# I NEQUALI TY AND THE CHOI CE OF THE PERSONAL TAX BASE 

Nigar Hashimzade University of Exeter

Gareth D. Myles
University of Exeter and I nstitute for Fiscal Studies

# Inequality and the Choice of the Personal Tax Base 

Nigar Hashimzade* University of Exeter<br>Gareth D. Myles ${ }^{\dagger}$<br>University of Exeter and Institute for Fiscal Studies

September 20, 2006


#### Abstract

It is possible to employ either income or expenditure as the base for personal taxation. A considerable literature has developed that investigates the relative efficiency of these bases. The answer is usually in favor of the expenditure tax since it encourages capital accumulation through the avoidance of the double taxation of saving. In contrast, the literature is almost silent on the relative equity of the two bases. We investigate the redistributive consequences of the choice in models with two sources of heterogeneity: skill in employment and lump-sum endowment. The Gini coefficient is used to measure the degreee of equity achieved by the tax bases in static and dynamic settings. Income taxes and expenditure taxes that generate equal welfare (or equal revenue) are compared. In the static economy the income tax leads to lower inequality except when skill and endowment are negatively correlated. Inequality is always lower with the income tax in the dynamic economy. These results support the choice of income as the base for personal taxation if reduction in inequality is a priority of policy.


Acknowledgements: The paper has emerged from discussions of the Mirrlees Review of UK Taxation. We wish to thank members of the Review, especially Richard Blundell, Stephen Bond, and James Mirrlees.

[^0]
## 1 Introduction

The argument for employing expenditure, rather than income, as the base for taxation has a long and distinguished history. There are claims that it can be traced back to Hobbes (1660) (see Batina and Ihori, 2000), but certainly both Mill (1888) and Ramsey (1927) argued that saving should be exempt from taxation - precisely the feature that distinguishes an expenditure base from an income base. Some of the strongest arguments in favor of an expenditure tax were made by Kaldor (1955) and the Meade (1978) review of taxation in the UK. The use of an expenditure tax was also proposed in the US (see US Treasury Department, 1942). Despite this, expenditure taxation has been adopted in just two countries (India and Sri Lanka), and then only briefly. Most academic discussion has focussed on the efficiency benefit of expenditure taxation, with little said on the equity aspects. One exception is Kaldor (1955), but his discussion of "taxable capacity" does not correspond well to modern concepts of tax theory. The contribution of this paper is to provide an assessment of the relative success of expenditure taxation and income taxation in achieving equity objectives.

There are numerous versions of the general concept of an expenditure tax. These include any tax on consumption such as the value added tax and the sales tax, more direct expenditure taxes such as the cashflow tax (where income minus net additions to wealth forms the tax base), and the X-tax of Bradford (1986). Many reform proposals argue for an integrated treatment of the corporation and the individual which can be achieved with an expenditure base. For example, the full-fledged cash flow expenditure tax would not tax the corporation, but for individuals equity purchases are subtracted from income, dividends received are added to income and the expenditure tax levied on the cashflow basis. It has been claimed that since expenditure is not observed the tax liability under an expenditure tax cannot be calculated. The Meade Review (1978) demonstrated convincingly that this was not the case by describing the income adjustment method: the level of expenditure is obtained by summing all incomes and subtracting savings. This ensures there is no need to evaluate wealth or assess increases in the value of wealth, permits progressiveness in the marginal tax rate, and makes the expenditure tax implementable.

Why might an expenditure tax be preferred to an income tax? At least three reasons are normally cited. First, an expenditure tax treats saving favorably relative to an income tax. Second, an expenditure tax allows integration of the taxation of the personal and corporate sectors. Third, an expenditure tax removes the need to distinguish between capital gains and income. The second and third points may have relevance from an administrative perspective. From the perspective of economic analysis the major point is the first: an expenditure tax avoids the double taxation of savings. If income is the basis for taxation savings are taxed when the income is initially earned, and then again when income is obtained from the savings. This double taxation provides a disincentive to saving and, it is generally claimed, reduces the aggregate level of saving and hence of national income. If expenditure forms the base for taxation savings
are only taxed when expenditures are made. This eliminates the disincentive to save. This point is developed further in Auerbach (2006).

The literature analyzing which tax base is preferable has focused this on efficiency argument. The relative efficiency of the two tax bases has been addressed in a range of models. The starting point for understanding much of the analysis is the result of Chamley (1986) and Judd (1985) that the long-run tax on capital should be zero. The inefficiency of the tax on capital income is emphasized by the welfare calculations of Chamley (1981) and his observation that the replacement of a capital tax by a lump-sum tax leads to an increase in consumption and welfare. These results point to an efficiency advantage for the expenditure tax rather than the income tax. Further results have been obtained from simulations using overlapping generations economies. Altig et al. (2003) employ the Auerbach-Kotlikoff model (overlapping generations with each consumer living 55 years) and show that both flat tax and a consumption tax raise national income compared to a proportional income tax. Furthermore, the consumption tax raises national income by more. Endogenous growth models have also displayed the property that a switch from an income tax to an expenditure tax raises the growth rate (see the survey in Myles, 2000).

The literature has much less to say on the equity implications on the choice between the tax bases. Comments have been made about the morality issues (an income base taxes what you put into the economy, an expenditure base taxes what you take out) but this is not what normally determines the choice between tax instruments. One approach to an assessment of the equity implication has been to calculate the effects of reform using data from tax filing. Feenberg et al. (1997) contrast income taxes and retail sales tax with various exemptions and find that the average tax burden rises for most of the low income groups when a retail sales tax is used. However, such analysis does not take account of re-optimization by consumers or equilibrium adjustments.

The theory of equity requires taxes to compensate for differences in lifetime utility generated as the consequence of unchangeable characteristics that are not the result of economic choices. For example, the Mirrlees (1971) model of income taxation summarized such characteristics in the level of skill. Having a higher level of skill raises economic opportunities and therefore puts high skill consumers in a potentially better situation - but it remains a choice whether to take advantage of the opportunity. The first-best tax system involves a lumpsum tax levied on the value of these unchangeable characteristics to provide compensation for those less fortunate in the allocation (i.e. those with lower skill in the Mirrlees's model). From this perspective an income tax can be viewed as an approximation to the first-best lump-sum tax on earning potential. The potential for greater earned income is not the only source of inequality. Inequality can also be due to unearned wealth (such as bequests). It is worth noting that unearned wealth is not subject to tax when income is chosen as the tax base. In contrast, an expenditure tax does tax this wealth when it is consumed. The taxes, therefore, have different impacts on the two sources of inequality and it may be thought that the expenditure tax will be more redistributive since it taxes both sources (earned and unearned) of finance for
consumption. Some theoretical work on these lines has been undertaken by Correira (2005) who combines the two sources of inequality. Her main result is to show that an increase in the consumption tax with a corresponding reduction in the income tax raises efficiency.

What we do in this paper is build upon the recognition of the two sources of inequality and their interaction with the choice of tax base in achieving redistribution. This is undertaken by extending the standard model of income taxation to incorporate variation in initial wealth and variation in skill across the population. We then contrast the success of the income tax to that of the expenditure tax in reducing inequality. There is clearly an open question here about which will be the most successful. The income tax does not tax initial wealth but the expenditure tax is a blunter tool when it comes to taxing skill. A priori, there is apparently a trade-off between the relative benefits of the two instruments.

It is helpful at this point to describe how we implement the analysis. The idea of judging the redistributive success of alternative tax instruments is clearly open to a number of potential interpretations. To make the idea concrete a framework for coherent comparison has to be developed. What we choose to do is to measure inequality using the Gini coefficient applied to lifetime income. We then contrast the value of the Gini achieved by income taxation to that achieved by expenditure taxation. To ensure comparability we make these comparisons at both equal levels of welfare and at equal levels of government revenue. That is, we set the income tax rate, compute the level of welfare (or revenue) and then find the expenditure tax rate that generates the same level of welfare (or revenue). The Gini coefficients are then computed and contrasted. These comparisons are made in both a static economy and a dynamic economy.

The second section of the paper presents the comparison in a static economy which is an extension of the standard Mirrlees's framework. Section 3 contrasts the two taxes in an overlapping generations model with bequests. Conclusions are given in Section 4.

## 2 Static Economy

This section contrasts the success at achieving equity of the income and expenditure taxes in a static economy. The economy has two periods and a population of consumers who differ in income and initial endowment. In the first period each consumer makes a labor supply decision and allocates income between consumption and saving. Consumers are retired in the second period and finance consumption from saving. The economy is static, but the fact that saving plays a key role in smoothing consumption across the lifecycle allows the effect of income and expenditure taxes to be distinguished.

### 2.1 Model

Consumers are differentiated by two characteristics: initial endowment and skill in employment. The level of skill is measured by the wage rate received. The endowment of consumer $h$ is denoted $e_{h}$ and the wage received per unit of labor supply is denoted $w_{h}$. A consumer is described by the pair $\left\{e_{h}, w_{h}\right\}$.

The initial endowment can take one of two values, $e_{L}$ and $e_{H}$, with $e_{L}<e_{H}$. $e_{L}$ is called the low endowment and $e_{H}$ the high endowment. The level of skill can also take two values. The low skill level is $w_{L}$ and the high skill level is $w_{H}$, with $w_{L}<w_{H}$. The economy, therefore, has four types of consumer. The labelling of these types is summarized in Table 1.

|  | $e_{L}$ | $e_{H}$ |
| :--- | :--- | :--- |
| $w_{L}$ | $L L$ | $L H$ |
| $w_{H}$ | $H L$ | $H H$ |

Table 1: Types and labeling
The population size is fixed so it is the proportion of each type that is relevant for measuring welfare and inequality. Let $p_{h}$ denote the proportion of population that is of type $h, h \in\{L L, L H, H L, H H\}$. By definition $\sum_{h} p_{h}=1$. Using the labelling of types we have

$$
\begin{equation*}
\sigma_{e}^{2}=\sum p_{h} e_{h}^{2}-\left(\sum p_{h} e_{h}\right)^{2} \tag{1}
\end{equation*}
$$

Similarly,

$$
\begin{equation*}
\sigma_{w}^{2}=\sum p_{h} w_{h}^{2}-\left(\sum p_{h} w_{h}\right)^{2} \tag{2}
\end{equation*}
$$

and

$$
\begin{equation*}
\sigma_{e w}=\sum p_{h} e_{h} w_{h}-\sum p_{h} e_{h} \sum p_{h} w_{h} \tag{3}
\end{equation*}
$$

The correlation between endowment and skill plays a key role in the interpretation of our results. The correlation coefficient is defined by $\rho=\frac{\sigma_{e w}}{\sigma_{e} \sigma_{w}}$.

Each consumer lives for two periods. They work during the first period of life and are retired in the second period. In the absence of taxation the firstand second-period budget constraints for a consumer of type $h$ are

$$
\begin{equation*}
x_{h}^{1}+s_{h}=\ell_{h} w_{h}+e_{h}, \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
x_{h}^{2}=(1+r) s_{h}, \tag{5}
\end{equation*}
$$

where $\ell_{h}$ is labor supply, $s_{h}$ is saving, and $r$ is the (fixed) interest rate. These per-period budget constraints combine to give the lifetime budget constraint

$$
\begin{equation*}
x_{h}^{1}+\frac{x_{h}^{2}}{1+r}=\ell_{h} w_{h}+e_{h} . \tag{6}
\end{equation*}
$$

The labor supply and consumption choices are made to maximize the utility function

$$
\begin{equation*}
U\left(x_{h}^{1}, x_{h}^{2}, \ell_{h}\right)=\ln \left(x_{h}^{1}\right)+(1-\alpha) \ln \left(1-\ell_{h}\right)+\delta \ln \left(x_{h}^{2}\right) . \tag{7}
\end{equation*}
$$

This specification of utility assumes that all consumers have the same preferences, so we abstract from the issue of capabilities affecting inequality (Foster and Sen, 1997). We adopt a specific functional form to permit the numerical comparison of the tax bases.

The degree of inequality in the population is measured by using the Gini coefficient applied to the discounted value of lifetime income,

$$
\begin{equation*}
G=1-\frac{1}{H^{2} \mu} \sum_{j} \sum_{k} \min \left\{\mathcal{I}_{j}, \mathcal{I}_{k}\right\}, \tag{8}
\end{equation*}
$$

where $\mathcal{I}_{j} \equiv \ell_{h} w_{h}+e_{h}+\frac{r s_{h}}{1+r}$. We interpret one tax system as being more successful in reducing inequality than an alternative system if it generates a lower value of the Gini coefficient. We are not the first to relate income taxation to economic indices. For example, Kanbur and Keen (1989) consider how the income tax should be chosen to minimize the value of a poverty or inequality measure. What has not been analyzed previously is how income and expenditure taxes perform as determined, in our case, by the value of the Gini with income taxation relative to the Gini with expenditure taxation. Using the population proportions the Gini coefficient can be written as

$$
\begin{align*}
G & =1-\frac{1}{H^{2} \mathcal{I}} \sum_{j} \sum_{k} H p_{j} p_{k} \min \left\{\mathcal{I}_{j}, \mathcal{I}_{k}\right\} \\
& =1-\frac{1}{H \mathcal{I}} \sum_{j} \sum_{k} p_{j} p_{k} \min \left\{\mathcal{I}_{j}, \mathcal{I}_{k}\right\} \tag{9}
\end{align*}
$$

where $\mathcal{I}=\sum_{j} p_{j} \mathcal{I}_{j}$.
From this point onward let the low wage be given by $w_{L}=0$ and the high wage by $w_{H}=w$. Also, we consider only tax systems that are linear. Hence, there is a constant marginal rate of tax and a common lump-sum subsidy for all consumers. This applies to the income tax and the expenditure tax.

### 2.2 Taxation

The introduction of income taxation modifies the budget constraints in the two periods of life to

$$
\begin{equation*}
x_{h}^{1}+s_{h}=e_{h}+w_{h} \ell_{h}(1-t)+g \tag{10}
\end{equation*}
$$

and

$$
\begin{equation*}
x_{h}^{2}=(1+r(1-t)) s_{h}+g, \tag{11}
\end{equation*}
$$

where $t$ is the tax rate and $g$ the lump-sum grant. Note that the endowment is not taxed since it is not income and that the treatment of interest income in (11) reflects the double taxation of saving. Combining these two budget constraints provides the lifetime budget constraint

$$
\begin{equation*}
x_{h}^{1}+\frac{x_{h}^{2}}{1+r(1-t)}=e_{h}+w_{h} \ell_{h}(1-t)+g \frac{2+r(1-t)}{1+r(1-t)} . \tag{12}
\end{equation*}
$$

The lifetime budget constraint reveals how second period incomes are discounted at the net-of-tax rate of interest. Using this budget constraint it is now possible to derive the optimal choices of each individual.

Consider first an individual of type $L L$ or $L H$ who has a low level of skill. Since $w_{h}=0$ for $h \in\{L L, L H\}$ it must be that $\ell_{h}$ is also zero. The consumption choices of a low skill consumer then solve the optimization problem

$$
\begin{equation*}
\max _{\left\{x_{h}^{1}, x_{h}^{2}\right\}} U_{h}=\alpha \ln \left(x_{h}^{1}\right)+\delta \ln \left(x_{h}^{2}\right) \quad \text { s.t. } \quad x_{h}^{1}+\frac{x_{h}^{2}}{1+r(1-t)}=e_{h}+g \frac{2+r(1-t)}{1+r(1-t)} . \tag{13}
\end{equation*}
$$

This optimization has the solution for the consumption levels in the two periods of life

$$
\begin{align*}
x_{h}^{1} & =\frac{\alpha}{\alpha+\delta}\left[e_{h}+g \frac{2+r(1-t)}{1+r(1-t)}\right] \\
x_{h}^{2} & =\frac{\delta}{\alpha+\delta}(1+r(1-t))\left[e_{h}+g \frac{2+r(1-t)}{1+r(1-t)}\right] \tag{14}
\end{align*}
$$

and the solution for saving

$$
\begin{equation*}
s_{h}=\frac{\delta}{\alpha+\delta} e_{h}+\frac{g}{1+r(1-t)}\left[\frac{\delta}{\alpha+\delta}(2+r(1-t))-1\right] . \tag{15}
\end{equation*}
$$

The high skill consumers will choose to supply a strictly positive amount of labor for tax rates below some strictly positive cut-off point. All the numerical computations that follow are for values below the cut-off. Hence, while recognizing that corner solutions can arise, we consider only interior solutions for high-skill consumers. For the consumers with $w_{h}=w>0$ this implies optimal choices are derived from

$$
\begin{equation*}
\max _{\left\{x_{h}^{1}, x_{h}^{2}, \ell_{h}\right\}} U_{h}=\alpha \ln \left(x_{h}^{1}\right)+(1-\alpha) \ln \left(1-\ell_{h}\right)+\delta \ln \left(x_{h}^{2}\right) \text { s.t. }(12) \tag{16}
\end{equation*}
$$

The resulting levels of consumption and labor supply are

$$
\begin{align*}
x_{h}^{1} & =\frac{\alpha}{1+\delta}\left[e_{h}+w(1-t)+g \frac{2+r(1-t)}{1+r(1-t)}\right]  \tag{17}\\
x_{h}^{2} & =\frac{\delta}{1+\delta}(1+r(1-t))\left[e_{h}+w(1-t)+g \frac{2+r(1-t)}{1+r(1-t)}\right],  \tag{18}\\
\ell_{h} & =1-\frac{1-\alpha}{1+\delta}\left[1+\frac{e_{h}}{w(1-t)}+\frac{g}{w(1-t)} \frac{2+r(1-t)}{1+r(1-t)}\right], \tag{19}
\end{align*}
$$

and the quantity of saving is

$$
\begin{equation*}
s_{h}=\frac{\delta}{1+\delta}\left[e_{h}+w(1-t)+g \frac{2+r(1-t)}{1+r(1-t)}\right]-\frac{g}{1+r(1-t)} \tag{20}
\end{equation*}
$$

The tax policy is assumed to be purely redistributive so the revenue raised by the government is returned as a lump-sum transfer to consumers. All consumers
receive the transfer regardless of income or endowment. Denote the transfer by $g$. The value of the transfer is calculated from the government's two-period budget constraint

$$
\begin{equation*}
g+\frac{g}{1+r}=t\left[\sum p_{h} w \ell_{h}+\frac{r}{1+r} \sum p_{h} s_{h}\right] \tag{21}
\end{equation*}
$$

In equilibrium $g$ is obtained by taking into account the dependence of the optimal choices of the consumers on the tax and transfer.

With expenditure taxation the budget constraints in the two periods of life become

$$
\begin{align*}
x_{h}^{1}(1+\tau)+s_{h} & =e_{h}+w_{h} \ell_{h}+g  \tag{22}\\
x_{h}^{2}(1+\tau) & =(1+r) s_{h}+g \tag{23}
\end{align*}
$$

where $\tau$ is the constant rate of expenditure taxation. Notice how the expenditure tax treats the endowment and labor income symmetrically, and the fact that income from saving is not taxed except as expenditure on consumption. Combining these two constraints into the lifetime budget constraint gives

$$
\begin{equation*}
x_{h}^{1}+\frac{x_{h}^{2}}{1+r}=\frac{1}{1+\tau}\left[e_{h}+w_{h} \ell_{h}+g \frac{2+r}{1+r}\right] . \tag{24}
\end{equation*}
$$

The optimal labor supply of an individual with a low skill level remains $\ell_{h}=0$. The choices of consumption and savings are given by

$$
\begin{align*}
x_{h}^{1} & =\frac{\alpha}{\alpha+\delta} \frac{1}{1+\tau}\left[e_{h}+g \frac{2+r}{1+r}\right]  \tag{25}\\
x_{h}^{2} & =\frac{\delta}{\alpha+\delta} \frac{1+r}{1+\tau}\left[e_{h}+g \frac{2+r}{1+r}\right] \tag{26}
\end{align*}
$$

and

$$
\begin{equation*}
s_{h}=\frac{\delta}{\alpha+\delta}\left[e_{h}+g \frac{2+r}{1+r}\right]-\frac{g}{1+r} . \tag{27}
\end{equation*}
$$

We again remain within the range of parameter values for which the labor supply of an individual with a high skill level is strictly positive. Consumption, labor supply and savings are then

$$
\begin{align*}
x_{h}^{1} & =\frac{\alpha}{(\alpha+\delta)(1+\tau)+1-\alpha}\left[e_{h}+w_{h}+g \frac{2+r}{1+r}\right]  \tag{28}\\
x_{h}^{2} & =\frac{\delta(1+r)}{(\alpha+\delta)(1+\tau)+1-\alpha}\left[e_{h}+w_{h}+g \frac{2+r}{1+r}\right]  \tag{29}\\
\ell_{h} & =1-\frac{1-\alpha}{(\alpha+\delta)(1+\tau)+1-\alpha}\left[1+\frac{e_{h}}{w_{h}}+\frac{g}{w_{h}} \frac{2+r}{1+r}\right], \tag{30}
\end{align*}
$$

and

$$
\begin{equation*}
s_{h}=\frac{\delta(1+\tau)}{(\alpha+\delta)(1+\tau)+1-\alpha}\left[e_{h}+w_{h}+g \frac{2+r}{1+r}\right]-\frac{g}{1+r} . \tag{31}
\end{equation*}
$$

The value of the transfer to every consumer is computed from the government budget constraint which, for expenditure taxation, is given by

$$
\begin{equation*}
g+\frac{g}{1+r}=\tau \sum p_{h}\left(x_{h}^{1}+\frac{x_{h}^{2}}{1+r}\right) \tag{32}
\end{equation*}
$$

### 2.3 Contrast

The intention is to contrast the success of the alternative tax bases at achieving redistribution. As noted in the Introduction we need to be careful in the way we conduct the comparison in order for the results to be meaningful. The process adopted is to set the income tax at a fixed rate and then compute the level of welfare this generates. The expenditure tax is then obtained that leads to the same level of welfare. The value of the Gini coefficient is then calculated for the two tax bases. This provides a comparison of the redistribution achieved for income and expenditure tax bases at an equal welfare level. The exercise is then repeated for a pair of taxes that generate identical levels of government revenue (and through the government budget constraint provide an identical value of the lump-sum transfer).

The second important aspect is to ensure that we make the comparison for a sufficiently wide range of the underlying parameters. Recall that the economy has both skill and endowment differences between consumers. Numerically testing a range of specifications revealed that the parameter that distinguishes different cases is the coefficient of correlation between skill and endowment. We therefore conduct our equal welfare (and equal revenue) comparisons for the full range of values of the correlation coefficient between -1 and 1 .

The details of our calculations are as follows. We consider two values of the income tax rate $(t=0.1$ and $t=0.3)$. We set the low wage and low endowment level at 0 . The high endowment is set at $e=1$. For each tax rate we consider high wages of $w=1$ and $w=5$. We set the probability of the wage-endowment pairs $(0,0)$ and $(1,1)$ at $p$ and the probability of the pairs $(0,1)$ and $(1,0)$ at $q$. The population variances of the wage and of the endowment are $(p+q)^{2}$ and the correlation between the two is $1-4 q$. Hence $q=0$ gives perfect positive correlation between endowment and skill, and $q=\frac{1}{2}$ gives perfect negative correlation. By varying $q$ between 0 and $\frac{1}{2}$ we are then able to cover the range of correlation coefficients.

The contrast between the two tax bases is illustrated in Figures 1 and 2 for $w=1$. The correlation coefficient is measured on the horizontal axis and the value of the Gini coefficient on the vertical axis. There is a single curve for the income tax $\left(G_{I}\right)$ and two curves $\left(G_{E W}\right.$ and $\left.G_{E R}\right)$ for the expenditure tax. $G_{E W}$ is the value of the Gini at the same welfare level as achieved by the income tax and $G_{E R}$ is the value of the Gini at the same revenue level as the income tax. The results show that when $e=w=1$ the expenditure tax generates a lower value of the Gini coefficient than the income tax (and hence achieves an equilibrium with less inequality) when there is a negative correlation between endowment and skill. When the correlation becomes sufficiently positive the


Figure 1: $t=0.1, e=1, w=1$
income tax generates a lower Gini coefficient. There is very little difference between the comparison with equal welfare and that with equal revenue. For these parameter values the choice between the two tax bases is dependent upon the value of the correlation coefficient.

The outcome of the comparison is different when $e<w$. This is illustrated in Figures 3 and 4 for $w=5$. In this case the income tax produces a lower value of the Gini index for the entire range of values for the correlation between skill and endowment. The reason for this change in outcome is that the increase in labor income relative to endowment income permits the income tax to be more successful at redistributing since it is levied on an increased proportion of total income.

The results show that the relative success of the two tax bases is dependent upon the relative values of labor income and initial endowment and the coefficient of correlation between these values. It is an interesting observation that the expenditure base only achieves a lower value of the Gini coefficient when there is negative correlation - in all other cases the income base is preferable. It is likely that the empirical evidence would determine that the correlation is positive in practice, thus providing a preference for the income base. Researching the evidence would also reveal that in practical terms initial endowments invariably arise from bequests. To interpret these within the static model seems to be pushing its interpretation too far. Instead, a better approach is to model bequests explicitly by adopting an intertemporal model that embodies a bequest motive.


Figure 2: $t=0.3, e=1, w=1$


Figure 3: $t=0.1, e=1, w=5$


Figure 4: $t=0.3, e=1, w=5$

## 3 Dynamic Economy

The analysis of the static economy has demonstrated how the correlation between endowment and skill affects the choice between the tax bases. In practice non-earned initial endowments arise primarily from bequests which, in turn, depend on the earning capacity of predecessors. This implies an endogenous correlation between skill and endowment related to the transmission mechanism of skill between generations. The static model captures some of the consequences of inequality but it does not reflect the fact that the endowments and skills are linked via the choices of dynasties of households.

It is therefore necessary to study a dynamic economy in which parents choose to leave bequests to their children. This allows the accumulation of wealth over time and the development of inequality, and the formation of an endogenous intertemporal link between skill and endowment. What is key in this economy is the mechanism by which skills are transmitted between generations. We now repeat the comparison of the tax bases in a dynamic economy where the transmission mechanism can be made explicit.

### 3.1 Model

We adopt an infinite-horizon economy that is populated by heterogenous agents. The agents have identical preferences but differ in skills and endowments. Each agent lives two periods. In the first period an agent receives an endowment and labor income which he divides between consumption and savings. In the second period he divides his savings between consumption and bequest. The bequest becomes the endowment of his descendant. There is no population growth, and the total size of the population is normalized to unity, with equal
proportions of young and old agents in every time period. With government intervention the agents pay tax and receive a transfer in every period. We consider two tax schemes, an income tax levied on labor income and interest income, and a consumption expenditure tax levied on consumption in every period. For simplicity, wages and the interest rate are exogenously fixed. The wage for skilled workers is normalized to unity, and that for unskilled workers is normalized to zero, so that in equilibrium unskilled workers do not supply labor.

The preferences of a consumer are described by the lifetime utility function

$$
\begin{equation*}
\mathcal{U}(\cdot)=U\left(x_{1}, \ell\right)+\delta V\left(x_{2}, b\right), \tag{33}
\end{equation*}
$$

where

$$
\begin{equation*}
U\left(x_{1}, \ell\right)=\alpha \ln x_{1}+(1-\alpha) \ln (1-\ell), \tag{34}
\end{equation*}
$$

and

$$
\begin{equation*}
V\left(x_{2}, b\right)=\beta \ln x_{2}+(1-\beta) \ln b \tag{35}
\end{equation*}
$$

with $x_{1}$ and $x_{2}$ consumption in the first and in the second period of life, respectively, $\ell$ labor supply, and $b$ the bequest. The probability that the descendant of a skilled worker is skilled is equal to $p_{s s}$, and the probability that the descendant of an unskilled worker is unskilled is equal to $p_{u u}$.

An infinitely lived government collects taxes and redistributes the revenues evenly among all agents in every period. We assume that the government can commit to a policy of a constant tax rate and a constant transfer. There is no borrowing constraints upon the government.

From the solution to each agent's optimization problem we can express bequest as a function of endowment. Because the bequest becomes the endowment of the next generation in a given dynasty, this function can be viewed as a law of motion of the endowment. The functional form of the law of motion depends on whether the bequestor is skilled or unskilled. Therefore, in every generation the law of motion of the endowment switches randomly between two regimes. The process of these random switches is a two-state Markov chain with the transition matrix

$$
P=\left[\begin{array}{ll}
p_{s s} & 1-p_{u u}  \tag{36}\\
1-p_{s s} & p_{u u}
\end{array}\right]
$$

The process is ergodic and irreducible if $p_{s s}<1, p_{u u}<1$, and $p_{s s}+p_{u u}>0$, with ergodic probabilities

$$
\pi=\left[\begin{array}{c}
\frac{1-p_{u u}}{2-p_{s s}-p_{u u}}  \tag{37}\\
\frac{1-p_{s}}{2-p_{s s}-p_{u u}}
\end{array}\right] \equiv\left[\begin{array}{c}
\pi_{1} \\
\pi_{2}
\end{array}\right] .
$$

The ergodic probabilities can be interpreted as unconditional probabilities of being in each regime (see Hamilton, 1994, Ch. 22). Hence, in the long run on average $\pi_{1}$ agents are skilled and $\pi_{2}=1-\pi_{1}$ are unskilled. Any initial distribution of endowments in the long run converges to a bimodal distribution, with peaks at the stationary points of the two regimes. This is illustrated in


Figure 5: Convergence to steady state bequests

Figure 5 where $e_{u}$ and $e_{s}$ are the long-run bequests of the unskilled and skilled respectively. The government will run an "on average" balanced budget if it computes the amount of transfer taking $\pi_{1}$ skilled and $\pi_{2}$ unskilled agents, with corresponding stationary endowments, as the tax base.

### 3.2 Taxation

With an income tax the first- and second-period budget constraints of an agent with endowment $e$ are

$$
\begin{align*}
& x_{1}+s \leq e+w \ell(1-t)+g  \tag{38}\\
& x_{2}+b \leq s(1+r(1-t))+g \tag{39}
\end{align*}
$$

The solution to the optimization problem of a skilled agent is

$$
\begin{align*}
x_{1}^{s} & =\frac{\alpha}{1+\delta}\left[e+w(1-t)+g\left(1+\frac{1}{1+r(1-t)}\right)\right]  \tag{40}\\
x_{2}^{s} & =\frac{\delta \beta}{1+\delta}\left[e+w(1-t)+g\left(1+\frac{1}{1+r(1-t)}\right)\right][1+r(1-t)]  \tag{41}\\
\ell^{s} & =1-\frac{\alpha}{1-\alpha} \frac{x_{1}^{s}}{w(1-t)},  \tag{42}\\
s^{s} & =\frac{\delta}{\alpha} x_{1}^{s}-\frac{g}{1+r(1-t)},  \tag{43}\\
b^{s} & =\frac{\delta(1-\beta)}{1+\delta}\left[e+w(1-t)+g\left(1+\frac{1}{1+r(1-t)}\right)\right][1+r(1-t)] \tag{44}
\end{align*}
$$

The wage of unskilled agents is set at zero so they supply no labor. The
optimal choices of an unskilled agent are

$$
\begin{align*}
x_{1}^{u} & =\frac{\alpha}{\alpha+\delta}\left[e+g\left(1+\frac{1}{1+r(1-t)}\right)\right]  \tag{45}\\
x_{2}^{u} & =\frac{\delta \beta}{\alpha+\delta}\left[e+g\left(1+\frac{1}{1+r(1-t)}\right)\right][1+r(1-t)]  \tag{46}\\
\ell^{u} & =0  \tag{47}\\
s^{u} & =\frac{\delta}{\alpha} x_{1}^{u}-\frac{g}{1+r(1-t)},  \tag{48}\\
b^{u} & =\frac{\delta(1-\beta)}{\alpha+\delta}\left[e+g\left(1+\frac{1}{1+r(1-t)}\right)\right][1+r(1-t)] . \tag{49}
\end{align*}
$$

The long-run average tax revenue is

$$
\begin{equation*}
T R=t\left[\pi_{1}\left(w \ell^{s}+r s^{s}\right)+\pi_{2} r s^{u}\right] \tag{50}
\end{equation*}
$$

and the transfer is computed from the government budget constraint $T R=g$. The budget need not balance every period so we are implicitly assuming that the government can borrow and lend at the rate of interest $r$.

With the expenditure tax the first- and second-period budget constraints of an agent with endowment $e$ are

$$
\begin{align*}
x_{1}(1+\tau)+s & \leq e+w \ell+g  \tag{51}\\
x_{2}(1+\tau)+b & \leq s(1+r)+g \tag{52}
\end{align*}
$$

The solution to the optimization problem of a skilled agent is

$$
\begin{align*}
x_{1}^{s} & =\frac{\alpha}{1+\delta} \frac{1}{1+\tau}\left[e+w+g\left(1+\frac{1}{1+r}\right)\right]  \tag{53}\\
x_{2}^{s} & =\frac{\delta \beta}{1+\delta} \frac{1}{1+\tau}\left[e+w+g\left(1+\frac{1}{1+r}\right)\right](1+r)  \tag{54}\\
\ell^{s} & =1-\frac{\alpha}{1-\alpha} \frac{x_{1}^{s}(1+\tau)}{w}  \tag{55}\\
b^{s} & =\frac{\delta(1-\beta)}{1+\delta}\left[e+w+g\left(1+\frac{1}{1+r}\right)\right](1+r) \tag{56}
\end{align*}
$$

Using the fact that unskilled agents supply no labor,

$$
\begin{align*}
x_{1}^{u} & =\frac{\alpha}{\alpha+\delta} \frac{1}{1+\tau}\left[e+g\left(1+\frac{1}{1+r}\right)\right]  \tag{57}\\
x_{2}^{u} & =\frac{\delta \beta}{\alpha+\delta} \frac{1}{1+\tau}\left[e+g\left(1+\frac{1}{1+r}\right)\right]  \tag{58}\\
b^{u} & =\frac{\delta(1-\beta)}{\alpha+\delta}\left[e+g\left(1+\frac{1}{1+r}\right)\right](1+r) \tag{59}
\end{align*}
$$

The long-run average tax revenue is

$$
\begin{equation*}
T R=\tau\left[\pi_{1}\left(x_{1}^{s}+x_{2}^{s}\right)+\pi_{2}\left(x_{1}^{u}+x_{2}^{u}\right)\right] \tag{60}
\end{equation*}
$$

and the transfer is computed from $T R=g$.

### 3.3 Contrast

This section contrasts the two tax bases. This is done in two ways. First, we consider the dynamic evolution of the economy beginning from an arbitrary assignment of initial endowments. Second, we analyze the instantaneous stationary equilibrium with population proportions equal to the ergodic probabilities. In both cases we focus upon the value of the Gini coefficient using equal welfare and equal revenue comparisons.

Figures 6-9 depict the value of the Gini coefficient for income taxation and the two Gini coefficients for expenditure taxation: one expenditure Gini is at the same welfare level as for the income tax, the other Gini for expenditure is at the same government revenue level. Two income tax rates are considered ( $t=0.2$ and $t=0.4$ ) and two different probabilities for a high-skill parent to have a high-skill offspring ( $p_{s s}=0.2$ and $p_{s s}=0.8$ ). The ergodic probabilities that these generate imply long-run average proportions for high-skill of $\frac{5}{13}$ and $\frac{5}{7}$ respectively.

These simulations compute the Gini coefficient for one hundred generations of one hundred families, with zero initial endowment and a uniform distribution of skills ( 0 or 1 with equal probability) for the families in the first generation. We plot only from generation ten since by this point the effect of the assumptions on initial distribution has disappeared. In every period the agents (families) choose their optimal consumption, leisure and bequest given their endowment and skills (wage income). The bequest becomes the endowment of the agent's offspring, whose skill is determined randomly, according to (36). The economy does not reach a steady state since there is always randomness in the ability of offspring.

What is observed in all the figures is that the Gini for income taxation is on average below the Ginis for expenditure taxation. Increasing $t$ and reducing $p_{s s}$ emphasizes this effect. The results confirm the observation made in the static setting that the income tax leads to a lower value of the Gini. It should be observed that in this model for $p_{s s}=0.8$ there is a positive correlation between wage income and endowment driven by the fact that high-skill parents leave a higher bequest and are more likely to have high-skill offspring. In contrast, for $p_{s s}=0.2$ the correlation between wage income and endowment is negative. In all cases in the long-run equilibrium the average endowment of both skilled and unskilled is less than the wage of skilled. Hence, the outcome in the dynamic economy (lower Gini with the income tax) is consistent with the one in the static economy.

The results for the cross-section, "stationary" analysis confirm the observations from the dynamic process. In Figures 10-11 we plot the Gini for an economy with the (instantaneous) proportion of skilled agents equal to $\pi_{1}$, the ergodic probability, or the long-run average proportion of skilled, for a fixed $p_{s s}$, and $p_{u u}$ varying from 0.01 to 0.99 . In every case the Gini for the income tax is below the two Ginis for the expenditure tax. This emphasizes that the few cases in which the expenditure base is observed to produce a lower Gini than the income base during the dynamic evolution are consequences of particular


Figure 6: $t=0.2, p_{s s}=0.2, p_{u u}=0.5$


Figure 7: $t=0.4, p_{s s}=0.2, p_{u u}=0.5$


Figure 8: $t=0.4, p_{s s}=0.8, p_{u u}=0.5$
random realizations of the economy. The long-run stationary outcome confirms that the expected position is for the income base to ensure a lower value of the Gini.

## 4 Conclusions

The intention of the paper was to contrast the relative success of alternative bases for personal taxation. In a static model with inequality arising from skill in employment and from initial endowment the income base performed better in all cases considered if there was positive correlation between the sources of inequality. The expenditure base only bettered the income base when there was negative correlation and a low level of income from employment. These results were strengthened in the dynamic model. The income base performed better except for a small number of realizations of the economy, and was clearly better in the long-run equilibrium. If the choice over the tax base rests on the reduction of inequality these results provide evidence in favor of an income base.

It seems natural to question the extent to which policy recommendations can be drawn from these stylized models. Both models capture the fact that inequality of income has two dimensions - earned and unearned - and the dynamic model also involves accumulation of inequality over time through the role of bequests. We would agree that the static model is limited by the lack


Figure 9: $t=0.4, p_{s s}=0.8, p_{u u}=0.5$
of transmission of inequality across generations. For this reason we prefer to focus upon the outcome of the dynamic model. Reassuringly, the results of the dynamic model not only support those from the static model but are actually more decisive. The income tax performed better than the expenditure tax for all the parameter combinations considered (many of which have not been reported in the paper). The dynamic model was simplified by the assumption of a fixed interest rate but this can be rationalized by assuming a small open economy or a constant marginal product for capital. The advantage remains that it avoided intermixing issues of redistribution and dynamic inefficiency. We therefore feel that our conclusions on the advantage of the income tax are robust.

## References

Altig, D., Auerbach, A., Kotlikoff, L., Smetters, K., and Walliser, J. (2001) "Simulating tax reform in the United States", American Economic Review, 91, 574-595.

Auerbach, A.J. (2006)"The choice between income and consumption taxes: a primer", University of California.

Batina, R.G. and Ihori, I. (2000) Consumption Tax Policy and the Taxation of Capital Income, Oxford: Oxford University Press.

Bradford, D.F. (1986), Untangling the Income Tax, Cambridge: Harvard University Press.


Figure 10: $p_{s s}=0.2$

Chamley, C. (1986) "Optimal taxation of capital income in a general equilibrium with infinite lives", Econometrica, 54, 607-622.

Chamley, C. (1981) "The welfare cost of capital income taxation in a growing economy", Journal of Political Economy, 89, 468-496.

Correira, I.H. (2005) "Consumption taxes and redistribution", CEPR Discussion Paper No. 5280.

Feenberg, D., Mitrusi, A., and Poterba, J. (1997) "Distributional effects of adopting a national retail sales tax", in Tax Policy in the Economy 11, ed. J. Poterba.

Foster, J.E. and Sen, A. (1997) On Economic Inequality, Oxford: Clarendon Paperbacks.

Hall, R. and Rabushka, A. (1985) The Flat Tax, Stanford: Hoover Institution Press.

Hamilton, J.D. (1994) Time Series Analysis, Princeton: Princeton University Press.

Hobbes, T. (1660) Leviathan (reprinted by Penguin Classics, 1985).
Jones, L.E., Manuelli, R.E. and Rossi, P.E. (1993) "Optimal taxation in models of endogenous growth", Journal of Political Economy, 101, 485-517.

Judd, K.L. (1985) "Redistributive taxation in a simple perfect foresight model", Journal of Public Economics, 28, 59-83.

Kaldor, N. (1955) An Expenditure Tax, London: George Allen and Unwin.


Figure 11: $p_{s s}=0.8$

Kanbur, R. and Keen, M. (1989) "Poverty, incentives, and linear income taxation" in Dilnot, A. and Walker, I. (eds.) The Economics of Social Security, Oxford: Clarendon Press.

Kay, J.A. and King, M.A. (1990) The British Tax System, Oxford: Oxford University Press.

King, R.G. and Rebelo, S. (1990) "Public policy and endogenous growth: developing neoclassical implications", Journal of Political Economy, 98, S126 S150.

Meade, J.E. (1978) The Structure and Reform of Direct Taxation, London: George Allen and Unwin.

Mill, J.S. (1888) Principles of Political Economy, New York: Appleton.
Myles, G.D. (2000) "Taxation and economic growth", Fiscal Studies, 21, 141-168.

US Treasury Department (1942) Proposal for a "consumption expenditure tax", www.taxhistory.org/Civilization/Documents/Spending/hst9369/9369-1.htm.


[^0]:    *Nigar Hashimzade, Department of Economics, University of Exeter, Exeter EX4 4RJ, UK, n.hashimzade@ex.ac.uk.
    ${ }^{\dagger}$ Gareth D. Myles, Department of Economics, University of Exeter, Exeter EX4 4RJ, UK, g.d.myles@ex.ac.uk.

