Euler Equation Estimation on Micro Data

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PRELIMINARY

Motivation (1) Euler Equations

- Models of economic agents' dynamic optimization problems are work horses of modern macroeconomics, public finance, etc.
- Since Hall (1978), first order conditions (Euler Equations) used to test models and estimate preference parameters
    - life-cycle model of consumption (Attanasio et. al. 1999)
    - investment behaviour of firms (Bond and Meghir 1994, or Mulligan, 2004)
    - asset pricing (Mehra and Prescott, 1985)
- Euler equation approach removes need to
    - model agents' expectations
    - observe wealth
        - contrast to estimation of consumption function

Motivation (2) Problems with Euler Equation Estimation

- Estimation on aggregate data leads to biased parameter estimates and false rejections of the underlying models (Attanasio and Weber, 1993)
- Measurement error is endemic to micro (survey) data; in the presence of measurement error, standard non-linear GMM methods yield inconsistent estimates (Amemiya 1986)
- Standard solution: apply standard linear IV and GMM techniques to a first-, or possibly second-order approximation to the Euler equation
- New problem: higher order terms that are ignored in the approximation are potentially correlated with the typical instruments (lagged variables)

Motivate (3) Existing Literature

A series of papers:

1. Solve and simulate a life-cycle consumption model
2. Monte-carlo experiments with the simulated data

- **Ludvigson and Paxson (2001)** use a second order approximation to estimate the relative prudence parameters in an environment with a fixed interest rate and impatient agents. Conclude the estimation strategy doesn't work.
- **Carroll (2001) "Death to the log-linearized Euler Equation..")** reaches the same conclusion for the elasticity of intertemporal substitution (EIS) in an environment with cross-sectional variation in interest rates
- **Attanasio and Low (2004)** argue that with sufficiently long sample periods and enough time-series variation in the intertemporal price (interest rate), good estimates of the EIS can be obtained with linearized Euler equations

Our Contribution

- Another Monte Carlo study
- Attempt to reconcile the conclusions reached by different researchers
- Emphasize that key differences in assumed economic environments; previous papers emphasized differences in available data.
    - We solve and simulate 6 different variants of a life-cycle consumption model with CRRA preferences, (aggregate) interest rate uncertainty, and an uninsurable idiosyncratic income risk
- Working hypothesis:
    - problems with instrument validity are related to the severity of the approximation bias
    - the severity of approximation bias is determined by the curvature of the underlying policy functions
    - the curvature of the policy functions follows from assumptions made in specifying the economic environment

Our Contribution (2)

- We are assessing not only instrument validity but also instrument relevance
- Yogo (2004) discusses instrument relevance for consumption Euler equations estimated on aggregate data
- To our knowledge, has not been taken up in the micro literature.
- We preliminary results suggest two tradeoffs:
  - The instruments typically used to estimate consumption Euler equations tend to be strongly relevant in economic environments where they are less valid, and possibly weak in environments in environments where they have greater predictive power
  - Using a second-order approximation reduces problems with instrument validity, but may introduce weak instrument problems

Outline of the Talk

1. Motivation
2. The Econometrics of Euler Equation Estimation
3. Models, Variants and Simulation Details
4. Monte Carlo Results
5. Conclusions

The Econometrics of Euler Equation Approximation

First order condition (no liquidity constraints):

$$U'(C_{t-1}) = \beta E_{t-1}[(1+R_t)U'(C_t)] \qquad (1)$$

Sub-utility function is the iso-elastic form:

$$U(C_t) = \frac{(C_t)^{1-\gamma}}{1-\gamma} \qquad (2)$$

- $(\frac{1}{\gamma})$ is the Elasticity of Intertemporal Substitution (EIS) (γ is the coefficient of relative risk aversion and $\gamma+1$ is the coefficient of relative prudence.)

Exact Euler equation:

$$(\frac{C_t}{C_{t-1}})^{-\gamma}(1+R_t)\beta = \varepsilon_t \text{ with } E_{t-1}(\varepsilon_t) = 1 \qquad (3)$$

- ε_t expectation error (the innovation in discounted marginal utility), orthogonal to variables in the information set at time *t-1*

Multiplicative measurement error:

$$C_t^0 = C_t \eta_t \tag{4}$$

$$\left(\frac{C_t^0}{C_{t-1}^0}\right)^{-\gamma}(1+R_t)\beta = \left(\frac{\eta_t}{\eta_{t-1}}\right)^{-\gamma}\varepsilon_t \tag{5}$$

- The composite error term does not have a conditional expectation of unity, even η_t and ε_t are independent:

$$E_{t-1}\left[\left(\frac{\eta_t}{\eta_{t-1}}\right)^{-\gamma}\varepsilon_t\right] = E_{t-1}\left(\frac{\eta_t}{\eta_{t-1}}\right)^{-\gamma} E_{t-1}(\varepsilon_t) = E_{t-1}\left(\frac{\eta_t}{\eta_{t-1}}\right)^{-\gamma} \neq 1$$

- Runkle (1991) estimates 76% of the variation in the growth rate of food consumption in the PSID is noise.
- Alan and Browning (forthcoming) obtain an even higher estimate of 86%.

(Log) Linearized Euler Equation:

$$\Delta log c_{h,t} = \alpha + \frac{1}{\gamma} log(1 + R_t) + \frac{\gamma+1}{2} \left(\Delta log c_{h,t}\right)^2 + e_{h,t} \qquad (6)$$

- α contains the discount rate and the unconditional means of the third (and higher order) moments of the expectation error ε_t
- The residual term e_t contains the measurement error and also the time varying component of approximation error.
- The time varying component of approximation error consist of time varying components of the higher conditional moments (conditional on past information) of the expectation error
- Literature uses twice lagged instruments because the measurement error induces a MA(1) component in the residuals

Models

Basic setup

- Agents face two types of income shocks, permanent and transitory.

$$Y_{h,t} = P_{h,t} U_{h,t} \tag{7}$$

- $U_{h,t}$ is an iid lognormal transitory shock with unit mean and constant variance $\left(e^{\sigma_u^2} - 1\right)$
- $P_{h,t}$ is permanent income which follows a log random walk process:

$$P_{h,t} = G P_{h,t-1} Z_{h,t} \tag{8}$$

- $Z_{h,t}$ is an iid lognormal permanent shock with unit mean and constant variance $\left(e^{\sigma_z^2} - 1\right)$
- Innovations to income are independent across individuals; we abstract from aggregate shocks to income
- The real rate follows an AR(1) process (aggregate shock)

Table 1: Parameter Values

Parameter	Value
Coefficient of Risk Aversion (γ)	4
Discount Rate $\left(\delta=(\frac{1}{\beta})\text{-}1\right)$	0.03 and 0.07
Standard Deviation of Permanent Income Shocks (σ_z)	0.05
Standard Deviation of Transitory Income Shocks (σ_u)	0.1
Unconditional Mean of Interest Rate Process (μ)	0.03
AR(1) Coefficient of Interest Rate Process (ρ)	0.6
Standard Deviation of Interest Rate Process (σ_ε)	0.025
Probability of Zero Income (Models 3 and 4)	0.01

Variants

- The 6 models (or environments) we study differ by
  - degree of impatience (discount rate)
  - the presence or absence of a borrowing constraint (apart from the natural borrowing constraint).
- Model 1 is the environment studied by Attanasio and Low (2004). Model 2 is similar but agents are *more* impatient.
- In Model 3 and Model 4 transitory income shocks, with some small probability (0.01), can take a '0' value in any period. This addition to the model strengthens agents' precautionary motive. Model 4 is very like the "buffer-stock" model studied by Carroll (2001). Model 3 is similar, but agents are *less* impatient.
- Model 5 and 6 follow Deaton (1991): individuals are explicitly prevented from borrowing. Agents are more impatient in Model 6.
  - Motivated by the observation that there are asset levels of zero are observed in the real data

Table 2: Variants

Model	Impatient	Patient	Borrowing Constraint	
1	-	Yes	No	Attanasio and Low
2	Yes	-	No	
3	-	Yes	Implicit	
4	Yes	-	Implicit	Carroll
5	-	Yes	Explicit	
6	Yes	-	Explicit	

Simulation and Experiments

- Models solved by standard methods
- A simulated population of 10,000 individuals is generated after solving each life cycle model
- Generated 80 periods of consumption paths for ex-ante identical consumers then removed the first 20 periods and the last 20 periods
- Monte Carlo experiments performed using the simulated consumption paths of 1000 individuals (observed 40 periods) drawn from the population of 10,000 individuals with replacement

Predicting Failure

- Hypothesis: efficacy of linearized Euler equation estimation depends on the *effective* curvature of policy functions (which in turn depends on parameter values).
  - More curvature means more approximation error

$$\Delta log c_{h,t} = \alpha + \frac{1}{\gamma} log(1 + R_t) + \frac{\gamma+1}{2} \left(\Delta log c_{h,t}\right)^2 + e_{h,t} \quad (9)$$

- Policy functions have similar shape; what is really different is where agents locate in the state space (asset levels)
- We use the simulated data to characterize each variant's "***effective curvature***"

- We measure the effective curvature by weighting the curvature of underlying policy function (at a given age) at every point by the ex-post density of normalized cash-on-hand.
- Estimate (non-parametrically):

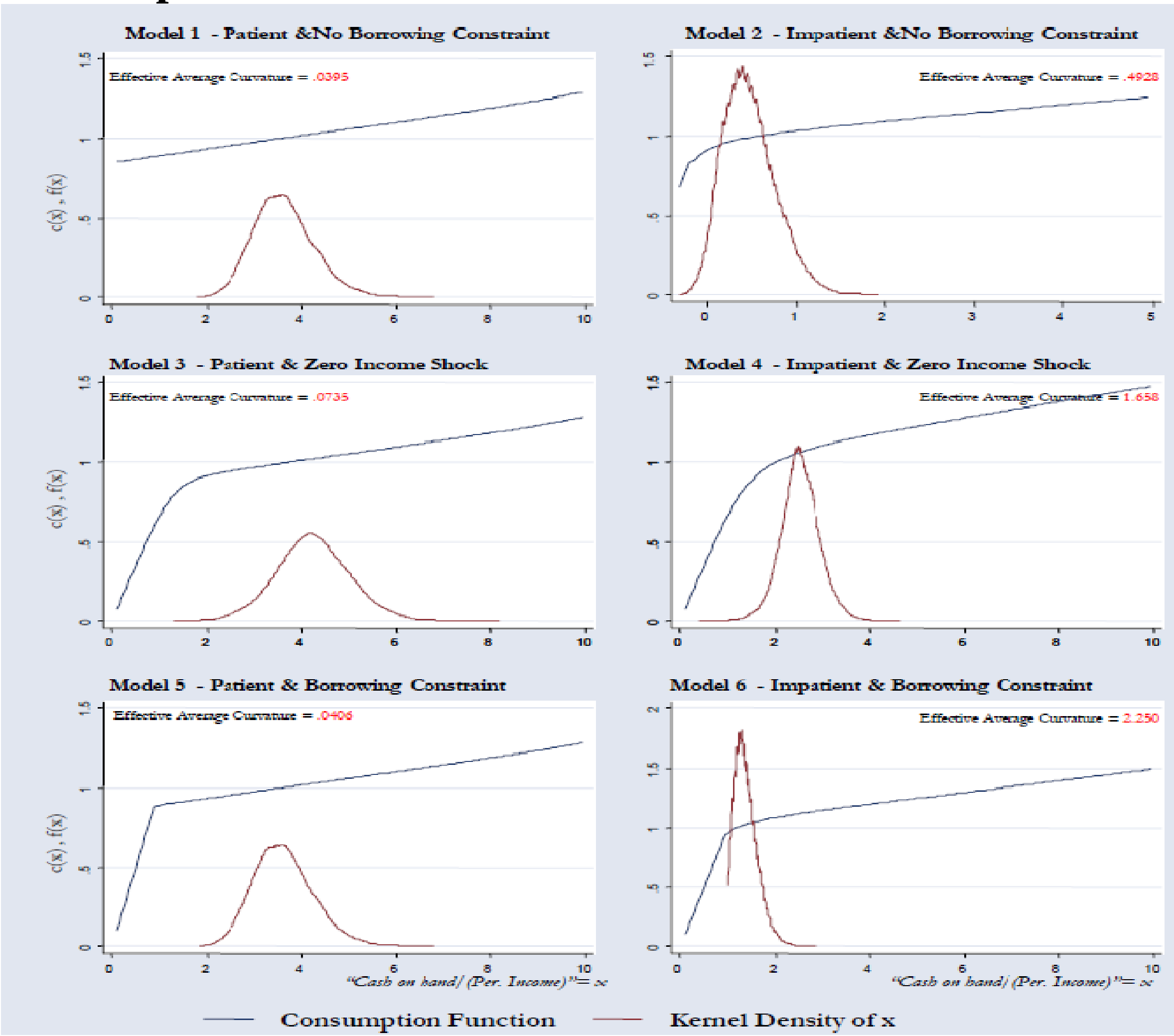
$$E\left[c_i \mid x_i, A_i\right] = g_A(x_i)$$

- Calculate curvature measure $\left(-\frac{\widehat{g}''(x)}{\widehat{g}'(x)}x\right)$ at every x point.
- Take the weighted average of this measure:

$$\text{Effective Average Curvature} = -\int \widehat{f}(x)\left(-\frac{\widehat{g}''(x)}{\widehat{g}'(x)}x\right) \quad (10)$$

- Like an average derivative

Figure 1: Consumption Functions and Simulated Distribution of Cash-on-Hand



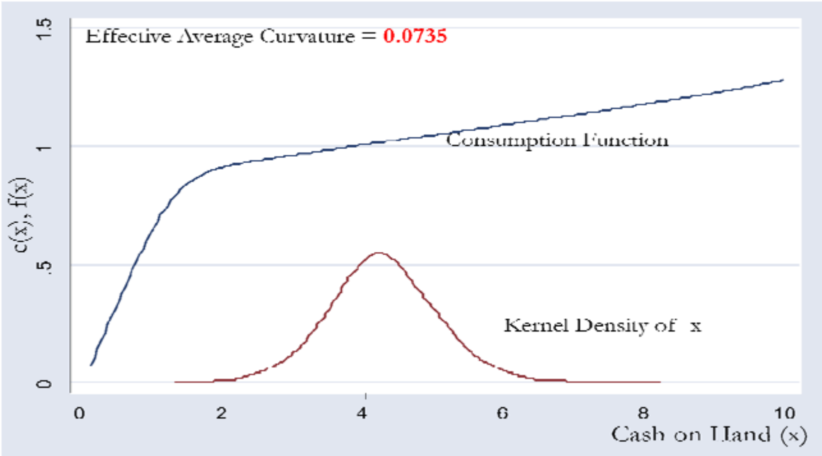
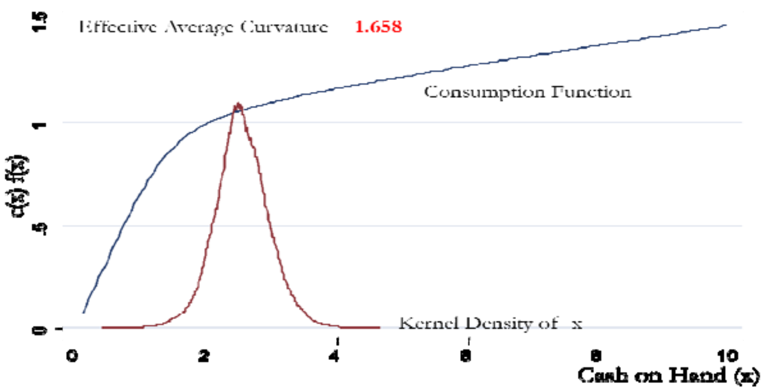
Model 3	Model 4
	

Table 3: Moments of Expectation Errors

Model	Effective Curvature	Var	Skw	Krt
1	0.0395	0.03	0.522	3.55
2	0.4928	0.061	1.01	6.11
3	0.0735	0.039	14.56	2233
4	1.658	0.237	55.38	5788
5	0.0407	0.03	0.523	3.55
6	2.250	0.047	0.914	5.8

- Suggests the link between effective curvature and approximation bias through the distribution of expectation errors
- As the effective curvature increases higher order moments tend to increase

Results

Instrument Validity and Relevance

- Construct the true residuals of the first and second order log-linearized models using the true parameter values.

$$\Delta logc_{h,t} - \frac{1}{4} log(1 + R_t) = \alpha + e_{h,t} \qquad (11)$$

$$\Delta logc_{h,t} - \frac{1}{4} log(1 + R_t) - \frac{4+1}{2} \left(\Delta logc_{h,t}\right)^2 = \alpha + e_{h,t} \qquad (12)$$

- Instrument validity: regression of true residuals on instruments
- Instrument relevance: F statistic, Cragg-Donald statistic (minimum eigen value)
- Instruments: lagged interest rate, lagged consumption growth and lagged income (Attanasio and Low, 2004)
- Add lagged consumption growth squared to the instrument set for the second order approximation.

Table 4:

- Patient agents, low effective curvature, no evidence that instruments are invalid
- Impatient agents, high effective curvature, instruments invalid
- Deaton variants (5,6) are an exception: dropping observations with zero assets at t-1.
- Instrument relevance not an issue for the first-order approximations (Stock, Wright and Yogo suggest F>10 rule of thumb).

Table 4: Instrument Validity and Relevance Results, First Order Approximation

$$\Delta log C_{t+1}^{h} = \alpha + \frac{1}{\gamma} log(1 + R_{t+1}) + e_{t+1}^{h}$$

	Instrument Validity $\alpha + e_{t+1}^{h} = \Delta log C_{t+1}^{h} - \frac{1}{4} log(1 + R_{t+1})$ Mean t stat				Instrument Relevance F [10% , 90%]
	$\log(1+R_t)$	$\Delta \log C_t^h$	$\log y_t^h$	Mean R^2	
Model 1	-0.916	-0.459	0.655	0.00004	7438
(0.0395)					[7431 , 7447]
Model 2	-1.042	-6.746	-4.774	0.00175	7457
(0.4928)					[7442 , 7478]
Model 3	0.448	-0.754	0.619	0.00004	7362
(0.0735)					[7321 , 7402]
Model 4	-0.928	-18.254	1.58	0.00914	7362
(1.658)					[7320 , 7402]
Model 5	-0.957	-0.476	0.673	0.00004	7438
(0.0406)					[7431 , 7447]
Model 6	-1.614	-1.424	-1.074	0.00015	6962
(2.250)					[6853 , 7076]

Table 5: Second Order Approximations

- Reduce approximation error
- Direct estimation of the relative prudence parameter (potentially a test of CRRA)
- At low effective curvature, instruments are valid but may be weak
- Instruments may have more predictive power when effective curvature is higher, but are then more often invalid

Table 5: Instrument Validity and Relevance Results, Second Order Approximation

$$\Delta logC_{t+1}^{h} = \alpha + \frac{1}{\gamma} log(1+R_{t+1}) + \frac{\gamma+1}{2}\left(\Delta logC_{t+1}^{h}\right)^2 + e_{t+1}^{h}$$

	Instrument Validity $\alpha + e_{t+1}^{h} = \Delta logC_{t+1}^{h} - \frac{1}{4} log(1+R_{t+1}) - \frac{4+1}{2}\left(\Delta logC_{t+1}^{h}\right)^2$ Mean t stat					Instrument Relevance Cragg-Donald statistic
	$\log(1+R_t)$	$\Delta \log C_t^h$	$\log y_t^h$	$\left(\Delta \log C_t^h\right)^2$	Mean R^2	[10% , 90%]
Model 1	-2.002	-0.537	-0.149	-0.41	0.000127	7.49
(0.0395)						[1.774 , 15.329]
Model 2	-1.817	-4.209	-2.41	2.917	0.000929	92.843
(0.4928)						[50.507 , 163.971]
Model 3	-0.417	0.276	-0.208	-4.316	0.004666	409.125
(0.0735)						[3.289 , 5308]
Model 4	0.001	-3.062	-0.103	-4.811	0.00277	2008.187
(1.658)						[555.01 , 5400]
Model 5	-2.024	-0.544	-0.105	-0.521	0.000133	7.457
(0.0406)						[1.999 , 15.525]
Model 6	-0.971	-0.917	-0.938	0.34	0.000075	2.636
(2.250)						[.156 , 9.437]

Estimates of the EIS

- Small sample results from the GMM and LIML estimation of first and second order approximate Euler equations
- Report mean parameter estimate, mean standard error, and % of confidence intervals containing the true value
- Table 6: first order approximate Euler equations
  - Estimates much better when agents are patient (effective curvature is low).
- Table 7: second order approximations
  - In addition to EIS we also estimate the prudence parameter (the true value is 2.5).
  - Results for the EIS are quite similar to those from the first order approximation.
  - Results for the prudence parameter are quiet different across models

Table 6: Monte Carlo Results for the First Order Approximation

Estimates of the EIS using First Order Approximation

$$\Delta log C_{t+1}^{h} = \alpha + \frac{1}{\gamma} log(1 + R_{t+1}) + e_{t+1}^{h}$$

True Value of EIS $\left(\frac{1}{\gamma}\right)$ is 0.25

	Linear GMM	**LIML**
Model 1	.239	.2389
(0.0395)	(.012)	(.0119)
	83.1	83.4
Model 2	.2341	.2389
(0.4928)	(.0169)	(.0167)
	85.2	90.8
Model 3	.2545	.2546
(0.0735)	(.0117)	(.0117)
	93.2	93
Model 4	.2237	.2203
(1.658)	(.0151)	(.0148)
	59.9	48.5
Model 5	.2385	.2384
(0.0406)	(.012)	(.0119)
	84.7	84.8
Model 6	.2246	.2247
(2.250)	(.0156)	(.0153)
	64.9	63.8

Table 7: Monte Carlo Results for the Second Order Approximation

Estimates of the EIS and Prudence Using Second Order Approximation

$$\Delta logC_{t+1}^{h} = \alpha + \frac{1}{\gamma} log(1 + R_{t+1}) + \frac{\gamma + 1}{2} \left(\Delta logC_{t+1}^{h}\right)^{2} + e_{t+1}^{h}$$

	EIS		**Prudence**	
Real Value	$\left(\frac{1}{\gamma}\right) = 0.25$		$\left(\frac{\gamma+1}{2}\right) = 2.5$	
	Linear GMM	LIML	Linear GMM	LIML
Model 1	0.2295	0.23	1.705	1.532
(0.0395)	(0.0207)	(0.023)	(3.069)	(3.558)
	83.6	83.8	95.7	95.3
Model 2	0.2051	0.205	5.928	5.969
(0.4928)	(0.0195)	(0.0196)	(0.7667)	(0.6943)
	36	36.8	0.2	0.2
Model 3	0.2459	0.2458	1.973	2.021
(0.0735)	(0.0147)	(0.0145)	(1.705)	(1.669)
	93.1	92.7	79.6	76
Model 4	0.2463	0.2446	2.526	2.341
(1.658)	(0.0198)	(0.0185)	(0.3673)	(0.0708)
	92.3	90.1	78.9	19
Model 5	0.2296	0.2303	1.646	1.505
(0.0406)	(0.0207)	(0.0224)	(3.095)	(3.448)
	82.1	82.9	96.2	95.3
Model 6	0.2489	0.2593	6.49	9.856
(2.250)	(0.0326)	(0.8925)	(6.397)	(167)
	93.8	96.6	99.2	98.6

Additional comments and experiments

- Lower risk aversion (and hence lower cash on hand) can make things worse (experiments with crra = 2)
- Shorter panels (14 years rather than 40) can make things worse (and note that we don't have aggregate income shocks)

Conclusion

- Apparent success/failure of approximate Euler equations estimation related to assumptions about economic environment/parameter values, in a fairly coherent way
- There seem to be environments where approximate Euler equation estimation can yield good estimates, as well as environments in which the approximate Euler equation works quite badly)
- Effective curvature good predictor of failure within classes of models, less good for very different models
- Not just instrument validity but also instrument relevance may be a problem (particularly for 2nd order approximation)
- Where does this leave us?
    - Refine effective curvature measure/look for other ways to generalize results
    - More carefully look at weak instrument problems?
    - Impose restrictions?
    - Emerging evidence that key parameters heterogeneous (Alan and Browning, forthcoming, Guvenen, JME 2006) may be insurmountable problem for Euler Equation approach?