{is}(s - t),$$

where τ is the corporate tax rate, r is the risk-free real interest rate (assumed to equal 3 percent), π^e is the expected inflation rate, and $DEP_{is}(a)$ is the depreciation allowance permitted for an asset of age a.

We discuss below how we construct V^E and $\hat{V}$. Unless noted otherwise, the remaining components of Q_{it} come from Compustat data. The value of debt is the sum of short-term debt (data item 34) and long-term debt (data item 9), both measured at book value at the start of period t. The present value of the depreciation allowances on investment made before period t is calculated using the method in Salinger and Summers (1983). Current assets is total current assets (data item 4) at the start of period t, which is the sum of short-term cash and marketable securities, inventories, accounts receivable, and other current assets. The replacement cost of the capital stock is calculated using the standard perpetual inventory method with the initial observation set equal to the book value of the firm's first reported net stock of property, plant, and equipment (data item 8) and an industry-level rate of economic depreciation constructed from Hulten and Wykoff (1981).

Among the remaining components of tax-adjusted Q, the data on expected inflation are the annual averages of the monthly expectations in the Livingston Survey, administered by the Federal Reserve Bank of Philadelphia. The tax parameters (A, k, τ, and DEP) are updated from those used in Cummins *et al.* (1994); we construct firm-specific investment tax credits and depreciation allowances to reflect the asset composition of the firm's two-digit SIC sector.

A.2 Construction of V^E and $\hat{V}$

V^E, which replaces the fundamental valuation of the firm V to form Q^E, is the sum of the market value of common equity at the start of period t and the market value of preferred stock. The market value of common equity is defined as the number of common shares outstanding multiplied by the share price at the end of the previous financial year, and the market value of preferred stock is defined as the firm's preferred dividend payout divided by S&P's preferred dividend yield, obtained from Citibase.

$\hat{V}$, which replaces the fundamental valuation of the firm V to form $\hat{Q}$, is constructed in the following way. Let $E_t^c\left[\Pi_{i,t+s}^a\right]$ denote the analysts' consensus forecast of earnings for firm i in period $t+s$, based on information available at the beginning of period t. Let GR_{it} denote firm i's five-year expected growth rate of profits, also based on beginning-of-period t information. We use forecasts issued at the beginning of the period to be consistent with our measurement of the stock market valuation of the firm, V_t^E, used in Q_t^E. In practice, few analysts provide annual forecasts beyond two years ahead so we calculate the implied level of profits for periods after $t+1$ by growing out the average of $E_t^c\left[\Pi_{it}^a\right]$ and $E_t^c\left[\Pi_{i,t+1}^a\right]$ at the rate GR_{it}. Let this average of the two earnings forecasts be $E_t^c\left[\overline{\Pi}_{it}^a\right]$. In principle, the horizon for calculating $\hat{V}$ should be infinity. Since the analysts estimate GR over a horizon of five years, we set the forecast horizon to five years. We then calculate a terminal value correction to account for the firm's expected profits beyond year five. The correction assumes that the growth rate for earnings beyond this five year horizon is equal to that expected for the economy. Specifically, the last year of expected earnings is turned into a growth perpetuity by dividing it by $\bar{r}-\bar{g}$, where we assume that $\bar{r}$ is the mean nominal interest rate for the sample period as a whole (about 15 percent, which includes a constant 8 percent risk premium) and

$\bar{g}$ is the mean nominal growth rate of the economy for the sample period as a whole (about 6 percent).

The resulting sequence of expected profits defines $\hat{V}_{it}$:

$$\hat{V}_{it} = E_t^c\Bigg[\Pi_{it}^a + \tilde{\beta}_t\Pi_{i,t+1}^a + \tilde{\beta}_t^2(1+GR_{it})\overline{\Pi}_{it}^a + \tilde{\beta}_t^3(1+GR_{it})^2\overline{\Pi}_{it}^a + \tilde{\beta}_t^4(1+GR_{it})^3\overline{\Pi}_{it}^a + \tilde{\beta}_t^5\frac{(1+GR_{it})^3\overline{\Pi}_{it}^a}{\bar{r}-\bar{g}}\Bigg].$$

The discount factor $\tilde{\beta}_t$ reflects a static expectation of the nominal interest rate over this five year horizon. That is, we use the thirty-year Treasury bond interest rate in year t (plus a fixed 8 percent risk premium).

We conducted an exhaustive battery of robustness checks on the effect of different definitions of $\hat{V}$. For example, the key features of the empirical results presented in Tables 1 and 2 are robust to lengthening the forecast horizon, using different types of terminal value corrections, using a firm-specific risk-premium to construct the discount factor $\tilde{\beta}_t$, and using lower or declining risk-premia. This robustness is not surprising if the kind of measurement error introduced by using these estimates of $\hat{V}$ to measure the firm's fundamental value V is indeed orthogonal to our set of instruments.

Descriptive statistics for the variables used in our empirical analysis reported in section 4 are in Table A1, which precedes the empirical results.

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Table A1: Descriptive Statistics for Variables Used in Empirical Analysis

Variable	mean	standard deviation	first quartile	median	third quartile
$(I/K)_t$	0.163	0.091	0.098	0.148	0.208
Q_t^E	1.47	2.17	-0.011	0.713	2.12
$\hat{Q}_t$	0.42	1.13	-0.354	0.083	0.828
$(CF/K)_t$	0.196	0.141	0.094	0.170	0.267
$(S/K)_t$	2.27	1.94	0.758	1.79	3.17
$\Delta(I/K)_t$	-0.007	0.083	-0.034	-0.002	0.025
ΔQ_t^E	0.025	0.870	-0.251	0.015	0.307
$\Delta\hat{Q}_t$	0.009	0.435	-0.144	0.001	0.163
$\Delta(CF/K)_t$	-0.001	0.086	-0.024	0.002	0.026
$\Delta(S/K)_t$	-0.023	0.353	-0.125	-0.006	0.093

The sample contains firms with at least five years of complete Compustat and I/B/E/S data. The number of firms in this sample is 1066, for a total of 11,431 observations, and the sample period is 1982–99.

Table 1: GMM Estimates of First-Differenced Dynamic Investment Equations: Comparing Market- and Analyst-Based Measures of Q with Different Instrumental Variables

Parameter	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Q_t^E	0.017	0.020	0.022	—	—	—	0.009
	(0.002)	(0.005)	(0.007)				(0.008)
$\hat{Q}_t$	—	—	—	0.033	0.097	0.110	0.104
				(0.003)	(0.014)	(0.019)	(0.020)
ρ	0.389	0.366	0.397	0.324	0.295	0.246	0.253
	(0.052)	(0.058)	(0.057)	(0.052)	(0.060)	(0.052)	(0.057)
	DIAGNOSTIC TESTS (p-VALUES)						
Second-Order Serial Correlation	0.692	0.490	0.507	0.891	0.714	0.549	0.655
Sargan Test	0.009	0.007	0.005	0.059	0.100	0.470	0.534
Common Factor Restriction	0.013	0.418	0.173	0.150	0.563	0.191	0.499
	INSTRUMENTAL VARIABLES						
	$Q_t^E, \mathbf{Z}_{t-3}$ $Q_{t-1}^E, \mathbf{Z}_{t-4}$ Q_{t-2}^E	$Q_{t-3}^E, \mathbf{Z}_{t-3}$ $Q_{t-4}^E, \mathbf{Z}_{t-4}$	$\mathbf{Z}_{t-3}$ $\mathbf{Z}_{t-4}$	$\hat{Q}_t, \mathbf{Z}_{t-3}$ $\hat{Q}_{t-1}, \mathbf{Z}_{t-4}$ $\hat{Q}_{t-2}$	$\hat{Q}_{t-3}, \mathbf{Z}_{t-3}$ $\hat{Q}_{t-4}, \mathbf{Z}_{t-4}$	$\mathbf{Z}_{t-3}$ $\mathbf{Z}_{t-4}$	$\mathbf{Z}_{t-3}$ $\mathbf{Z}_{t-4}$

The dependent variable is the first-difference of the ratio of investment to capital, $(I/K)_t$. Year dummies are included (but not reported) in all specifications. Robust standard errors on coefficients are in parentheses.

The sample contains firms with at least five years of complete Compustat and I/B/E/S data. The number of firms in this sample is 1066, for a total of 7167 observations, and the estimation period is 1986–99.

The instrumental variables in $\mathbf{Z}$ are I/K and CF/K. Year dummy variables are also included as instruments in all specifications.

The test of the overidentifying restrictions, called a Sargan test, is asymptotically distributed as $\chi^2_{(n-p)}$, where n is the number of instruments and p is the number of parameters. The test for second-order serial correlation in the first-differenced residuals is asymptotically distributed as N(0,1) under the null of no serial correlation. The common factor test is asymptotically distributed as χ^2_r, where r is the number of non-linear common factor restrictions.

Table 2: GMM Estimates of First-Differenced Dynamic Investment Equations: Additional Sensitivity Tests

Parameter	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Q_t^E	0.023 (0.007)	0.025 (0.007)	0.043 (0.017)	—	—	—	0.010 (0.007)	0.013 (0.007)	-0.006 (0.026)
$\hat{Q}_t$	—	—	—	0.099 (0.020)	0.095 (0.021)	0.124 (0.034)	0.090 (0.020)	0.080 (0.022)	0.121 (0.045)
$(CF/K)_{t-1}$	0.218 (0.089)	—	—	0.110 (0.100)	—	—	0.123 (0.101)	—	—
$(S/K)_{t-1}$	—	0.076 (0.020)	—	—	0.018 (0.024)	—	—	0.031 (0.024)	—
$(Q_t^E)^2$	—	—	-0.002 (0.001)	—	—	—	—	—	0.001 (0.002)
$(\hat{Q}_t)^2$	—	—	—	—	—	-0.003 (0.005)	—	—	-0.004 (0.006)
ρ	0.356 (0.043)	0.318 (0.048)	0.242 (0.029)	0.238 (0.047)	0.234 (0.043)	0.197 (0.047)	0.230 (0.048)	0.246 (0.050)	0.188 (0.035)
	Diagnostic Tests (p-values)								
Second-Order Serial Correlation	0.973	0.631	0.145	0.590	0.851	0.316	0.689	0.713	0.320
Sargan Test	0.141	0.134	0.115	0.751	0.408	0.478	0.707	0.481	0.519
Common Factor Restriction	0.041	0.289	0.307	0.376	0.506	0.561	0.653	0.753	0.906
Wald Test	—	—	—	—	—	—	0.330	0.385	0.391
	Instrumental Variables								
	$\mathbf{Z}_{t-3}$ $\mathbf{Z}_{t-4}$	$\mathbf{Z}_{t-3}$ $\mathbf{Z}_{t-4}$	$\mathbf{Z}_{t-3}$ $\mathbf{Z}_{t-4}$	$\mathbf{Z}_{t-3}$ $\mathbf{Z}_{t-4}$	$\mathbf{Z}_{t-3}$ $\mathbf{Z}_{t-4}$	$\mathbf{Z}_{t-3}$ $\mathbf{Z}_{t-4}$	$\mathbf{Z}_{t-3}$ $\mathbf{Z}_{t-4}$	$\mathbf{Z}_{t-3}$ $\mathbf{Z}_{t-4}$	$\mathbf{Z}_{t-3}$ $\mathbf{Z}_{t-4}$

See notes to Table 1. The Wald test is a test of the null hypothesis that all coefficients except those on $\hat{Q}_t$, ρ, and the year dummies are jointly equal to zero.

Table 3: GMM Estimates of First-Differenced Dynamic Investment Equations: Semi-log Model

Parameter	
$\ln q_t^E$	0.082 (0.021)
ρ	0.302 (0.067)
	DIAGNOSTIC TESTS (p-VALUES)
Second-Order Serial Correlation	0.836
Sargan Test	0.123
Common Factor Restriction	0.404
	INSTRUMENTAL VARIABLES
	I/K_{t-3}, $\ln q_{t-3}^E$ I/K_{t-4}, $\ln q_{t-4}^E$

See notes to Table 1.