Consumer Demand with Unobserved Stockpiling and Intertemporal Price Discrimination

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Abstract

We construct a tractable structural dynamic model of consumption, purchase and stocks by consumers for whom stockpiling is unobserved and for whom preferences are isolastic and affected by independent and identically distributed shocks. Consumers purchase in stores which they meet randomly and which are supposed to maximize short run profits. We show that a two-price mixed strategy by stores satisfies conditions for an equilibrium in which consumers and stores coordinate their expectations on this stationary solution. We derive a simple and tractable estimation method using log linearized demand equations and equilibrium conditions. We estimate parameters using scanner data registering soda purchases by French consumers during 2005-2007.

Keywords: Stocks, consumer demand, dynamic choice, intertemporal discrimination

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1 Introduction[1]

Common economic sense suggests that consumers stock products according to their durability. It is difficult to imagine that consumers stock durable goods like furniture or cars in anticipation of price increases while it would be very costly to stock sizeable quantities of perishable goods like butter or yoghurt for long. This is why the most likely storable goods are semi-durables like beverages, cans, cereals, office supplies and the like although their storage involves a cost.

Similarly, the rationale of firms for proposing sales or promotions seem to differ according to the durability of the product. Sales aim at acting on renewal purchases for durable goods or sales are related to supply side constraints for perishable goods and are likely to be explained by reasons on the firms' side (Aguirregabiria, 1999). In contrast, the mechanism on which sellers of semi-durables may rely is that consumers, when facing promotions or sales, purchase more than usual and stock the surplus that they would not consume in the current period. In other words, some firms might poach future demand from other firms at the cost of low current prices. This is the economic mechanism of intertemporal price discrimination that we model in this paper and that we apply in an empirical model of long histories of purchases of semi-durables. Specifically, if prices are low, purchases consist of consumption and stocks and if prices are high, purchases consist of consumption net of destocking. This implies that own price elasticities estimated in static models are downward biased (Hendel and Nevo, 2006a).

This paper makes three contributions. First, we aim at developing dynamic structural models that dispense with the computer intensive dynamic mixed discrete and continuous choice models (Erdem, Imai and Keane, 2003, Hendel and Nevo, 2006b). Second, following up on Hendel and Nevo (2013), we seek to integrate demand and supply in a single tractable equilibrium setting. Third, we estimate the parameters of such a model using purchases of semi-durables such as coca-cola or other sodas using French scanner data of purchases over a period of two years. Specifically, we provide a simple estimation method for price elasticities in those dynamic models.

We now return to these contributions in more detail.

The first building block is a structural dynamic model of decisions by (a continuum of) consumers facing firms with which they interact randomly. The dynamics is generated by the accumulation of

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stocks whose costs are assumed to be iceberg costs. Purchasing one unit of the product delivers a stock of $1-\delta$ unit only next period. Preferences of consumers are assumed isoelastic and independent and identically distributed preference shocks across consumers and time are multiplicative. Imposing that these shocks are bounded from below leads to a very tractable model in which stocks, if any, are depleted in a single period. We also provide an extension of our basic set-up in which consumers might also have no taste for the product at some periods. This generates a low decay of stocks over time. This framework enables us to write analytically consumption, purchases and stockpiling as functions of current prices and of the marginal value of end-of-period stocks.

Moreover, we use an equilibrium framework which assumes stationarity, mainly because we cannot observe the level of stocks. Our definition of equilibrium also imposes that at any period t, consumers and firms believe that the support of the distribution of prices at the previous period, was reduced to two prices, high and low, as in Salop and Stiglitz (1982), and that consumers and firms expect that the support of prices in the future will also be reduced to these same two prices. Furthermore, they also believe that the probability of facing a low price at past and future periods is constant. We derive from these premises the stationary marginal value of stocks and the optimal level of stocks.

Second, we show that the mixed price strategy consisting of two prices played randomly is indeed a best response of firms to consumer behavior in period t, holding beliefs about the previous and future periods constant. To keep the set-up simple, firms are assumed to maximize short-run profits and consumers are assumed to meet them randomly. The two-price solution(s) that we derive is in this sense, a stationary equilibrium of this game. We prove existence when the consumer discount factor is close to 1 and when the iceberg cost is close to zero. The region in which equilibria exist is enlarged if the price elasticity of the product increases and if the distribution of preferences becomes less dispersed.

Third, we derive an estimable demand model by log-linearization, the equilibrium equations providing two additional estimating equations. We estimate parameters using a method of moments estimator under non linear constraints imposed by minimum distance.

We use scanner data on purchases of Coca-Cola of French consumers over 2005-2007. Coca-Cola is by far the main brand for colas in France. In our sample, it represents almost 84% of sales value (average over 3 years), more than 66% of quantity market shares and almost 70% of purchases. Pepsi has only between 6% and 7% market share. We thus focus on Coca Cola sales and estimate our model controlling for periods of no purchases by households. We show that using the demand side given by household purchases per week results in a biased estimation of preference parameters. Using the specification of our model and equilibrium supply conditions, we are able to identify those preferences and estimate parameters by non linear GMM. Estimation results are quite precise and show that stockpiling by consumers is significant in this market.

Literature review: Boizot, Robin and Visser (2001) analyze a continuous time model in which a single good is consumed by a constant amount over time. They derive predictions on durations since last sales and until next sales (see also Hendel and Nevo, 2006a). Specifically, during sales, duration until next purchase is longer. Duration elapsed since previous purchase is shorter during sales. The purchase probability in a non-sale period is smaller after a sale period than after a non-sale period i.e. prices at t and $t-1$ affect demand without any habit formation motive.

These reduced form results which are insightful are completed by several structural analyzes.

Erdem et al. (2003) develop also a mixed discrete and continuous dynamic model under the assumption that brands are used proportionately to meet some exogenous usage requirement. They show how to estimate their dynamic model of consumer brand and quantity choice dynamics under price uncertainty and find that estimates of demand own and crossprice elasticities are very sensitive to how households form expectations of future prices and to the stochastic price process.

Hendel and Nevo (2006b) develop a mixed discrete and continuous dynamic model in which quantity and brand choices are separable. Hendel and Nevo (2006b) show that estimating static demand functions overestimate elasticities by 30% and underestimate cross-price elasticities by a factor of 5. They also show that the price-cost margin is underestimated and the effect of mergers is mispredicted. Perrone (2010) also proposes a fully dynamic model with unobserved heterogeneity. She uses the argument that high-price periods "break" the dynamics by making stocks equal to zero and shows that price elasticities are overestimated by 10 to 200% using French data.

Hendel and Nevo (2013) were the first to link an empirical demand model in which there are sales to the pricing policies of firms that justify the existence of sales. In their set-up in which there are multiple goods, the number of storage periods is fixed arbitrarily with full depreciation after some time. Consumers perfectly foresight prices either because firms play a two-price strategy or because firms use pricing cycles. In contrast to our paper, there are no preference shocks although there is discrimination between price insensitive consumers (non stockers) and price sensitive consumers (stockers).

Section 2 sets up the consumer side, by deriving first the static conditions for purchases, stocks and consumption and second the dynamic conditions leading to the determination of the marginal value of stocks and optimal stockpiling. This leads to a tractable system of expected demands as a function of previous and current prices. Section 3 derives firms's strategies in the case profits are short-run. We also prove that a stationary equilibrium in which all agents coordinate on the same two-price distribution function exists under some conditions on the discount rate and the cost of stocking. We also provide extensions to this basic result by allowing consumers to have no taste for the product at some periods. Section 4 presents a descriptive analysis of our data, the empirical strategy and the estimation method and the results. Section 5 concludes.

2 Demand Model

Consider a forward looking household or consumer, who can purchase and stockpile a single good at each period. In this section, we do not index variables by household identity, h. The quantity purchased at each period t is denoted x_t and c_t denotes the consumption level during the same period. We denote i_{t-1} the level of stocks at the beginning of period t and assume that the evolution of stocks satisfies:

$$i_t = (1-\delta)i_{t-1} + x_t - c_t \tag{1}$$

in which δ is an iceberg cost for stocking (the more durable the good the lower δ). The price of purchases is equal to p_t, and the period t consumer utility, depending on consumption and purchases, is given by

$$u_t\left(c_t\right) - \alpha p_t x_t$$

where the household marginal utility of income α is heterogenous across households. The utility function u_t is an increasing and strictly concave function of c_t and is affected by preference shocks, η_t. For making the algebra simpler, we assume that it is isoelastic and that heterogeneity in preferences is multiplicative:

$$u_t(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}\eta_t \tag{2}$$

in which σ is the relative risk aversion parameter and $0 < \sigma < 1$. Preference shocks η_t are distributed on $[\underline{\eta}, +\infty)$, where $\underline{\eta} > 0$ and are supposed to be independently and identically distributed over time. Because of the lower positive bound, marginal utility $u_t'(c_t) = c_t^{-\sigma}\eta_t$ is uniformly bounded away from zero. Importantly, we shall analyze in an extension later on, the case in which households can have no taste for the product, $\eta_t = 0$, with a strictly positive probability.

The intertemporal consumer objective at time t consists in the following expected discounted utility

$$E \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[u_t\left(c_\tau\right) - \alpha p_\tau x_\tau \right]$$

in which β is the discount rate.

Within a period t, the timing of the game between consumers and firms is the following:

1. The consumer starts with a stock of $(1-\delta)i_{t-1}$ inherited from the previous period.

2. Stores (or firms) set prices, p_t.

3. Consumer chooses store and discovers price p_t in this store.

4. Consumer possibly purchases, x_t.

5. Given previous stocks and purchases, the household consumes, c_t and stocks i_t.

In this section, we take the price setting by firms as given and study the reaction function of consumers to prices. We nonetheless have to be more precise about the dynamics of prices if consumers (and firms) rationally expect price dynamics.

For simplicity, we assume first that each consumer chooses the store at which he does all his purchases independently of the particular product we look at or that this product is "small" among all products. His choice of store is thus independent of prices p_t. Second, we assume that the store choice is independent over time.[2] We further strengthen these conditions by assuming that they are common knowledge.

The consequences are that prices that the consumer faces at each period are independent over time. Indeed, denoting F_t the store choice at time t by the consumer, the conditional probability distribution of prices that each consumer faces is equal to :

$$\begin{aligned}
\Pr(p_t &\leq p \mid p_{t-1}) = \sum_{f,f'} \Pr(p_t \leq p, F_t = f, F_{t-1} = f' \mid p_{t-1}) \\
&= \sum_{f,f'} \Pr(p_t \leq p \mid F_t = f, F_{t-1} = f', p_{t-1}) \Pr(F_t = f \mid F_{t-1} = f', p_{t-1}) \Pr(F_{t-1} = f' \mid p_{t-1}) \\
&= \sum_{f,f'} \Pr(p_t \leq p \mid F_t = f) \Pr(F_t = f) \Pr(F_{t-1} = f' \mid p_{t-1}),
\end{aligned}$$

[2] This is a stronger assumption whose relaxation is left for future research.

because (i) the store choice of consumers is independent over time, $\Pr(F_t = f \mid F_{t-1} = f', p_{t-1}) = \Pr(F_t = f)$ (ii) firm f knows that store choices by consumers are independent over time and set prices independently of previous choices and prices $\Pr(p_t \leq p \mid F_t = f, F_{t-1} = f', p_{t-1}) = \Pr(p_t \leq p \mid F_t = f)$. This implies that:

$$\begin{aligned}\Pr(p_t &\leq p \mid p_{t-1}) = \sum\nolimits_f \Pr(p_t \leq p \mid F_t = f) \Pr(F_t = f) \sum\nolimits_{f'} \Pr(F_{t-1} = f' \mid p_{t-1}), \\ &= \sum\nolimits_f \Pr(p_t \leq p \mid F_t = f) \Pr(F_t = f) = \Pr(p_t \leq p).\end{aligned}$$

2.1 Bellman and optimality equations

As consumers believe that all future uncertainty related to prices, p_{t+k}, and preferences, η_{t+k} is independent over time, the only state variable is the level of stocks, $(1-\delta)i_{t-1}$ inherited at the beginning of each period t. The maximization of the expected discounted utility can thus be written as the solution to the following Bellman equation:

$$W_t(i_{t-1}) = \max_{c_t, x_t} \left\{ u_t\left(c_t\right) - \alpha p_t x_t + \beta \mathbb{E}_t W_{t+1}(i_t) \right\} \tag{3}$$

subject to equation (1) and positivity constraints, $i_t \geq 0$, $x_t \geq 0$, where we use the notation $\mathbb{E}_t\left[.\right]$ for the expectation operator with respect to future prices and preference shocks and conditional on the information available at period t. Assume that $W_{t+1}(i_t)$ is an increasing and concave differentiable function of its first argument. We shall prove in Appendix A.6 after deriving the necessary conditions that our setting entails that $W_t(i_{t-1})$ is also an increasing and concave differentiable function.

Define the expected marginal value of stocks $\lambda_t\left(i_t\right)$ at the end of period t as:

$$\lambda_t\left(i_t\right) = \beta \mathbb{E}_t \left[\frac{\partial W_{t+1}\left(i_t\right)}{\partial i_t} \right]$$

It is a non-increasing function of i_t because of the concavity of W_{t+1} in its first argument.[3]

The first order conditions of program (3) are:

$$\begin{cases} u_t'\left(c_t\right) = & \lambda_t\left(i_t\right) + \mu_t \\ \alpha p_t = & \lambda_t\left(i_t\right) + \mu_t + \psi_t, \end{cases} \tag{4}$$

[3] We will also assume that :

$$\lim_{i_t \longrightarrow \infty} \lambda_t(i_t) = 0.$$

in which μ_t (resp. ψ_t) is the Lagrange multiplier associated to constraint $i_t \geq 0$ (resp. $x_t \geq 0$).

Furthermore, dynamic optimality yields an Euler-type equation:

$$\lambda_t(i_t) = \beta(1-\delta)\mathbb{E}_t\left[\lambda_{t+1}(i_{t+1}) + \mu_{t+1}\right]. \tag{5}$$

In order to solve this consumer problem, we will first study the static necessary conditions of optimality and characterize optimal purchase and stock decisions of the consumer given the endogenous marginal value $\lambda_t(i_t)$. We then turn to the dynamic optimality condition (5) and impose stationarity on this dynamic decision process.

2.2 Static Necessary Conditions at Period t

Using the static necessary conditions of optimal consumer behavior expressed in equation (4), we can derive optimal purchase and consumption behavior at each period, depending on price, stocks and on the shape of the marginal value of stocks function. Denoting the inverse marginal utility $u_t'^{-1}(m) = (m/\eta_t)^{-\frac{1}{\sigma}}$, we summarize these results in the following proposition,

Proposition 1 *At a given period t, optimal stocks i_t^*, consumption c_t^* and purchase x_t^* will depend on current stocks i_{t-1}, the marginal value function of stocks, $\lambda_t(i_t)$ and price p_t as follows:*

- *If $\lambda_t(0) \leq \alpha p_t < u_t'((1-\delta)i_{t-1})$ then*

$$\begin{cases} x_t^* = u_t'^{-1}(\alpha p_t) - (1-\delta)i_{t-1} \\ c_t^* = u_t'^{-1}(\alpha p_t) \\ i_t^* = 0 \end{cases} \tag{I.a}$$

- *If $\lambda_t(0) < u_t'((1-\delta)i_{t-1}) < \alpha p_t$ then*

$$\begin{cases} x_t^* = 0 \\ c_t^* = (1-\delta)i_{t-1} \\ i_t^* = 0 \end{cases} \tag{II.a}$$

- *If $\alpha p_t < \lambda_t(0)$ and $\alpha p_t \leq \lambda_t(\max\left\{(1-\delta)\,i_{t-1} - u_t'^{-1}(\alpha p_t), 0\right\}$ then*

$$\begin{cases} x_t^* = u_t'^{-1}(\alpha p_t) + i_t^* - (1-\delta)i_{t-1} \\ c_t^* = u_t'^{-1}(\alpha p_t) \\ i_t^* > 0 \text{ solution of } \lambda_t(i_t^*) = \alpha p_t \end{cases} \tag{I.b}$$

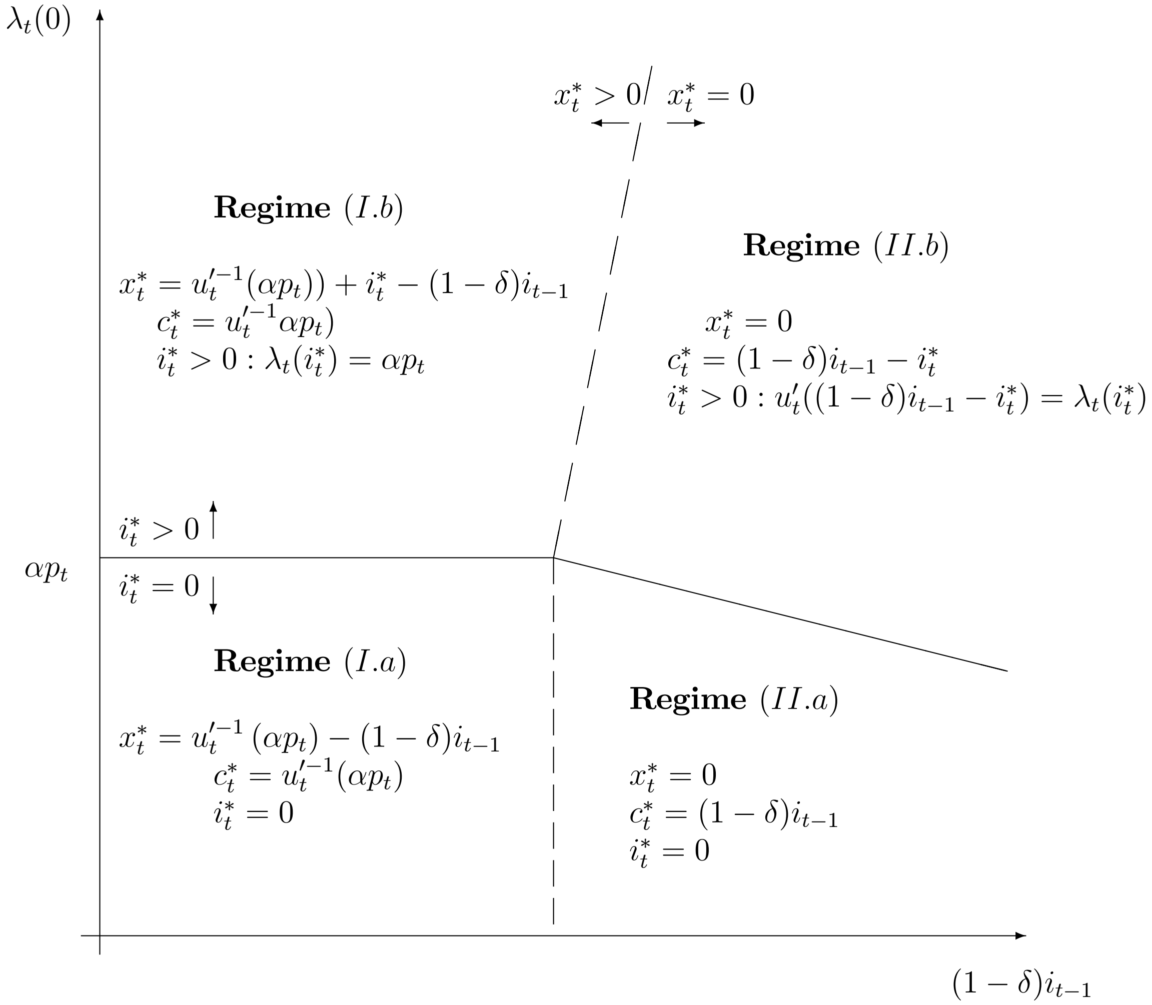
- *If* $u_t'((1-\delta)i_{t-1}) \leq \lambda_t(0)$ *and* $\lambda_t(0) \geq \lambda_t(\max\{(1-\delta)i_{t-1} - u_t'^{-1}(\alpha p_t), 0\})$ *then*

$$\begin{cases} x_t^* = 0 \\ c_t^* = (1-\delta)i_{t-1} - i_t^* \\ i_t^* > 0 \text{ solution of } u_t'((1-\delta)i_{t-1} - i_t^*) = \lambda_t(i_t^*) \end{cases} \tag{II.b}$$

Proof. See Appendix A.1. Note that the solution i_t^* might not be unique depending on the strict monotonicity of $\lambda_t(i_t)$, itself an endogenous object. ∎

The different cases are summarized in Figure 1.

Figure 1: Choice space



In regime (I.a), previous stocks are fully used for consumption and in addition purchases are made so that the marginal utility of consumption equals its price (up to the marginal value of money) although no stocks are constituted because their marginal value ($\lambda_t(0)$) is lower than the marginal value of purchasing at the current product price. The absence of stocking is also true in regime (II.a) although purchases are now equal to zero and previous stocks are fully consumed because the marginal value of stocks and prices are too high. In contrast, regime (I.b) exhibits positive stocks and positive purchases because the marginal value of stocks is high and previous stocks are low. Finally, Regime (II.b) corresponds to another "autarkic" case as regime (II.a) in which consumers do not purchase and consume only out of their previous stocks. In a sense that will be made precise later, previous stocks are too large in this regime.

We now turn to the determination of the marginal value of stocks, $\lambda_t(i_t)$, which is determined by the dynamic optimality equation (5).

2.3 Stationarity and Consumer Dynamic Choices

2.3.1 Firm decision process and stationarity conditions

For simplicity, we assume that firms maximize static conditions only and that firms' marginal costs in a period are constant over time and equal to κ. Because we cannot observe stocks that consumers make and only their purchases, we proceed by imposing stationary conditions on consumer and firm decisions.

Definition 1 *Consumer decisions are stationary if the distribution function of (c_t, i_t, x_t) conditional on i_{t-1} and p_t is independent of time and firm decisions are stationary if the distribution of prices is independent of time.*

We further restrict the set of equilibria on which consumers and firms are coordinating. Since profits should be non negative, this restricts the set of prices that firms choose to be such that $p_t \geq \kappa$. Salop and Stiglitz (1982) show in a simpler but similar model that firms have never an interest in using more than a two point distribution of prices. From now on, we will assume that agents coordinate their expectations on the belief that the stationary price distribution is $\{p_L, p_U\}$ in which $p_L < p_U$ and that the probability that p_L is played is denoted π.

We shall adopt the following definition of an equilibrium:

Definition 2 $\{p_L, p_U, \pi\}$ *is an equilibrium if decisions are stationary and:*

(1) Given that $\{p_L, p_U, \pi\}$ was played at period $t-1$ and holding their expectations of future prices fixed and equal to $\{p_L, p_U, \pi\}$, consumers play their best responses, $x_t^*(p_t)$, $c_t^*(p_t)$ and $i^*(p_t)$ to prices p_t set by firms at period t.

(2) Given that $\{p_L, p_U, \pi\}$ was played at period $t-1$ and holding their expectations of future prices fixed and equal to $\{p_L, p_U, \pi\}$, maximizing profit firms set their best response at period t as equal to the mixed strategy in $\{p_L, p_U\}$ in which p_L is played with probability π.

We now derive the conditions that need to be satisfied and show that there exists a non empty set of parameters such that the two-point price distribution is indeed an equilibrium. It does not mean that other equilibria or types of equilibria do not exist.

In summary, in order to derive the out-of-equilibrium behavior of consumers, we derive demands at period t when consumers believe that $\{p_L, p_U, \pi\}$ was played at period $t-1$ and hold expectations of future prices fixed and equal to $\{p_L, p_U, \pi\}$.

2.3.2 The existence of stocks

In the various period t regimes described in Section 2.2, a pivotal quantity at any given period t is the marginal value of stocks when stocks are zero, $\lambda_t(0)$, the current price being p_t. At a stationary equilibrium, the marginal value of stocks function $\lambda_t(.)$ does not depend on time and we have:

$$\lambda_t(0) = \lambda(0).$$

The next Lemma relates this quantity to the expectation of future price which is also independent of time.

Lemma 1 *We have:*

$$\lambda(0) = \beta(1-\delta)\alpha\mathbb{E}(p_{t+1}). \tag{6}$$

Proof. We want to evaluate the marginal value $\lambda_t = \lambda(0)$ at the values $i_t = 0$. Static conditions above imply that purchases at period $t+1$, x_{t+1}, are positive since stocks i_t at the beginning of the period are zero and thus the multiplier associated to positive purchase, $\psi_{t+1} = 0$. Therefore by the second equation in (4):

$$\lambda_{t+1} + \mu_{t+1} = \alpha p_{t+1}$$

so that equation (6) holds because of the Euler equation (5). ■

Lemma 1 says that at any given period, the marginal value of stocks when the consumer has no stock is equal to the discounted expected future price times the marginal utility of income times the rate of conservation of stocks.

We will denote, p^a, the expected price at the stationary equilibrium which is equal to the weighted combination of low and high prices in which the weight is π:

$$p^a = \mathbb{E}(p_{t+1}) = \pi p_L + (1-\pi)p_U,$$

since consumers' and firms' beliefs are coordinated on this equilibrium.

Conditions in Proposition 1 show that if stocks i_{t-1} are zero, whether case (I.a) in which no stocks are made or case (I.b) in which some stocks are made applies, depends on the condition that the marginal utility of one unit purchase at current price, αp_t, is larger or smaller than the marginal value of stocks when they are zero, $\lambda(0)$. Using the marginal value of zero stock given by Lemma 1, we obtain the following Lemma:

Lemma 2 *We have the two following properties:*

$$\lambda(0) < \alpha p_U$$

$$\lambda(0) < \alpha p_L \text{ iff } \delta > 1 - \frac{p_L}{\beta p^a}.$$

Proof. Using Lemma 1 yields that:

$$\lambda(0) = \beta(1-\delta)\alpha\mathbb{E}(p_{t+1}) < \beta(1-\delta)\alpha p_U < \alpha p_U,$$

since $\beta(1-\delta) < 1$. In contrast, the difference between $\lambda(0)$ and αp_L depends on the parameters and using Lemma 1:

$$\lambda(0) - \alpha p_L = \beta(1-\delta)\alpha p^a - \alpha p_L < 0$$

$$\Leftrightarrow (1-\delta) < \frac{p_L}{\beta p^a}.$$

■

Lemma 2 states that when the price is at its upper bound, p_U, the marginal value of stocks when the consumer holds no stocks is always lower than the marginal utility of buying one unit of the good. This implies that the consumer never increases the level of stocks when prices are high. When the price is at its lower bound p_L, the marginal value of stocks when the consumer holds no stocks is always lower than the marginal utility of buying one unit of the good if and only if the iceberg cost of stocks, δ, is too high i.e. greater than a threshold which is a function of the discount factor and the distribution of prices (which is endogenous). If this condition holds, this would imply that the agent would always be in the static condition (I.a) if i_{t-1} is close to zero so that a no stock stationary equilibrium arises.

This benchmark case is summarized in the next proposition:

Proposition 2 *If* $\delta \in (1 - \frac{p_L}{\beta p^a}, 1]$ *then at the stationary equilibrium there are no stocks, purchase equals consumption and:*

$$\forall t,\; i_t = 0, \quad c_t = x_t = u_t'^{-1}(\alpha p_t).$$

Proof. See Appendix A.2 ■

The economic interpretation of Proposition 2 is simple. In spite of fluctuation of prices, the cost of stocking is too high or prices are not differentiated enough to make stocks valuable. In this case, the consumer always prefers not to stock and purchases the optimal consumption level. It is true this condition depends on endogenous prices p_L and p_U that are determined below but it anticipates the result that the two-price equilibrium exists if the cost, δ, is not too large or the discount rate, β, is not too low.

2.3.3 The stationary distribution of stocks

We now continue with the more interesting case in which the cost of stocks is not too high so that consumers may prefer to hold stocks. The optimal purchase and stock decisions of the consumer will depend on the price process. We thus maintain the assumption from now on that:

Condition E(xistence of stocks): $\quad p_L \leq \beta(1-\delta)p^a.$

and will check below that it holds given the deep parameters.

In the following, we will use a function that summarizes the distribution function of preference shocks as:

$$f(a) = \mathbb{E}([\eta_t a - 1]\,\mathbf{1}\{\eta_t a - 1 \leq 0\}) \text{ for } a > 0,$$

whose properties are given by:

Lemma 3 *Function* $f(.)$ *is continuous, increasing, concave and such that:*

$$f(a) \leq 0, \lim_{a \to 0} f(a) = -1 \textit{ and } f(a) = 0 \textit{ for } a \geq \frac{1}{\underline{\eta}}.$$

It is differentiable almost everywhere, its derivative is CADLAG[4] *and* $f'(0) = 1$. *Furthermore, right derivatives are such that:*

$$\lim_{\varepsilon \downarrow 0} f'(a+\varepsilon) > 0 \text{ if } a < \frac{1}{\underline{\eta}}.$$

Moreover, $f^{-1}(a)$ *exists everywhere for* $a < \frac{1}{\underline{\eta}}$ *and* $f^{-1}(0)$ *is the set* $[\frac{1}{\underline{\eta}}, \infty)$.

Proof. The first properties are straightforward by applying the definition of $f(a)$. Differentiability holds at all points at which the density of η is not a mass point. At all points the right derivative is:

$$f'(a) = \mathbb{E}(\eta_t \mathbf{1}\{\eta_t \leq \frac{1}{a}\}),$$

and this is why $f'(a)$ inherits the CADLAG property from distribution functions. It is positive whenever $a < \frac{1}{\underline{\eta}}$. At the points of differentiability, $f"(a) \leq 0$ and f is concave. ■

In the following, we focus our analysis on the case in which stocks deplete quickly in a single period.[5] It is interesting that this stems from a primitive condition on the distribution of preferences and the bound, $\underline{\eta}$, described in the following Proposition. Denoting, $i(p_t)$, the stationary value of stocks whose expression will be derived below, the following necessary conditions on the optimal purchase, consumption and stockpiling decisions:

Proposition 3 *Under Condition E that* $p_L \leq \beta(1-\delta)p^a$, *and under the condition that:*

$$\underline{\eta} \geq \beta(1-\delta)\alpha p_a((1-\delta)i(\kappa))^{\sigma}, \tag{7}$$

in which $\underline{\eta}$ *is the lower bound for preferences and* κ *the marginal cost, we necessarily have, at a stationary equilibrium indexed by* p^a, *that:*

(i) If $p_t > \beta(1-\delta)p^a$ *and* $\eta_t > \frac{\alpha p_t}{((1-\delta)i_{t-1})^{-\sigma}}$,

$$c_t = u_t'^{-1}(\alpha p_t), \quad x_t = c_t - (1-\delta)i_{t-1}, \quad i_t = 0.$$

(ii) Else if $p_t > \beta(1-\delta)p^a$ *and* $\eta_t \in [\frac{\lambda(0)}{((1-\delta)i_{t-1})^{-\sigma}}, \frac{\alpha p_t}{((1-\delta)i_{t-1})^{-\sigma}}]$,

$$x_t = 0, \quad c_t = (1-\delta)i_{t-1}, \quad i_t = 0.$$

[4] CADLAG or RCLL for Right Continuous with Left Limits.

[5] The general case is significantly more involved and although interesting, is left for future research. Insights derived from this restricted case are already quite rich as further extensions of our set up to the case in which consumers could have no taste for the product are proved below.

(iii) Else if $\kappa \leq p_t < \beta(1-\delta)p^a$:

$$x_t = u_t'^{-1}(\alpha p_t) + i(p_t) - (1-\delta)i_{t-1}, \quad c_t = u_t'^{-1}(\alpha p_t), \quad i_t = i(p_t) \equiv \lambda^{-1}(\alpha p_t).$$

Furthermore, if $p_t = \beta(1-\delta)p^a$, the consumer is indifferent between the strategies defined in cases (ii) and (iii) above.

Proof. See Appendix A.3

Proposition 3 shows that the condition E for consumers to stock is satisfied and preference shocks have a lower bound that is not too small, the consumers will not hold stocks at the end of the period if the current price is larger that the discounted expected future price and will purchase if and only if the preference shock is large enough given current stocks and price. In this condition of high price, the consumers will consume to equalize marginal utility of consumption with marginal value of purchases if the preference shock is not too low, and will consume the stock if the preference shocks is too low given current price and stocks. If the price is lower than the expected discounted future price, the consumers will hold optimal stocks at the end of the period, consume to equalize marginal utility of consumption with marginal value of purchases and purchase to replete optimal stocks given the current price. ■

2.3.4 The marginal value of stocks and the optimal stocks

Proposition 3 enables us to derive the marginal value of stocks, $\lambda(i_t)$ using equation (5):

Lemma 4

$$\lambda(i_t) = \lambda(0) + \beta(1-\delta)\alpha p_U f(\frac{((1-\delta)i_t)^{-\sigma}}{\alpha p_U}),$$

Proof. See Appendix A.4 ■

Some characteristics of this marginal value function are worth noting. First, as $f(a) = 0$ for any $a \geq \frac{1}{\underline{\eta}}$, $\lambda(i_t)$ is constant and equal to $\lambda(0)$ over the range $[0, i^{(0)} \equiv \frac{1}{1-\delta}(\frac{\alpha p_U}{\underline{\eta}})^{-\frac{1}{\sigma}}]$. Second, as $f(.)$ is a negative continuous function and strictly increasing when $a < \frac{1}{\underline{\eta}}$, $\lambda(i_t)$ is a strictly decreasing continuous function when $i_t \geq i^{(0)}$. Finally, because $f(0) = -1$, and $\lambda(0) = \beta(1-\delta)\alpha p^a < \beta(1-\delta)\alpha p_U$, $\lambda(i_t)$ is negative when i_t tends to ∞. Therefore, this function satisfies the postulated conditions for the value function and solutions to equation $\lambda(i_t) = \alpha p_t$ exist.

In particular, the optimal solution for $i(p_t)$ in Proposition 3 is obtained as:

Lemma 5

$$\begin{aligned} i(p_t) &= \frac{1}{1-\delta}\left[\alpha p_U f^{-1}\left(\frac{\frac{p_t}{\beta(1-\delta)}-p^a}{p_U}\right)\right]^{-\frac{1}{\sigma}} \quad \textit{when } p_t < \beta(1-\delta)p^a \qquad (8) \\ &= 0 \textit{ when } p_t > \beta(1-\delta)p^a \end{aligned}$$

Moreover, if $p_t = \beta(1-\delta)p^a$*, the consumer is indifferent between any values belonging in the range* $[0, \frac{1}{1-\delta}(\frac{\alpha p_U}{\underline{\eta}})^{-\frac{1}{\sigma}}]$.

Proof. See Appendix A.5. ■

As f^{-1} is increasing, the optimal stock is decreasing in prices when $p_t < \beta(1-\delta)p^a$ and a continuous function of prices except at the value $p_t = \beta(1-\delta)p^a$ at which it has a jump downward towards $i(p_t) = 0$ for $p_t < \beta(1-\delta)p^a$. We delay the discussion of the variation of this function as a function of δ or prices, p^a and p_U since the latter are endogenous and decided by the firm. Before analyzing firm decisions, we finish this section by stating the purchase equations that are anticipated and used by the firms to compute their optimal decisions.

2.3.5 Expected purchases

As the equilibrium played at the previous period $t-1$ is supposed to be a two-point distribution function for prices, whose support is $\{p_L, p_U\}$, stocks inherited from the previous period, can only take two values, 0 and $i(p_L)$, when $p_L < \beta(1-\delta)p^a$ as defined by equation (8). When $p_L = \beta(1-\delta)p^a$, however, the consumer is indifferent between a range of values defined in Lemma 5 and can play mixed strategies. We will assume that there exists a parameter ρ that indexes the mean optimal stock in the population and therefore :[6]

$$i(\beta(1-\delta)p^a) = \frac{1}{1-\delta}\left[\alpha p_U f^{-1}(0)\right]^{-\frac{1}{\sigma}} = \frac{\rho}{1-\delta}\left[\frac{\alpha p_U}{\underline{\eta}}\right]^{-\frac{1}{\sigma}}. \qquad (9)$$

[6] Another way to justify the existence of ρ is to assume that $f^{-1}(0)$ can take any value in $[\frac{1}{\underline{\eta}}, +\infty)$ that is $\frac{\rho^{-\sigma}}{\underline{\eta}}$ for any $\rho \in [0,1]$.

Purchases at equilibrium and off-equilibrium, which matter for firms' price strategies, are thus given as a corollary to Proposition 3 by:

$$\begin{cases} x_t\left(p_U,p_t\right) = \begin{cases} u_t'^{-1}(\alpha p_t) = \alpha p_t^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}} & \text{if } p_t > \beta(1-\delta)p^a, \\ \alpha p_t^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}} + i(p_t) & \text{if } p_t < \beta(1-\delta)p^a \end{cases} \\ x_t\left(p_L,p_t\right) = \begin{cases} \alpha p_t^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}} - (1-\delta)i(p_L) & \text{if } p_t > \beta(1-\delta)p^a \text{ and } \eta_t > \frac{\alpha p_t}{((1-\delta)i(p_L))^{-\sigma}}, \\ 0 & \text{if } p_t > \beta(1-\delta)p^a \text{ and } \eta_t \leq \frac{\alpha p_t}{((1-\delta)i(p_L))^{-\sigma}}, \\ \alpha p_t^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}} + i(p_t) - (1-\delta)i(p_L) & \text{if } p_t < \beta(1-\delta)p^a. \end{cases} \end{cases} \tag{10}$$

3 Supply and stationary equilibrium

We now turn to the study of firm pricing when consumers can stockpile as determined in Section 2. The resulting firm price strategy has two points of support and choose $\{p_L, p_U\}$ and the probability, π, of playing p_L. Salop and Stiglitz (1982) show in a simpler but similar model that firms have never an interest in using more than a two points distribution of prices. We exhibit conditions under which a two-point of support distribution is an equilibrium of the game.

3.1 Firm decisions and equilibrium

We assume that expectations are common across firms and consumers, and thus that the expected price in the market at time $t+1$ is p^a, the weighted combination of p_L and p_U. We also have that $\pi = \Pr(p_{t-1} \leq \beta(1-\delta)p^a)$ is the probability perceived by firms that the consumer was facing a low price at the previous period.

As firms face many consumers, we suppose that firms integrate out the consumer specific parameters, α, their marginal value of money and η_t, their preference shocks, as if firms were facing a continuum of consumers. This integration is summarized by new parameters, $k = \mathbb{E}(\alpha^{-\frac{1}{\sigma}})\mathbb{E}(\eta_t^{1/\sigma})$ and $\underline{\nu} = \frac{\underline{\eta}^{1/\sigma}}{\mathbb{E}(\eta_t^{1/\sigma})} < 1$ in which integration is supposed to be taken over the continuum of households.

The following necessary conditions on the reaction function by the firms are:

Lemma 6 *When the firm chooses to set high prices, p_U, we have that:*

$$p_U = \frac{\kappa}{1 - \sigma(1 - \pi\rho\underline{\nu})}, \tag{11}$$

and her expected profits is:

$$\Pi(p_U) = k(1-\pi)\left[p_U - \kappa\right](p_U)^{-\frac{1}{\sigma}}\left[1 - \pi\rho\underline{\nu}\right],$$

When the firm chooses to set low prices, p_L, we have that:

$$p_L = \beta(1-\delta)p^a \tag{12}$$

and:

$$\Pi(p_L) = k(p_L - \kappa)\left[p_L^{-\frac{1}{\sigma}} + (\frac{1}{1-\delta} - \pi)\rho\underline{\nu}p_U^{-\frac{1}{\sigma}}\right].$$

Proof. See Appendix B.1. ■

Some comments are in order. First the low price is the minimal price at which consumers stock. This is due to the fact that a further lowering of prices would only affect stocks at the second order while affecting profits by a first order term. Second, the upper price, p_U, is NOT the price maximizing the corresponding profit function since there are dynamic externalities that imply that the price is lower than the static monopoly price, $\frac{\kappa}{1-\sigma}$. The second element of the low price profit function describes these dynamic externalities due to stocking behavior and the weighting of the upper profit function by the factor $(1-\pi)$ is another consequence.

In Definition 2, we defined a price dispersed stationary equilibrium as the set of (π, p_L, p_U) which solve equations (11), (12) and equalize profits at the two prices:

$$\Pi(p_L) = \Pi(p_U). \tag{13}$$

It is rather straightforward to show that all quantities are homogenous in prices and we can thus redefine the equilibrium in terms of the two quantities $(\pi, \frac{p_L}{p_U})$. We obtain the following proposition:

Proposition 4 *There exists a neighborhood of $(\beta, \delta) = (1, 0)$ in which there exists a mixed strategy equilibrium $(\pi, \frac{p_L}{p_U})$. It is defined by the two supply equations:*

$$(\frac{p_L}{p_U} - (1 - \sigma + \sigma\pi\rho\underline{\nu}))\left[(\frac{p_L}{p_U})^{-\frac{1}{\sigma}} + (\frac{1}{1-\delta} - \pi)\rho\underline{\nu}\right] = \sigma(1 - \pi\rho\underline{\nu})^2 \tag{14}$$

$$\frac{p_L}{p_U} = \beta(1-\delta)(\pi\frac{p_L}{p_U} + (1-\pi)) \tag{15}$$

Proof. See Appendix B.2 ■

Lemma 6 also leads to new expressions for demands. As prices in $\{p_L, p_U\}$ only are played, we have:

$$\begin{cases} x_t(p_U, p_U) = \alpha p_U^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}} \\ x_t(p_U, p_L) = \alpha p_L^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}} + i(p_L) \\ x_t(p_L, p_U) = \begin{cases} \alpha p_U^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}} - (1-\delta) i(p_L) & \text{if } \eta_t > \frac{\alpha p_U}{((1-\delta)i(p_L))^{-\sigma}} \\ 0 \quad \text{if } \eta_t \leq \frac{\alpha p_U}{((1-\delta)i(p_L))^{-\sigma}} \end{cases} \\ x_t(p_L, p_L) = \alpha p_L^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}} + \delta i(p_L). \end{cases}$$

Furthermore, using:

$$i(p_L) = \frac{\rho}{1-\delta} (\frac{\alpha p_U}{\underline{\eta}})^{-\frac{1}{\sigma}},$$

and noting that the condition for the third equation writes $\eta_t > \rho \underline{\eta}$ which is true by construction, the fourth equation never applies and we end up with:

$$\begin{cases} x_t(p_U, p_U) = u_t'^{-1}(\alpha p_U) = \alpha p_U^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}}, \\ x_t(p_U, p_L) = \alpha p_L^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}} + \frac{\rho}{1-\delta} (\frac{\alpha p_U}{\underline{\eta}})^{-\frac{1}{\sigma}}, \\ x_t(p_L, p_U) = \alpha p_U^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}} - \rho (\frac{\alpha p_U}{\underline{\eta}})^{-\frac{1}{\sigma}}, \\ x_t(p_L, p_L) = \alpha p_L^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}} + \delta \frac{\rho}{1-\delta} (\frac{\alpha p_U}{\underline{\eta}})^{-\frac{1}{\sigma}}. \end{cases} \tag{16}$$

3.2 Extension

The previous results make the assumption that consumers purchase at any period and this is far from true in the empirical application that we analyze below. This is why we now extend the set-up above to the case in which we allow consumers to have no tastes for the product in period t so that $\eta_t = 0$ with probability ω while with probability $1-\omega$ they have preferences $\eta_t > \underline{\eta}$ as above. The derivations in this case are very close to what was obtained before and we summarize these results as:

Proposition 5 *Denote* $\theta(\pi) = \frac{1}{1-\omega(1-\pi)(1-\delta)}$. *The upper price is now given by:*

$$p_U = \frac{\kappa}{1 - \sigma(1 - \pi\theta(\pi)\rho\underline{\nu})}.$$

and the two supply equations by:

$$(\frac{p_L}{p_U} - (1 - \sigma + \sigma\pi\theta(\pi)\rho\underline{\nu})) \left[(1-\omega)(\frac{p_L}{p_U})^{-\frac{1}{\sigma}} + (\frac{1}{1-\delta} - \pi\theta(\pi))\rho\underline{\nu} \right] = (1-\omega)\sigma(1 - \pi\rho\underline{\nu}\theta(\pi))^2$$

and

$$(1 - \beta(1-\delta)\omega(1-\pi))\frac{p_L}{p_U} = \beta(1-\delta)(\pi\frac{p_L}{p_U} + (1-\pi)(1-\omega)),$$

Proof. See Appendix B.3 ■

This is this form of the model to which we now turn and estimate using purchase data.

4 Empirical analysis

4.1 Data and Descriptive Statistics

The data on purchases we are using, are from the Kantar World Panel in France over the period January 2005 to December 2007. For each household we observe purchases of food items made and brought into the home. Those purchases are by definition made for future consumption either before the next purchase decision or after the next purchase in which case it means the consumer is stockpiling. We observe neither consumption nor stockpiling but purchases only. Very detailed information on each item purchased at the bar code level provides us with exact characteristics such as price, brand, pack size and the retailer chain where the item was purchased. We also have information on the demographic composition of households and their location. These data are similar to the US data used by Hendel and Nevo (2006a, 2006b).

We analyze the household purchase behavior of Coca Cola ("Coke") only, as it is the most popular soft drink in France. We first present descriptive statistics about the market for cola soft drinks. As shown in Table 1 below, Coke is by far the main brand for colas in France. In our sample, it represents almost 84% of sales value (averaged over 3 years), more than 66% of market shares in volume and almost 70% of purchases. Pepsi's market share is between 6% and 7% only.

Table 1: Cola Soft Drinks Market Shares

Manufacturer	Value Market Share	Quantity Market Share	Purchase Frequency
Coca-cola company	83.9%	66.2%	69.5%
Pepsico Inc.	6.3%	7.3%	6.0%
Lidl (Hard Discount Brand)	2.9%	9.7%	6.4%
Leclerc (Store Brand)	1.3%	2.8%	3.3%
Other brands	5.5%	14.0%	14.8%

During the three years of data, we observe the exhaustive purchases of soft drinks of 11 183 households. Looking at cola soft drinks purchases, as seen in Table 1, the two main brands are

Coca-Cola and Pepsi but Coca-Cola is much more dominant. Moreover, brand preference seems quite strong and very few consumers seem to substitute one with another. Actually, among the 11,183 households surveyed in these home scan data, 9,509 never bought any Pepsi while only 541 never bought any Coke, and only 1,496 households bought at least once of each brand during the three year period of study. If one considers shorter periods than the full three years of data, the overlap between Coke and Pepsi consumers is even smaller.

We define in our model below the standard purchase period as a week as most households purchase food items in supermarkets once a week. Looking at weekly purchases presented in Table 2, only 0.26% of households purchase both Coke and Pepsi during the same week while the purchase frequency of Coke only is 12.45% and the one for Pepsi only is 0.77%. Moreover, the transition probabilities between Coke and Pepsi in successive weeks reported in Table 2 show that very few consumers switch from Coke to Pepsi (0.06%) or from Pepsi to Coke (0.05%). Substitution between these two main brands of cola soft drinks seems very weak among French consumers.

Table 2: Brand Purchase Transition Matrix

Transitions			Week t		
Week $t-1$	None	Pepsi	Coke	Coke&Pepsi	Total
None	77.78%	0.60%	8.46%	0.11%	86.96%
Pepsi	0.62%	0.23%	0.05%	0.01%	0.91%
Coke	8.45%	0.05%	3.40%	0.04%	11.94%
Coke&Pepsi	0.12%	0.01%	0.04%	0.02%	0.19%
Total	86.97%	0.90%	11.95%	0.26%	100%

We thus consider purchases of Coke only as it seems a market by itself. Of course different packages exist with different pack sizes and different pack types (mostly cans or bottles). Bottles account for 78% of purchases and have quite a different average unit price. The mean price per liter of cans is almost 20% higher than the mean price of bottles and the distributions of price per liter of cans and bottles hardly overlap. The 75% percentile of the price of bottles of Coke is below the 5% percentile of the price of cans.

Table 3 reports the transition matrix between purchases of bottles or cans. The transition between purchases in cans and in bottle is very low. For example, if a can of coke was bought in week t, the probability to buy a bottle of Coke the following week is equal to 4.3% only while the probability of buying a can again is equal to 15%. The odds ratio of buying a can versus a bottle in t while buying a bottle in $t-1$ is very low (0.05). These descriptive statistics led us to consider that the markets for bottles and cans are different enough and for simplicity we do not consider cans in our demand model.

Table 3: Pack Type Transition Matrix of Coke weekly purchases

Transitions			Week t		
Week $t-1$	None	Bottle	Can	Bottle+Can	Total
None	75.60%	7.51%	2.56%	0.26%	85.93%
Bottle	7.51%	2.75%	0.14%	0.07%	10.47%
Can	2.57%	0.14%	0.48%	0.04%	3.21%
Bottle+Can	0.26%	0.06%	0.04%	0.03%	0.39%
Total	85.93%	10.46%	3.22%	0.39%	100.00%

As bottles of 1.5 liters and 2 liters have similar price distributions while bottles of 0.5 liter have much higher prices (even higher than cans) and a small market share, we also consider bottles of 1.5 liters and 2 liters only. They constitute the bulk of the market by accounting for 77% purchases and almost 88% of total volume sold in bottles.

We can now turn to descriptive statistics about households weekly purchases of bottles of 1.5 and 2 liters. Table 4 below reports summaries of the distribution of weekly quantities purchased (in liters) conditional on purchase among all households from 2005 to 2007. It shows that half of the sample bought less than two bottles of 1.5 liters although the distribution is very skewed to the right because the 75^{th} percentile is equal to 6 liters and the 90 percentile to almost 10 liters. Given that the frequency of purchase is equal to 15.7% over the period, it shows that purchases are very far from constant. This empirical fact seems to indicate that consumers don't buy for immediate consumption only and steers us towards modelling the stockpiling behavior of households.

Table 4: Descriptive Statistics

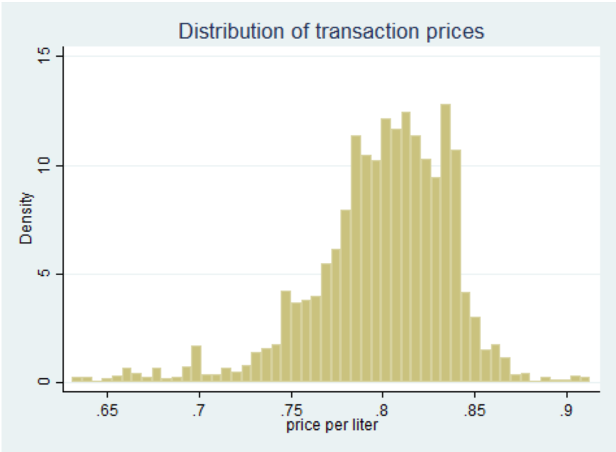
Quantile	1%	5%	10%	25%	50%	75%	90%	95%	99%
Quantity purchased (liters/week, if any purchase)	1.5	1.5	1.5	1.65	3	6	9.72	11.5	20
Price per liter	0.65	0.68	0.70	0.76	0.81	0.84	0.86	0.87	0.91
Price per liter[7]	0.66	0.73	0.75	0.78	0.81	0.83	0.85	0.86	0.88

As our data come from home scans, we exhaustively observe household purchases but of course, we don't observe prices when households do not purchase. From the model, we infer that prices are likely to be high when households do not purchase. For the sake of roughly correcting the distortion

[7] Prices are imputed in week t when the household does not purchase and the procedure is described in the text.

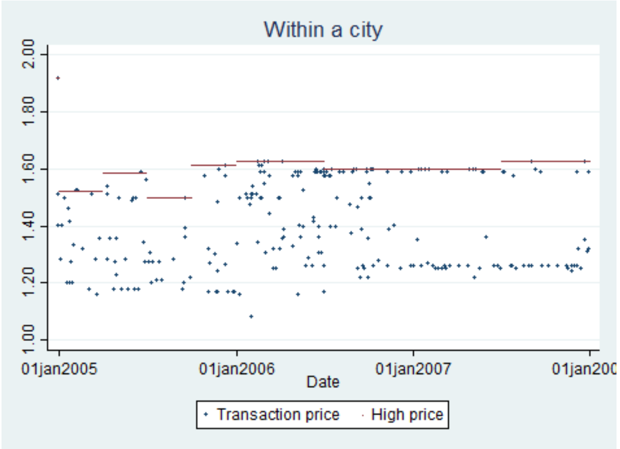
in the price distribution because of missing prices, we first implement an approximate imputation of unobserved prices although we will take into account this issue of unobservability, in a more sophisticated way, in our econometric estimates.

The imputation procedure goes as follows. We consider the 90^{th} percentile of observed transaction prices by municipality and week and we do an analysis-of-variance having additive week and municipality effects. Imputed prices when missing are equal to the resulting predicted prices. The next graph displays the distribution of prices. It has several modes that are partly due to regional variation.

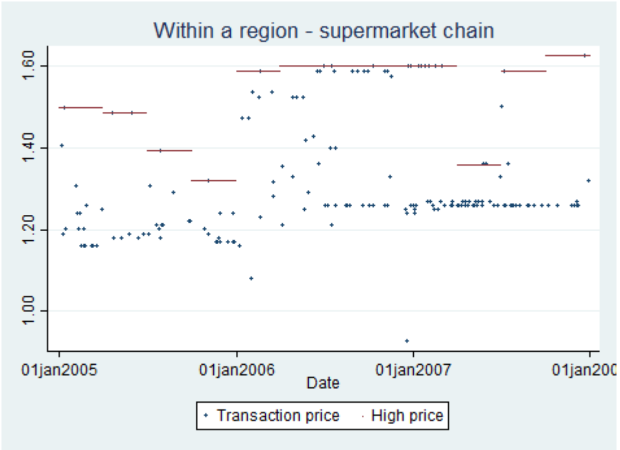


Graph 1: Distribution of prices per liter

Zooming in at the municipality, region or supermarket level, we find evidence of a discrete distribution of prices that seems to have a two-point support. Graph 2, displaying the price distribution in a given city, shows that the price distribution has several mass points that seem to change over time. These mass points correspond to different supermarket chains. However, looking at Graph 3 in which the price distribution over time within a given supermarket chain but across a whole region, there are less mass points but still more than two. This is evidence that supermarket chains also change the distribution of prices across their stores in different cities. This can reflect different demand characteristics but probably mainly different cost shifters correlated with the location of stores.

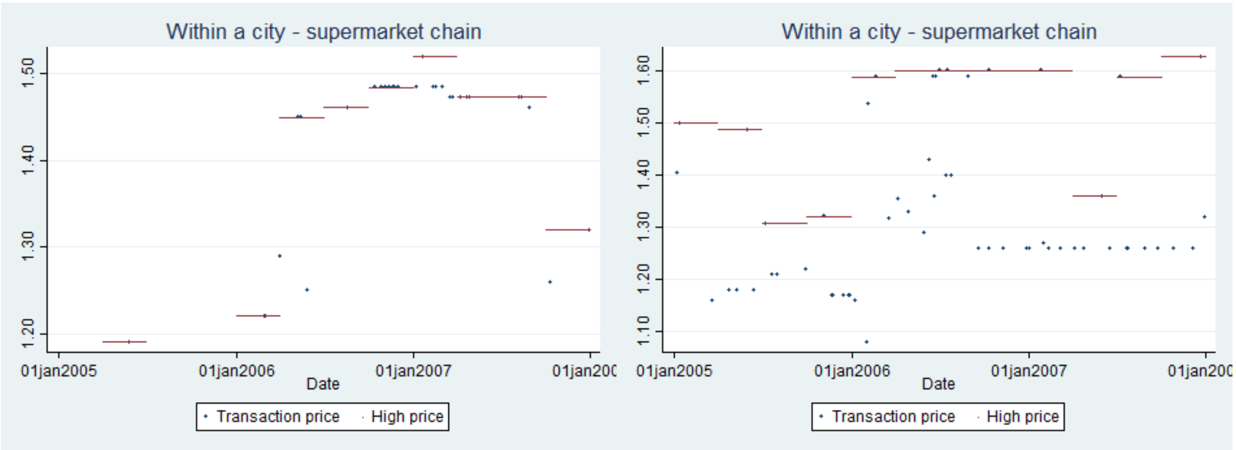


Graph 2: Transaction prices and imputed high prices within a city



Graph 3: Transaction prices and imputed high prices within a region - supermarket chain

Finally, Graph 4 displays the distribution of prices for two supermarket chains and cities. It shows that the distribution of prices has fewer mass points and restricting to few months periods, gives evidence of a two point pattern. This evidence is consistent with our model showing that for a given cost firms choose optimally two points of support for the price of Coke.



Graph 4: Transaction prices and imputed high prices within a city - supermarket chain for two chains

For our first descriptive purpose, we look at the transition matrix of prices between two successive weeks and at the conditional purchase behavior of households. Table 5 reports the transition matrix by deciles of deviations from the city-week average of prices. While the diagonal of the transition matrix by deciles somewhat dominates off-diagonal cells, it is far from being a diagonal matrix.

Table 5: Transition matrix of price deciles

Price Decile	Price Decile (week t)									
(week $t-1$)	1	2	3	4	5	6	7	8	9	10
1	2.25	1.11	0.81	0.68	0.75	0.67	0.71	0.92	0.92	1.05
2	1.16	2.07	1.00	0.63	0.60	0.64	0.61	1.07	1.20	1.07
3	0.79	0.93	1.33	1.06	1.03	1.01	1.04	1.16	0.90	0.79
4	0.69	0.66	1.02	1.56	1.40	1.43	1.45	0.56	0.56	0.68
5	0.72	0.67	0.97	1.46	1.54	1.44	1.45	0.60	0.53	0.64
6	0.68	0.68	1.03	1.43	1.42	1.56	1.43	0.60	0.56	0.65
7	0.70	0.61	1.04	1.43	1.48	1.42	1.53	0.63	0.51	0.69
8	0.88	1.13	1.15	0.60	0.58	0.63	0.61	2.10	1.43	0.90
9	0.97	1.15	0.90	0.52	0.57	0.54	0.53	1.35	2.09	1.37
10	1.04	1.02	0.80	0.66	0.67	0.68	0.67	1.02	1.30	2.09

Note: Each cell denotes the percentage probability of the transition between the row decile to the column decile of deviations of prices from their city-week average.

We then analyze the purchase decisions of households depending on the level of prices in the low and high states. For the sake of robustness, we use various definitions. First, we define the high (respectively low) state as observing a price above (resp. below) the 90^{th} percentile that the household is ever paying during the three years of data (definition (i) in Table 6). Second, the high state can be defined as when the price paid is higher than the 95^{th} percentile of prices by city (definition (ii) in Table 6). Finally, we use a third definition where after regressing log prices on week fixed effects, we define the high state as a price above the predicted average price plus 0.77%[8] (definition (iii) in Table 6).

[8] This parameter denoted χ is estimated by our econometric procedure presented below and leads to a 87% probability that the price is high in a given week.

Table 6: Transition matrix of prices and conditional purchases (with two consecutive purchases)

		Current state			
State	Lagged	Low		High	
definition	state	Quantity	Obs.	Quantity	Obs.
(i)					
	Low	5.14	27,890	4.12	9,697
	High	5.29	2,657	3.74	2,715
(ii)					
	Low	4.96	38,482	3.57	2,762
	High	4.93	586	3.36	1,129
(iii)					
	Low	4.73	25,381	4.49	9,626
	High	6.20	4,138	4.88	3,854

Notes: Quantity is in liters per week. The quantity column is the average quantity purchased in liters.

Table 6 is obtained on the selected sample of households-weeks where the household purchased in two consecutive weeks. Interestingly, we can see that the quantity purchased in case of low state is higher than in the case of high state whatever the past state and moreover that it is higher if the past state was high than when the past state was low. This is consistent with the prediction that households need to replete their stocks.

4.2 Econometric estimation

We start by adding measurement errors, $\exp(\xi_t)$, in the system of purchases described by equation (16):

$$\begin{cases} x_t\,(p_U, p_U) = \alpha p_U^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}} \exp(\xi_t), \\ x_t\,(p_U, p_L) = (\alpha p_L^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}} + \frac{\rho}{1-\delta} (\frac{\alpha p_U}{\underline{\eta}})^{-\frac{1}{\sigma}}) \exp(\xi_t), \\ x_t\,(p_L, p_L) = (\alpha p_L^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}} + \delta \frac{\rho}{1-\delta} (\frac{\alpha p_U}{\underline{\eta}})^{-\frac{1}{\sigma}}) \exp(\xi_t), \\ x_t\,(p_L, p_U) = (\alpha p_U^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}} - \rho (\frac{\alpha p_U}{\underline{\eta}})^{-\frac{1}{\sigma}}) \exp(\xi_t). \end{cases}$$

We also log-linearize this system of equations and the result is described as:

Lemma 7 *Setting* $\underline{\nu} = \frac{\underline{\eta}^{\frac{1}{\sigma}}}{E(\eta_t^{\frac{1}{\sigma}})}$, *and* $\widetilde{\alpha} = \frac{-\ln \alpha}{\sigma} + \ln(E(\eta_t^{\frac{1}{\sigma}}))$, *we obtain*

$$\begin{cases} \ln x_t\,(p_U, p_U) = \widetilde{\alpha} - \frac{1}{\sigma} \ln p_U + \epsilon_t^{(UU)}, \\ \ln x_t\,(p_U, p_L) = \widetilde{\alpha} - \frac{1}{\sigma} \ln p_L + \ln \left[1 + \frac{1}{1-\delta} (\frac{p_L}{p_U})^{\frac{1}{\sigma}} \rho \underline{\nu}\right] + \epsilon_t^{(UL)}, \\ \ln x_t\,(p_L, p_L) = \widetilde{\alpha} - \frac{1}{\sigma} \ln p_L + \ln \left[1 + \frac{\delta}{1-\delta} (\frac{p_L}{p_U})^{\frac{1}{\sigma}} \rho \underline{\nu}\right] + \epsilon_t^{(LL)}, \\ \ln x_t\,(p_L, p_U) = \widetilde{\alpha} - \frac{1}{\sigma} \ln p_U + \ln(1 - \rho \underline{\nu}) + \epsilon_t^{(LU)}. \end{cases}$$

in which $\epsilon_t^{(\cdot)}$ are defined in the proof.

Proof. See Appendix C.1. ■

4.2.1 Using demand equations

We consider the extension of our model with zero taste shocks defined in Section 3.2. As determined in equations (B.38) and (B.39) in the Appendix, it allows us to obtain the following log linearized demand system for households purchases that we now index by h and t, and where arguments are the history of prices from current week to the last previous week with purchase and observed price:[9]

$$
\begin{aligned}
\ln x_{ht}\left(p_U, p^m, ., p^m, p_U\right) &= \begin{cases} \widetilde{\alpha}_h - \frac{1}{\sigma}\ln p_U + \epsilon_{ht}^{(UU)}, & \text{w.p. } 1-\omega \\ -\infty & \text{w.p. } \omega \end{cases} \\
\ln x_{ht}\left(p_U, p^m, ., p^m, p_L\right) &= \begin{cases} \widetilde{\alpha}_h - \frac{1}{\sigma}\ln p_L + \ln\left[1 + \frac{1}{1-\delta}(\frac{p_L}{p_U})^{\frac{1}{\sigma}}\rho\underline{\nu}\right] + \epsilon_{ht}^{(UL)}, & \text{w.p. } 1-\omega \\ \widetilde{\alpha}_h + \ln\left[\frac{1}{1-\delta}p_U^{-\frac{1}{\sigma}}\rho\underline{\nu}\right] + \xi_{ht}, & \text{w.p. } \omega \end{cases} \\
\ln x_{ht}\left(p_L, p^m, ., p^m, p_L\right) &= \begin{cases} \widetilde{\alpha}_h - \frac{1}{\sigma}\ln p_L + \ln\left[1 + \frac{(1-(1-\delta)^{m+1})}{1-\delta}(\frac{p_L}{p_U})^{\frac{1}{\sigma}}\rho\underline{\nu}\right] + \epsilon_{ht}^{(LL)} & \text{w.p. } 1-\omega \\ \widetilde{\alpha}_h + \ln\left[\frac{(1-(1-\delta)^{m+1})}{1-\delta}p_U^{-\frac{1}{\sigma}}\rho\underline{\nu}\right] + \xi_{ht}, & \text{w.p. } \omega \end{cases} \\
\ln x_{ht}\left(p_L, p^m, ., p^m, p_U\right) &= \begin{cases} \widetilde{\alpha}_h - \frac{1}{\sigma}\ln p_U + \ln(1-(1-\delta)^m\rho\underline{\nu}) + \epsilon_{ht}^{(LU)}, & \text{w.p. } 1-\omega \\ -\infty & \text{w.p. } \omega \end{cases}
\end{aligned}
$$

in which p^m denotes a period in which the consumer is not purchasing and in consequence in which the price is missing. Variable m denotes the number of such missing weeks (i.e., $m = 0$ means the consumer purchased two consecutive weeks). Moreover, denoting τ_t at period t the duration in weeks elapsed since consumer's last purchase, we have $\tau_t = m + 1$ if there are $m + 1$ weeks since when the consumer purchased and thus m weeks for which we have some missing price information at period $t-1, .., t-m$.

[9] in which w.p. is a shortcut for "with probability"

Then, using that $\epsilon_{ht}^{(UU)}$, $\epsilon_{ht}^{(UL)}$, $\epsilon_{ht}^{(LL)}$, $\epsilon_{ht}^{(LU)}$ are uncorrelated with prices and preference shocks as shown in the proof of Lemma 7, the previous demand equations yield:

$$\begin{aligned}
E\left[\ln x_{ht}|p_{t-\tau_t}=p_U,\tau_t=m+1,p_t=p_U,x_t>0\right] &= \widetilde{\alpha}_h-\frac{1}{\sigma}\ln p_U \\
E\left[\ln x_{ht}|p_{t-\tau_t}=p_U,\tau_t=m+1,p_t=p_L\right] &= \widetilde{\alpha}_h+\alpha_{UL}-\frac{(1-\omega)}{\sigma}\ln p_L \\
E\left[\ln x_{ht}|p_{t-\tau_t}=p_L,\tau_t=m+1,p_t=p_L\right] &= \widetilde{\alpha}_h+\alpha_{LL}^m-\frac{(1-\omega)}{\sigma}\ln p_L \\
E\left[\ln x_{ht}|p_{t-\tau_t}=p_L,\tau_t=m+1,p_t=p_U,x_t>0\right] &= \widetilde{\alpha}_h+\alpha_{LU}^m-\frac{1}{\sigma}\ln p_U
\end{aligned} \tag{17}$$

in which

$$\begin{aligned}
\alpha_{UL} &= (1-\omega)\ln\left[1+\frac{1}{1-\delta}(\frac{p_L}{p_U})^{\frac{1}{\sigma}}\rho\underline{\nu}\right]+\omega\ln\left[\frac{1}{1-\delta}p_U^{-\frac{1}{\sigma}}\rho\underline{\nu}\right] \\
\alpha_{LL}^m &= (1-\omega)\ln\left[1+\frac{(1-(1-\delta)^{m+1})}{1-\delta}(\frac{p_L}{p_U})^{\frac{1}{\sigma}}\rho\underline{\nu}\right]+\omega\ln\left[\frac{(1-(1-\delta)^{m+1})}{1-\delta}p_U^{-\frac{1}{\sigma}}\rho\underline{\nu}\right] \\
\alpha_{LU}^m &= \ln(1-(1-\delta)^m\rho\underline{\nu})
\end{aligned}$$

In these demand functions, α_{UL} represents the average excess purchase in case of low price versus high price if the last purchase was made in the high price regime, whenever it was done. α_{LL}^m represents the average excess purchase, compared to the case when current and last purchases were done in the high price regime, if the price is low and the last purchase was done m weeks before in a low price regime. α_{LU}^m is negative and represents the average reduced purchase when the current price if high, if the last purchase was done m weeks before in a low price regime rather than made any time before under the high price regime.

Remark that this demand model allows to identify σ, ω and (over)identify δ and $\rho\underline{\nu}$ using

$$\begin{aligned}
\delta &= 1-\frac{1-\exp\left(\alpha_{LU}^{m+1}\right)}{1-\exp\left(\alpha_{LU}^m\right)} \\
\rho\underline{\nu} &= \frac{\left(1-\exp\left(\alpha_{LU}^m\right)\right)^{m+1}}{\left(1-\exp\left(\alpha_{LU}^{m+1}\right)\right)^m}
\end{aligned}$$

for all m.

Defining the high state as in the definition (i) in Table 6, that is when the price paid by the household is larger than the 90^{th} percentile of prices this household paid over the whole period, we can estimate the above demand model by linear regression with household fixed effects and identify some of the parameters of the model. Results of this estimation are presented in Table 7 in Appendix C.2. Using definition (*ii*) or (*iii*) of Table 6 brings similar results. In column (1) we estimate the model without taking into account missing prices in previous periods but only conditional on the current and lagged states (as predicted by our model as if households were purchasing at every period). In this Table 7, the estimated price coefficient in the low price regime is larger in absolute value than the high price estimate while the model predicts that it should be lower or equal. Moreover, $\widehat{\alpha_{UL}}$ is negative while it should be positive and $\widehat{\alpha_{LU}^0}$ is positive while it should be negative. However, when we estimate the above demand model allowing for a maximum of M periods at which households do not purchase so that using M maximum lags in purchase history is sufficient, varying M in 1, 2, 5, 10, 20 as shown in columns (2) to (6), we then obtain that the low price coefficient estimate is lower in absolute value than the high price one and that $\widehat{\alpha_{UL}}$ is positive.

While this first estimation procedure does not use supply side equilibrium conditions and also uses a crude definition of high prices states, we obtain estimates of most parameters that do not reject the model. Indeed we cannot reject that $\sigma \in (0,1)$ with estimates between 0.3 and 0.5, as well as $\omega \in (0,1)$ with estimates around 0.08-0.10 (meaning that consumers may have no taste for the product in around 8-10% of weeks). Other parameters are not precisely estimated. However, when significant, we still have positive coefficients $\widehat{\alpha_{LU}^m}$ while the model predicts they should be negative.

We now extend the estimation in order to take into account equilibrium conditions and imperfect observability of price regimes.

4.2.2 Using demand and equilibrium conditions

We now use the equilibrium conditions on firms' profits in addition to demand equations. We also allow for imperfect observability of states as follows. We define a model of prices such that for each household, the price is high or low according to the following regimes:

$$\begin{cases} \text{Regime } L \text{ if} & \log p_{ht} \leq \chi + \log \chi_h \\ \text{Regime } U \text{ if} & \log p_{ht} > \chi + \log \chi_h \end{cases}$$

in which χ is a threshold parameter common to all consumers and $\log \chi_h$ is simply the mean log price paid by household h during the whole time period allowing variation in the environments faced by each household. Then, for any value of the threshold parameter, χ, we can define high and low states according to whether p_{ht} is below or above the threshold $s_h(\chi) \equiv \chi_h \exp(\chi)$.

From the supply side, we know by Proposition 5 that the mixed strategy equilibrium of firms implies that expected profits in case of low and high prices should be equal. Denoting $\phi = \frac{p_L}{p_U}$, this implies the following constraint

$$\phi - (1 - \sigma + \sigma\pi\theta\rho\underline{\nu})) \left[(1-\omega)\phi^{-\frac{1}{\sigma}} + (\frac{1}{1-\delta} - \pi\theta)\rho\underline{\nu} \right] = (1-\omega)\sigma(1 - \pi\rho\underline{\nu}\theta)^2$$

in which

$$\theta = \frac{1}{1 - \omega(1-\pi)(1-\delta)}$$

Moreover, we must also have that low prices are equal to marginal value of stocks such that

$$(1 - \beta(1-\delta)\omega(1-\pi))\phi = \beta(1-\delta)(\pi\phi + (1-\pi)(1-\omega))$$

or equivalently

$$\pi = \frac{1 - \omega + \omega\phi - \frac{\phi}{\beta(1-\delta)}}{1 - \omega + \omega\phi - \phi}$$

Finally, there is also a constraint on $\underline{\eta}$ that we do not use as it depends on the distribution of preferences through function $f(.)$.

Using notation τ_t for the duration in weeks defined in the previous section, conditional on the parameter χ, demand equations derived from equation (17) are

$$\begin{aligned} \ln x_{ht} &= \widetilde{\alpha}_h - \frac{1}{\sigma} \ln p_{ht} 1_{\{p_{ht} > s_h(\chi)\}} - \frac{1-\omega}{\sigma} \ln p_{ht} 1_{\{p_{ht} < s_h(\chi)\}} + \alpha_{UL} 1_{\{p_{ht} > s_h(\chi), p_{ht-\tau_{ht}} < s_h(\chi)\}} \\ &+ \sum_{m=1,..,M} \alpha_{LL}^m 1_{\{p_{ht} < s_h(\chi), p_{ht-m} < s_h(\chi), \tau_{ht} = m\}} + \sum_{m=1,..,M} \alpha_{LU}^m 1_{\{p_{ht} > s_h(\chi), p_{ht-m} < s_h(\chi), \tau_{ht} = m\}} + \varepsilon_{ht} \end{aligned} \quad (18)$$

whose within estimation delivers the following estimates

$$\widehat{\widetilde{\alpha}_h}(\chi), \widehat{\alpha_{UL}}(\chi), \widehat{\alpha_{LL}^m}(\chi), \widehat{\alpha_{LU}^m}(\chi), \widehat{\left(\frac{1-\omega}{\sigma}\right)}(\chi), \widehat{\left(\frac{1}{\sigma}\right)}(\chi)$$

We then estimate parameters $(\sigma, \delta, \beta, \chi, \phi, (\rho\underline{\nu}), \omega)$ by GMM using the following moments conditions

$$E\left(\widehat{\alpha_{UL}}(\chi) - (1-\omega)\ln(1+\frac{1}{1-\delta}\phi^{\frac{1}{\sigma}}\rho\underline{\nu}) - \omega\ln(\frac{1}{1-\delta}(\widehat{p_U}(\chi))^{-\frac{1}{\sigma}}\rho\underline{\nu})\right) = 0 \quad \text{(M1)}$$

$$E\left(\widehat{\alpha_{LL}^m}(\chi) \text{-} (1\text{-}\omega)\ln[1+\frac{(1\text{-}(1\text{-}\delta)^{m+1}}{1-\delta}\phi^{\frac{1}{\sigma}}\rho\underline{\nu}]\text{-}\omega\ln(\frac{(1\text{-}(1\text{-}\delta)^{m+1}}{1-\delta}(\widehat{p_U}(\chi))^{-\frac{1}{\sigma}}\rho\underline{\nu})\right) = 0 \quad \text{(M2)}$$

$$E\left(\widehat{\alpha_{LU}^m}(\chi) - \ln(1-(1-\delta)^m\rho\underline{\nu})\right) = 0 \quad \text{(M3)}$$

$$E\left((1\text{-}\omega)\sigma(1\text{-}\widehat{\pi}(\chi)\rho\underline{\nu}\theta)^2\text{-}\phi\text{+}(1\text{-}\sigma\text{+}\sigma\widehat{\pi}(\chi)\theta\rho\underline{\nu}))[(1\text{-}\omega)\phi^{-\frac{1}{\sigma}}+(\frac{1}{1\text{-}\delta}\text{-}\widehat{\pi}(\chi)\theta)\rho\underline{\nu}]\right) = 0 \quad \text{(M4)}$$

$$E\left((1\text{-}\beta(1\text{-}\delta)\omega(1-\widehat{\pi}(\chi)))\phi - \beta(1\text{-}\delta)(\widehat{\pi}(\chi)\phi + (1-\widehat{\pi}(\chi))(1\text{-}\omega))\right) = 0 \quad \text{(M5)}$$

$$E\left(\frac{1}{HT}\sum\nolimits_{h,t}1_{\{x_{ht}=0\}} - (1-\widehat{\pi}(\chi))\omega\right) = 0 \quad \text{(M6)}$$

$$E\left(\left(\widehat{\left(\frac{1-\omega}{\sigma}\right)}(\chi)\right)/\left(\widehat{\left(\frac{1}{\sigma}\right)}(\chi)\right) - (1-\omega)\right) = 0 \quad \text{(M7)}$$

$$E\left(\left(1-\exp\left(\widehat{\alpha_{LU}^{m+1}}(\chi)\right)\right)/\left(1-\exp\left(\widehat{\alpha_{LU}^m}(\chi)\right)\right) - (1-\delta)\right) = 0 \quad \text{(M8)}$$

$$E\left(\left(\left(1-\exp\left(\widehat{\alpha_{LU}^m}(\chi)\right)\right)^{m+1}\right)/\left((1-\exp\left(\widehat{\alpha_{LU}^{m+1}}(\chi)\right))^m\right) - (\rho\underline{\nu})\right) = 0 \quad \text{(M9)}$$

for $m = 1, .., M$ and in which:

$$\widehat{\pi}(\chi) \equiv \frac{1}{HT}\sum\nolimits_{h,t}1_{\{p_{ht}\leq s_h(\chi)\}}$$

$$\widehat{p_U}(\chi) = \frac{\sum_{h,t}1_{\{p_{ht}>s_h(\chi)\}}p_{ht}}{\sum_{h,t}1_{\{p_{ht}>s_h(\chi)\}}}$$

Moments (M1) to (M3) correspond to equilibrium conditions relating demand parameters to the empirical mean price in the high price regime. Moment (M4) relates the mixed strategy equilibrium condition to the empirical probability of low price regime. Moment (M5) relates the marginal value of stocks that depends on the empirical probability of low price regime and other parameters. Moment (M6) relates the empirical probability of low price regime and the probability of low preference shock to the empirical probability of purchase. Moment (M7), (M8), (M9) imposes theoretical restrictions across parameters of the demand function.

We perform the estimation with varying values of lags M from 1 to 3 in the demand function and also use different moment conditions. We always use moments (M1) to (M6) and then add moments (M7), (M8), (M9) as shown in Tables 8 and 9. The results show that the preference parameter σ is precisely estimated between 0.34 and 0.46 according to the number of lags allowed and the moments used. Estimates of the parameter β are precise and show that β is between 0.89 and 1. The parameter ω is also precisely estimated, always in the range of 0.18-0.51 meaning that consumers have no taste for the product between 18% and 51% of weeks. The iceberg cost parameter δ is also precisely estimated, varying across specifications between 0.07 and 0.40. The parameter $\rho\underline{\nu}$ is also precisely estimated as well as the threshold parameter χ. Finally, the parameter ϕ that is the ratio of the low price to the high price is a bit less well estimated and varies between 0.45 and 0.58 with standard errors that vary across specifications. Then, using the estimated parameters, one can compute the equilibrium probability of sales (low price), which varies across specifications between 9% and 22%.

Table 8: GMM estimation results

Moments used	$M1-M6$	$M1-M6$	$M1-M6$	$M1-M7$	$M1-M7$	$M1-M7$
Lags M	1	2	3	1	2	3
Parameters						
σ	0.4667	0.4054	0.4305	0.4018	0.3433	0.3963
	0.0000	0.0022	0.0007	0.0007	0.0019	0.0017
δ	0.2727	0.2065	0.2654	0.0708	0.1515	0.2742
	0.0000	0.0020	0.0009	0.0013	0.0028	0.0025
$\rho\underline{\nu}$	0.2118	0.2118	0.2194	0.1987	0.2101	0.1945
	0.0000	0.0003	0.0003	0.0002	0.0012	0.0009
$\phi = \frac{p_L}{p_U}$	0.4565	0.4961	0.4623	0.5437	0.5565	0.5018
	0.0000	1.1538	5.6458	0.0221	0.6567	1.3014
β	0.9870	0.9975	0.9994	0.8907	0.9907	0.9910
	0.0000	0.0015	0.0014	0.0015	0.0044	0.0044
χ	0.0717	0.0377	0.0522	0.0485	0.0144	0.0276
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ω	0.4703	0.5129	0.5003	0.4782	0.5043	0.4033
	0.0000	0.0011	0.0008	0.0007	0.0033	0.0021
N	29,508	43,431	50,929	29,508	43,431	50,929
$P(p_{ht} < s_h(\widehat{\chi}))$	0.0908	0.1484	0.1181	0.1281	0.2242	0.1717
$P(x_{ht} = 0)$	0.4718	0.5091	0.5091	0.4718	0.5091	0.5091

Note: Standard errors under parameters estimates.

Table 9: GMM estimation results

GMM estimation results						
Moments used	$M1-M8$	$M1-M8$	$M1-M8$	$M1-M9$	$M1-M9$	$M1-M9$
Lags M	1	2	3	1	2	3
Parameters						
σ	0.4018	0.4170	0.3893	0.4018	0.4134	0.4121
	0.0007	0.0014	0.0018	0.0007	0.0015	0.0021
δ	0.0708	0.2191	0.3356	0.0708	0.2422	0.4017
	0.0013	0.0029	0.0015	0.0013	0.0033	0.0016
$\rho\underline{\nu}$	0.1987	0.1940	0.1655	0.1987	0.1842	0.1274
	0.0002	0.0008	0.0014	0.0002	0.0008	0.0013
$\phi = \frac{p_L}{p_U}$	0.5437	0.5144	0.5839	0.5437	0.4898	0.5423
	0.0221	0.2504	0.0568	0.0221	0.2871	0.0781
β	0.8907	0.9646	1.0000	0.8907	0.9582	1.0000
	0.0015	0.0040	0.0000	0.0015	0.0048	0.0000
χ	0.0485	0.0374	0.0353	0.0485	0.0374	0.0381
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ω	0.4782	0.4194	0.1885	0.4782	0.4132	0.1753
	0.0007	0.0015	0.0016	0.0007	0.0016	0.0017
N	29,508	43,431	50,929	29,508	43,431	50,929
$P(p_{ht} < s_h(\widehat{\chi}))$	0.1281	0.1492	0.1517	0.1281	0.1492	0.1450
$P(x_{ht} = 0)$	0.4718	0.5091	0.5091	0.4718	0.5091	0.5091

Note: Standard errors under parameters estimates.

In appendix, Tables 10 and 11 provide the results of the same estimation on the demand for "Orangina" soft drink, while tables 12 and 13 show the results on Pepsi. In both case, we can see consistent estimates of the model parameters, showing that the model is not only fitting well the demand for Coke.

5 Conclusion

We constructed a tractable structural dynamic model of consumption, purchase and stocks by consumers for whom stockpiling is unobserved, preferences are isolastic and affected by independent and identically distributed shocks. Consumers purchase in stores which they meet randomly and which are supposed to maximize short run profits. We show that a two-price mixed strategy by stores satisfies conditions for an equilibrium in which consumers and stores coordinate their expectations on this stationary solution. We derive a simple and tractable estimation method using log linearized demand equations and equilibrium conditions. We estimate parameters using scanner data registering soda purchases by French consumers during 2005-2007.

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A Proofs of Section 2

A.1 Proof of Proposition 1

Whether multiplier ψ_t is equal to zero or not defines two cases which are further subdivided into two sub-cases according to whether the other multiplier μ_t is equal to zero or not :

1. Let $\psi_t = 0$: then $x_t^* \geq 0$, and by equation (4), $u_t'(c_t^*) = \alpha p_t = \lambda_t + \mu_t$. Define two further sub-cases:

   (a) Let $\lambda_t(0) < \alpha p_t$: as λ_t is non-increasing in i_t, it implies that $\mu_t > 0$ and $i_t^* = 0$. Therefore:

   $$x_t^* = c_t^* - (1-\delta)i_{t-1} = u_t'^{-1}(\alpha p_t) - (1-\delta)i_{t-1}.$$

   Compliance with the condition $\psi_t = 0$ applies if $u_t'^{-1}(\alpha p_t) - (1-\delta)i_{t-1} \geq 0$. This condition implies $(1-\delta)i_{t-1} \leq u_t'^{-1}(\alpha p_t)$ and therefore $u_t'((1-\delta)i_{t-1}) \geq \alpha p_t$. These conditions and results are summarized in regime (I.a).

   (b) Let $\lambda_t(0) \geq \alpha p_t$: as $\mu_t \geq 0$ and λ_t is non-increasing in i_t, it implies that $i_t^* \geq 0$ and $\mu_t = 0$, and that i_t^* is the (possibly non unique) solution to

   $$\lambda_t(i_t^*) = \alpha p_t,$$

   which exists as λ_t is continuous and $\lambda_t(+\infty) = 0$. Thus, the solution is:

   $$x_t^* = c_t^* + i_t^* - (1-\delta)i_{t-1} = u_t'^{-1}(\alpha p_t) + i_t^* - (1-\delta)i_{t-1}$$

   To comply with the condition, $\psi_t = 0$, we have $x_t^* \geq 0$ and therefore:

   $$i_t^* \geq (1-\delta)i_{t-1} - u_t'^{-1}(\alpha p_t) \Longrightarrow \alpha p_t \leq \lambda_t(\max\left\{(1-\delta)i_{t-1} - u_t'^{-1}(\alpha p_t), 0\right\}).$$

   These conditions and results are summarized in regime (I.b).

2. If $\psi_t > 0$: therefore, $x_t^* = 0$, and by equation (4), $u_t'(c_t^*) = \alpha p_t - \psi_t < \alpha p_t$. The problem to solve is now single dimensional:

   $$u_t'(c_t^*) = \lambda_t(i_t^*) + \mu_t \quad , c_t^* = (1-\delta)i_{t-1}^* - i_t^*.$$

   which implies

   $$u_t'((1-\delta)i_{t-1} - i_t^*) = \lambda_t(i_t^*) + \mu_t$$

   There are again two sub-cases:

(a) Let $\lambda_t(0) < u'_t\left((1-\delta)i_{t-1}\right)$: because $\lambda_t(i_t)$ is non-increasing, $\mu_t > 0$ and $i_t^* = 0$, $c_t^* = (1-\delta)i_{t-1}$. The condition that $\psi_t > 0$ is satisfied if $u'_t\left((1-\delta)i_{t-1}\right) < \alpha p_t$. These conditions and results are summarized in regime (II.a)

(b) Let $\lambda_t(0) \geq u'_t\left((1-\delta)i_{t-1}\right)$: then $\mu_t = 0$. Define $i_t^* < (1-\delta)i_{t-1}$ as the value satisfying:

$$u'_t\left((1-\delta)i_{t-1} - i_t^*\right) = \lambda_t(i_t^*) \tag{A.19}$$

which exists and is unique because, when i_t^* increases from zero to $(1-\delta)i_{t-1}$, the left hand side increases between $u'_t\left((1-\delta)i_{t-1}\right)$ and $u'_t(0) = +\infty$ (if $\eta_t \geq \underline{\eta} > 0$) and the right hand side is non-increasing from $\lambda_t(0)$ to zero. Then

$$c_t^* = (1-\delta)i_{t-1} - i_t^*.$$

The condition that $\psi_t > 0$ is satisfied is $u'_t(c_t^*) < \alpha p_t$. This happens when $i_t^* = (1-\delta)i_{t-1} - c_t^* < (1-\delta)i_{t-1} - u_t'^{-1}(\alpha p_t)$ and therefore when:

$$\lambda_t(0) \geq \lambda_t(i_t^*) > \lambda_t(\max\left\{(1-\delta)i_{t-1} - u_t'^{-1}(\alpha p_t), 0\right\}).$$

A.2 Proof of Proposition 2

Using the condition on δ and Lemma 2, we have:

$$\lambda(0) < \alpha p_t \text{ if } p_t \in \{p_L, p_U\}.$$

Assume first that (i) i_{t-1} is such that $(1-\delta)i_{t-1} \leq u_t'^{-1}(\alpha p_t) = (\alpha p_t/\eta_t)^{-\frac{1}{\sigma}}$. Then we are in the static case (I.a) and $i_t = 0$, $c_t = u_t'^{-1}(\alpha p_t) = (\alpha p_t/\eta_t)^{-\frac{1}{\sigma}}$, $x_t = (\alpha p_t/\eta_t)^{-\frac{1}{\sigma}} - (1-\delta)i_{t-1}$. Then, at period $t+1$, $(1-\delta)i_t = 0 \leq u_t'^{-1}(\alpha p_{t+1})$, implying that $i_{t+1} = 0$, $c_{t+1} = u_t'^{-1}(\alpha p_{t+1})$, $x_{t+1} = u_t'^{-1}(\alpha p_{t+1})$ and this also applies to every subsequent period.

Assume second that (*ii*) $u_t'^{-1}(\alpha p_t) = (\alpha p_t/\eta_t)^{-\frac{1}{\sigma}} < (1-\delta)i_{t-1} \leq u_t'^{-1}(\lambda(0))$. This is static case (II.a) and $i_t = 0$, $c_t = (1-\delta)i_{t-1}$, $x_t = 0$. Therefore, at time $t+1$, we are in case (i) above and $i_{t+1} = 0$ as well as for any period after $t+1$.

Assume finally that (iii) $(1-\delta)i_{t-1} > u_t'^{-1}(\lambda(0)) = (\lambda(0)/\eta_t)^{-\frac{1}{\sigma}}$. This is static case (II.b). Then $i_t(i_{t-1})$ is defined by equation (A.19) and this implies that $i_t(i_{t-1}) < (1-\delta)i_{t-1}$. The probability of this case (iii) is equal to 0 if $i_{t-1} = 0$ since $\lambda(0) > 0$ and $\eta_t > 0$ and if $i_{t-1} \neq 0$:

$$\Pr\{(1-\delta)i_{t-1} > (\lambda(0)/\eta_t)^{-\frac{1}{\sigma}}\} = \Pr\{\frac{((1-\delta)i_{t-1})^{-\sigma}}{\lambda(0)} > \eta_t\}.$$

As $i_t = 0$ (cases (i) and (ii)) or $i_t < (1-\delta)i_{t-1}$, this last probability converges to zero when t tends to ∞ whatever the initial level of stocks is. Therefore, stocks i_t converge in probability towards 0 and the unique stationary solution is given as in the Proposition.

A.3 Proof of Proposition 3

The proof proceeds in several steps. As preliminaries, define $i(p_t)$ the non-increasing function such that:

$$\lambda(i(p_t)) = \alpha p_t,$$

and more generally, if $\lambda()$ is constant over some range:

$$i(p_t) = \lambda^{-1}(\alpha p_t) \in \{i : \lambda(i) = \alpha p_t\}.$$

The selection of any particular value has no importance at the moment and this issue will be solved later on.

As $i(p_t)$ is non increasing and the price is bounded from below by the marginal cost κ, we have:

$$i(p_t) \leq i(\kappa). \tag{A.20}$$

As $\delta < 1 - \frac{p_L}{\beta p^a}$, we also have by Lemma 2 that:

$$\lambda(0) < \alpha p_U \text{ and } \lambda(0) > \alpha p_L \Leftrightarrow p_L < \beta(1-\delta)p^a < p_U.$$

We first show that stocks are bounded from above at the stationary equilibrium:

Lemma 8

$$i_t \leq \max(i(\kappa), (1-\delta)i_{t-1}).$$

Proof. *(a)* Suppose $p_t > \beta(1-\delta)p^a$: As the marginal value of stocks $\lambda(0)$ is smaller than the marginal utility of one unit purchase, αp_t, stocks necessarily decline over time as in the proof of Proposition 2 and we proceed likewise.

Assume first that *(i)* i_{t-1} *is such that* $(1-\delta)i_{t-1} \leq u_t'^{-1}(\alpha p_t)$. Then we are in static case (I.a) and $i_t = 0$, $c_t = u_t'^{-1}(\alpha p_t)$. This then also applies to $t+1$ and every subsequent period. Assume second that *(ii)* $u_t'^{-1}(\alpha p_t) < (1-\delta)i_{t-1} \leq u_t'^{-1}(\lambda(0))$. This is static case (II.a) and $i_t = 0$, $c_t = (1-\delta)i_{t-1}$, $x_t = 0$. Therefore, at time $t+1$, we are in case *(i)* and $i_{t+1} = 0$ as well as for any period after $t+1$. Assume finally that *(iii)* $(1-\delta)i_{t-1} > u_t'(\lambda(0))$. This is static case (II.b). Then $i_t(i_{t-1})$ is defined by equation (A.19) and this implies that $i_t(i_{t-1}) < (1-\delta)i_{t-1}$. In all cases:

$$i_t \leq (1-\delta)i_{t-1}. \tag{A.21}$$

(b) Suppose $p_t < \beta(1-\delta)p^a$: As the marginal value of stocks is higher than the marginal utility of purchase, i.e. $\lambda(0) > \alpha p_L$, static conditions developed in the previous Section show that cases (I.b) or (II.b) apply.

In case (I.b) that is if $(1-\delta)i_{t-1} \leq i_t + u_t'^{-1}(\alpha p_t)$, current stocks are not too large, purchases are positive and we have $i_t = i(p_t)$. Otherwise, if current stocks are large enough, $(1-\delta)i_{t-1} >$

$i(p_t) + u_t'^{-1}(\alpha p_t)$ then $(1-\delta)i_{t-1} > u_t'^{-1}(\alpha p_t)$ and we are in case (II.b). In this case, $i_t^* < (1-\delta)i_{t-1}$ and stocks are necessarily decreasing.[10]

Summarizing both cases, using equation (A.21) when $p_t > \beta(1-\delta)p^a$, we have that:

$$i_t \le \max(i(p_t), (1-\delta)i_{t-1}) \le \max(i(\kappa), (1-\delta)i_{t-1}),$$

by equation (A.20). ■

A consequence of this Lemma is that stocks cannot be greater than $i(\kappa)$ at the stationary equilibrium since if they are larger, they would necessarily decrease in finite time to a lower value than $i(\kappa)$ whatever the price is.

Thus at any time, $i_{t-1} \le i(\kappa)$:

$$\begin{aligned} u_t'((1-\delta)i_{t-1}) &\ge u_t'((1-\delta)i(\kappa)) = ((1-\delta)i(\kappa))^{-\sigma}\eta_t, \\ &\ge ((1-\delta)i(\kappa))^{-\sigma}\underline{\eta}, \\ &\ge \beta(1-\delta)\alpha p_a = \lambda(0), \end{aligned}$$

in which we have used the isoelastic utility specification and the bound condition (7) on preference shocks.

This means that the probability of case (II.b) is zero at the stationary equilibrium since $u_t'((1-\delta)i_{t-1}) \ge \lambda(0)$. The other static cases are defined as a function of $\lambda(0)$, αp_t and $\eta_t^* = u_t'((1-\delta)i_{t-1}) = ((1-\delta)i_{t-1})^{-\sigma}\eta_t$ by:

(i) Regime I.a : $\alpha p_t > \eta_t^* \ge \lambda(0)$.

(ii) Regime II.a: $\eta_t^* \ge \alpha p_t > \lambda(0)$

(iii) Regime I.b: $\eta_t^* \ge \lambda(0) > \alpha p_t$.

These three cases correspond to the three lines of Proposition 3. Consumption, purchases and stocks are then obtained as in Proposition 1.

Finally, when $\alpha p_t = \lambda(0)$, the consumer is at the limit between Regimes II.a and I.b.

A.4 Proof of Lemma 4

By equation (5) we have:

$$\lambda(i_t) = \beta(1-\delta)E(\lambda(i_{t+1}) + \mu_{t+1}).$$

We distinguish between the three cases of Proposition 3 and then combine cases.

[10] Note that by equation (A.19), the value i_t^* is given by:

$$u_t'((1-\delta)i_{t-1} - i_t^*) = \lambda(i_t^*). \tag{A.22}$$

Because we are in the case $(1-\delta)i_{t-1} > i(p_t) + u_t'^{-1}(\alpha_i p_t)$ then:

$$u_t'((1-\delta)i_{t-1} - i(p_t)) < \alpha_i p_t = \lambda(i(p_t)),$$

by definition of $i(p_t)$. Therefore, stocks at $t-1$ are very large i.e. $(1-\delta)i_{t-1} > i_t^* > i(p_t)$ since the LHS of equation (A.22) is strictly increasing in i_t^* and the RHS is non-increasing in this value.

(i) When $p_{t+1} < \beta(1-\delta)p^a$, then $\lambda(i_{t+1}) + \mu_{t+1} = \alpha p_{t+1}$ using the static conditions (I.b).
(ii) When $p_{t+1} > \beta(1-\delta)p^a$ and $\eta_{t+1} > \frac{\alpha p_{t+1}}{((1-\delta)i_t)^{-\sigma}}$, then:

$$\lambda(i_{t+1}) + \mu_{t+1} = u'_{t+1}(c_{t+1}) = \alpha p_{t+1}.$$

(iii) When $p_{t+1} > \beta(1-\delta)p^a$ and $\frac{\lambda(0)}{((1-\delta)i_t)^{-\sigma}} \leq \eta_{t+1} \leq \frac{\alpha p_{t+1}}{((1-\delta)i_t)^{-\sigma}}$ then:

$$\lambda(i_{t+1}) + \mu_{t+1} = u'_{t+1}(c_{t+1}) = (1-\delta)i_t.$$

Note that $\frac{\lambda(0)}{((1-\delta)i_t)^{-\sigma}} \leq \eta_{t+1}$ is always satisfied under condition (7) as shown at the end of the proof of Proposition 3.

Using this remark and combining cases, we have:

$$\begin{aligned}
\lambda(i_t) &= \beta(1-\delta)\mathbb{E}\left[\alpha p_{t+1}\mathbf{1}\{p_{t+1} < \beta(1-\delta)p^a\}\right. \\
&\quad +\alpha p_{t+1}\mathbf{1}\{p_{t+1} > \beta(1-\delta)p^a, \eta_{t+1} > \frac{\alpha p_{t+1}}{((1-\delta)i_t)^{-\sigma}}) \\
&\quad \left.+u'_t((1-\delta)i_t)\,\mathbf{1}\left\{p_{t+1} > \beta(1-\delta)p^a, \eta_{t+1} \leq \frac{\alpha p_{t+1}}{((1-\delta)i_t)^{-\sigma}}\right\}\right] \\
&= \beta(1-\delta)\mathbb{E}(\alpha p_{t+1}) \\
&\quad +\beta(1-\delta)\mathbb{E}(\left[((1-\delta)i_t)^{-\sigma}\eta_{t+1} - \alpha p_{t+1}\right]\mathbf{1}\left\{p_{t+1} > \beta(1-\delta)p^a, \eta_{t+1} \leq \frac{\alpha p_{t+1}}{((1-\delta)i_t)^{-\sigma}}\right\}), \\
&= \lambda(0) + \beta(1-\delta)\gamma(i_t).
\end{aligned}$$

Thus by definition:

$$\gamma(i_t) = \mathbb{E}(\left[((1-\delta)i_t)^{-\sigma}\eta_{t+1} - \alpha p_{t+1}\right]\mathbf{1}\left\{p_{t+1} > \beta(1-\delta)p^a, \eta_{t+1} \leq \frac{\alpha p_{t+1}}{((1-\delta)i_t)^{-\sigma}}\right\} \leq 0.$$

Moreover, the stationary equilibrium of Definition 2 holds constant the expectations of consumers concerning future prices. The distribution of prices has therefore a support with two points $\{p_L, p_U\}$ and by condition E, $\beta(1-\delta)p^a$ is in between these two points. Then:

$$\mathbf{1}\left\{p_{t+1} > \beta(1-\delta)p^a, \eta_{t+1} \leq \frac{\alpha p_{t+1}}{((1-\delta)i_t)^{-\sigma}}\right\} = \mathbf{1}\left\{p_{t+1} = p_U, \eta_{t+1} \leq \frac{\alpha p_U}{((1-\delta)i_t)^{-\sigma}}\right\}$$

so that:

$$\begin{aligned}\gamma(i_t) &= \mathbb{E}(\left[((1-\delta)i_t)^{-\sigma}\eta_{t+1} - \alpha p_U\right]\mathbf{1}\left\{\eta_{t+1} \leq \frac{\alpha p_U}{((1-\delta)i_t)^{-\sigma}}\right\}), \\ &= \alpha p_U \mathbb{E}(\left[\frac{((1-\delta)i_t)^{-\sigma}}{\alpha p_U}\eta_{t+1} - 1\right]\mathbf{1}\left\{\eta_{t+1} \leq \frac{\alpha p_U}{((1-\delta)i_t)^{-\sigma}}\right\}, \\ &= \alpha p_U f(\frac{((1-\delta)i_t)^{-\sigma}}{\alpha p_U}).\end{aligned}$$

by the definition of $f(.)$.

A.5 Proof of Lemma 5

We seek the optimal solution for $i(p_t)$ which leads to the complete characterization of optimal decisions by Proposition 3.

By definition $\lambda(i(p_t)) = \alpha p_t$ when $p_t < \beta(1-\delta)p^a$. Therefore:

$$\lambda(0) + \beta(1-\delta)\gamma(i(p_t)) = \beta(1-\delta)\alpha p_a + \beta(1-\delta)\alpha p_U f(\frac{((1-\delta)i(p_t))^{-\sigma}}{\alpha p_U}) = \alpha p_t$$

and:

$$f(\frac{((1-\delta)i(p_t))^{-\sigma}}{\alpha p_U}) = \frac{\frac{p_t}{\beta(1-\delta)} - p^a}{p_U}.$$

The reciprocal of this equation is the result reported in Lemma 5 when $p_t \neq \beta(1-\delta)p^a$. Note that it is well defined only if:

$$\frac{\frac{p_t}{\beta(1-\delta)} - p^a}{p_U} > -1 \iff \frac{p_t}{\beta(1-\delta)} > p^a - p_U,$$

which in the 2-point of support case, yields:

$$\frac{p_t}{\beta(1-\delta)} > \pi(p_L - p_U),$$

which is always true since the RHS is negative and the LHS positive.

When $p_t = \beta(1-\delta)p^a$, the previous expression is ambiguous since as noted above $f^{-1}(0)$ is the whole range $[\frac{1}{\underline{\eta}}, +\infty)$ and therefore $i(\beta(1-\delta)p^a)$ is the whole range :

$$[0, \frac{1}{1-\delta}(\frac{\alpha p_U}{\underline{\eta}})^{-\frac{1}{\sigma}}].$$

The consumer is indifferent between all the values in this range. The two bounds correspond to the limit of the cases (ii) and (iii) in Proposition 3.

A.6 Proof

Assume that $W_{t+1}(i_t)$ is an increasing and concave differentiable function of its argument. We shall prove that $W_t(i_{t-1})$ is then also an increasing and concave differentiable function.

We have,

$$W_t(i_{t-1}) = \max_{c_t,x_t} \{u_t(c_t) - \alpha p_t x_t + \beta E_t W_{t+1}(i_t)\}$$

subject to the constraints of accumulation of stocks and non-negativity of purchases.

We have shown in Lemma 4 that the marginal value of stocks:

$$\lambda(i_{t-1}) = \frac{\partial W_t(i_{t-1})}{\partial i_{t-1}}$$

is a positive and non-increasing function of its argument so that $W_t(i_{t-1})$ is then also an increasing and concave differentiable function. Furthermore, $\lambda(i_{t-1}) < 0$ when $i_{t-1} \to \infty$.

B Proofs of Section 3

B.1 Proof of Lemma 6

B.1.1 High price profits

From the system of demand (10) and given that the expected probability that a consumer has faced p_L in the previous period is π, we get that expected demands when a high price, $p_t > \beta(1-\delta)p^a$, is played, are equal to :

$$D_U(p_t) = \left[\begin{array}{c} (1-\pi)\mathbb{E}(\alpha^{-\frac{1}{\sigma}})p_t^{-\frac{1}{\sigma}}\mathbb{E}(\eta_t^{\frac{1}{\sigma}}) + \\ \pi\mathbb{E}\left[(\alpha^{-\frac{1}{\sigma}}p_t^{-\frac{1}{\sigma}}\eta_t^{\frac{1}{\sigma}} - (1-\delta)i(p_L))\mathbf{1}\{\alpha^{-\frac{1}{\sigma}}p_t^{-\frac{1}{\sigma}}\eta_t^{\frac{1}{\sigma}} - (1-\delta)i(p_L) \geq 0\}\right] \end{array} \right]$$

Recomposing, this yields:

$$\begin{aligned} D_U(p_t) &= \left[\begin{array}{c} E(\alpha^{-\frac{1}{\sigma}})p_t^{-\frac{1}{\sigma}}E(\eta_t^{\frac{1}{\sigma}}) - \pi(1-\delta)i(p_L) \\ -\pi\mathbb{E}\left[(\alpha^{-\frac{1}{\sigma}}p_t^{-\frac{1}{\sigma}}\eta_t^{\frac{1}{\sigma}} - (1-\delta)i(p_L))\mathbf{1}\{\alpha^{-\frac{1}{\sigma}}p_t^{-\frac{1}{\sigma}}\eta_t^{\frac{1}{\sigma}} - (1-\delta)i(p_L) < 0\}\right] \end{array} \right] \\ &= \left[\begin{array}{c} E(\alpha^{-\frac{1}{\sigma}})p_t^{-\frac{1}{\sigma}}E(\eta_t^{\frac{1}{\sigma}}) - \pi(1-\delta)i(p_L) \\ -\pi(1-\delta)i(p_L)\mathbb{E}\left[(\frac{\alpha^{-\frac{1}{\sigma}}p_t^{-\frac{1}{\sigma}}\eta_t^{\frac{1}{\sigma}}}{(1-\delta)i(p_L)} - 1)\mathbf{1}\{\alpha^{-\frac{1}{\sigma}}p_t^{-\frac{1}{\sigma}}\eta_t^{\frac{1}{\sigma}} - (1-\delta)i(p_L) < 0\}\right] \end{array} \right], \\ &= E(\alpha^{-\frac{1}{\sigma}})p_t^{-\frac{1}{\sigma}}E(\eta_t^{\frac{1}{\sigma}}) - \pi(1-\delta)i(p_L) - \pi(1-\delta)i(p_L)f_{1/\sigma}(\frac{\alpha^{-\frac{1}{\sigma}}p_t^{-\frac{1}{\sigma}}}{(1-\delta)i(p_L)}), \end{aligned}$$

in which we have generalized the definition of function f to:

$$f_{1/\sigma}(a) = \mathbb{E}\left[(a\eta_t^{\frac{1}{\sigma}} - 1)\mathbf{1}\{a\eta_t^{\frac{1}{\sigma}} - 1 < 0\}\right].$$

Note that the argument of this function using equation (8) is equal to:

$$\frac{\alpha^{-\frac{1}{\sigma}} p_t^{-\frac{1}{\sigma}}}{(1-\delta)i(p_L)} = \frac{\alpha^{-\frac{1}{\sigma}} p_t^{-\frac{1}{\sigma}}}{\left[\alpha p_U f^{-1}\left(\frac{\frac{p_L}{\beta(1-\delta)} - p^a}{p_U}\right)\right]^{-\frac{1}{\sigma}}} = \left[\frac{p_U}{p_t} f^{-1}\left(\frac{\frac{p_L}{\beta(1-\delta)} - p^a}{p_U}\right)\right]^{\frac{1}{\sigma}},$$

and therefore, setting $k_\alpha = E(\alpha^{-\frac{1}{\sigma}})$:

$$D_U(p_t) = k_\alpha \left[p_t^{-\frac{1}{\sigma}} E(\eta_t^{\frac{1}{\sigma}}) - \pi \left[p_U f^{-1}\left(\frac{\frac{p_L}{\beta(1-\delta)} - p^a}{p_U}\right)\right]^{-\frac{1}{\sigma}} \left[1 + f_{1/\sigma}\left(\left[\frac{p_U}{p_t} f^{-1}\left(\frac{\frac{p_L}{\beta(1-\delta)} - p^a}{p_U}\right)\right]^{\frac{1}{\sigma}}\right)\right]\right],$$

and the profit when the price is high is therefore

$$\Pi_U(p_t) = (p_t - \kappa) D_U(p_t).$$

Lemma 9 *$D_U(p_t)$ is decreasing in prices.*

Proof. As $f_{1/\sigma}$ is concave, $f_{1/\sigma}$ is twice differentiable almost everywhere. We have, setting $k_\alpha = 1$ for simplicity in the proof, and

$$\theta = \left[p_U f^{-1}\left(\frac{\frac{p_L}{\beta(1-\delta)} - p^a}{p_U}\right)\right]^{\frac{1}{\sigma}}$$

that the derivative of the demand function is:

$$\begin{aligned}
\frac{\partial D_U}{\partial p_t} &= k_\alpha \left[-\frac{1}{\sigma} p_t^{(-\frac{1}{\sigma}-1)} \mathbb{E}(\eta_t^{1/\sigma}) + \frac{\pi}{\theta} \frac{\partial f_{1/\sigma}}{\partial a} \frac{\theta}{\sigma} p_t^{(-\frac{1}{\sigma}-1)}\right], \\
&= -\frac{1}{\sigma} p_t^{(-\frac{1}{\sigma}-1)} k_\alpha \left[E(\eta_t^{1/\sigma}) - \pi \frac{\partial f_{1/\sigma}}{\partial a}\right] < 0.
\end{aligned}$$

since $E(\eta_t^{1/\sigma}) > 0$, $\pi < 1$ and by definition:

$$\frac{\partial f_{1/\sigma}}{\partial a} = \mathbb{E}\left[\eta_t^{1/\sigma} \mathbf{1}\{\eta_t^{1/\sigma} < 1/a\}\right] \in [0, E(\eta_t^{1/\sigma})].$$

We can also compute the second derivative (almost everywhere) that will help us later to prove that the maximizing argument of the profit function is unique:

$$\begin{aligned}\frac{\partial^2 D_U}{\partial p_t^2} &= k_\alpha\left[\frac{1}{\sigma}(\frac{1}{\sigma}+1)p_t^{(-\frac{1}{\sigma}-2)}\left[E(\eta_t^{1/\sigma})-\pi\frac{\partial f_{1/\sigma}}{\partial a}\right]-\frac{1}{\sigma}p_t^{(-\frac{1}{\sigma}-1)}\pi\frac{\partial^2 f_{1/\sigma}}{\partial a^2}\frac{\theta}{\sigma}p_t^{(-\frac{1}{\sigma}-1)}\right], \quad \text{(B.23)}\\ &= k_\alpha\frac{1}{\sigma}p_t^{(-\frac{1}{\sigma}-2)}\left[(\frac{1}{\sigma}+1)(E(\eta_t^{1/\sigma})-\pi\frac{\partial f_{1/\sigma}}{\partial a})-\frac{\pi\theta}{\sigma}p_t^{(-\frac{1}{\sigma})}\frac{\partial^2 f_{1/\sigma}}{\partial a^2}\right].\end{aligned}$$

■

Call $\tilde{p}_U$ an argument, possibly not unique, that maximizes the profit function with respect to p_t over a compact range of prices $[\kappa, p_U^{\max}]$ and that satisfies the first order condition:

$$D_U(p_t)+(p_t-\kappa)\frac{\partial D_U}{\partial p_t}=0.$$

As a matter $\tilde{p}_U$ is itself a function of p_L and p_U. A solution in price p_U, possibly non unique, thus satisfies the fixed point equation, $\tilde{p}_U = p_U$. Nonetheless, before solving this equation, we first determine p_L as a function of p_U.

B.1.2 Low price profits

Using the system of demand (10) and assuming that a low price is played, we get that expected demands are equal to:

$$\begin{aligned}D_L(p_t) &= (1-\pi)(\mathbb{E}(\alpha^{-\frac{1}{\sigma}})p_t^{-\frac{1}{\sigma}}\mathbb{E}(\eta_t^{\frac{1}{\sigma}})+i(p_t))+\pi(\mathbb{E}(\alpha^{-\frac{1}{\sigma}})p_t^{-\frac{1}{\sigma}}\mathbb{E}(\eta_t^{\frac{1}{\sigma}})+i(p_t)-(1-\delta)i(p_L)),\\ &= \mathbb{E}(\alpha^{-\frac{1}{\sigma}})p_t^{-\frac{1}{\sigma}}\mathbb{E}(\eta_t^{\frac{1}{\sigma}})+i(p_t)-\pi(1-\delta)i(p_L),\\ &= k_\alpha\left[p_t^{-\frac{1}{\sigma}}\mathbb{E}(\eta_t^{\frac{1}{\sigma}})+\frac{1}{1-\delta}\left[p_U f^{-1}\left(\frac{\frac{p_t}{\beta(1-\delta)}-p^a}{p_U}\right)\right]^{-\frac{1}{\sigma}}-\pi\left[p_U f^{-1}\left(\frac{\frac{p_L}{\beta(1-\delta)}-p^a}{p_U}\right)\right]^{-\frac{1}{\sigma}}\right].\end{aligned}$$

Note that:

$$D_L(p_t)-D_U(p_t)=k_\alpha\left[\begin{array}{c}\frac{1}{1-\delta}\left[p_U f^{-1}\left(\frac{\frac{p_t}{\beta(1-\delta)}-p^a}{p_U}\right)\right]^{-\frac{1}{\sigma}}\\ +\pi\left[p_U f^{-1}\left(\frac{\frac{p_L}{\beta(1-\delta)}-p^a}{p_U}\right)\right]^{-\frac{1}{\sigma}} f_{1/\sigma}\left(\left[\frac{p_U}{p_t}f^{-1}\left(\frac{\frac{p_L}{\beta(1-\delta)}-p^a}{p_U}\right)\right]^{\frac{1}{\sigma}}\right)\end{array}\right]$$

and the difference in profit is therefore:

$$\begin{aligned}
\Pi_L(p_t) - \Pi_U(p_t) &= (p_t-\kappa)(D_L(p_t) - D_U(p_t)), \\
&= k_\alpha \left[\begin{array}{c} \frac{(p_t-\kappa)}{1-\delta}\left[p_U f^{-1}\left(\frac{\frac{p_t}{\beta(1-\delta)}-p^a}{p_U}\right)\right]^{-\frac{1}{\sigma}} \\ +(p_t-\kappa)\pi\left[p_U f^{-1}\left(\frac{\frac{p_L}{\beta(1-\delta)}-p^a}{p_U}\right)\right]^{-\frac{1}{\sigma}} f_{1/\sigma}\left(\left[\frac{p_U}{p_t} f^{-1}\left(\frac{\frac{p_L}{\beta(1-\delta)}-p^a}{p_U}\right)\right]^{\frac{1}{\sigma}}\right). \end{array} \right], \\
&= k_\alpha \left[\begin{array}{c} \frac{(p_t-\kappa)}{1-\delta}\left\{\left[p_U f^{-1}\left(\frac{\frac{p_t}{\beta(1-\delta)}-p^a}{p_U}\right)\right]^{-\frac{1}{\sigma}} - \left[p_U f^{-1}\left(\frac{\frac{p_L}{\beta(1-\delta)}-p^a}{p_U}\right)\right]^{-\frac{1}{\sigma}}\right\} \\ +(p_t-\kappa)\left[p_U f^{-1}\left(\frac{\frac{p_L}{\beta(1-\delta)}-p^a}{p_U}\right)\right]^{-\frac{1}{\sigma}} \left\{\frac{1}{1-\delta} + \pi f_{1/\sigma}\left(\left[\frac{p_U}{p_t} f^{-1}\left(\frac{\frac{p_L}{\beta(1-\delta)}-p^a}{p_U}\right)\right]^{\frac{1}{\sigma}}\right)\right\}. \end{array} \right]
\end{aligned}$$

As this is a regime in which $p_t \leq \beta(1-\delta)p^a < p_U$, the concave function $\Pi_U(p_t)$ is necessarily increasing in p_t over this range of prices. The first term on the RHS is increasing in p_t when $p_t < p_L$ because $f^{-1}(.)$ is increasing and the second term is decreasing when $p_t > p_L$. The third term is increasing in p_t because $f^{-1}\left(\frac{\frac{p_L}{\beta(1-\delta)}-p^a}{p_U}\right) > 0$ and:

$$\frac{1}{1-\delta} + \pi f_{1/\sigma}\left(\left[\frac{p_U}{p_t} f^{-1}\left(\frac{\frac{p_L}{\beta(1-\delta)}-p^a}{p_U}\right)\right]^{\frac{1}{\sigma}}\right) > 0,$$

since $f_{1/\sigma} \in [-1, 0]$. Thus, the derivative of profits when $p_t = p_L$ is strictly positive and thus the constraint $\frac{p_t}{\beta(1-\delta)} - p^a \leq 0$ binds at the optimum.

Therefore $p_L = \beta(1-\delta)p_a$. In this case, the expression of the profit above need to be slightly modified to take into account that the optimal stock is given by equation (9) and therefore:

$$\begin{aligned}
\Pi(p_L) &= k_\alpha(p_L-\kappa)\left\{p_L^{-\frac{1}{\sigma}}\mathbb{E}(\eta_t^{\frac{1}{\sigma}}) + \frac{\rho}{1-\delta}\left[p_U f^{-1}(0)\right]^{-\frac{1}{\sigma}} - \rho\pi\left[p_U f^{-1}(0)\right]^{-\frac{1}{\sigma}}\right\}, \\
&= k_\alpha(p_L-\kappa)\left[p_L^{-\frac{1}{\sigma}}\mathbb{E}(\eta_t^{\frac{1}{\sigma}}) + (\frac{1}{1-\delta}-\pi)\rho\left[\frac{p_U}{\underline{\eta}}\right]^{-\frac{1}{\sigma}}\right], \\
&= k_\alpha\mathbb{E}(\eta_t^{\frac{1}{\sigma}})(p_L-\kappa)\left[p_L^{-\frac{1}{\sigma}} + (\frac{1}{1-\delta}-\pi)\rho\frac{\underline{\eta}^{\frac{1}{\sigma}}}{\mathbb{E}(\eta_t^{\frac{1}{\sigma}})}p_U^{-\frac{1}{\sigma}}\right], \\
&= k(p_L-\kappa)\left[p_L^{-\frac{1}{\sigma}} + (\frac{1}{1-\delta}-\pi)\rho\underline{\nu}p_U^{-\frac{1}{\sigma}}\right].
\end{aligned}$$

using $k = k_\alpha\mathbb{E}(\eta_t^{\frac{1}{\sigma}})$ and $\underline{\nu} = \frac{\underline{\eta}^{\frac{1}{\sigma}}}{E(\eta_t^{1/\sigma})}$. With these results, we can now return to the determination of p_U.

B.1.3 Determination of p_U

As $p_L = \beta(1-\delta)p_a$, we have:

$$D_U(p_t) = k\left[p_t^{-\frac{1}{\sigma}}E(\eta_t^{\frac{1}{\sigma}}) - \pi\rho\left[\frac{p_U}{\underline{\eta}}\right]^{-\frac{1}{\sigma}}\left[1 + f_{1/\sigma}\left(\frac{\left[\frac{p_U}{p_t\underline{\eta}}\right]^{\frac{1}{\sigma}}}{\rho}\right)\right]\right].$$

The first order condition for the maximization of this profit function is:

$$D_U(p_t) + (p_t - \kappa)k_\alpha\left(-\frac{1}{\sigma}p_t^{(-\frac{1}{\sigma}-1)}\left[E(\eta_t^{1/\sigma}) - \pi\frac{\partial f_{1/\sigma}}{\partial a}\right]\right) = 0. \quad \text{(B.24)}$$

Setting $p_t = p_U$ yields:

$$D_U(p_U) + (p_U - \kappa)k_\alpha\left(-\frac{1}{\sigma}p_U^{(-\frac{1}{\sigma}-1)}\left[E(\eta_t^{1/\sigma}) - \pi\frac{\partial f_{1/\sigma}}{\partial a}\right]\right) = 0,$$

in which $a = \frac{1}{\underline{\eta}^{\frac{1}{\sigma}}\rho}$. We have:

$$p_U^{-\frac{1}{\sigma}}E(\eta_t^{\frac{1}{\sigma}}) - \pi\rho\left[\frac{p_U}{\underline{\eta}}\right]^{-\frac{1}{\sigma}}\left[1 + f_{1/\sigma}\left(\frac{1}{\underline{\eta}^{\frac{1}{\sigma}}\rho}\right)\right] + (p_U - \kappa)\left(-\frac{1}{\sigma}p_U^{(-\frac{1}{\sigma}-1)}\left[E(\eta_t^{1/\sigma}) - \pi\mathbb{E}\left[\eta_t^{1/\sigma}\mathbf{1}\{\eta_t^{1/\sigma} \leq \underline{\eta}^{\frac{1}{\sigma}}\rho\}\right]\right]\right) = 0$$

since by definition:

$$\frac{\partial f_{1/\sigma}}{\partial a} = \mathbb{E}\left[\eta_t^{1/\sigma}\mathbf{1}\{\eta_t^{1/\sigma} \leq 1/a\}\right].$$

We shall assume that $\rho < 1$ so that:

$$f_{1/\sigma}\left(\frac{1}{\underline{\eta}^{\frac{1}{\sigma}}\rho}\right) = 0 \text{ and } \frac{\partial f_{1/\sigma}}{\partial a} = \mathbb{E}\left[\eta_t^{1/\sigma}\mathbf{1}\{\eta_t^{1/\sigma} \leq \underline{\eta}^{\frac{1}{\sigma}}\rho\}\right] = 0. \quad \text{(B.25)}$$

Thence we have:

$$
\begin{aligned}
p_U^{-\frac{1}{\sigma}} E(\eta_t^{\frac{1}{\sigma}}) - \pi\rho \left[\frac{p_U}{\underline{\eta}}\right]^{-\frac{1}{\sigma}} + (p_U - \kappa)\left(-\frac{1}{\sigma} p_U^{(-\frac{1}{\sigma}-1)} E(\eta_t^{1/\sigma})\right) &= 0,\\
p_U(E(\eta_t^{\frac{1}{\sigma}}) - \pi\rho\underline{\eta}^{\frac{1}{\sigma}}) - (p_U - \kappa)\frac{1}{\sigma} E(\eta_t^{1/\sigma}) &= 0,\\
p_U((1-\frac{1}{\sigma})E(\eta_t^{\frac{1}{\sigma}}) - \pi\rho\underline{\eta}^{\frac{1}{\sigma}}) + \kappa\frac{1}{\sigma}E(\eta_t^{1/\sigma}) &= 0,\\
p_U((1-\frac{1}{\sigma}) - \pi\rho\frac{\underline{\eta}^{\frac{1}{\sigma}}}{E(\eta_t^{1/\sigma})}) + \kappa\frac{1}{\sigma} &= 0,\\
p_U((1-\frac{1}{\sigma}) - \pi\rho\underline{\nu}) + \kappa\frac{1}{\sigma} &= 0
\end{aligned}
$$

denoting $\underline{\nu} = \frac{\underline{\eta}^{\frac{1}{\sigma}}}{E(\eta_t^{1/\sigma})}$. This implies that:

$$
p_U = \frac{\kappa}{1 - \sigma(1 - \pi\rho\underline{\nu})}. \tag{B.26}
$$

In turn this implies that:

$$
\begin{aligned}
\Pi_U(p_U) &= k_\alpha (p_U - \kappa) p_U^{-\frac{1}{\sigma}} \left[E(\eta_t^{\frac{1}{\sigma}}) - \pi\rho\underline{\eta}^{\frac{1}{\sigma}}\right],\\
&= k(p_U - \kappa) p_U^{-\frac{1}{\sigma}} \left[1 - \pi\rho\underline{\nu}\right].
\end{aligned}
$$

Note that p_U is NOT the maximum of this profit function since there are dynamic externalities that imply that the price is lower than the monopolistic price, $\frac{\kappa}{1-\sigma}$.

Finally, we need to check that the solution to equation (B.24) is a maximum that is:

$$
\left.\frac{\partial^2 \Pi_U(p_t)}{\partial p_t^2}\right|_{p_t = p_U} < 0.
$$

We use the second order derivative of demands given in equation (B.23) and replace $p_t = p_U$:

$$
\left.\frac{\partial^2 D_U}{\partial p_t^2}\right|_{p_t = p_U} = k_\alpha \frac{1}{\sigma} p_U^{(-\frac{1}{\sigma}-2)} \left[(\frac{1}{\sigma}+1)(E(\eta_t^{1/\sigma}) - \pi\frac{\partial f_{1/\sigma}}{\partial a}) - \frac{\pi\theta}{\sigma} p_U^{(-\frac{1}{\sigma})} \frac{\partial^2 f_{1/\sigma}}{\partial a^2}\right].
$$

By the same argument as in the above equation (B.25) we have when p_t is in the neighborhood of p_U:

$$
\frac{\partial f_{1/\sigma}}{\partial a} = 0, \frac{\partial^2 f_{1/\sigma}}{\partial a^2} = 0
$$

so that the arguments for the concavity of the profit function goes as follows:

$$
\begin{aligned}
\left.\frac{\partial^2 \Pi_U(p_t)}{\partial p_t^2}\right|_{p_t=p_U} &= 2\left.\frac{\partial D_U}{\partial p_t}\right|_{p_t=p_U} + (p_U-\kappa)\left.\frac{\partial^2 D_U}{\partial p_t^2}\right|_{p_t=p_U}, \\
&= \left[2\left(-\frac{1}{\sigma}p_U^{(-\frac{1}{\sigma}-1)}E(\eta_t^{1/\sigma})\right) + \frac{1}{\sigma}(p_U-\kappa)p_U^{(-\frac{1}{\sigma}-2)}(\frac{1}{\sigma}+1)(E(\eta_t^{1/\sigma})\right]k_\alpha, \\
&= \frac{1}{\sigma}p_U^{(-\frac{1}{\sigma}-2)}E(\eta_t^{1/\sigma})\left[-2p_U+(\frac{1}{\sigma}+1)(p_U-\kappa)\right]k_\alpha, \\
&\propto \left[(\frac{1}{\sigma}-1)p_U-(\frac{1}{\sigma}+1)\kappa)\right], \\
&= \frac{p_U}{\sigma}\left[1-\sigma-(1+\sigma)(1-\sigma+\sigma\pi\rho\underline{\nu})\right] \\
&= \frac{p_U}{\sigma}\left[-\sigma(1-\sigma)-(1+\sigma)\sigma\pi\rho\underline{\nu})\right] < 0
\end{aligned}
$$

if $\sigma \in (0,1)$.

Remark: This proof is slightly more involved if $\rho = 1$ and all the mass is concentrated at $\underline{\eta}$ in the distribution of η_t. We assume away this case.

B.2 Proof of Proposition 4

The equalization of profits lead to:

$$\Pi_L(p_L) = k(p_L-\kappa)\left[p_L^{-\frac{1}{\sigma}} + (\frac{1}{1-\delta}-\pi)\rho\underline{\nu}p_U^{-\frac{1}{\sigma}}\right] = k(p_U-\kappa)p_U^{-\frac{1}{\sigma}}\left[1-\pi\rho\underline{\nu}\right] = \Pi_U(p_U),$$

and therefore:

$$
\begin{aligned}
(p_L-\kappa)\left[p_L^{-\frac{1}{\sigma}}+(\frac{1}{1-\delta}-\pi)\rho\underline{\nu}p_U^{-\frac{1}{\sigma}}\right] &= (p_U-\kappa)p_U^{-\frac{1}{\sigma}}\left[1-\pi\rho\underline{\nu}\right], \\
\Longleftrightarrow (p_L-(1-\sigma+\sigma\pi\rho\underline{\nu})p_U)\left[p_L^{-\frac{1}{\sigma}}+(\frac{1}{1-\delta}-\pi)\rho\underline{\nu}p_U^{-\frac{1}{\sigma}}\right] &= p_U(1-(1-\sigma+\sigma\pi\rho\underline{\nu}))p_U^{-\frac{1}{\sigma}}\left[1-\pi\rho\underline{\nu}\right] \\
\Longleftrightarrow (\phi-(1-\sigma+\sigma\pi\rho\underline{\nu}))\left[\phi^{-\frac{1}{\sigma}}+(\frac{1}{1-\delta}-\pi)\rho\underline{\nu}\right] &= \sigma(1-\pi\rho\underline{\nu})^2 \qquad \text{(B.27)}
\end{aligned}
$$

in which we have set $\phi = \frac{p_L}{p_U}$.

The second equation is provided by:

$$\alpha p_L = \beta(1-\delta)p^a = \alpha\beta(1-\delta)(\pi p_L + (1-\pi)p_U),$$

and therefore:

$$\phi = \beta(1-\delta)(\pi\phi+(1-\pi)) \Longrightarrow \pi = \frac{1-\frac{\phi}{\beta(1-\delta)}}{1-\phi}. \qquad \text{(B.28)}$$

We shall study below the existence of a solution to equations (B.27) and (B.28) but we first analyze conditions under which the constraint (7) is satisfied.

B.2.1 The constraint on $\underline{\eta}$

To derive these equations, we assumed that:

$$\begin{aligned}\underline{\eta} &\geq \beta(1-\delta)\alpha p_a((1-\delta)i(\kappa))^\sigma = \frac{\beta(1-\delta)p_a}{p_U f^{-1}\left(\frac{\frac{\kappa}{\beta(1-\delta)}-p^a}{p_U}\right)}, \\ \iff f^{-1}\left(\frac{\frac{\kappa}{\beta(1-\delta)}-p^a}{p_U}\right) &\geq \frac{\beta(1-\delta)p_a}{p_U\underline{\eta}}. \end{aligned} \tag{B.29}$$

We have that:

$$\begin{aligned} p_L &= \beta(1-\delta)p^a = \beta(1-\delta)(\pi p_L + (1-\pi)p_U) \\ &\iff p_L = \frac{\beta(1-\delta)(1-\pi)}{1-\beta(1-\delta)\pi}p_U \end{aligned}$$

so that equation (B.29) can be rewritten as:

$$\begin{aligned} f^{-1}\left(\frac{\frac{1-\sigma+\sigma\pi\rho\underline{\nu}}{\beta(1-\delta)}p_U - \frac{(1-\pi)}{1-\beta(1-\delta)\pi}p_U}{p_U}\right) &\geq \frac{1}{\underline{\eta}}\frac{\beta(1-\delta)(1-\pi)}{1-\beta(1-\delta)\pi}, \\ \iff f^{-1}\left(\frac{1-\sigma+\sigma\pi\rho\underline{\nu}}{\beta(1-\delta)} - \frac{(1-\pi)}{1-\beta(1-\delta)\pi}\right) &\geq \frac{1}{\underline{\eta}}\frac{\beta(1-\delta)(1-\pi)}{1-\beta(1-\delta)\pi}, \\ \iff \frac{1-\sigma+\sigma\pi\rho\underline{\nu}}{\beta(1-\delta)} - \frac{(1-\pi)}{1-\beta(1-\delta)\pi} &\geq f\left(\frac{1}{\underline{\eta}}\frac{\beta(1-\delta)(1-\pi)}{1-\beta(1-\delta)\pi}\right). \end{aligned}$$

The constraint can be expressed as:

$$\frac{1-\sigma}{\beta(1-\delta)} \geq -\frac{\sigma\pi\rho\underline{\nu}}{\beta(1-\delta)} + \frac{(1-\pi)}{1-\beta(1-\delta)\pi} + f\left(\frac{\beta(1-\delta)}{\underline{\eta}}\frac{(1-\pi)}{1-\beta(1-\delta)\pi}\right)$$

in which the term $\frac{1}{\underline{\eta}}\frac{\beta(1-\delta)(1-\pi)}{1-\beta(1-\delta)\pi} < \frac{1}{\underline{\eta}}$ and therefore, $f(\frac{1}{\underline{\eta}}\frac{\beta(1-\delta)(1-\pi)}{1-\beta(1-\delta)\pi}) < 0$. The right hand side is decreasing with π because $\frac{(1-\pi)}{1-\beta(1-\delta)\pi}$ is decreasing in π and $f()$ is increasing. The strongest constraint is then obtained when $\pi = 0$ and the right hand side is:

$$\frac{1-\sigma}{\beta(1-\delta)} \geq 1 + f\left(\frac{\beta(1-\delta)}{\underline{\eta}}\right) < 1,$$

which does not preclude any value for σ.

As $\phi = \frac{p_L}{p_U} = \frac{\beta(1-\delta)(1-\pi)}{1-\beta(1-\delta)\pi}$ we can also rewrite the constraint as:

$$\begin{aligned}\frac{1-\sigma}{\beta(1-\delta)} &\geq -\frac{\sigma\pi\rho\underline{\nu}}{\beta(1-\delta)} + \frac{\phi}{\beta(1-\delta)} + f(\frac{\phi}{\underline{\eta}}),\\ &\iff 1-\sigma+\sigma\pi\rho\underline{\nu} \geq \phi + \beta(1-\delta)f(\frac{\phi}{\underline{\eta}})\\ &\iff \pi \geq \frac{1}{\sigma\rho\underline{\nu}}\left(\sigma - 1 + \phi + \beta(1-\delta)f(\frac{\phi}{\underline{\eta}})\right)\end{aligned}$$

B.2.2 The supply equations

Equations (B.27) and (B.28) provide a system of 2 equations in 2 unknowns (ϕ, π). We can rewrite equation (B.27) as:

$$(\phi-(1-\sigma))\left[\phi^{-\frac{1}{\sigma}} + (\frac{1}{1-\delta} - \pi)\rho\underline{\nu}\right] - \sigma\pi\rho\underline{\nu}(\phi^{-\frac{1}{\sigma}} + \frac{\rho\underline{\nu}}{1-\delta}) + \sigma(\pi\rho\underline{\nu})^2 = \sigma(1-2\pi\rho\underline{\nu}) + \sigma(\pi\rho\underline{\nu})^2$$

which is linear in π. Define $\phi_0(\phi) = \phi^{-\frac{1}{\sigma}} + \frac{\rho\underline{\nu}}{1-\delta}$ and write:

$$(\phi-(1-\sigma))(\phi_0(\phi) - \pi\rho\underline{\nu}) - \sigma\pi\rho\underline{\nu}\phi_0(\phi) = \sigma(1-2\pi\rho\underline{\nu}),$$

so that:

$$(\phi-(1-\sigma))\phi_0(\phi) - \sigma = \pi\rho\underline{\nu}\left[\phi-(1-\sigma)+\sigma\phi_0(\phi)-2\sigma\right]$$

and therefore:

$$\pi\rho\underline{\nu} = \frac{\sigma-(\phi-(1-\sigma))\phi_0(\phi)}{1-\phi-\sigma(\phi_0(\phi)-1)}$$

Thus:

$$\pi_1(\phi) = \frac{1}{\rho\underline{\nu}}\frac{\sigma-(\phi-(1-\sigma))\phi_0(\phi)}{1-\phi-\sigma(\phi_0(\phi)-1)}.$$

The second equation is also a function of π :

$$\pi_2(\phi) = \frac{1-\frac{\phi}{\beta(1-\delta)}}{1-\phi}. \tag{B.30}$$

Furthermore since $\phi = \frac{p_L}{p_U} \in (0,1)$:

$$\begin{aligned}0 &< \pi_2(\phi) < 1 \Leftrightarrow 0 < 1 - \frac{\phi}{\beta(1-\delta)} < 1-\phi,\\ &\iff \phi < \beta(1-\delta) < 1.\end{aligned}$$

Moreover:

$$\phi = \frac{p_L}{p_U} > \frac{\kappa}{p_U} = 1 - \sigma + \sigma\pi\rho\underline{\nu} > 1 - \sigma.$$

so that:

$$1 - \sigma < \phi < \beta(1 - \delta).$$

B.2.3 Existence

We shall assume in the rest of the proof that we are in a neighborhood of $\beta = 1$ and $\delta = 0$. Let $\varepsilon_\beta = 1 - \beta > 0$ and $\delta > 0$ be small quantities so that first-order Taylor expansions with respect to them are legitimate.

Set:

$$\begin{aligned}
\varphi(\phi) &= (1-\phi)(1-\phi-\sigma(\phi_0(\phi)-1))\rho\underline{\nu}(\pi_1(\phi)-\pi_2(\phi)), \\
&= (\sigma - (\phi-(1-\sigma))\phi_0(\phi))(1-\phi) - \rho\underline{\nu}(1 - \frac{\phi}{\beta(1-\delta)})(1-\phi-\sigma(\phi_0(\phi)-1)).
\end{aligned}$$

As:

$$\phi_0(\phi) = \phi^{-\frac{1}{\sigma}} + \frac{\rho\underline{\nu}}{1-\delta}$$

is decreasing with its argument, we have $\phi_0(\phi) \geq 1 + \frac{\rho\underline{\nu}}{1-\delta}$ in the range of interest $[1-\sigma, \beta(1-\delta)]$.

We have:

$$\begin{aligned}
\varphi(1-\sigma) &= \sigma^2 - \rho\underline{\nu}(1 - \frac{1-\sigma}{\beta(1-\delta)})(\sigma - \sigma(\phi_0(\phi)-1)) \\
&= \sigma(\sigma - \rho\underline{\nu}(1 - \frac{1-\sigma}{\beta(1-\delta)})(2-\phi_0(1-\sigma)), \\
&= \sigma(\sigma - \rho\underline{\nu}(1-(1-\sigma)(1+\varepsilon_\beta+\delta))(2-\phi_0(1-\sigma)) + o(\varepsilon_\beta, \delta) \\
&= \sigma(\sigma - \rho\underline{\nu}(\sigma-(1-\sigma)(\varepsilon_\beta+\delta))(2-\phi_0(1-\sigma)) + o(\varepsilon_\beta, \delta) \\
&= \sigma^2(1-\rho\underline{\nu}(2-\phi_0(1-\sigma)) + O(\varepsilon_\beta, \delta)
\end{aligned}$$

Therefore if $2 - \phi_0(1-\sigma) < 0$, $\varphi(1-\sigma) > 0$. Else if $2 - \phi_0(1-\sigma) > 0$ and as $\rho\underline{\nu} < 1$:

$$\begin{aligned}
\varphi(1-\sigma) &> \sigma^2(\phi_0(1-\sigma)-1) + O(\varepsilon_\beta, \delta) \\
&= \sigma^2\rho\underline{\nu} + O(\varepsilon_\beta, \delta) > 0
\end{aligned} \tag{B.31}$$

which proves that $\varphi(1-\sigma) > 0$ in a neighborhood of $\beta = 1$ and $\delta = 0$.

We also have:

$$\varphi(\beta(1-\delta)) = (\sigma - (\beta(1-\delta)-(1-\sigma))\phi_0(\beta(1-\delta)))(1-\beta(1-\delta)).$$

Using $\beta(1-\delta) = 1 - (\varepsilon_\beta + \delta) + o(\varepsilon_\beta, \delta)$:

$$\begin{aligned}\phi_0(\beta(1-\delta)) &= (1-(\varepsilon_\beta+\delta))^{-\frac{1}{\sigma}} + \rho\underline{\nu}(1+\delta) + o(\varepsilon_\beta,\delta),\\ &= 1 + \frac{\varepsilon_\beta+\delta}{\sigma} + \rho\underline{\nu}(1+\delta) + o(\varepsilon_\beta,\delta),\end{aligned}$$

which yields

$$\begin{aligned}\varphi(\beta(1-\delta)) &= (\sigma + (\varepsilon_\beta + \delta - \sigma)(1 + \frac{\varepsilon_\beta+\delta}{\sigma} + \rho\underline{\nu}(1+\delta)))(\varepsilon_\beta+\delta) + o(\varepsilon_\beta,\delta),\\ &= (\sigma(-\rho\underline{\nu}) + (\varepsilon_\beta+\delta)(1+\rho\underline{\nu}) - \sigma(\frac{\varepsilon_\beta+\delta}{\sigma} + \rho\underline{\nu}\delta))(\varepsilon_\beta+\delta) + o(\varepsilon_\beta,\delta),\\ &= -\sigma\rho\underline{\nu}(\varepsilon_\beta+\delta) + o(\varepsilon_\beta,\delta). \qquad \text{(B.32)}\end{aligned}$$

Therefore $\varphi(\beta(1-\delta)) < 0$ in a neighborhood of $\beta = 1$ and $\delta = 0$.

It remains to be seen if $1-\phi-\sigma(\phi_0(\phi)-1)$ keeps the same sign over the range of ϕ. Its derivative is equal to

$$-1 - \sigma\phi_0'(\phi) = -1 - \sigma(-\frac{1}{\sigma}\phi^{-\frac{1}{\sigma}-1}) = -1 + \phi^{-\frac{1}{\sigma}-1} \geq 0 \text{ if } \phi < 1,$$

so that its maximum is bounded by the value at $\phi = 1$ which is equal to $-\sigma\frac{\rho\underline{\nu}}{1-\delta}$. Therefore,

$$1 - \phi - \sigma(\phi_0(\phi) - 1) < 0 \text{ for } \phi \in [1-\sigma, \beta(1-\delta)]. \qquad \text{(B.33)}$$

In summary we have that:

$$\pi_1(\phi) - \pi_2(\phi) = \frac{\varphi(\phi)}{(1-\phi)(1-\phi-\sigma(\phi_0(\phi)-1))\rho\underline{\nu}} \qquad \text{(B.34)}$$

takes negative values when $\phi = 1-\sigma$ because of equations (B.31) and (B.33) and takes positive values when $\phi = \beta(1-\delta)$ because of equations (B.32) and (B.33). It is continuous since $\varphi(\phi)$ and the denominator in equation (B.34) are continuous over this range of values and the denominator is not equal to zero. Then there exists $\phi^* \in [1-\sigma, \beta(1-\delta)]$ such that:

$$\pi^* = \pi_1(\phi^*) = \pi_2(\phi^*) \in (0,1).$$

Remark that the validity conditions on ε_β and δ given in equations (B.31) and (B.32) are weaker the larger σ and the larger $\rho\underline{\nu}$.

B.3 Proof of Proposition 5

Assume that preference shocks η_t have a probability ω to take value 0 and a probability $(1-\omega)$ to take values above $\underline{\eta}> 0$. The Bellman equation writes as a weighted combination of:

$$W_t^{(+)}(i_{t-1}) = \max_{c_t, x_t} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma}\eta_t - \alpha p_t x_t + \beta E_t W_{t+1}(i_t) \right\}$$

$$W_t^{(0)}(i_{t-1}) = \max_{x_t} \left\{ -\alpha p_t x_t + \beta E_t W_{t+1}(i_t) \right\}.$$

under the constraints:

$$i_t = (1-\delta)i_{t-1} + x_t - c_t, i_t \geq 0, x_t \geq 0,$$

The analysis of the static case in case preference shocks are positive is the same as in Section 2.

B.3.1 Static conditions

When preference shocks are zero, we write the Lagrangian as:

$$W_t^{(0)}(i_{t-1}) = \max_{x_t} \left\{ -\alpha p_t x_t + \beta E_t W_{t+1}(i_t) + \mu_t i_t + \psi_t x_t \right\},$$

the multipliers μ_t and ψ_t being associated to the two constraints. We get the first order condition with respect to x_t as:

$$-\alpha p_t + \lambda_t(i_t) + \mu_t + \psi_t = 0.$$

Note however that the constraint on i_t cannot be binding since the control variable cannot but increase stocks from its value $(1-\delta)i_{t-1}$. We can then set $\mu_t = 0$. We thus have two cases:

(i) The constraint on x_t is not binding that is $\psi_t = 0$ and $x_t^* \geq 0$. Optimal stocks are given by:

$$\alpha p_t = \lambda_t(i_t^*),$$

this implies that $\lambda_t(0) \geq \alpha p_t$. We have, $x_t^* = i_t^* - (1-\delta)i_{t-1}$ which is realizable when $(1-\delta)i_{t-1} \leq i_t^*$. If $(1-\delta)i_{t-1} > i_t^*$ the constraint on x_t is binding.

(ii) The constraint on x_t is binding that is $\psi_t > 0$ and $x_t^* = 0$. Then $i_t = (1-\delta)i_{t-1}$ and $\lambda_t(i_t) = \alpha p_t - \psi_t < \alpha p_t$.

In summary:

(i) If $\lambda_t((1-\delta)i_{t-1}) < \alpha p_t$ then $i_t = (1-\delta)i_{t-1}$ and $x_t^* = 0$

(ii) Else if $\lambda_t((1-\delta)i_{t-1}) \geq \alpha p_t$ then $\alpha p_t = \lambda_t(i_t^*)$ and $x_t^* = i_t^* - (1-\delta)i_{t-1}$.

The dynamic equation remains similar when we set $\mu_t = 0$:

$$\begin{aligned} \frac{\partial W_t^{(0)}(i_{t-1})}{\partial i_{t-1}} &= \beta(1-\delta)E_t \frac{\partial W_{t+1}(i_t)}{\partial i_t}, \\ &\Longrightarrow \lambda_{t-1} = \beta(1-\delta)E(\lambda_t + \mu_t) \end{aligned}$$

B.3.2 Dynamics

We now have:

$$\lambda(i_t) = \beta(1-\delta)\left\{\omega E(\lambda(i_{t+1}) \mid \eta_{t+1}=0) + (1-\omega)E(\lambda(i_{t+1}) + \mu_{t+1} \mid \eta_{t+1} > \underline{\eta})\right\}. \tag{B.35}$$

No initial stocks First assume that $i_t = 0$. When preference shocks are positive, purchases are necessarily positive and the expected marginal value of stocks is equal to the expected price weighted by the marginal value of money:

$$E(\lambda(i_{t+1}) + \mu_{t+1} \mid \eta_{t+1} > \underline{\eta}) = E(\alpha p_{t+1}) = \alpha p^a.$$

When preference shocks are zero, we derive from the static conditions in the previous section that:

$$\begin{aligned} E(\lambda(i_{t+1}) \quad \mid \quad & \eta_{t+1}=0) = E\left[\alpha p_{t+1}\mathbf{1}\left\{\lambda(0) \geq \alpha p_{t+1}\right\} + \lambda(0)\mathbf{1}\left\{\lambda(0) < \alpha p_{t+1}\right\}\right] \\ = \quad & \alpha p^a + E\left[(\lambda(0) - \alpha p_{t+1})\mathbf{1}\left\{\lambda(0) < \alpha p_{t+1}\right\}\right]. \end{aligned}$$

Using equation (B.35) we get:

$$\lambda(0) = \beta(1-\delta)\alpha p^a + \beta(1-\delta)\omega E\left[(\lambda(0) - \alpha p_{t+1})\mathbf{1}\left\{\lambda(0) < \alpha p_{t+1}\right\}\right]$$

If there are stocks in equilibrium and all agents coordinate on the expectations that future prices belong in the set $\{p_L, p_U\}$ and the low price has probability $\pi \in (0,1)$ then $\alpha p_L < \lambda(0) < \alpha p_U$ and

$$\lambda(0) = \beta(1-\delta)\alpha p^a + \beta(1-\delta)\omega(1-\pi)(\lambda(0) - \alpha p_U) \tag{B.36}$$

so that :

$$\lambda(0) = \frac{\beta(1-\delta)\alpha(p^a - \omega(1-\pi)p_U)}{1-\beta(1-\delta)\omega(1-\pi)}.$$

Since $p_U > \beta(1-\delta)p^a$, we have:

$$\frac{\partial\lambda(0)}{\partial\omega} < 0,$$

because zero preference shocks decreases the value of stocking.

Positive initial stocks Returning to the general argument, the second term of equation (B.35) is the usual term and we can write:

$$E(\lambda(i_{t+1}) + \mu_{t+1} \mid \eta_{t+1} > 0) = \beta(1-\delta)\alpha p^a + \beta(1-\delta)\gamma(i_t),$$

in which

$$\begin{aligned}\gamma(i_t) &= \mathbb{E}(\left[((1-\delta)i_t)^{-\sigma}\eta_{t+1} - \alpha p_{t+1}\right]\mathbf{1}\left\{\alpha p_{t+1} > \lambda(0), \eta_{t+1} \leq \frac{\alpha p_{t+1}}{((1-\delta)i_t)^{-\sigma}}\right\} \mid \eta_{t+1} > \underline{\eta}) \leq 0,\\ &= \alpha p_U f(\frac{((1-\delta)i_t)^{-\sigma}}{\alpha p_U}).\end{aligned}$$

The first term of equation (B.35) can be written using the static conditions given in the previous section:

$$\begin{aligned}E(\lambda(i_{t+1}) \quad &| \quad \eta_{t+1} = 0) = E\left[\alpha p_{t+1}\mathbf{1}\left\{\lambda((1-\delta)i_t) > \alpha p_{t+1}\right\} + \lambda((1-\delta)i_t)\mathbf{1}\left\{\lambda((1-\delta)i_t) \leq \alpha p_{t+1}\right\}\right],\\ &= \quad E\left[\alpha p_{t+1} + (\lambda((1-\delta)i_t) - \alpha p_{t+1})\mathbf{1}\left\{\lambda((1-\delta)i_t) \leq \alpha p_{t+1}\right\}\right].\end{aligned}$$

If all agents coordinate on the expectations that future prices belong in the set $\{p_L, p_U\}$ then $\alpha p_L \leq \lambda(0) < \alpha p_U$. Therefore, for any i_t we have $\lambda((1-\delta)i_t) \leq \alpha p_U$ since $\lambda(.)$ is non increasing and $\lambda((1-\delta)i_t) \geq \alpha p_L$ if $(1-\delta)i_t \leq i(p_L)$. As $(1-\delta)i_t > i(p_L)$ is never played at the future equilibrium, we have:

$$E(\lambda(i_{t+1}) \mid \eta_{t+1} = 0) = \alpha p^a + (1-\pi)\left(\lambda((1-\delta)i_t) - \alpha p_U\right).$$

Using results from the positive preference shocks regime we can write:

$$\lambda(i_t) = \beta(1-\delta)\alpha p^a + \beta(1-\delta)(\omega(1-\pi)\left(\lambda((1-\delta)i_t) - \alpha p_U\right) + (1-\omega)\gamma(i_t))$$

Subtracting equation (B.36) that defines $\lambda(0)$, we get:

$$\lambda(i_t) = \lambda(0) + \beta(1-\delta)(\omega(1-\pi)((\lambda((1-\delta)i_t) - \lambda(0)) + (1-\omega)\gamma(i_t)).$$

This equation can be written as:

$$\Delta_k = \beta(1-\delta)(\omega(1-\pi)\Delta_{k+1} + (1-\omega)\gamma((1-\delta)^k i_t))$$

in which for any $k \geq 0$:

$$\Delta_k = \lambda((1-\delta)^k i_t) - \lambda(0),$$

and therefore:

$$\begin{aligned}\lambda(i_t) &= \lambda(0) + \beta(1-\delta)(1-\omega)\sum_{k=0}^{\infty}\gamma((1-\delta)^k i_t)\left[\beta(1-\delta)\omega(1-\pi)\right]^k,\\ &= \lambda(0) + \beta(1-\delta)(1-\omega)\alpha p_U\sum_{k=0}^{\infty} f(\frac{((1-\delta)^k i_t)^{-\sigma}}{\alpha p_U})\left[\beta(1-\delta)\omega(1-\pi)\right]^k,\\ &= \lambda(0) + \beta(1-\delta)(1-\omega)\alpha p_U\widetilde{\gamma}(i_t)\end{aligned}$$

in which:

$$\widetilde{\gamma}(i_t) = \sum_{k=0}^{\infty} f(\frac{((1-\delta)^{k+1}i_t)^{-\sigma}}{\alpha p_U})\,[\beta(1-\delta)\omega(1-\pi)]^k \leq f(\frac{((1-\delta)i_t)^{-\sigma}}{\alpha p_U})$$

the bound being the first term in the series and obtained since $f < 0$. If $\omega = 0$, the bound is attained and we get the usual expression.

Furthermore as $|f| < 1$and $\beta(1-\delta)\omega(1-\pi)$ the infinite series is convergent for any i_t and $\widetilde{\gamma}(i_t)$ inherits the differentiability, invertibility and concavity properties of f.

Optimal stocks Note first that if for $i_t^{(0)}$, we have:

$$f(\frac{((1-\delta)i_t^{(0)})^{-\sigma}}{\alpha p_U}) = 0$$

then for any $i_t < i_t^{(0)}$ we have

$$f(\frac{((1-\delta)i_t)^{-\sigma}}{\alpha p_U}) = 0$$

since f is increasing in its argument. This implies in particular that:

$$\forall k \geq 0, f(\frac{((1-\delta)^k i_t^{(0)})^{-\sigma}}{\alpha p_U}) = 0$$

Thus the equation $\widetilde{\gamma}(i(p_L)) = 0$ has the solution corresponding to the minimum value which makes function f equal to zero and therefore:

$$\frac{((1-\delta)i(p_L))^{-\sigma}}{\alpha p_U} = \frac{1}{\underline{\eta}}$$

the usual solution when $\omega = 0$.

The optimal stock is now obtained by setting:

$$\lambda(i_t) = \alpha p_t$$

and thus:

$$\begin{aligned} \beta(1-\delta)(1-\omega)\alpha p_U \widetilde{\gamma}(i_t) &= \alpha p_t - \lambda(0) \\ &= \alpha p_t - \frac{\beta(1-\delta)\alpha(p^a - \omega(1-\pi)p_U)}{1-\beta(1-\delta)\omega(1-\pi)} \end{aligned}$$

so that:

$$\widetilde{\gamma}(i_t) = \frac{p_t}{\beta(1-\delta)(1-\omega)p_U} - \frac{(p^a - \omega(1-\pi)p_U)}{(1-\omega)(1-\beta(1-\delta)\omega(1-\pi))p_U}.$$

and:

$$i_t = \widetilde{\gamma}^{-1}\left(\frac{p_t}{\beta(1-\delta)(1-\omega)p_U} - \frac{(p^a - \omega(1-\pi)p_U)}{(1-\omega)(1-\beta(1-\delta)\omega(1-\pi))p_U}\right)$$

By the above, stocks are now larger than in the positive preference case.

We have seen in Section B.1 that maximizing profits leads the firm to make the consumer indifferent between owning or not stocks, and thus to set the argument of the previous function to 0 and thus:

$$p_L = \frac{\beta(1-\delta)(p^a - \omega(1-\pi)p_U)}{1-\beta(1-\delta)\omega(1-\pi)}. \tag{B.37}$$

We then obtain that the optimal stocks are $\rho i(p_{L)}$.

B.3.3 Demands

When preference shocks are positive at two successive dates (with probability $(1-\omega)^2$), we have the usual demands given by equation (10) in which we have used equation (B.37):

$$\begin{aligned}
x_t(p_U, p_U) &= \alpha^{-\frac{1}{\sigma}} p_U^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}}, \\
x_t(p_U, p_L) &= \alpha^{-\frac{1}{\sigma}} \left[p_L^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}} + \frac{\rho}{1-\delta} p_U^{-\frac{1}{\sigma}} \underline{\eta}^{\frac{1}{\sigma}} \right], \\
x_t(p_L, p_L) &= \alpha^{-\frac{1}{\sigma}} \left[p_L^{-\frac{1}{\sigma}} \eta_t^{\frac{1}{\sigma}} + \frac{\rho\delta}{1-\delta} p_U^{-\frac{1}{\sigma}} \underline{\eta}^{\frac{1}{\sigma}} \right], \\
x_t(p_L, p_U) &= \alpha^{-\frac{1}{\sigma}} p_U^{-\frac{1}{\sigma}} (\eta_t^{\frac{1}{\sigma}} - \rho \underline{\eta}^{\frac{1}{\sigma}}).
\end{aligned}$$

If preference shocks are positive last period and zero at the current period we have:

$$\begin{aligned}
x_t(p_U, p_U) &= 0, \\
x_t(p_U, p_L) &= \alpha^{-\frac{1}{\sigma}} \left[\frac{\rho}{1-\delta} p_U^{-\frac{1}{\sigma}} \underline{\eta}^{\frac{1}{\sigma}} \right], \\
x_t(p_L, p_L) &= \alpha^{-\frac{1}{\sigma}} \left[\frac{\rho\delta}{1-\delta} p_U^{-\frac{1}{\sigma}} \underline{\eta}^{\frac{1}{\sigma}} \right], \\
x_t(p_L, p_U) &= 0.
\end{aligned}$$

This provides the case with missing prices. We can then impute p_U if we do not observe purchases. It means that if prices are low, stocks are put at their optimal values. If prices are high, existing stocks are depleted in the regime p_L, p_U from $i(p_L)$ to $(1-\delta)i(p_L)$. The imputation of p_U has no effect on demands apart from assigning these observations to a regime of high prices.

It means that we can chain over the zero purchase dates, say p^m. We distinguish two cases according to whether preferences are positive or not at period t :

1. Preferences are positive with probability $1-\omega$ and:

$$\begin{aligned} x_t\left(p_U,\ \overbrace{p^m,.,p^m}^{m\text{ missing dates}},p_U\right) &= \alpha^{-\frac{1}{\sigma}}p_U^{-\frac{1}{\sigma}}\eta_t^{\frac{1}{\sigma}}, \\ x_t\left(p_U,p^m,.,p^m,p_L\right) &= \alpha^{-\frac{1}{\sigma}}\left[p_L^{-\frac{1}{\sigma}}\eta_t^{\frac{1}{\sigma}}+\frac{\rho}{1-\delta}p_U^{-\frac{1}{\sigma}}\underline{\eta}^{\frac{1}{\sigma}}\right], \\ x_t\left(p_L,p^m,.,p^m,p_L\right) &= \alpha^{-\frac{1}{\sigma}}\left[p_L^{-\frac{1}{\sigma}}\eta_t^{\frac{1}{\sigma}}+\frac{\rho(1-(1-\delta)^{m+1})}{1-\delta}p_U^{-\frac{1}{\sigma}}\underline{\eta}^{\frac{1}{\sigma}}\right], \\ x_t\left(p_L,p^m,.,p^m,p_U\right) &= \alpha^{-\frac{1}{\sigma}}p_U^{-\frac{1}{\sigma}}(\eta_t^{\frac{1}{\sigma}}-\rho(1-\delta)^m\underline{\eta}^{\frac{1}{\sigma}}). \end{aligned} \tag{B.38}$$

2. Preferences are zero with probability ω and:

$$\begin{aligned} & x_t\left(p_U,\ \overbrace{p^m,.,p^m}^{m\text{ missing dates}},p_U\right)=0 \\ & x_t\left(p_U,p^m,.,p^m,p_L\right)=\alpha^{-\frac{1}{\sigma}}\left[\frac{\rho}{1-\delta}p_U^{-\frac{1}{\sigma}}\underline{\eta}^{\frac{1}{\sigma}}\right]\exp(\xi_{it}), \\ & x_t\left(p_L,p^m,.,p^m,p_L\right)=\alpha^{-\frac{1}{\sigma}}\left[\frac{\rho(1-(1-\delta)^{m+1})}{1-\delta}p_U^{-\frac{1}{\sigma}}\underline{\eta}^{\frac{1}{\sigma}}\right]\exp(\xi_{it}), \\ & x_t\left(p_L,p^m,.,p^m,p_U\right)=0 \end{aligned} \tag{B.39}$$

B.3.4 Profits

Consumer segmentation At period t, the firm faces different subpopulations of consumers according to the realizations of their preference shocks and the prices they faced during the previous periods.

- Some consumers may have purchased in period $t-1$ (i.e. the number of missing prices before t is $m=0$) and this happens because in the previous period, they have had positive preference shocks or because they have had no taste for the product but faced low prices. This happens with probability $(1-\omega+\omega\pi)$. Among those consumers, the probabilities of high and low prices at period $t-1$ are:

$$\begin{aligned} \Pr(p_{t-1} &= p_L \mid m=0)=\frac{(1-\omega)\pi+\omega\pi}{1-\omega+\omega\pi}=\frac{\pi}{1-\omega+\omega\pi}, \\ \Pr(p_{t-1} &= p_U \mid m=0)=\frac{(1-\omega)(1-\pi)}{1-\omega+\omega\pi}. \end{aligned}$$

- Other consumers may have purchased in period $t-m-1$ but not after until period t (i.e. the number of missing prices before t is m). This happens because in period $t-m-1$, they have had positive preference shocks or because they have had no taste for the product but faced low prices whereas in the intermediate periods they had no taste and faced high prices. This happens with probability $(1-\omega+\omega\pi)(\omega(1-\pi))^m$.Among those consumers, the probabilities of high and low prices at period $t-1$ are the same as above since the no purchase periods do not matter:

$$\begin{aligned}
\Pr(p_{t-m-1} &= p_L \mid m) = \frac{\pi}{1-\omega+\omega\pi}, \\
\Pr(p_{t-m-1} &= p_U \mid m) = \frac{(1-\omega)(1-\pi)}{1-\omega+\omega\pi}.
\end{aligned}$$

High price profits Using the system of demand (B.38) and (B.39) and assuming that a high price is played, we get that demands are equal to the sum of expected demands obtained through the segmentation of consumers given above. Specifically we have that consumers purchasing when prices are high consist of two sub-populations whose probabilities are:

$$\begin{aligned}
\Pr(\eta_t &> 0, p_{t-m-1} = p_U, m) = \Pr(\eta_t > 0, p_t = p_U)(1-\omega)(\omega(1-\pi))^m, \\
&= (1-\omega)(1-\omega)(1-\pi)(\omega(1-\pi))^m \\
\Pr(\eta_t &> 0, p_{t-m-1} = p_L, m) = \Pr(\eta_t > 0, p_t = p_U)\pi(\omega(1-\pi))^m,
\end{aligned}$$

and in consequence if prices are set to p_U, the expected demand is:

$$D_U = (1-\omega)\left[\begin{array}{c} (1-\omega)(1-\pi)\sum_{m=0}^{\infty}(\omega(1-\pi))^m E(\alpha^{-\frac{1}{\sigma}})p_U^{-\frac{1}{\sigma}}E(\eta_t^{\frac{1}{\sigma}})+ \\ \pi\sum_{m=0}^{\infty}(\omega(1-\pi))^m E(\alpha^{-\frac{1}{\sigma}})p_U^{-\frac{1}{\sigma}}(E(\eta_t^{\frac{1}{\sigma}})-\rho(1-\delta)^m\underline{\eta}^{\frac{1}{\sigma}}). \end{array}\right]$$

Normalizing $E(\alpha^{-\frac{1}{\sigma}}) = k_\alpha$ and recomposing, this yields:

$$\begin{aligned}
D_U &= k_\alpha(1-\omega)\left[\frac{(1-\omega)(1-\pi)+\pi}{1-\omega+\omega\pi}E(\eta_t^{\frac{1}{\sigma}})p_U^{-\frac{1}{\sigma}} - \rho\pi p_U^{-\frac{1}{\sigma}}\underline{\eta}^{\frac{1}{\sigma}}\sum_{m=0}^{\infty}(\omega(1-\pi))^m(1-\delta)^m\right], \\
&= k_\alpha(1-\omega)p_U^{-\frac{1}{\sigma}}\left[E(\eta_t^{\frac{1}{\sigma}}) - \frac{\rho\pi\underline{\eta}^{\frac{1}{\sigma}}}{1-\omega(1-\pi)(1-\delta)}\right], \\
&= k(1-\omega)p_U^{-\frac{1}{\sigma}}\left[1-\pi\rho\underline{\nu}\theta(\pi)\right],
\end{aligned}$$

in which we have set:

$$\theta(\pi) = \frac{1}{1-\omega(1-\pi)(1-\delta)}.$$

Following the same steps as the ones before equation (B.26) we obtain:

$$p_U = \frac{\kappa}{1-\sigma(1-\pi\rho\underline{\nu}\theta(\pi))}.$$

and the profit is :

$$\Pi_U = k(1-\omega)(p_U-\kappa)p_U^{-\frac{1}{\sigma}}\left[1-\rho\pi\underline{\nu}\theta(\pi)\right].$$

Low price profits Using the systems of demand (B.38) and (B.39) and assuming that a low price is played, we get that demands are equal to the sum of expected demands obtained through the segmentation of consumers given above:

$$\begin{array}{rcl} D_L & = & k(1-\omega)\left[\begin{array}{c}(1-\omega)(1-\pi)\sum_{m=0}^{\infty}(\omega(1-\pi))^m(p_L^{-\frac{1}{\sigma}}E\eta_t^{\frac{1}{\sigma}}+\frac{\rho}{1-\delta}p_U^{-\frac{1}{\sigma}}\underline{\eta}^{\frac{1}{\sigma}})+\\ \pi\sum_{m=0}^{\infty}(\omega(1-\pi))^m(p_L^{-\frac{1}{\sigma}}E(\eta_t^{\frac{1}{\sigma}})+\rho\frac{1-(1-\delta)^{m+1}}{1-\delta}p_U^{-\frac{1}{\sigma}}\underline{\eta}^{\frac{1}{\sigma}})\end{array}\right] \\ & & +k\omega\left[\begin{array}{c}(1-\omega)(1-\pi)\sum_{m=0}^{\infty}(\omega(1-\pi))^m\frac{\rho}{1-\delta}p_U^{-\frac{1}{\sigma}}\underline{\eta}^{\frac{1}{\sigma}}+\\ \pi\sum_{m=0}^{\infty}(\omega(1-\pi))^m\rho\frac{1-(1-\delta)^{m+1}}{1-\delta}p_U^{-\frac{1}{\sigma}}\underline{\eta}^{\frac{1}{\sigma}}\end{array}\right]\end{array}$$

Recomposing, this yields:

$$\begin{aligned} D_L &= k_\alpha(1-\omega)\left[p_L^{-\frac{1}{\sigma}}E(\eta_t^{\frac{1}{\sigma}})+\frac{\rho}{1-\delta}p_U^{-\frac{1}{\sigma}}\underline{\eta}^{\frac{1}{\sigma}}-\rho\pi p_U^{-\frac{1}{\sigma}}\underline{\eta}^{\frac{1}{\sigma}}\sum_{m=0}^{\infty}(\omega(1-\pi))^m(1-\delta)^m\right]+ \\ &\quad +k_\alpha\omega\left[\frac{\rho}{1-\delta}p_U^{-\frac{1}{\sigma}}\underline{\eta}^{\frac{1}{\sigma}}-\rho\pi p_U^{-\frac{1}{\sigma}}\underline{\eta}^{\frac{1}{\sigma}}\sum_{m=0}^{\infty}(\omega(1-\pi))^m(1-\delta)^m\right], \\ &= k\left[(1-\omega)p_L^{-\frac{1}{\sigma}}+\frac{\rho}{1-\delta}p_U^{-\frac{1}{\sigma}}\underline{\nu}-\rho\pi p_U^{-\frac{1}{\sigma}}\underline{\nu}\frac{1}{1-\omega(1-\pi)(1-\delta)}\right], \\ &= k\left[(1-\omega)p_L^{-\frac{1}{\sigma}}+(\frac{1}{1-\delta}-\pi\theta(\pi))\rho\underline{\eta}^{\frac{1}{\sigma}}p_U^{-\frac{1}{\sigma}}\right], \end{aligned}$$

by defining θ as above. The profit becomes:

$$\Pi_L = k(p_L-\kappa)\left[(1-\omega)p_L^{-\frac{1}{\sigma}}+(\frac{1}{1-\delta}-\pi\theta(\pi))\rho\underline{\nu}p_U^{-\frac{1}{\sigma}}\right].$$

Equalizing profits The equalization of profits lead to:

$$\begin{aligned} (p_L-\kappa)\left[(1-\omega)p_L^{-\frac{1}{\sigma}}+(\frac{1}{1-\delta}-\pi\theta(\pi))\rho\underline{\nu}p_U^{-\frac{1}{\sigma}}\right] &= (1-\omega)(p_U-\kappa)p_U^{-\frac{1}{\sigma}}\left[1-\rho\pi\underline{\nu}\theta(\pi)\right], \\ \Longleftrightarrow (p_L-(1-\sigma+\sigma\pi\rho\underline{\nu}\theta)p_U)\left[(1-\omega)p_L^{-\frac{1}{\sigma}}+(\frac{1}{1-\delta}-\pi\theta)\rho\underline{\nu}p_U^{-\frac{1}{\sigma}}\right] &= p_U(1-\omega)(1-(1-\sigma+\sigma\pi\rho\underline{\nu}\theta))p_U^{-} \\ \Longleftrightarrow (\phi-(1-\sigma+\sigma\pi\rho\underline{\nu}\theta))\left[(1-\omega)\phi^{-\frac{1}{\sigma}}+(\frac{1}{1-\delta}-\pi\theta)\rho\underline{\nu}\right] &= (1-\omega)\sigma(1-\pi\rho\underline{\nu}\theta)^2. \end{aligned}$$

The second equation is provided by:

$$\alpha_i p_L = \lambda(0)$$

and therefore:

$$p_L = \frac{\beta(1-\delta)(p^a - \omega(1-\pi)p_U)}{1-\beta(1-\delta)\omega(1-\pi)}.$$

This yields:

$$\begin{aligned}(1-\beta(1-\delta)\omega(1-\pi))p_L &= \beta(1-\delta)(\pi p_L + (1-\pi)p_U - \omega(1-\pi)p_U) \\ &\iff (1-\beta(1-\delta)\omega(1-\pi))\phi = \beta(1-\delta)(\pi\phi + (1-\pi)(1-\omega)).\end{aligned}$$

the second equation on the supply side.

C Proofs of Section 4

C.1 Proof of Lemma 7

By setting $\varepsilon_t = \eta_t^{\frac{1}{\sigma}}$ and $\underline{\varepsilon} = \underline{\eta}^{\frac{1}{\sigma}}$, we have the following demand equations

$$\begin{cases} x_t\,(p_U, p_U) = \alpha^{\frac{1}{\sigma}} p_U^{-\frac{1}{\sigma}} \varepsilon_t \exp(\xi_t), \\ x_t\,(p_U, p_L) = (\alpha^{\frac{1}{\sigma}} p_L^{-\frac{1}{\sigma}} \varepsilon_t + \frac{\rho}{1-\delta} \alpha p_U^{-\frac{1}{\sigma}} \underline{\varepsilon}) \exp(\xi_t), \\ x_t\,(p_L, p_L) = (\alpha^{\frac{1}{\sigma}} p_L^{-\frac{1}{\sigma}} \varepsilon_t + \delta\frac{\rho}{1-\delta} \alpha p_U^{-\frac{1}{\sigma}} \underline{\varepsilon}) \exp(\xi_t), \\ x_t\,(p_L, p_U) = (\alpha^{\frac{1}{\sigma}} p_U^{-\frac{1}{\sigma}} \varepsilon_t - \rho\alpha p_U^{-\frac{1}{\sigma}} \underline{\varepsilon}) \exp(\xi_t). \end{cases}$$

Taking logs:

$$\begin{cases} \log x_t\,(p_U, p_U) = -\frac{1}{\sigma}\ln\alpha - \frac{1}{\sigma}\ln p_U + \log(\varepsilon_t) + \xi_t, \\ \log x_t\,(p_U, p_L) = -\frac{1}{\sigma}\ln\alpha - \frac{1}{\sigma}\ln p_L + \log(\varepsilon_t + \frac{\rho}{1-\delta}(\frac{p_L}{p_U})^{\frac{1}{\sigma}}\underline{\varepsilon}) + \xi_t, \\ \log x_t\,(p_L, p_L) = -\frac{1}{\sigma}\ln\alpha - \frac{1}{\sigma}\ln p_L + \log(\varepsilon_t + \frac{\rho\delta}{1-\delta}(\frac{p_L}{p_U})^{\frac{1}{\sigma}}\underline{\varepsilon}) + \xi_t, \\ \log x_t\,(p_L, p_U) = -\frac{1}{\sigma}\ln\alpha - \frac{1}{\sigma}\ln p_U + \log(\varepsilon_t - \rho\underline{\varepsilon}) + \xi_t. \end{cases}$$

We log linearize the terms $\log(\varepsilon_t)$ around $\log(E(\varepsilon_t))$ that is:

$$\begin{aligned} \log(E\varepsilon_t + (\varepsilon_t - E\varepsilon_t)) &= \log(E\varepsilon_t) + \frac{\varepsilon_t - E\varepsilon_t}{E\varepsilon_t}, \\ \log(\varepsilon_t + \frac{\rho}{1-\delta}(\frac{p_L}{p_U})^{\frac{1}{\sigma}}\underline{\varepsilon}) &= \log(E\varepsilon_t + \frac{\rho}{1-\delta}(\frac{p_L}{p_U})^{\frac{1}{\sigma}}\underline{\varepsilon}) + \frac{\varepsilon_t - E\varepsilon_t}{E\varepsilon_t + \frac{\rho}{1-\delta}(\frac{p_L}{p_U})^{\frac{1}{\sigma}}\underline{\varepsilon}}, \\ \log(\varepsilon_t + \frac{\rho\delta}{1-\delta}(\frac{p_L}{p_U})^{\frac{1}{\sigma}}\underline{\varepsilon}) &= \log(E\varepsilon_t + \frac{\rho\delta}{1-\delta}(\frac{p_L}{p_U})^{\frac{1}{\sigma}}\underline{\varepsilon}) + \frac{\varepsilon_t - E\varepsilon_t}{E\varepsilon_t + \frac{\rho\delta}{1-\delta}(\frac{p_L}{p_U})^{\frac{1}{\sigma}}\underline{\varepsilon}}, \\ \log(\varepsilon_t - \rho\underline{\varepsilon}) &= \log(E\varepsilon_t - \rho\underline{\varepsilon}) + \frac{\varepsilon_t - E\varepsilon_t}{E\varepsilon_t - \rho\underline{\varepsilon}}. \end{aligned}$$

By Setting $\underline{\nu}= \frac{\varepsilon}{E\varepsilon_t} = \frac{\eta^{\frac{1}{\sigma}}}{E\eta_t^{\frac{1}{\sigma}}}$, and $\widetilde{\alpha} = \frac{-\ln\alpha}{\sigma} + \log(E\varepsilon_t)$ we obtain the system of equations displayed in Lemma 7 in which:

$$\begin{aligned}
\epsilon_t^{(UU)} &= \frac{\varepsilon_t - E\varepsilon_t}{E\varepsilon_t}, \\
\epsilon_t^{(UL)} &= \frac{\frac{\varepsilon_t}{E\varepsilon_t} - 1}{1 + \frac{\rho}{1-\delta}(\frac{p_L}{p_U})^{\frac{1}{\sigma}}\underline{\nu}}, \\
\epsilon_t^{(LL)} &= \frac{\frac{\varepsilon_t}{E\varepsilon_t} - 1}{1 + \frac{\rho\delta}{1-\delta}(\frac{p_L}{p_U})^{\frac{1}{\sigma}}\underline{\nu}}, \\
\epsilon_t^{(LU)} &= \frac{\frac{\varepsilon_t}{E\varepsilon_t} - 1}{1 - \rho\underline{\nu}},
\end{aligned}$$

all of mean zero by construction.

C.2 Tables

Table 7 (part 1):

	(1)	(2)	(3)	(4)	(5)	(6)
Variables	$\ln x_{ht}$	$\ln x_{ht}$	$\ln x_{ht}$	$\ln x_{ht}$	$\ln x_{ht}$	$\ln x_{ht}$
$\ln p_U$	-1.974***	-3.148***	-3.072***	-3.039***	-3.048***	-3.081***
	(0.341)	(0.124)	(0.116)	(0.107)	(0.103)	(0.101)
$\ln p_L$	-2.796***	-2.816***	-2.771***	-2.805***	-2.823***	-2.867***
	(0.0865)	(0.0630)	(0.0577)	(0.0519)	(0.0500)	(0.0486)
α_{UL}	-0.156***	0.0383**	0.0356**	0.0333**	0.0328**	0.0282**
	(0.0603)	(0.0169)	(0.0159)	(0.0148)	(0.0145)	(0.0142)
α_{LL}^0	-0.211***	-0.0217	-0.0230*	-0.0259*	-0.0266**	-0.0290**
	(0.0592)	(0.0141)	(0.0138)	(0.0134)	(0.0133)	(0.0132)
α_{LL}^1		0.00564	0.00226	-0.000236	-0.000682	-0.00347
		(0.0147)	(0.0144)	(0.0140)	(0.0139)	(0.0139)
α_{LL}^2			0.0123	0.00628	0.00506	0.00233
			(0.0159)	(0.0155)	(0.0155)	(0.0154)
α_{LL}^3				-0.0136	-0.0141	-0.0160
				(0.0171)	(0.0171)	(0.0170)
α_{LL}^4				-0.00545	-0.00586	-0.0103
				(0.0191)	(0.0191)	(0.0190)
α_{LL}^5				0.0166	0.0159	0.0118
				(0.0219)	(0.0219)	(0.0219)
α_{LL}^6					0.0117	0.00798
					(0.0247)	(0.0246)
α_{LL}^7					-0.0406	-0.0411
					(0.0265)	(0.0264)
α_{LL}^8					-0.0449	-0.0456
					(0.0293)	(0.0292)
α_{LL}^9					-0.0163	-0.0194
					(0.0333)	(0.0332)
α_{LL}^{10}						-0.0267
						(0.0348)
α_{LL}^{11}						0.0122
						(0.0418)
α_{LL}^{12}						0.0432
						(0.0427)
α_{LL}^{13}						-0.137***
						(0.0492)
α_{LL}^{14}						-0.0226
						(0.0466)
α_{LL}^{15}						-0.0182
						(0.0509)
α_{LL}^{16}						0.0418
						(0.0555)
α_{LL}^{17}						0.0540
						(0.0575)
α_{LL}^{18}						-0.0742
						(0.0629)
α_{LL}^{19}						-0.0329
						(0.0660)

Table 7 (part 2):						
	(1)	(2)	(3)	(4)	(5)	(6)
Variables	$\ln x_{ht}$	$\ln x_{ht}$	$\ln x_{ht}$	$\ln x_{ht}$	$\ln x_{ht}$	$\ln x_{ht}$
α_{LU}^{0}	0.0669***	0.0706***	0.0734***	0.0853***	0.0883***	0.0902***
	(0.0180)	(0.0160)	(0.0157)	(0.0153)	(0.0152)	(0.0152)
α_{LU}^{1}		0.0806***	0.0775***	0.0849***	0.0873***	0.0887***
		(0.0186)	(0.0182)	(0.0179)	(0.0178)	(0.0178)
α_{LU}^{2}			0.0493**	0.0573***	0.0604***	0.0616***
			(0.0220)	(0.0216)	(0.0216)	(0.0216)
α_{LU}^{3}				0.0412*	0.0458*	0.0476*
				(0.0249)	(0.0248)	(0.0248)
α_{LU}^{4}				0.0833***	0.0847***	0.0850***
				(0.0301)	(0.0301)	(0.0300)
α_{LU}^{5}				-0.0226	-0.0210	-0.0221
				(0.0361)	(0.0360)	(0.0360)
α_{LU}^{6}					0.0398	0.0456
					(0.0383)	(0.0382)
α_{LU}^{7}					0.0674	0.0677
					(0.0440)	(0.0439)
α_{LU}^{8}					0.0379	0.0352
					(0.0483)	(0.0483)
α_{LU}^{9}					-5.66e-06	0.00525
					(0.0574)	(0.0572)
α_{LU}^{10}						0.0346
						(0.0539)
α_{LU}^{11}						0.0335
						(0.0610)
α_{LU}^{12}						0.0631
						(0.0706)
α_{LU}^{13}						0.0220
						(0.0791)
α_{LU}^{14}						-0.0241
						(0.0694)
α_{LU}^{15}						0.0627
						(0.0728)
α_{LU}^{16}						-0.0599
						(0.0844)
α_{LU}^{17}						-0.135
						(0.0961)
α_{LU}^{18}						0.0755
						(0.0959)
α_{LU}^{19}						0.167
						(0.102)
Year FE	yes	yes	yes	yes	yes	yes
Observations	13,403	23,501	27,731	33,837	36,830	39,133
R-squared	0.094	0.099	0.097	0.103	0.104	0.108
Number of hh	1,322	1,468	1,468	1,468	1,468	1,468

Note: Selected households doing at least 10 purchases in the 3 year period.

Table 10: GMM estimation results (Orangina)

Moments used	$M1-M6$	$M1-M6$	$M1-M6$	$M1-M7$	$M1-M7$	$M1-M7$
Lags M	1	2	3	1	2	3
Parameters						
σ	0.4425	0.4028	0.4192	0.3923	0.3781	0.3413
	0.0000	0.0036	0.0039	0.0059	0.0043	0.0067
δ	0.2625	0.2690	0.3204	0.2075	0.2342	0.2674
	0.0000	0.0025	0.0030	0.0075	0.0045	0.0071
$\rho\underline{\nu}$	0.2179	0.2235	0.2349	0.2120	0.2117	0.2065
	0.0000	0.0011	0.0013	0.0029	0.0020	0.0031
$\phi = \frac{p_L}{p_U}$	0.4130	0.4230	0.3988	0.4516	0.4311	0.4920
	0.0000	497.93	3010.9	0.7191	0.6420	1.2815
β	0.9778	1.0000	1.0000	0.9599	0.9604	0.9696
	0.0000	0.0042	0.0041	0.0131	0.0098	0.0141
χ	0.0449	0.0333	0.0357	0.0343	0.0323	0.0264
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ω	0.5262	0.5339	0.5202	0.5145	0.5149	0.4252
	0.0000	0.0024	0.0025	0.0057	0.0043	0.0056
N	3,584	5,827	7,417	3,584	5,827	7,417
$P\left(p_{ht} < s_h\left(\widehat{\chi}\right)\right)$	0.0681	0.0819	0.0747	0.0820	0.0836	0.0911
$P(x_{ht} = 0)$	0.6258	0.6577	0.6805	0.6258	0.6577	0.6805

Note: Standard errors under parameters estimates.

Table 11: GMM estimation results (Orangina)

Moments used	$M1-M8$	$M1-M8$	$M1-M8$	$M1-M9$	$M1-M9$	$M1-M9$
Lags M	1	2	3	1	2	3
Parameters						
σ	0.3923	0.8070	0.5977	0.3923	0.5356	0.5492
	0.0059	0.0088	0.0087	0.0059	0.0092	0.0095
δ	0.2075	0.5780	0.4598	0.2075	0.5578	0.4394
	0.0075	0.0116	0.0088	0.0075	0.0089	0.0076
$\rho\underline{\nu}$	0.2120	0.2398	0.1522	0.2120	0.1201	0.2209
	0.0029	0.0085	0.0077	0.0029	0.0045	0.0043
$\phi = \frac{p_L}{p_U}$	0.4516	0.4016	0.4786	0.4516	0.3153	0.3598
	0.7191	1.8930	4.2682	0.7191	394.85	3.4540
β	0.9599	1.0000	0.9582	0.9599	0.9991	0.9277
	0.0131	0.0000	0.0236	0.0131	0.0264	0.0269
χ	0.0343	0.0881	0.0389	0.0343	0.0339	0.0389
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ω	0.5145	0.0810	0.0900	0.5145	0.2883	0.3433
	0.0057	0.0077	0.0059	0.0057	0.0065	0.0064
N	3,584	5,827	7,417	3,584	5,827	7,417
$P\left(p_{ht} < s_h\left(\widehat{\chi}\right)\right)$	0.0820	0.0369	0.0696	0.0820	0.0815	0.0697
$P(x_{ht} = 0)$	0.6258	0.6577	0.6805	0.6258	0.6577	0.6805

Note: Standard errors under parameters estimates.

Table 12: GMM estimation results (Pepsi)

Moments used	$M1-M6$	$M1-M6$	$M1-M6$	$M1-M7$	$M1-M7$	$M1-M7$
Lags M	1	2	3	1	2	3
Parameters						
σ	0.4013	0.4020	0.4331	0.4663	0.4001	0.3901
	0.0000	0.0023	0.0020	0.0081	0.0070	0.0037
δ	0.0075	0.0134	0.2039	0.0000	0.0039	0.1049
	0.0000	0.0046	0.0028	0.0000	0.0155	0.0070
$\rho\underline{\nu}$	0.2001	0.2010	0.2252	0.2009	0.2001	0.2062
	0.0000	0.0010	0.0010	0.0051	0.0029	0.0021
$\phi = \frac{p_L}{p_U}$	0.5701	0.5674	0.4800	0.8282	0.5725	0.5201
	0.0000	0.4318	1.9187	0.2110	5.0213	0.1924
β	0.8829	0.8855	0.9957	0.8784	0.8814	0.9151
	0.0000	0.0045	0.0045	0.0148	0.0143	0.0096
χ	0.0401	0.0399	0.0365	0.0233	0.0399	0.0362
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ω	0.5001	0.5015	0.5558	0.2260	0.4998	0.5025
	0.0000	0.0026	0.0025	0.0066	0.0072	0.0050
N	2,540	3,993	4,988	2,540	3,993	4,988
$P\left(p_{ht} < s_h\left(\widehat{\chi}\right)\right)$	0.0835	0.0756	0.0776	0.1323	0.0756	0.0778
$P(x_{ht}=0)$	0.5720	0.6130	0.6421	0.5720	0.6130	0.6421

Note: Standard errors under parameters estimates.

Table 13: GMM estimation results (Pepsi)

Moments used	$M1-M8$	$M1-M8$	$M1-M8$	$M1-M9$	$M1-M9$	$M1-M9$
Lags M	1	2	3	1	2	3
Parameters						
σ	0.4663	0.4012	0.4050	0.4663	0.4018	0.4047
	0.0081	0.0063	0.0048	0.0081	0.0064	0.0049
δ	0.0000	0.0430	0.0374	0.0000	0.0442	0.0351
	0.0000	0.0086	0.0079	0.0000	0.0088	0.0081
$\rho\underline{\nu}$	0.2009	0.2015	0.2011	0.2009	0.2014	0.2004
	0.0051	0.0030	0.0033	0.0051	0.0029	0.0031
$\phi = \frac{p_L}{p_U}$	0.8282	0.5512	0.5502	0.8282	0.5510	0.5527
	0.2110	0.3042	0.3372	0.2110	0.3084	0.3602
β	0.8784	0.8902	0.8863	0.8784	0.8902	0.8860
	0.0148	0.0118	0.0136	0.0148	0.0122	0.0142
χ	0.0233	0.0389	0.0399	0.0233	0.0389	0.0399
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ω	0.2260	0.4944	0.4943	0.2260	0.4935	0.4935
	0.0066	0.0064	0.0068	0.0066	0.0064	0.0070
N	2,540	3,993	4,988	2,540	3,993	4,988
$P\left(p_{ht} < s_h\left(\widehat{\chi}\right)\right)$	0.1323	0.0784	0.0704	0.1323	0.0784	0.0704
$P(x_{ht}=0)$	0.5720	0.6130	0.6421	0.5720	0.6130	0.6421

Note: Standard errors under parameters estimates.