Mobility and the Lifetime Distributional Impact of Tax and Transfer Reforms

Peter Levell, Barra Roantree, and Jonathan Shaw[+,*]

The distributional impact of proposed reforms plays a central role in public debates around tax and transfer policy. We show that accounting for realistic patterns of mobility in employment, earnings and household circumstances over the life-cycle greatly affects our assessment of the distributional effects of tax and transfer reforms. We focus on three reforms modelled in the UK context: (i) changes to out-of-work versus in-work benefits, (ii) adjustments to income tax rates, and (iii) reforms to indirect taxation. In all three cases, the long-run distributional impact differs to that implied by a standard cross-section analysis in important ways. (JEL D31, H20, H24)

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[+] Institute for Fiscal Studies, 7 Ridgmount Street, London WC1E7AE, United Kingdom and University College London, 30 Gordon Street, London WC1H0AX, United Kingdom (Levell: peter_l@ifs.org.uk, Roantree: Barra_r@ifs.org.uk, Shaw: j.shaw@ifs.org.uk). The research reported in this paper was not the result of a for-pay consulting relationship. Further, none of the authors nor their respective institutions have a financial interest in the topic of the paper that might constitute a conflict of interest.

The distributional impact of proposed reforms plays a central role in public debates around tax and transfer policy. These impacts are typically assessed by comparing the impact of tax changes across different income groups. Reforms are classified as progressive if they result in a greater proportional gain, or a smaller proportional loss, for households with low incomes, and regressive if the other way around.[1] Such analyses however suffer from an important limitation. The incomes used to rank households are almost always assessed over a period of a year or less, and other household characteristics are only observed in cross-section. However individuals' circumstances – and therefore tax and transfer payments – can vary a great deal across life: for example those with low current incomes may have high lifetime incomes and vice versa, meaning the distributional impact of reforms assessed using a snapshot in time may give a partial or misleading picture.

The contribution of this paper is to show that, accounting for realistic patterns of mobility in employment, earnings and household circumstances, the long-run distributional impact of tax and transfer policies differs in important ways compared to a standard cross-sectional analysis. Alongside income mobility, we also consider the joint dynamics of employment status, moves between part-time and full-time work, housing tenure, health status and family formation. All of these are key determinants of taxes and transfers, and so need to be modelled simultaneously to assess the impact of the most interesting reforms. We focus on three reforms modelled in the UK context: (i) changes to out-of-work versus in-work benefits, (ii) adjustments to income tax rates, and (iii) reforms to indirect taxation. These reforms are either the subject of intense policy debate in many countries or have been proposed as potential efficiency-improving changes (see for example Mirlees, 2011).

We have three main findings. First, we find that increases to in-work benefits are just as good as increases to out-of-work benefits at targeting the lifetime poor because the lifetime poor spend the majority of working life in work. This stands in contrast to a snapshot analysis, which suggests increases to out-of-work benefits are considerably more progressive than increases to in-work benefits. This finding alters the apparent equity-efficiency trade off between these two sets of policies in favour of in-work benefits, which tend to have less distortionary effects on labour supply (Eissa and Liebman, 1996) even if they do not reach those with the lowest current incomes. Second, higher rates of income tax remain an effective

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[1] For recent examples see Congressional Budget Office (2015) which looks at the impact of possible changes to the Supplemental Nutritional Program, and HM Treasury (2015) which undertakes an assessment of the impact of the UK 2015 Budget.

way of targeting the lifetime rich because the higher persistence of incomes towards the top of the distribution means that high current incomes tend to be indicative of high lifetime incomes. Third, while increases to the standard rate of VAT appear strongly regressive in the cross-section, they are almost distributionally neutral over the lifetime because income and expenditure are much closer to being equal.

Over the last few years, our knowledge of the nature of household mobility in and out of work and up and down the income distribution has grown considerably. Recent work on earnings dynamics has for example shown that negative earnings shocks are more persistent for high-earners, while positive shocks are more persistent for low earners (Guvenen et al., 2015). Moreover, the variance of shocks to income has also been found to vary with age (Blundell, 2014). Incorporating these more realistic income dynamics has been shown to have important implications for understanding the variance of lifetime earnings (Altonji, Smith and Vidangos, 2013), observed patterns of wealth inequality (Castañeda, Díaz-Giménez and Ríos-Rull, 2003; De Nardi, Fella and Gonzalo Paz-Pardo, 2016), and consumption decisions (Arellano, Blundell and Bonhomme, 2015).

The relatively small literature that has explored the distributional impact of reforms over longer periods (c.f. Davies, St-Hilaire and Whalley, 1984; Poterba, 1989; Lyon and Schwab, 1995; Fullerton and Rogers, 1991; Caspersen and Metcalf, 1994; Metcalf, 1999) has made much simpler assumptions about the income process. In the main it has adopted simple (typically ARIMA) processes without the important features identified in the more recent literature to model income dynamics and simulate tax payments.[2] It has also tended to neglect mobility in other variables (such as family composition, health and housing tenure) and focus on the distributional impact of indirect tax reforms, leaving unaddressed the longer run distributional impact of direct tax and in-work transfer reforms that have dominated the policy agenda in recent decades. Our work addresses these shortcomings.

In addition, we contribute to a growing literature recognising that tax and transfer policy often has important long-run implications for individual welfare and behaviour in addition to its short-run impacts: for example Hoynes, Schanzenbach and Almond (2016), who investigate the long-run effects of Food Stamps on health and economic outcomes in the US; Blundell et al. (2016), who look at how welfare programmes subsidising the wages of low-earning individuals affect the careers of women; and Dahl, Kostøl and Mogstad (2014), who

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[2] Davies, St-Hilaire and Whalley (1984) rank individuals by income across age groups and link them under different assumptions about the degree of mobility. Poterba (1989) uses consumption expenditures as a proxy for lifetime income.

consider the effects of participation in welfare programmes on the participation of subsequent generations.

The ideal dataset for our analysis would be a long-running panel from birth until death that includes all the necessary information for computing taxes and transfers. Sadly, such a dataset does not exist. Even in countries where large administrative datasets are available, they typically do not cover the whole of life and often do not include all the variables needed to calculate taxes and transfers. We therefore adopt an alternative approach, constructing a simulated cohort born 1945–54 that contains all the key characteristics for calculating tax and transfer payments from the start of adult life until death. The life courses for each of these individuals are simulated using processes estimated from an 18-wave panel survey, combined with information from a much longer-running cross-sectional survey. This approach allows us to model outcomes for a particular cohort while minimising the risks of conflating age, period and cohort effects that can arise when generalising processes estimated using a relatively short panel. Our long-run simulations allow us to extend the work of Roantree and Shaw (2014), who consider outcomes over the panel's 18-year horizon at most.

Our results show that a lifetime assessment of tax reforms contains important insights that may be missed in simple cross-sectional analyses. Nevertheless, we do not argue that a lifetime analysis of the kind implemented here provides a definitive measure of the welfare effects of tax and benefit changes. In particular, in the presence of credit market imperfections, redistribution towards periods of temporarily low incomes may be welfare improving. While important, this must be set against the equally extreme assumption of zero smoothing that is implicitly embedded in snapshot analyses. We view the two approaches as complementing one another.

The remainder of this paper is structured as follows. In the next section, we describe how we construct our simulated lifetime profiles. Section II presents some descriptive statistics on lifetime incomes and how they compare to cross-sectional measures. This forms the backdrop to our results on the effects of tax and transfer reforms on lifetime and cross-sectional inequality in Section III. Section IV concludes.

I. Methodology

In this section we describe the data we use and how our lifetime processes are estimated and simulated for use in our tax policy analysis. Our aim throughout is to simulate life profiles that are representative of individuals born in the United Kingdom in the period 1945-54

(roughly corresponding to the 'baby-boom' cohort). Space constraints mean that we do not provide full details of our method and how we have validated it in the main text. We refer readers interested in additional details of our approach to Levell and Shaw (2015) and to the technical appendix that accompanies this paper.

A. Data

We rely primarily on two datasets: the British Household Panel Survey (BHPS) and the Living Costs and Food Survey (LCFS).

The BHPS is a panel survey that ran for 18 waves from 1991 to 2008, collecting a wide range of demographic and socio-economic information. The survey followed individuals and their descendants over successive waves. The original sample comprised around 10,000 individuals in 5,500 households and was nationally representative. Booster samples were introduced for Scotland and Wales in 1999. In each wave, the survey aimed to interview all individuals aged 16+ in each household, including children who reached adulthood after the survey began and adults who moved into households that were previously surveyed.

The Living Costs and Food Survey (LCFS) is the latest name for a long-running, annual (for most of its history), cross-sectional survey of household spending patterns in the United Kingdom. It was known as the Family Expenditure Survey (FES) between 1957 and 2001 and the Expenditure and Food Survey (EFS) between 2001 and 2008. For simplicity in what follows we shall refer to all these surveys simply as the LCFS. The LCFS collects data on household incomes from various sources over the past 12 months, employment, family characteristics (including years of education from 1978 onwards) and expenditures. We make use of the LCFS between 1968 and 2012.

B. Overview of Simulation Approach

The first step in our approach is to estimate the conditional probabilities associated with different transitions at each age using the BHPS panel data. The processes we model are those that are central to determining taxes and benefits: mortality, partnering, separation, child arrival and departure, movements into and out of disability, movements in and out of employment, movements between full-time and part-time work, movements across ranks in the earnings and rent distributions, movements into and out of rented accommodation, and movements between local property tax bands (known in the United Kingdom as council tax). We do not model capital or asset incomes, or receipts of transfers and inheritances from other

households. We also ignore benefits in kind, such as health and education spending. In most cases, we estimate transition probabilities using binary and multivariate logit models with a detailed set of covariates. A summary of the exact specifications we use in the estimation stage is set out in the technical appendix.

Once we have obtained estimates, the next step is to use these to simulate a set of lifetime profiles. These simulations allow for contemporaneous correlations between outcomes, but, as it is not possible to determine all variables of the system simultaneously, variables must be determined in a sequential manner. The sequence we adopt is as follows. First we determine whether or not the agent lives or dies in the period. We then assign births to individuals according to probabilities of child arrival that we have estimated, and determine whether children between ages 16 and 18 leave the household. Individuals then partner or separate. Childbirth is determined prior to partnering so that it will depend on lagged rather than current partner status (thus allowing for a nine month gestation period). We then determine whether or not individuals receive disability benefits, before assigning an employment status, and a location in the earnings distribution. We impose that all those who are disabled are unemployed. Finally we determine whether or not the individual is a renter and the household's council tax band, before incrementing individuals' ages and repeating the process. The order imposed necessarily entails assumptions about the way in which outcomes are determined. For example, since child arrival and departure are determined before partnering and separation, the number of children an individual has this period can affect their probability of being in a couple this period, but not vice versa.

We start simulating from 1960 when the baby-boomers are in childhood. Initial conditions (education levels, likelihood of being a renter and so on) are set using data on the baby-boom cohort from the LCFS. We simulate 5,000 lifecycles. Consumption and private pension profiles are imputed to individuals once the simulations are complete. Consumption is imputed on a year-by-year basis using regressions run on the cross-sectional LCFS. For private pensions we make use pension incomes projected for future years that were calculated using the English Longitudinal Study of Ageing for real world members of the baby-boom cohort (details of the methodology to construct these profiles can be found in Crawford, 2012 and an example of their use in Banks, Emmerson and Tetlow, 2014). These are calculated given observed pension wealth and for different possible retirement dates for each individual. The profiles are matched to our simulated individuals within cells defined by cohort, year, age and sex on estimated ranks in the private pension wealth distribution.

For some of our variables, the process of simulation is quite straightforward. Probabilities are estimated and transitions then drawn from the relevant distribution. For other variables further explanation is required. In what follows we give more detail on how we model employment and earnings (for details of how we have modelled other variables, and how we impute pensions and consumption, we refer the reader to the technical appendix). Before doing this however, we discuss how we ensure that the transition probabilities estimated using our panel data for the period 1991-2008 are consistent with the experiences of our cohort of interest as described by the cross-sectional data.

C. Scaling Transition Probabilities

Typically two different approaches are used to construct lifetime profiles from panels that cover a relatively short period of time. One is to estimate processes using a sample that pools together data on different cohorts (see for example Falkingham and Hills, 1995). Another is to adopt a splicing approach that joins individuals observed at different ages with the same short period together to form a complete lifecycle (Bovenberg, Hansen and Sørensen, 2008).[3] The downside of both of these approaches is that processes for earlier ages will be estimated using data taken from cohorts that are younger than the baby-boom cohort and those for later in life from older cohorts. This creates difficulties in interpreting the resulting estimates, since the simulated life-cycles may not correspond to the actual experiences of any real-world cohort of individuals. In addition, the approach does not make use of the information we do have on the evolution of average employment, fertility and so on across years from longer-running cross-sectional surveys.

To make best use of the data we have available, we adopt a different approach. We first estimate transition probabilities for each of the relevant variables using panel data as we described above, and then we adjust these probabilities so that the resulting cross-sectional averages for our simulated cohort match those in the cross-sectional data for the same cohort from 1960 onwards.[4]

To be more precise we aim to find transition probabilities for, say, employment such that we move from the (observed) cross-sectional distribution given at time *t-1* to the cross-

[3] For a comparison of our current method with that of the splicing approach using in Bovenberg, Hansen and Sørensen (2008) see Levell and Shaw (2015).
[4] The latest year we observe individuals in 2012. For years beyond this we project variables using a combination of data taken from older cohorts, estimated rates of change by age, and external forecasts.

sectional distribution at time t. For a general binary outcome for some individual i in year t, y_{it}, let

$$(1) \quad p_{it} = Prob(y_{it} = 1)$$

$$(2) \quad \pi_{it}^{01} = Prob(y_{it} = 1 | y_{i,t-1} = 0)$$

$$(3) \quad \pi_{it}^{11} = Prob(y_{it} = 1 | y_{i,t-1} = 1)$$

Then by the law of total probability, we get the flow equation

$$(4) \quad p_{it} = \pi_{it}^{01}(1 - p_{i,t-1}) + \pi_{it}^{11} p_{i,t-1}$$

It is clear that π_{it}^{01} and π_{it}^{11} cannot be uniquely identified with knowledge of p_{it} and $(1 - p_{it-1})$ alone. This is because a variety of transition probabilities could in principle satisfy (4). Cross-sectional data alone is therefore not sufficient to identify these and panel data is required. However, the transition probabilities that we have estimated are taken need not satisfy this expression – not least because they are partly based on data for different cohorts and years. Let these probabilities be denoted by $\hat{\pi}_{it}^{01}$ and $\hat{\pi}_{it}^{11}$. We proceed by choosing values for π_{it}^{01} and π_{it}^{11} so as to minimise the average distance to our estimated transition probabilities

$$(5) \quad \left(\frac{1}{N}\sum_i \pi_{it}^{01} - \frac{1}{N}\sum_i \hat{\pi}_{it}^{01}\right)^2 + \left(\frac{1}{N}\sum_i \pi_{it}^{11} - \frac{1}{N}\sum_i \hat{\pi}_{it}^{11}\right)^2$$

subject to satisfying (4). Since our probabilities are estimated using logit models, we can equivalently achieve this by choosing z^1 and z^2 to minimise

$$(6) \quad \left(\frac{1}{N}\sum_i \Lambda(X_{it}\beta^{01} + z^1) - \frac{1}{N}\sum_i \Lambda(X_{it}\beta^{01})\right)^2 +$$

$$\left(\frac{1}{N}\sum_i \Lambda(X_{it}\beta^{11} + z^2) - \frac{1}{N}\sum_i \Lambda(X_{it}\beta^{11})\right)^2$$

again subject to satisfying (4). $\Lambda(.)$ is the cdf of the logistic function and β^{01} for example denotes the coefficients estimated for transitions from $y_{i,t-1} = 0$ to $y_{it} = 1$. Choosing terms to add to the index of individual's logistic functions rather than scaling overall probabilities ensures that all adjusted probabilities remain between zero and one.

The above scaling procedure is used to adjust transition probabilities for couple status, moving from being a renter to an owner (and vice versa) and employment. It was not found necessary to scale the probability of child arrival even though earlier cohorts tended to have more children. This is because the number of children among our simulated individuals tended to reach the right level once the probabilities for partnering and separation had been adjusted. We allow the scaling factors we choose to differ across subgroups of the population, allowing us to ensure that we match male and female employment rates implied by our

repeated cross sections. The precise subgroups used differ depending on the variable we are considering and are given in Table 1.

<div align="center">**TABLE 1 HERE**</div>

Scaling within subgroups means we will not necessarily exactly match the probability of being employed or in a couple in each year (since it is possible that the number of individuals with children differ from the numbers for the population for example). As we show in our technical appendix however, we match the cross-sectional probabilities extremely well.

The mortality rate is also scaled but using a simpler method. For this we take data from the UK Office for National Statistics Life Tables which provide average mortality rates for men and women at different ages for different birth years. We then use the difference between these and average within-sample mortality rates for individuals in the BHPS to scale mortality rates predicted using a logit regression on income, disability benefit receipt, education and couple status.

<div align="center">*D. Modelling Employment and Earnings*</div>

We model earnings and rent *ranks* rather than attempting to model the levels of these variables. The ranks are defined as relative positions in the distribution by age and year. Once our simulations are complete we are then able to 'fill-in' earnings and rent levels from the actual cross-sectional distributions of earnings as observed for our particular birth cohorts in the LCFS. By using this approach, we match exactly the distributions across individuals at a point in time in terms of means, variances and higher order moments by construction.

Modelling ranks in the way we do means that we assume that movements across the earnings distribution by age are the same across cohorts and periods. This assumption is not completely innocuous. One could for instance imagine that the degree of relative mobility has changed over time between cohorts or that recessions may differentially affect the relative positions of some individuals relative to others. However, if we had instead modelled earnings and rents levels, we would face the problem of having to disentangle age, period and cohort effects when predicting values from our panel data. Failure to specify the correct model in this regard would potentially severely bias the shape of our estimated age profiles. In addition, we would have faced the challenge of ensuring that our simulated individuals were subject to a realistic sequence of business cycles by for example modelling a separate process for period effects. Taking values from actual cross-sectional distributions as we do

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ensures that our simulated individuals automatically face a real-world process for aggregate shocks.

To model movements across the earnings distribution accurately, we must be careful to allow for key features of real world transitions. Standard ARIMA models of earnings dynamics do not allow for differing persistence in shocks between high- and low-income individuals, for negative shocks to be for example more persistent than positive shocks, or for there to be skew in the distribution of shocks that differs by age. They are also unlikely to capture the non-symmetric nature of mobility in the tails of the distribution well. Recent non-parametric modelling using administrative data has however shown that these factors are highly important (Guvenen et al., 2015).

To allow flexibly for these aspects of mobility, we adopt the approach of estimating transition matrices across earnings quantiles and part-time or full-time employment (for other examples see Buchinsky and Hunt, 1999; Bowlus and Robin, 2012). In particular, we proceed using the following three-step parametric approach.

1. **Determine employment status:** We estimate transition matrices for employment status separately for males and females and according to individuals' employment status over the previous two waves. The probabilities making up this matrix are estimated through a set of logit models which include several lags of employment status (and interactions thereof) to help us match the high persistence of employment status observed in the data. These transition probabilities are scaled so as to match the observed unemployment rates at different ages for the baby-boom cohort in the cross-sectional LCFS data (as discussed above).

2. **Place the individual in an earnings 'bin':** Once an individual's employment status is determined, we then place the individual in one of five possible bins: in part-time work, or in full-time work and in one of four different earnings quartiles. Distinguishing between part- and full-time work is important in our case as it determines eligibility for receipt of tax credits in the United Kingdom. We assume that part-time work corresponds to 20 hours per week and full-time work to 40 hours. To determine which bin an individual should be placed in we estimate multinomial logits from each of the six possible prior states *i* (which include unemployment)

$$(7) \qquad \Pr(i, j | X_{it}) = \frac{\exp(X_{it}\beta^{ij})}{\sum_{m=0}^{N} \exp(X_{it}\beta^{im})}$$

X is a set of covariates which includes a cubic in age, education, a dummy for whether individuals have children or not, and a dummy for whether they have children under the age of 5 (and various interactions of these) their current earnings rank (entering linearly) as well as five lags of full-time and employment status and lagged earnings quartiles. Including these lags relaxes what would otherwise be a Markov assumption that next period's transition depends only on current circumstances. Differences in the coefficients attached to these allow for differences in the persistence of earnings ranks across the distribution.

3. Determine the individual's precise earnings rank: The results from these models can be used to estimate the probability of moving between unemployment, part-time and full-time work and the different income quartiles. However, it does not place individuals precisely within these quartiles. One approach is to deal with this is to match simulated individuals to real-world individuals who made the same employment and income quintile transitions as they did and use these individuals' new ranks to determine the simulated individual's new locations (the approach adopted by Bowlus and Robin, 2012). We adopt an alternative parametric method. This involves predicting ranks using regressions of the following form
(8)

$$\Phi^{-1}(r_{it}) = $$
$$\sum_{\tau=1}^{4} \sum_{j} \delta_0^{\tau j} D_{Q_{t-\tau}=j}^i + \sum_{\tau=1}^{4} \sum_{j} \delta_1^{\tau j} D_{Q_{t-\tau}=j}^i \times r_{i,t-1} + \sum_{\tau=1}^{4} \sum_{j} \delta_2^{\tau j} D_{Q_{t-\tau}=j}^i \times r_{i,t-1}^2 \dots$$

where r_{ikt} is the within-bin rank of individual i in period *t* moving to bin *k*. $D_{Q_{t-\tau}=j}^i$ is a dummy which equals one if the individual was located in bin j in period $t - \tau$. $\Phi^{-1}(.)$ is the inverse of the CDF of the normal distribution. These regressions are run separately for each destination bin, allowing us to capture the asymmetric nature of persistence over the income distribution. They are also run separately for males and females. Linearly predicting $\Phi^{-1}(r_{ikt})$ (and then feeding this prediction through $\Phi(.)$) ensures that the predicted within-bin rank always lies between 0 and 1. The polynomial of past ranks (up to a cubic) included in this regression is also interacted with a cubic in age in order to help us match the differing persistence of earnings over the life-cycle. In our simulations, we add a normally distributed noise term with the variance of residuals seen in the data to the linear prediction made using (8). The raw ranks that we predict for our simulated individuals need not have a uniform distribution. If this were not corrected before imputing earnings, the quantiles of earnings for our simulated individuals would differ from those in our cross-sectional data. To make the

distribution uniform, we assign new ranks to individuals based on their relative positions within-sample as a final step. We see this additional step as analogous to our other cohort scaling adjustments.

Younger individuals (who do not have all the lags needed to be included in the regression models mentioned so far) have their status determined by a multinomial logit (across our five possible bins and unemployment) and a within-bin regression involving only one lag.

Once earnings ranks have been predicted, we then impute actual earnings levels using the distributions observed in the LCFS. For years when our cohort is not observed in the LCFS (prior to 1968 and after 2012) we uprate or downrate earnings distributions for the relevant ages from different cohorts using historical data on earnings growth or forecasts taken from the UK Office for Budget Responsibility.

This approach captures the persistence of earnings ranks well (subject to the caveat that we may understate the degree of persistence over very long horizons). Autocorrelations across different ages and different time horizons are shown for our data and the BHPS in the technical appendix. In addition we compare the persistence of employment in our simulations with our cohort of interest in the BHPS over a 10 year horizon. The proportions always employed and never employed within this period are very similar.

E. Taxes and Benefits

The rich set of outcomes we model gives us all the key variables we need to calculate tax and benefits for each of our simulated individuals. We assign tax and benefit payments to our simulated individuals using TAXBEN, a detailed tax and benefit microsimulation model for the United Kingdom. We model all major benefit payments – both universal and means-tested – including unemployment insurance, payments to families with children, disability benefits and tax credits.[5] We assume that all those eligible for benefits take them up. The taxes included are income tax, payroll taxes ("employee National Insurance contributions"), VAT, excise and fuel duties, and local property taxes ("council tax"). We do not model the effects of capital taxes, inheritance duties or business taxes which are difficult to attribute to individual households (such as corporation tax). For simplicity we do not model behavioural

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[5] More precisely, the benefits included are income support, jobseeker's allowance, housing benefit, council tax benefit, child benefit, family credit, tax credits (working families' tax credit, working tax credit and child tax credit), and the state pension.

responses that might result from our individuals being exposed to the tax systems we impose on them rather than the actual tax and benefit systems they faced.

II. Lifetime Incomes

To provide some background for interpreting our subsequent results, this section presents descriptive statistics for our simulated lifetime profiles and how these compare to statistics from a synthetic 2015/16 cross-section that is also based on our simulated lifecycles. This cross-section describes what the 2015/16 population would look like if all cohorts were the same as the baby-boom cohort. As a result, any differences relative to the lifetime will be due to the lifetime perspective. An alternative would have been to use a real-world cross-sectional distribution, but this would leave the reader in doubt as to whether our results were driven by the cross-sectional perspective or simple differences in the cohorts being considered. In Levell, Roantree and Shaw (2015), we examine how close our synthetic cross-section is to the 1978 and 2012 LCFS cross-sections. Results using the different approaches tell a very similar story.

The course of incomes over the lifecycle is shown in Figure 1. This plots mean annual earnings (not conditional on working) and mean private pensions (not conditional on receiving a pension) across life for men and women for our simulated profiles. We do not plot these against survey data, as average earnings match those in the LCFS by construction and private pension information is taken directly from our cohort of interest. The figure shows that, on average, earnings rise for men until the late 40s and then decline steadily thereafter. For women, earnings flatten off during the late 20s, associated with taking time out of the labour market for child-rearing. They then rise again, reaching a peak at around age 50 before falling again towards retirement. As earnings decline around retirement, individuals start receiving private pensions, though at a much lower level, on average, than earnings during working life.

FIGURE 1 HERE

In order to compare individuals with different lifetime incomes, we must first specify a way to convert income profiles across life into a single figure. One way to do this would be to set lifetime income equal to the present discounted sum of incomes across all ages. However, this definition could mean that those living long lives with low living standards in each period will appear to have higher incomes than those who living short lives but with higher

average incomes. Such outcomes are counterintuitive. Instead, we use a measure of the discounted average lifetime income

$$(9) \qquad Y_i = \frac{1}{\sum_{a=16}^{A_i}((1+r_a)^{a-16})}\sum_{a=16}^{A_i}\frac{y_a}{(1+r_a)^{a-16}}$$

where a is age, r_a is the nominal interest rate (taken from the yields on UK government consols) and A_i is the maximum age reached by individual i. This converts lifetime income into an annualised figure. Incomes are equivalised using the modified OECD scale in each year prior to doing this. The discounted value of lifetime income, Y_i, is then inflated using the Retail Price Index (RPI) to 2015 terms to ensure comparability across cohorts.

Figure 2 gives an indication of how persistent net income is over time in our simulations at different points in the income distribution. This shows that those lifetime poor (rich) spend quite a substantial fraction of life outside the poorest (richest) cross-sectional decile; in other words, there is a substantial degree of mobility around the distribution across life. It also shows that our simulations capture the non-symmetric nature of mobility. Persistence is greater at the top of the distribution than at the bottom: individuals in the richest lifetime decile spend more of their lives in the richest cross-sectional decile than individuals in the poorest lifetime decile spend in the poorest cross-sectional decile (34% compared to 21%). A similar result is obtained when we consider the BHPS data itself (see Figure 2.8 in Roantree and Shaw, 2014). Analysis using administrative data also indicates a similar difference in persistence across income groups, with the those in the top earnings quintile most likely to remain in their current relative position (Kopczuk, Saez and Song, 2007).

FIGURE 2 HERE

Figure 3 shows how employment varies across the net income distribution. The cross-section series shows the proportion of working-age individuals who are employed, split by cross-sectional net income decile. The lifetime series shows the average fraction of working life that individuals are employed, split by annualised lifetime net income decile. From this graph, it is clear that relatively few individuals in the bottom cross-sectional decile are employed (22%), but from a lifetime perspective, individuals in the bottom lifetime decile are employed for the majority of working life (an average of 63%). This has implications for the relative impact of in-work and out-of work benefits on lifetime inequality, which we discuss below.

FIGURE 3 HERE

Finally, Figure 4 plots the distributions for gross lifetime and cross-sectional incomes. The cross-sectional distribution exhibits positive skew, with a long tail of individuals with high incomes, while the lifetime distribution is more symmetric. This reflects the impact of income mobility on the lifetime distribution – a point we also return to in what follows.

FIGURE 4 HERE

III. Results

In this section we present results for the effect of the UK tax system on inequality and consider the impact of various reforms that have either been argued for on efficiency grounds or else play in an important role in current policy debates around the world.

The analysis below assumes that – barring the reforms we consider – individuals face the 2015/16 UK tax and benefit system for the entirety of their lives. This is because we are primarily interested in the characteristics of given tax and benefit systems from a lifetime perspective rather than, say, the experiences of a particular cohort under the systems they were actually exposed to. We hold behaviour fixed following reforms, meaning that labour supply and other variables are assumed to be the same as they were for the baby-boom cohort under all the different tax systems we consider. To determine individual incomes and tax and benefit payments we assume equal sharing of resources between members of couples.

In order to apply a given tax and benefit system to data from earlier or later years, we uprate in line with earnings. This brings us close to ensuring that the tax and benefit system raises the same revenue each year. An alternative would be to uprate in line with prices. Results from this alternative scheme tell a similar story (see Levell, Roantree and Shaw, 2015).

B. The Effects of the Tax and Benefit System on Inequality

Before discussing the impact of particular reforms, we start by showing how the 2015/16 system itself affects lifetime and cross-sectional inequality.

Table 2 shows gross and net income Gini coefficients calculated for the synthetic cross-section and on a lifetime basis for the baby-boom cohort. The first thing to notice is how much lower inequality is over the lifetime: the cross-section Gini coefficient for gross income is 0.493 compared with 0.258 across the whole of adult life. This indicates that a lot of the income inequality before taxes and benefits between individuals is temporary, either

reflecting the stage of life they are at (such as differences in work experience and family structure) or reflecting transitory shocks individuals experience (such as unemployment). Our figure for lifetime income inequality is roughly in line with related figures from other studies. Using German administrative data, Bönke, Corneo and Lüthen (2015) calculate a lifetime earnings inequality measure. They find that for the cohort of individuals born in 1949 it is 0.212 (calculated up to age 60).[6]

TABLE 2 HERE

The second thing to notice is that the tax and benefit system is effective at reducing inequality, but more so in the cross-section: when including the effect of indirect taxes, the cross-section Gini falls from 0.493 to 0.337, a reduction of 0.155 (or 31.4%) while the lifetime Gini falls from 0.258 to 0.195, a reduction of 0.046 (or 17.8%, which is just over half the corresponding cross-sectional fall).

Why should this be the case? The explanation is that much of the redistribution undertaken by the tax and benefit system is *intra*personal: redistribution between periods of life that nets off over the whole lifecycle. This sort of redistribution will be effective at reducing cross-sectional inequality but not lifetime inequality.

The proportion of redistribution that are intra or interpersonal can be independently quantified using a decomposition similar to Bovenberg, Hansen and Sørensen (2008). Let redistribution towards individual *i* at age *a* be given by

$$(10) \quad R_{i,a} = B_{i,a} - T_{i,a} - K_{i,a}$$

where $T_{i,a}$ is the taxes paid by individual *i* at age *a*, $B_{i,a}$ are the benefits received and $K_{i,a}$ is a 'no redistribution' baseline, all defined in PV 2015 terms. The baseline ensures that total redistribution across all individuals sums to zero even if the tax system raises net revenue overall. We consider two different definitions of a non-redistributive tax system to calculate $K_{i,a}$. Under the first, every individual's contribution is a constant amount in each period (a 'lump-sum baseline'). The second is where each individual pays a constant proportion of gross income in each period (a 'proportional baseline'). Redistribution towards or away from individuals is said to occur when net taxes paid are greater than or smaller than these baselines. Using these definitions it is possible to show (see the technical appendix) that total

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[6] The fact that our figure is slightly greater may partly be due to there being greater inequality in incomes than earnings. Bönke et al. find that inequality of long-run earnings are around two thirds of those for cross-sectional earnings. Our lifetime measure of inequality is less than half. Bönke et al. are however comparing within-cohort lifetime and cross-sectional inequality measures at different ages. Their measure of cross-sectional inequality is correspondingly lower.

redistribution can be decomposed into intra- and interpersonal components using the following relationship

(11) $$\underbrace{\sum_i \sum_a |R_{ia}|}_{\text{Total redistribution}} = \underbrace{\sum_i |R_i|}_{\substack{\text{Total} \\ \text{interpersonal}}} + \underbrace{2 \sum_i \min \{\sum_a [1(R_{ia} > 0) R_{ia}], - \sum_a [1(R_{ia} \leq 0) R_{ia}]\}}_{\text{Total intrapersonal}}.$$

where R_i is the sum of redistribution for individual i across life and $|\cdot|$ is the absolute value operator. When we calculate average proportions of intra- and interpersonal redistribution, we find that a majority of redistribution is intrapersonal i.e. across periods of life rather than across individuals. For the 1950, cohort these figures are 58.8% under the lump-sum baseline and 61.7% under a proportional baseline.[7] The reason for this is that most taxes and transfers are assessed over periods of a year or less, making it much easier to target outcomes over short horizons.

We now turn to using our simulations to answer questions on the lifetime impact of changing particular elements of the tax and benefit system, and how this might matter for more general design issues.

C. How Effective are Increases in In-work and Out-of-work Benefits at Targeting the Lifetime Poor?

Several countries provide significant income top-ups to low-income in-work families through tax credits. For example in the United States there is the Earned Income Tax Credit (EITC). In the United Kingdom these take the form of the Working Tax Credit (supplemented by Child Tax Credit for in and out-of-work families with children). Governments also attempt to help low-income households by removing low earning individuals from the direct tax net. Such reforms have the potential to provide help to low-income households while at the same time improving work incentives. However, in a cross-sectional analysis, they often appear less progressive than increases in out of work benefits, creating an equity-efficiency trade-off similar to that described for the EITC in Eissa and Hoynes (2011). How does a lifetime analysis affect this assessment?

To answer this question we compare reforms of these kinds in a cross-section and over the lifetime. We consider are three measures with similar cross-sectional revenue consequences (around £3 billion/$4.2 billion per year), namely:

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[7] Results from other birth cohorts are similar.

(i) an increase in out-of-work benefits: a 16.5% increase in the maximum income support, (income-based) jobseeker's allowance, pension credit and (non-contributory) employment support allowance awards;

(ii) an increase in in-work (i.e. work-contingent) benefits: an 18% increase in the maximum working tax credit award;

(iii) an income tax cut: a 4% increase in the income tax personal allowance. This is the threshold below which income tax is not paid. In 2015/16 this stood at £10,600 (around $16,000).

Reforms are applied using the 2015/16 tax and benefit system as a base.

Figure 5 shows the cross-sectional effect of the three reforms listed above. Unsurprisingly, the most progressive reform is the increase to out-of-work benefits: gains are concentrated in the bottom two income deciles, with the largest average gain experienced by the lowest income decile (5.0%), in which there is a high share of non-working individuals. Next most progressive is the increase to working tax credit. Here, the bottom four deciles are gainers, with gains peaking at an average of 1.7% in decile two. The bottom decile gains by less because fewer individuals here are entitled to working tax credit (because they are not in work). The least progressive (indeed a regressive) reform is the income tax cut in the form of an increase in the personal allowance. Gains are concentrated among the upper half of the income distribution, reflecting the fact that the poorest adults have income that is too low to benefit from the giveaway, while dual-income couples – who tend to have higher family incomes – can benefit twice over.

FIGURE 5 HERE

The pattern of gains over lifetime income is different, as shown in Figure 6. In particular, increases in out-of-work and in-work benefits are strongly progressive, with very similar distributional patterns, while the income tax cut (personal allowance increase) is close to distributionally neutral.

What is most interesting here is the fact that increases in out-of-work and in-work benefits have such a similar distributional pattern. This result stems from the pattern of worklessness over the lifecycle shown in Figure 3. While the poorest individuals in the cross-section are often out of work, this is often a temporary state and many of the poorest individuals in a lifetime sense move in and out of work. In addition, when in work, they are relatively likely

to be in low-paid work and therefore qualify for in-work support. This makes it possible to reach many of the lifetime poor through either out-of-work or in-work benefits.

FIGURE 6 HERE

The lifetime analysis is therefore relatively more favourable to a policy of increasing in-work benefits. The advantage of these sorts of payments is that – in general – they have much less of a negative impact on work incentives at the extensive margin (where labour supply responses may be particularly large, see Eissa and Liebman (1996) for a discussion in the context of the EITC). Out-of-work benefits reduce the net-financial gain to being in work, while in-work benefits in the United Kingdom are contingent on working a certain number of hours. Furthermore, as Blundell et al. (2015) highlight, the two types of benefits have different effects on incentives to accumulate human capital: while increasing out-of-work benefits provides a high level of insurance, they are associated with strong moral hazard effects and are less effective in improving overall welfare than in-work benefits.

Policymakers looking to target the lifetime poor might therefore favour doing so through in-work benefits. The disadvantage of such an approach is that it would do less to help the lifetime poor in the particular periods that they were not working, which could matter if they did not have access to savings or borrowing facilities. It would also do less to help the minority of the lifetime poor who do experience sustained periods without work. That said, recent experience suggests that the lifetime poor among younger cohorts may increasingly have substantial amounts of work over their lives but low levels of earnings rather than long periods out of the labour market (for cross-sectional evidence, see Belfield et al. (2014)).

D. Income Taxes and the Lifetime Rich

So far we have tended to find that measures that would otherwise be thought of as progressive appearing less redistributive once a longer-term view is taken. In light of this, we ask how well progressive income taxes target the lifetime rich. We consider increases in both the 'higher rate' of income tax which in 2015/16 applied at a rate of 40% to individual incomes above £31,786 (roughly $48,000) and the 'basic rate' of 20% applied to incomes between this and the personal allowance of £10,600 (around $16,000).[8]

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[8]The United Kingdom also has an additional rate of 45% which applies to incomes greater than £150,000 ($220,000). We would ideally also like to consider this but the survey data used to construct our simulations does not capture incomes well at the very top of the distribution and so we are unlikely to be able to model this accurately.

Figure 7 shows the distributional impact of a one percentage point increase in the higher rate of income tax, from both a cross-sectional and lifetime perspective. The reform is extremely progressive in the cross-section: the bottom four deciles are completely unaffected (because these individuals do not earn enough to pay the higher rate), and it is only the top two deciles that experience a hit to incomes of more than 0.1%, with losses peaking at 0.38% for the top decile. Over the lifetime, the reform remains strongly progressive, but there is slightly more of an impact further down the distribution. Those in the top four deciles experience a loss exceeding 0.1% but, as before, it is the very top decile that stands out, with a 0.23% fall. This reflects the greater persistence in earnings at the top of the distribution in our simulations (as we showed in figure 2). Thus, changes to the higher rate of income tax are reasonably effective at targeting the lifetime rich.

For comparison, we also present the distributional impact of a one percentage point increase in the basic rate of income tax (Figure 8). This shows that the cross-sectional impact is progressive: the bottom decile is largely unaffected (because most individuals in this decile do not earn enough to pay income tax) and the average loss peaks at 0.60% for the ninth decile. The top decile loses by slightly less because a smaller share of income for these individuals is subject to the basic rate. Over the lifetime, the impact remains progressive, but much less so because many more individuals will pay basic rate income tax at some point in life than in any one year. The bottom decile suffers a loss of 0.35%, rising to 0.53% for the top decile.

FIGURE 7 AND 8 HERE

E. Changes in VAT

The United Kingdom (along with other European countries) maintains different rates of VAT on different goods. Most expenditures are subject to the standard rate of VAT of 20%. Other expenditures such as domestic fuel are taxed at a reduced rate of 5%. Still others are exempt or zero-rated from VAT (the difference is that sellers of exempt goods are not able to claim back VAT charged on purchases of intermediate goods). These lower rates of VAT are typically charged on goods which are deemed to be economic necessities and which are disproportionately consumed by poorer households (such as food and children's clothing). However, a well known result in the public finance literature (Atkinson and Stiglitz, 1976; Kaplow, 2006) is that, given certain conditions, income taxes are less distortionary forms of

redistribution than differential commodity taxation. Here we consider whether such distributional concerns persist when things are considered in a lifetime perspective.

First we consider the distributional impact of VAT itself. Existing cross-sectional analysis (e.g. ONS, 2015) suggest that increases in the main rate of VAT are strongly regressive, with lower-income households losing much more as a percentage of their income than higher-income households. However, as has been argued by Mirrlees et al. (2011) among others, such a characterisation is misleading. It arises mainly because, at any given point in time, low-income households typically spend a lot relative to their incomes, and therefore pay a lot of VAT. However, households cannot spend more than their income indefinitely. Except for bequests given and received and the possibility of dying in debt, lifetime income and expenditure must be equal; households spending a lot relative to their income at any given point in time are often those experiencing only temporarily low incomes and either borrowing or running down their savings in order to maintain their expenditure smoothly at a level that more closely matches their lifetime resources. Such temporarily low incomes can arise for a variety of reasons: people who are temporarily unemployed, people with volatile income from self-employment, students, those taking time out of the labour market to raise children, retirees drawing on past savings, and so on.

Figure 9 shows that, on a cross-sectional basis, a one percentage point increase in the standard rate of VAT indeed appears strongly regressive: the lowest income decile loses, on average, 1.32% of its income, compared with 0.61% for the population as a whole. Over a lifetime, however, the effect is close to being neutral in distributional terms, with an average loss in the lowest lifetime income decile of 0.67% compared to 0.57% for the highest lifetime income decile and 0.61% for the population as a whole. This result is partly the result of the way we define lifetime income (ignoring inheritances and incomes other than earnings and private pensions). If a fuller definition of lifetime income were used which included all these things, and subtracted bequests and transfers given to other households, then a uniform VAT rate would of course be neutral by construction.[9]

FIGURE 9 HERE

Figure 10 shows the distributional impact of extending the main VAT rate to most zero- and reduced-rated goods. Levying or increasing VAT on those goods currently zero- or reduced-rated would be unambiguously regressive: on a cross-sectional basis, the average loss is 12.01% of income for the lowest income decile compared with 2.87% for the highest

[9] This (valuing transfers to others at less than dollar for dollar) is a measure of taxable capacity proposed by Kay (2010).

income decile, while over the lifetime the corresponding figures are 6.30% and 3.83%. This is because poorer households typically devote a larger share of their budgets to these items, and therefore will lose proportionally more from such a tax rise. However, Mirrlees et al. (2011) argue on efficiency grounds for such a reform and demonstrate that it is possible to design a set of direct tax cuts and benefit increases such that the overall package (including the flat-rate VAT) is revenue neutral, broadly distributionally neutral and would avoid worsening work incentives.

FIGURE 10 HERE

IV. Conclusion

In this paper, we have argued that snapshot analyses give only a partial impression of the distributional impact of tax and transfer reforms. Mobility in employment, earnings, housing tenure, health status and family composition means that longer-run reform impacts can look quite different. In particular, we found that increases to in-work benefits are just as good as increases to out-of-work benefits at targeting the lifetime poor because the lifetime poor spend the majority of working life in work. This stands in contrast to a snapshot analysis, which suggests increases to out-of-work benefits are considerably more progressive than increases to in-work benefits. We also showed how higher rates of income tax remain an effective way of targeting the lifetime rich because the greater persistence of incomes towards the top of the distribution means that high current incomes tend to be indicative of high lifetime incomes. Finally, while increases to the standard rate of VAT appear strongly regressive in the cross-section, they are almost distributionally neutral over the lifetime because income and expenditure are much closer to being equal.

We believe that lifetime analyses of this kind are an important complement to cross-sectional analyses. A purely static analysis can make measures appear more progressive or more regressive than they are over a longer horizon. This is not to say, however, that snapshot outcomes do not matter. A good reason why one might want to look at snapshot outcomes is if short-term hardship is particularly damaging to lifetime welfare. This might be the case if there are borrowing constraints or uncertainty, which mean that individuals are unable or unwilling to borrow to smooth out income fluctuations over time. An important avenue for future work is to explore the welfare implications of the reforms considered here.

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TABLES

Table 1: Cells within which Transition Probabilities are Scaled

State	Cells
Couple	Age, year, sex, number of children
Renter	Age, year
Employed	Age, year, sex, has children

Table 2: Gross and Net Income Gini Coefficients

Horizon	Gross income	Net income	Net income less in-direct taxes
Cross-section	0.493	0.298	0.337
Lifetime	0.258	0.195	0.212

Note: Taxes and benefits are calculated on an annual basis and are equivalised using the Modified OECD equivalence scale. The 'Net income' column excludes the effect of indirect taxes, while the 'Net income less indirect taxes' column subtracts them. Individuals face the 2015/16 tax and benefit system throughout life uprated in line with average earnings (AEI).

FIGURES

Figure 1: Mean Earnings and Pensions by Age and Sex
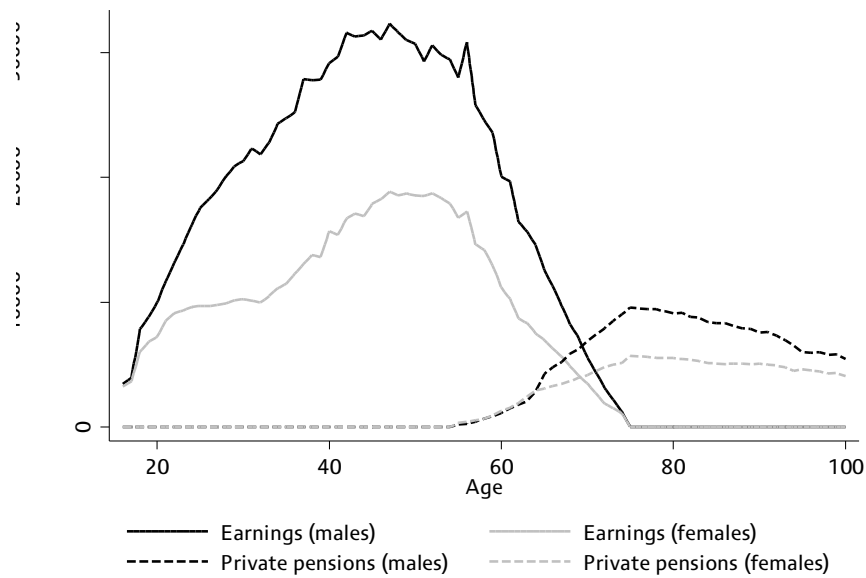
Note: Series show mean earnings and private pensions across life. Values are expressed in real 2015 terms (deflated by the Retail Prices Index, or RPI). Earnings are zero for the unemployed and for those not in receipt of a private pension.

Figure 2: Proportion of Life Spent in each Cross-Sectional Decile, by Lifetime Net Income Decile
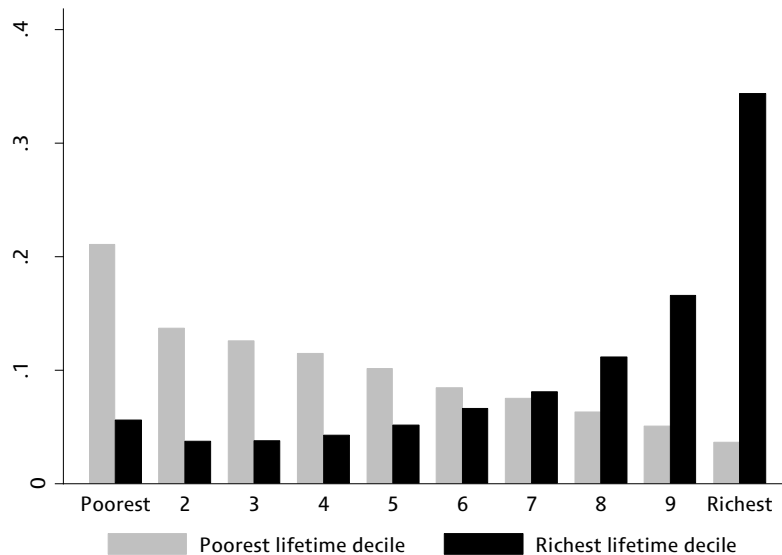

Note: The series show the proportion of life spent in each cross-section net income decile by individuals in the poorest/richest lifetime net income decile. Deciles are defined on equivalised net income ignoring indirect taxes (annualised net income for lifetime deciles).

Figrue 3: Employment among Working Age Individuals by Net Income Decile
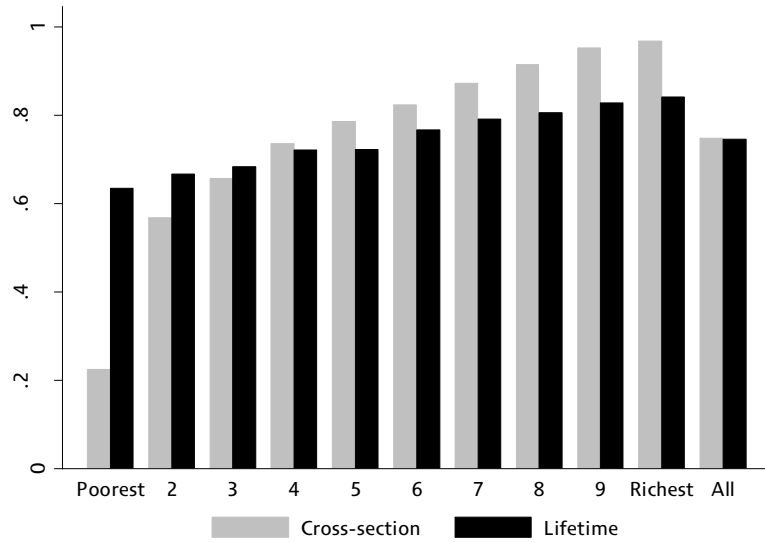
Note: The cross-section series shows the fraction of working-age individuals in each net income decile who are employed (deciles are defined using the whole population, not just working-age individuals). The lifetime series shows the average fraction of working life that individuals are employed for. Working age is defined as under 63 for women and under 65 for men. Deciles are defined on equivalised net income ignoring indirect taxes (annualised net income for lifetime deciles).

Figure 4: Gross Income Distributions
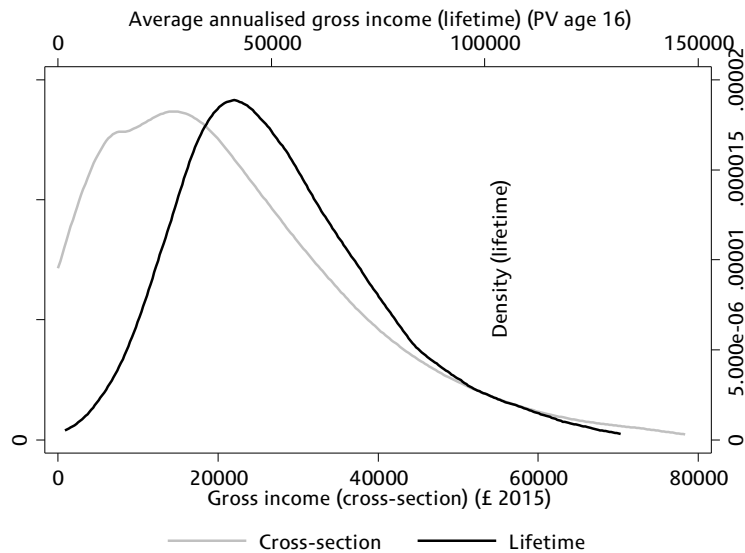
Note: The series show the densities of gross equivalised household incomes over the lifetime and in a cross-section. Lifetime incomes are expressed in annualised terns and are discounted to the year when individuals turned 16 (see equation (9)). We exclude the top 1% of incomes and those with zero incomes.

Figure 5: Cross-sectional Distributional Impact of Increases to Out-of-work and In-work Benefits and of the Income Tax Personal Allowance
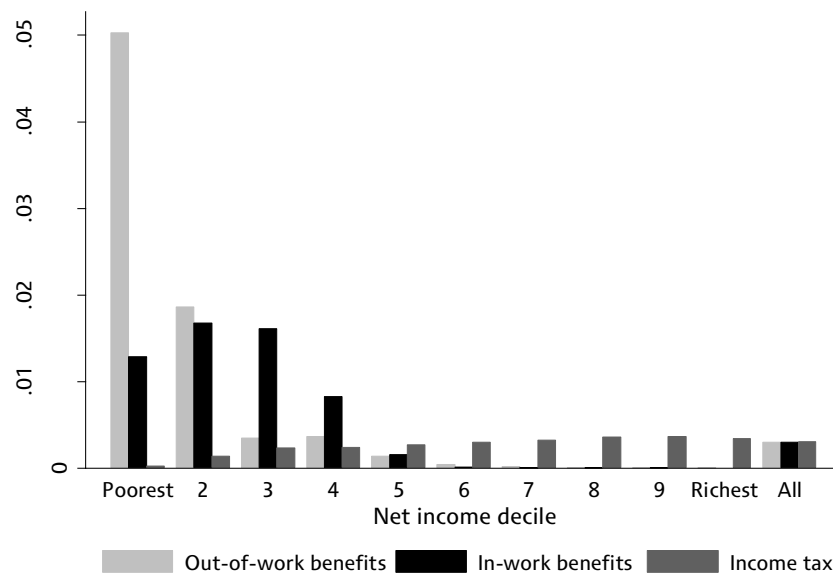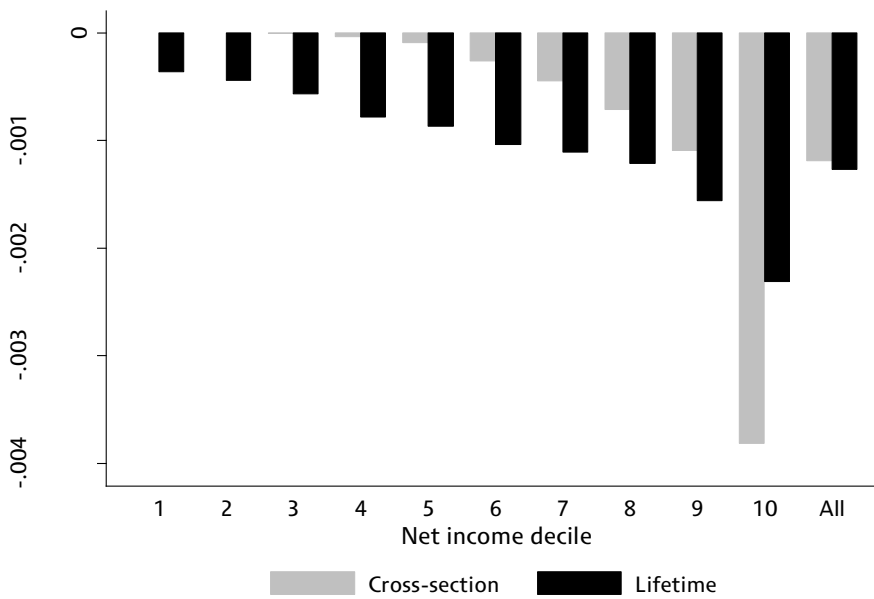

Note: Deciles are defined on the basis of cross-sectional equivalised net household income. The height of the bars is the gain or loss as a percentage of the relevant decile's total net (unequivalised) household income. The 'Out-of-work benefits' series shows the effect of a 16.5% increase in maximum income support, (income-based) jobseeker's allowance and (non-contributory) employment support allowance. The 'In-work benefits' series shows the effect of an 18% increase in maximum working tax credit. The 'Income tax' series shows the effect of a 4% increase in the income tax personal allowance. In all cases, the baseline tax and benefit system is the 2015/16 system.

Figure 6: Lifetime Distributional Impact of Increases to Out-of-work and In-work Benefits and of the Income Tax Personal Allowance
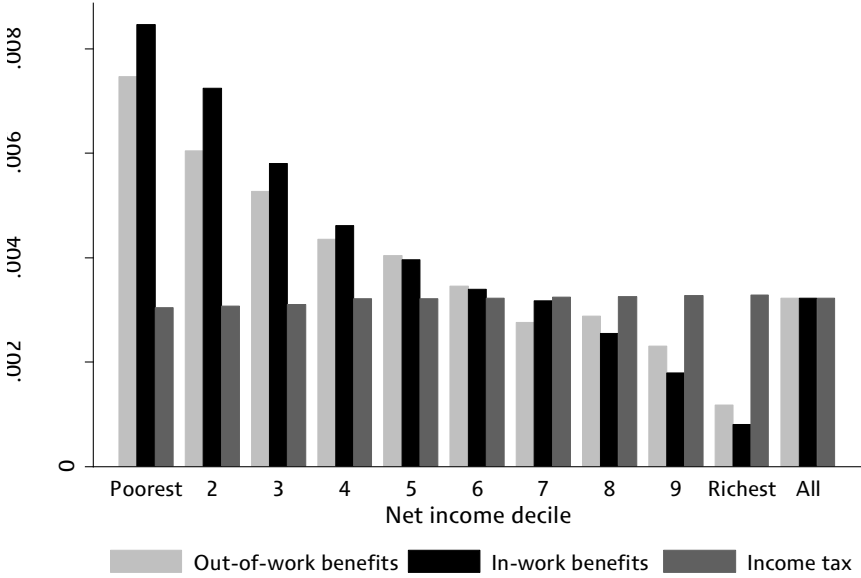

Note: The 'Out-of-work benefits' series shows the effect of a 16.5% increase in maximum income support, (income-based) jobseeker's allowance and (non-contributory) employment support allowance. The 'In-work benefits' series shows the effect of an 18% increase in maximum working tax credit. The 'Income tax' series shows the effect of a 4% increase in the income tax personal allowance. In all cases, the baseline tax and benefit system is the 2015/16 system. All individuals face the same system throughout life uprated in line with average earnings (AEI). To aid comparison, we have scaled gains proportionally such that the 'All' bars are the same across reforms.

Figure 7: Distributional Impact of a One Percentage Point Increase in the Higher Rate of Income Tax
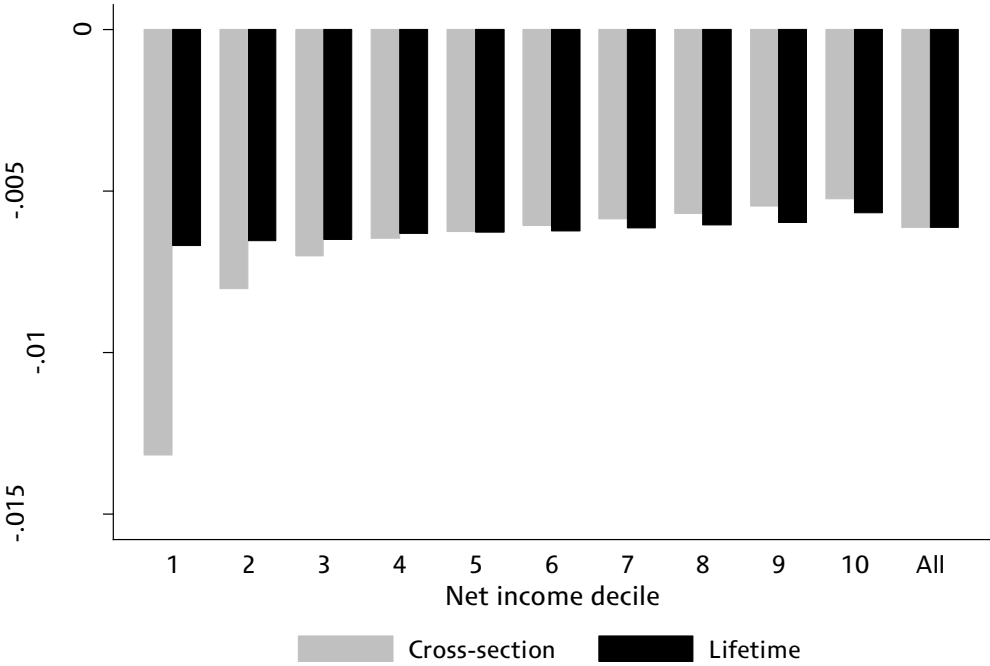

Note: Deciles are defined on the basis of equivalised net household income (cross-section income for the 'Cross-section' series and lifetime income for the 'Lifetime' series). The height of the bars is the gain or loss as a percentage of the relevant decile's total net (unequivalised) household income (cross-section income for the 'Cross-section' series and lifetime income for the 'Lifetime' series). The baseline tax and benefit system is the 2015/16 system. For the 'Lifetime' series, all individuals face the same system throughout life uprated in line with average earnings (AEI).

Figure 8: Distributional Impact of a One Percentage Point Increase in the Basic Rate of Income Tax
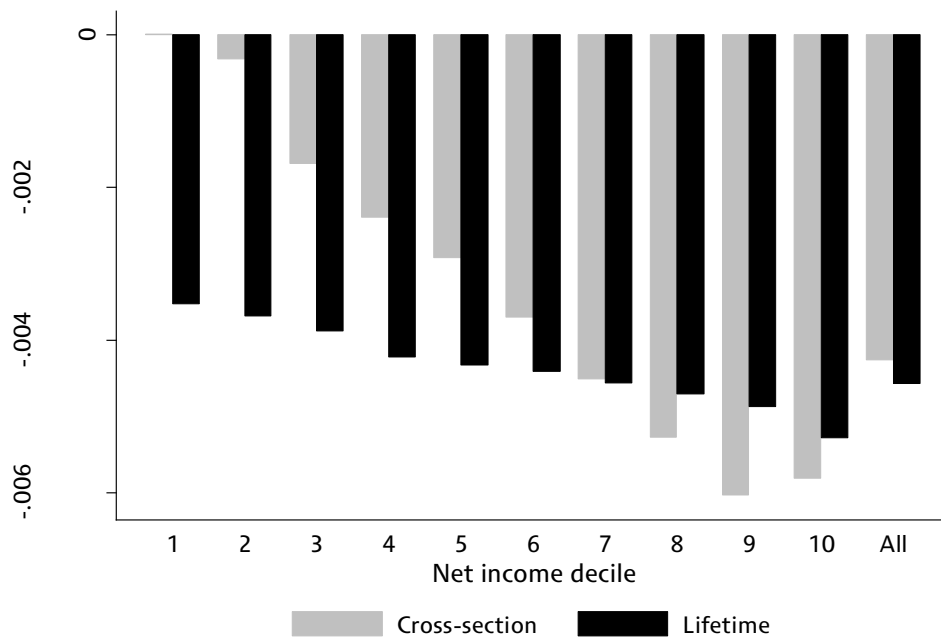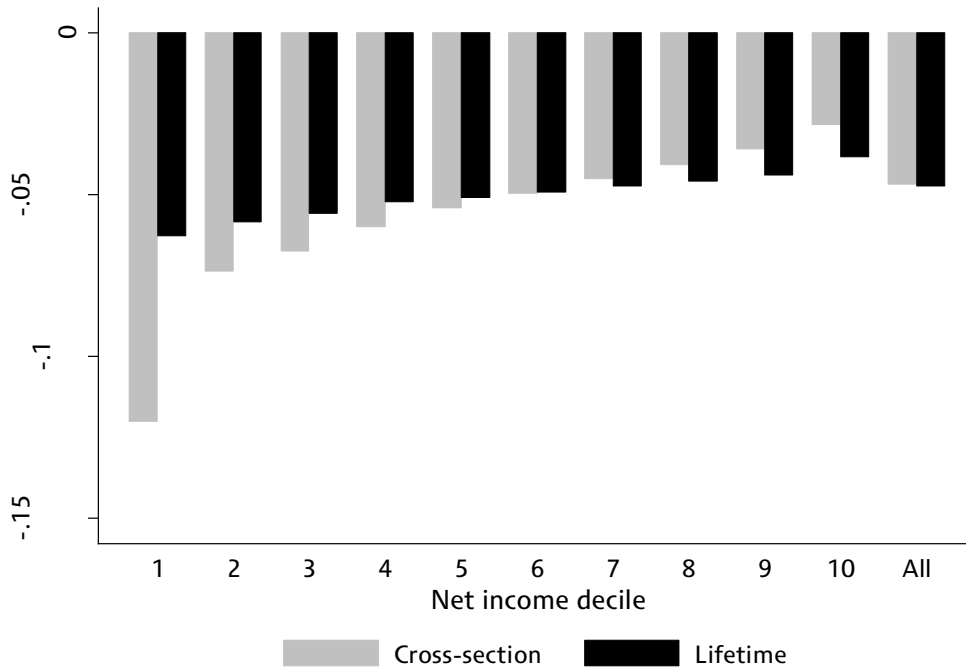

Note: Deciles are defined on the basis of equivalised net household income (cross section income for the 'Cross-section' series and lifetime income for the 'Lifetime' series). The height of the bars is the gain or loss as a percentage of the relevant decile's total net (unequivalised) household income (cross-section income for the 'Cross-section' series and lifetime income for the 'Lifetime' series). The baseline tax and benefit system is the 2015/16 system. For the 'Lifetime' series, all individuals face the same system throughout life uprated in line with average earnings (AEI).

Figure 9: Distributional Impact of an Increase in the Standard Rate of VAT

Note: Deciles are defined on the basis of equivalised net household income (cross-section income for the 'Cross-section' series and lifetime income for the 'Lifetime' series). The height of the bars is the gain or loss as a percentage of the relevant decile's total net (unequivalised) household income (cross-section income for the 'Cross-section' series and lifetime income for the 'Lifetime' series). The baseline tax and benefit system is the 2015/16 system. For the 'Lifetime' series, all individuals face the same system throughout life uprated in line with average earnings (AEI).

Figure 10: Distributional Impact of Extending the Standard Rate of VAT to Zero- and Reduced-rated Goods

Note: Deciles are defined on the basis of equivalised net household income (cross-section income for the 'Cross-section' series and lifetime income for the 'Lifetime' series). The height of the bars is the gain or loss as a percentage of the relevant decile's total net (unequivalised) household income (cross-section income for the 'Cross-section' series and lifetime income for the 'Lifetime' series). The baseline tax and benefit system is the 2015/16 system. For the 'Lifetime' series, all individuals face the same system throughout life uprated in line with average earnings (AEI).

Appendix A provides additional details on how we construct and validate our simulated lifetime profiles. Details on how we model employment and earnings as well as how we calculate taxes and benefits are given in the main body of the text. Appendix B gives the details of how our measure of intrapersonal redistribution was derived.

Appendix A: Methods and Validation

Tables A1 and A2 show the specifications we run to estimate transition probabilities for each of our processes. Section A.II then discusses how we model partnering behaviour. Section A.III discusses how we model rents, which bears some similarities to the way we model earnings. In Section A.IV we describe how we impute private pension profiles. Section A.V discusses the imputation of consumption. In Section A.VI we provide some statistics on how well our simulations capture important moments in the data.

Table A1: Estimation Equations for Demographics and Rents

Outcome	Method	Subsamples	Independent variables
Mortality	Logit		Cubic in age, dummy for receipt of disability benefits, couple status, education dummies and earnings quintile
Child arrival	LPM	Run separately for women in couples and single women	For childless women: quadratic in age, dummy for ever had kids, number of kids ever had
			For women in couples: as for childless but also banded number of kids (0,1,2, and 3 or more) in household, age of youngest child, age of youngest child interacted with age
Child departure	LPM	Run separately by age of child (16-19)	Dummies for mothers and fathers education
Partnering	Logit	Run separately for 3 education groups and sex	Quartic in age, dummy for employed last period, dummies for number of kids in household (0,1,and 2 or more), dummies for couple status in previous three periods, dummy for single status last period interacted with age
Separating	Logit	Run separately for own education and sex	Quartic in age, employed last period, partner employed last period, dummies for banded number of kids in household (0,1,and 2 or more), cubic in current relationship length, age of youngest child, dummy for education same as partner
Health (IB and DLA receipt)	Logit		For IB: quartic in age, 4 lags of employment status (interacted), 4 lags of IB status (interacted) earnings quartile last period
			For DLA: quartic in age, 4 lags of employment status (interacted), 4 lags of DLA status (interacted) earnings quartile last period and 2 lags of IB status
Renter (21 and over)	Logit	Run separately for current owners and current renters and for over and under 21s	Age of head of household, education of head of household, earnings quintile last period of head of household, banded number of kids (0,1,2 or 3 or more), couple status, relationship length dummy for rented last period, 4 lags of ownership status
Rank in rent distribution (21 and over)	OL	Run separately for owners, and renters in each of 5 rent quintiles	Age of head of household, education of head of household, earnings quintile last period of head of household, banded number of kids (0,1,2 or 3 or more), couple status, relationship length dummy for rented last period, 4 lags of ownership status
Renter status and rank (under 21)	MNL		Age of head of household, age of head of household squared
Council tax band	OL	Run separately for each of 8 possible prior bands	Cubic in age, banded number of children (0,1,2,3, 4 or more) renter status earnings quartile of household head, employment status

Notes: LPM = Linear probability model, OL = ordered logit, MNL = multinomial logit.

Table A2: Estimation Equations for Employment and Earnings

Outcome	Method	Subsamples	Independent variables
Employment (22 and over)	Logit	Run separately for males and females, by employment in prior wave and by employment 2 waves ago	Education dummies, quartic in age, age-education interactions, dummy for over state pension age, dummy for having kids, dummy for couple status, dummy for having kids under 5, kids under 5 interacted with cubic in age, 3 lags of full-time status, banded number of kids (0,1,2 and 3 or more), couple status, couple-age interaction, lagged full-time status, lagged earnings rank, dummies for earnings quartiles (and 5 lags), employment status 3, 4,5 and 6 waves ago (and interactions), lagged disability status
Earnings quartile and part-time/full-time status (22 and over)	MNL	Run separately for each of 5 possible prior states: in part-time work, in full-time work and in 4 earnings quartiles and separately for males and females	Education dummies, quartic in age, age-education interactions, dummy for over state pension age, dummy for has kids, couple status, dummy for kids under 5, 3 lags of full-time status, current earnings rank (and 3 lags), 3 status (interacted) lags of earnings quartile dummies, 3 lags of employment
Employment and earnings (under 22)	MNL	Run separately for each of 6 prior possible states: unemployment, in part-time work, in full-time work and in 4 earnings quartiles	Sex, education dummies, dummy for has kids and age
Earnings rank within `bin' (20 and over)	OLS	Run separately by prior state and sex	Cubics in 4 lagged (within bin) ranks interacted with cubic in age, education dummies, dummies for `bin' in previous 4 periods
Earnings rank within `bin' (under 20)	OLS	Run separately by prior state and sex	Cubics in lagged (within bin) ranks interacted with cubic in age, education dummies

Notes: LPM = Linear probability model, OL = ordered logit, MNL = multinomial logit.

A.II Partnering

Individuals select partners within our simulated sample. Thus all matches are assumed to take place within the same (nine year) birth cohort. We allow for assortative matching in the choice of partners on the basis of education level, such that university-educated individuals are more likely to match with other university-educated individuals than those with high school qualifications only. In order to implement this, we match potential partners based on an index that depends on education level and a random shock:

$$(1) \qquad I = ed_2 + \beta ed_3 + u$$

with $u \sim N(0, \sigma_u^2)$. The values of the unknown parameters β and σ_u^2 are chosen such that the distance between the simulated three-by-three matrix of education group against partner education group is as close to the empirical one as possible.

Which potential couples are realised, and which existing couples are dissolved, depends on partner arrival and departure probabilities estimated from our panel data and scaled so as to match the LCFS cross-sectional proportion of couples. Probabilities are scaled separately for those with and without children to allow for cohort differences in the partnering and separating behaviour of parents between our sample and the baby-boomers (in particular are large secular increase in the proportion of single parents). The lower of the two male and female probabilities are used to calculate the probability of separation for couples. This is to allow us to better match the persistence of couples observed in the data. New couples and newly single individuals do not return to the partnering market until the following period.

Each couple requires a male and a female, and so a mismatch in the numbers of each can lead to too few matches being formed relative to what our estimated probabilities would imply. To avoid this happening, probabilities of partnering are again scaled to achieve the expected number of matches. Matches can only occur between individuals who are both aged 16 or older.

We wish to allow for the fact that males in couples in the 1945-54 cohort seen in the LCFS are on average just over two years older than females. This is important because it has a knock-on effect on the ages at which children

are born. To achieve this, our simulated males are born in the years 1945-52 while females are born between 1947-1954. This means in each period that the marriage market will be composed of females that are on average two years younger than their male counterparts.

A.III Rents

For rental payments and ownership status, we adopt a very similar procedure to that for earnings. We first use the estimates of a logit to determine whether an individual is an owner or a renter. For those who are renters, we then use the estimates of an ordered logit to predict their rent quintile; controlling for education of the household head (assumed to be the male in any couple), a cubic in age for the household head, couple status, relationship length, banded number of children and several lags of past renter status and past quintiles of the rent distribution. Placement within rental quintiles is random. The variance of the rental distribution is not as great as that of earnings meaning the exact placement within quintiles matters less. If the lagged variables differ between two members of a couple, they are taken from the household head. For younger individuals for whom we do not have a complete set of lags (those under 21), we run a simpler multinomial logit to determine transitions across all the possible states.

The probability of owners becoming renters and renters become owners are scaled to match the LCFS proportions. For rental status this scaling is particularly important, as historically in the UK the proportion of renters was much higher than what we observe within the time frame covered by the BHPS panel.

A.IV Private Pensions

For private pensions we combine information from two datasets. The first consists of estimates of the discounted value of future private pension incomes for individuals in the BHPS survey from Disney et al. (2007). These estimates give the present value of future incomes for individuals if had they retired in 2001 or earlier, as well as projections for the future value of private pension wealth if individuals had continued in their present employment status until

state retirement age. They are calculated using information from the special module of questions on private pensions included in the 2001 wave of the survey.[10] The second is a set of predicted future private pension incomes for individuals seen in 2008 of the English Longitudinal Study of Ageing (ELSA). These include projected income streams conditional on individuals beginning to draw their private pensions in different years from 2008 onwards. The authors are indebted to Rowena Crawford, Soumaya Keynes and Gemma Tetlow for producing these projections and sharing them with us. Details of their methodology can be found in Crawford (2012) with an example of their use in Banks et al. (2014).

The approach we follow allows us to match real-world private pension income profiles to our simulated individuals on the basis of their labour market histories and other characteristics. We implement it in the following steps once our simulations have completed.

1. We first estimate a probability that a simulated individual will ever receive a private pension using an individual's characteristics in 2001. We do this by estimating a logit model in the BHPS for that year. This regresses a dummy for positive projected private pension wealth in 2001 on sex and education dummies (and interactions of these), dummies for the number of the previous five years the individual was employed and dummies for the individuals' decile of a five-year moving average of previous earnings ranks.

2. We then predict the 2001 private pension 'wealth' (defined in here as the discounted value of future private pension incomes) for those simulated individuals who are to receive private pensions. This is done using the results of a regression of pension wealth in 2001 on a cubic in age, education dummies (and interactions of these) sex, years employed and a moving average of past earnings in the BHPS to which we add a normally distributed noise term.

3. Finally we calculate the simulated individuals' ranks in this distribution within cells defined by age and year. We can then use these ranks to match individuals to a one of a set of future streams of private pension income from

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[10] The data itself has been deposited in the UK Data Archive.

the ELSA data within cells defined by cohort, sex and couple status in 2008 (or earlier if they retire before this). Ranks for our simulated individuals (estimated for 2001) are used to match individuals to private pension profiles ranked according to their present value in 2008.

An individual's retirement age is defined as the maximum of the final age at which they stopped working and 55. The ELSA data only predicts pension income for those who retire from 2008 onwards. For those who retire earlier than this, we deflate pension profiles associated with their retirement age using average earnings growth between 2008 and the year of their retirement. Earnings growth is what would determine private pension income for prior years from a defined benefit final salary scheme. The matching procedure works well, with on average 100 potential matches for each individual and an average distance between the ranks of donors and recipients of less than 1 percentage point.

A.V Consumption

Including consumption spending in our simulations is important because it will help us calculate the value of indirect taxes individuals pay at different life stages. Detailed consumption expenditure is necessary because different spending items are subject to different tax treatments in the UK. VAT is not charged on food for example, and a reduced rate is charged on energy spending. Consumption is imputed to our individuals separately by spending categories defined by tax treatment using regressions estimated in the LCFS over the period 1978-2012. We are not able to sort consumption by tax category before this. As with other national consumption surveys, spending as recorded by the LCFS has tended to fall over time relative to national account measures (Brewer and O'Dea, 2012; Barrett et al., 2015). To offset this we scale spending categories by a common factor for each individual in such a way that total spending matches the national accounts figures in each year. A similar (but smaller) scaling is also applied to average earnings.

The manner in which we impute consumption captures variation by income, age and demographics, but does not allow for autocorrelations in

shocks to spending. Other things equal, this will mean that those with volatile incomes in our data will have the same propensity to save as those with steady incomes. Such issues are unlikely to prevent us from drawing conclusions about the broad distributional impacts of consumption taxes over the lifetime. They will however have to be borne in mind when considering our results on the impacts of indirect taxation.

A.VI Validation

Figures A1-A6 show age profiles for males and females from our simulated individuals compared to those observed for the baby-boom cohort in the LCFS for couple status, employment, parenthood, single parenthood, number of children, and housing tenure. Despite our scaling procedure, cross-sectional averages for our simulations need not automatically match those in the LCFS, because the scaling only occurs within population subgroups. For instance, even if we accurately reproduced probabilities of being in a couple for those who have children and those who don't, the proportion of couples would not match those in the LCFS if we did not also have the correct probabilities of being a parent at each age. Nonetheless, the match between the simulated individuals and cross-sectional averages in the data is excellent for all variables and both sexes.

Age profiles show familiar "hump" shapes for parenthood and fertility. Employment tends to be higher for males than females and to decline with age. For females the age profile in employment shows a dip in the main child-rearing years just before age 30. The proportion of households who are renters declines steeply with age reflecting a secular increase in ownership rates over this period (partly driven by a series of "right to buy" reforms which allowed tenants in social housing to purchase their properties at reduced cost).

Some differences arise because of particular restrictions we impose. A difference in employment rates between the simulations and the data for younger ages is due to the fact that we impose that all those who have not completed full-time education are unemployed. A similar difference in the proportion of parents who are single in Figure A4 is due to the fact that, for

years when cohorts are unobserved, we set the marriage rate for under 18s to be zero.

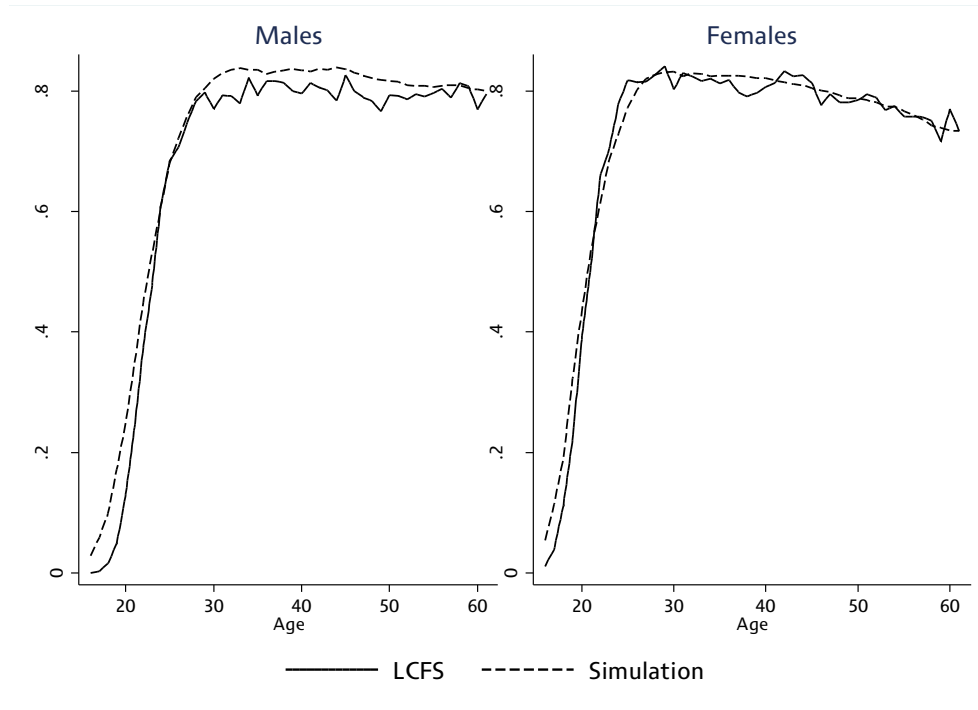**Figure A1: Proportion in Couples: Simulations versus Data**

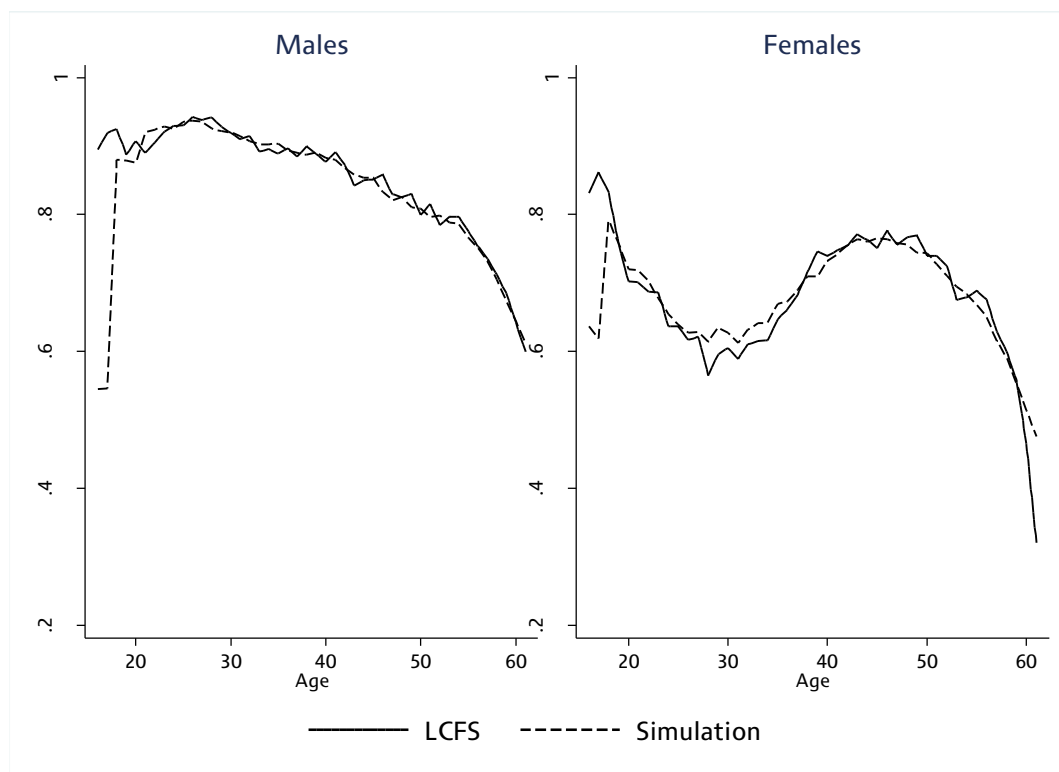**Figure A2: Employment: Simulations versus Data**

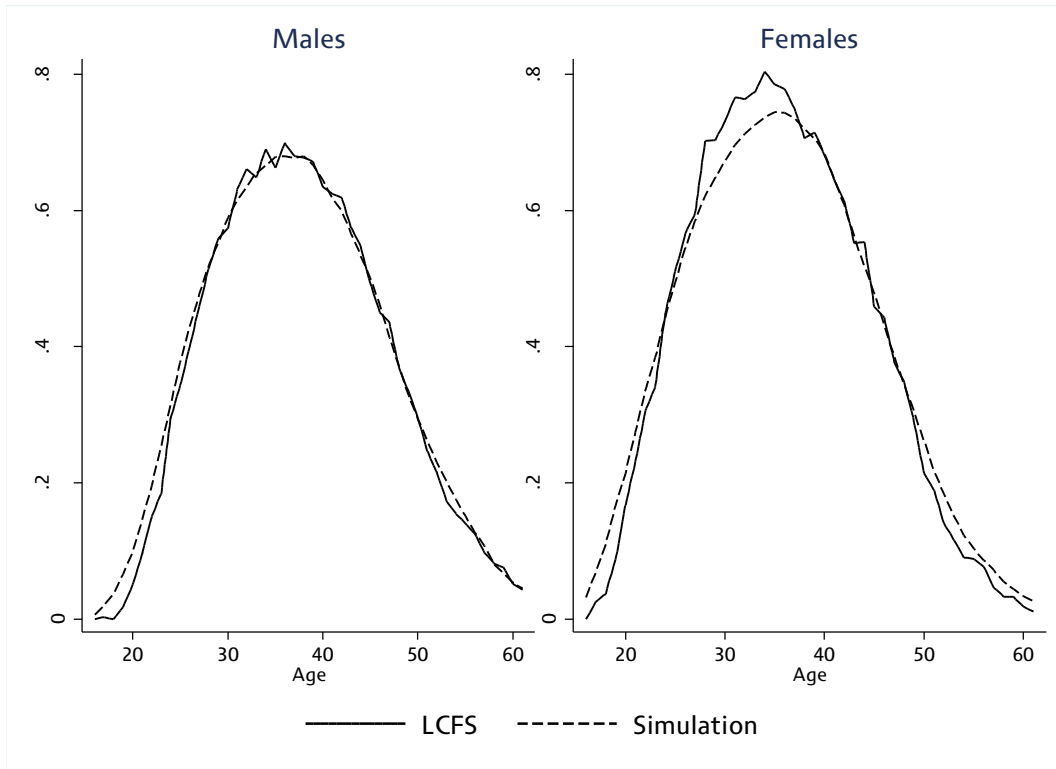**Figure A3: Proportion of Parents: Simulations versus Data**

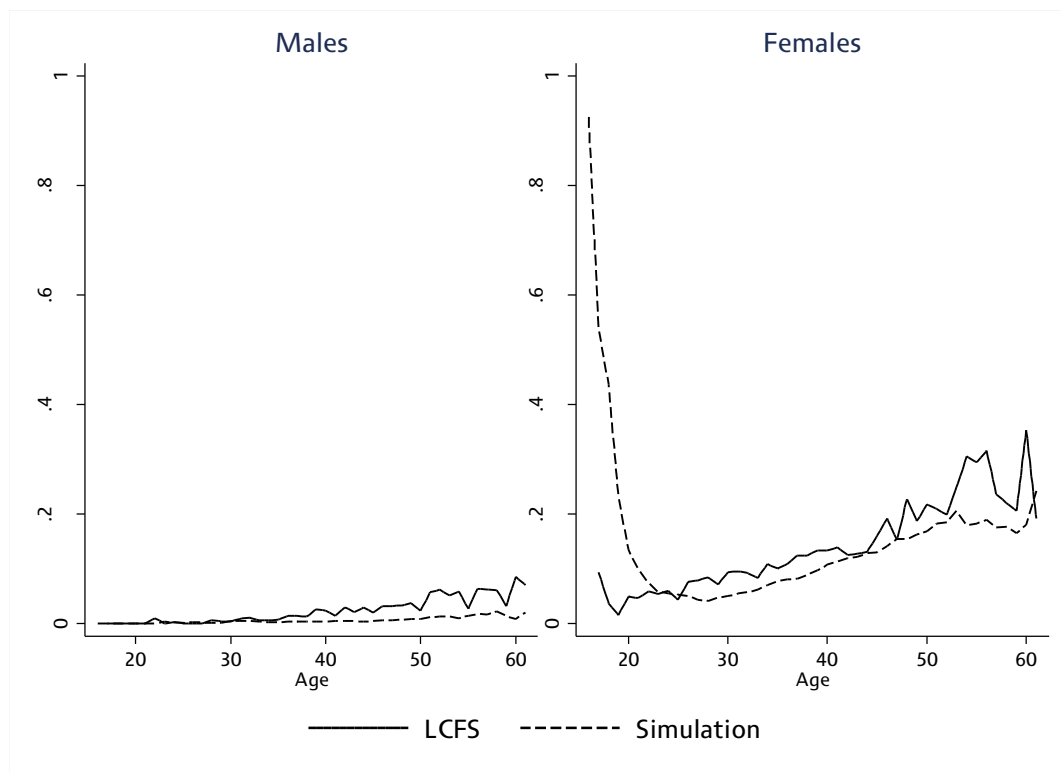**Figure A.4: Proportion of Parents who are Single: Simulations versus Data**
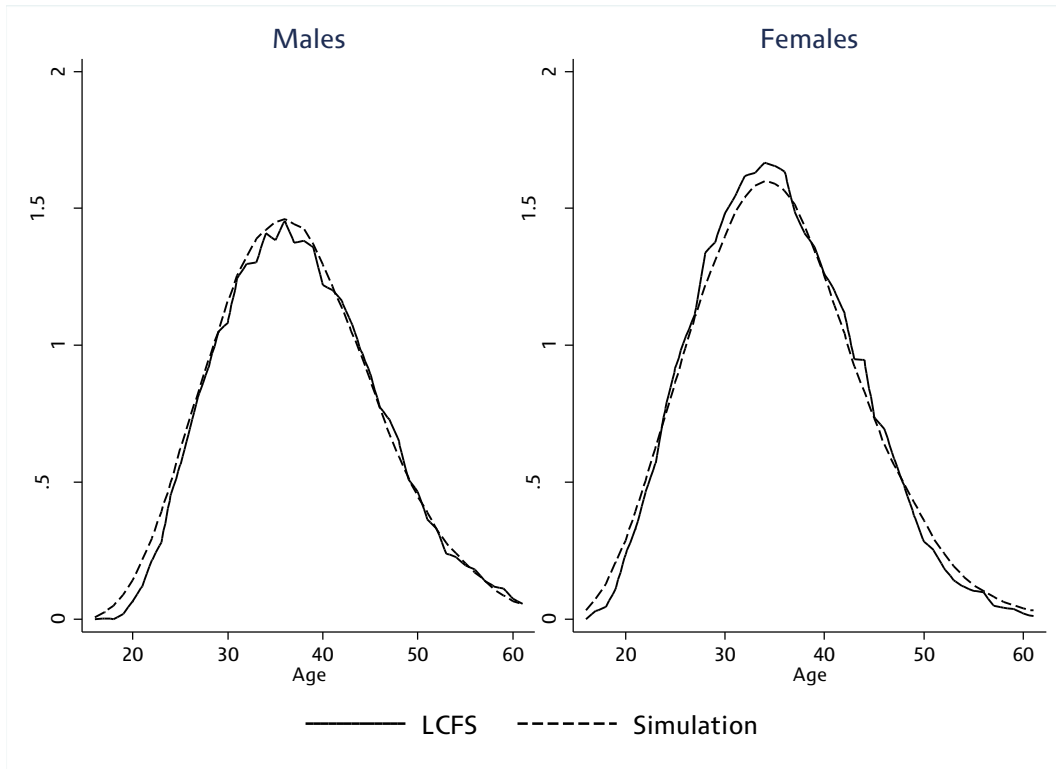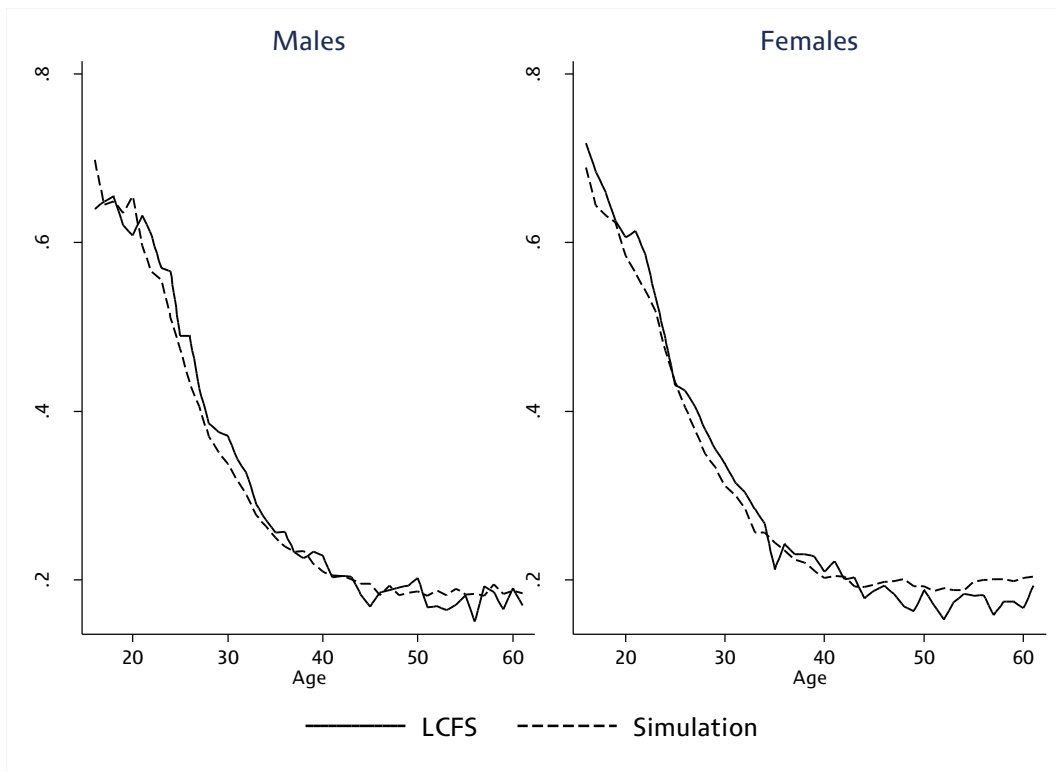
Figure A5: Number of Children: Simulations versus Data

Males

Females

Figure A6: Proportion of Renters: Simulations versus Data

Males

Females

46

Since we use our simulated individuals for distributional analysis of lifetime outcomes it is important that the persistence of variables such as income match those of the data, as well as cross-sectional average. Unfortunately we are not able to compare the persistence of variables for our simulated individuals directly with individuals from the baby-boom cohort throughout the whole life-cycle, because we do not have access to a panel dataset covering the whole of the adult life-cycle for the baby boomers. Instead, we plot autocorrelations for our spliced individuals against those individuals seen in the BHPS for the period 1991-2008. These may show whether the transitions we obtain are plausible even if they cannot be used for direct validation. Figures A7-A10 plot autocorrelations for 1 year ahead, 5 years ahead and 10 years ahead for males and females from ages 16-65 for couple status, parent status, employment and earnings ranks. The persistence of renter status is very different in the BHPS from the baby-boom cohort as a result of the much steeper declines experienced by the baby-boomers relative to those in later years. As a result autocorrelations for this variable are unlikely to be very informative and so we do not show them.

The processes experienced by our simulated individuals tend to have similar persistence to those observed in the BHPS, although there are differences. For example Figure A10 shows that for our simulated individuals, ranks in the earnings distribution are less persistent at middle age for longer horizons than earnings ranks in the BHPS, and a little more persistent at older ages. The difference is a little larger for males than for females. Employment, couple, and parent status have similar persistence in our simulations to the data for much of life- even over 10 a year horizon – but greater persistence at the end of life. This latter feature may well be due to differences between older individuals in the BHPS and in our cohort. The greater mobility of some variables (particularly earnings and employment) however likely reflects an understatement of persistence resulting from our use of a relatively short panel. This will be an important caveat to consider when interpreting the results.

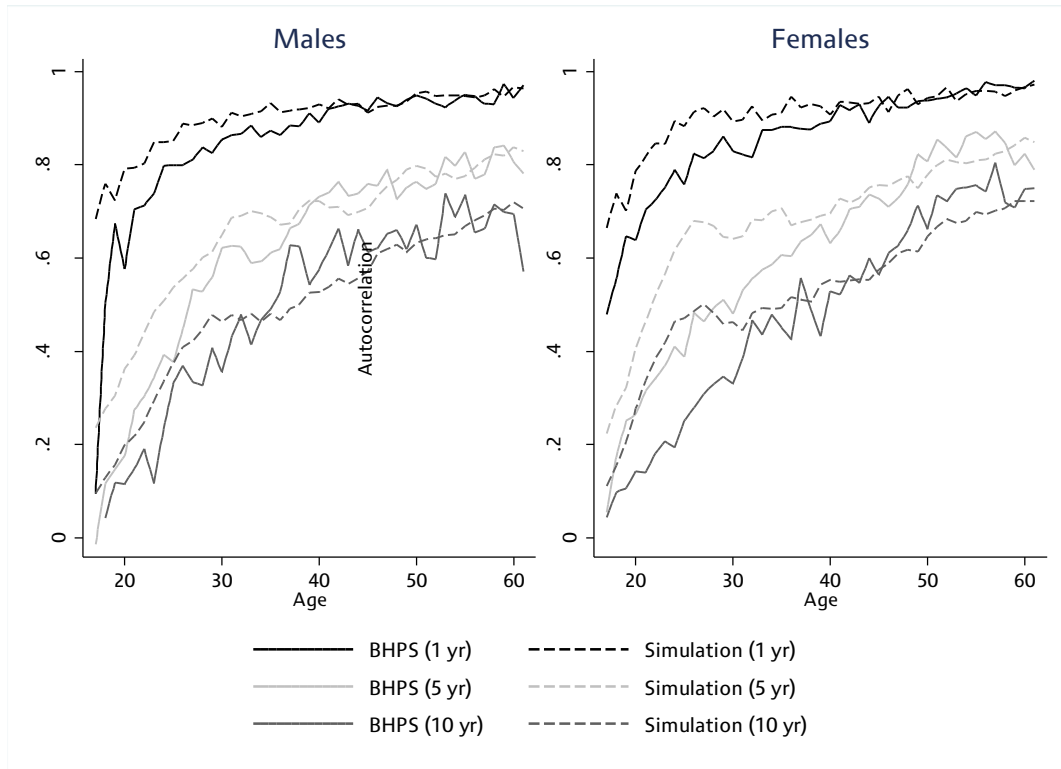**Figure A7: Autocorrelations Couple: Simulations versus Data**

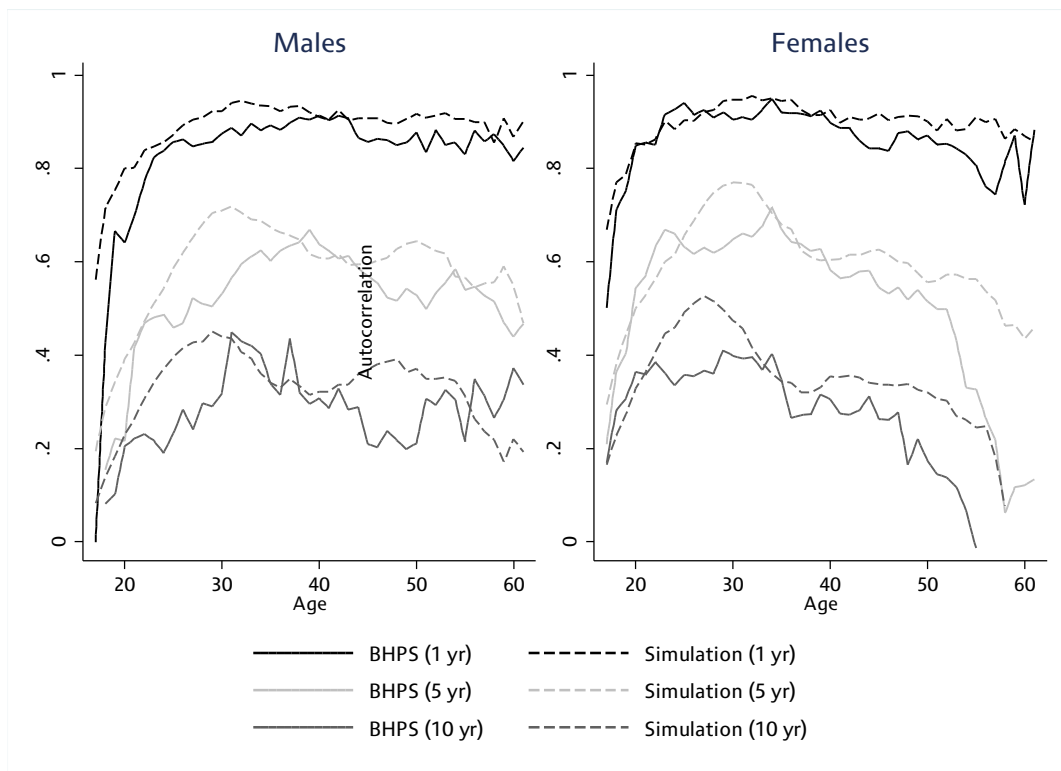**Figure A8: Autocorrelations Parent: Simulations versus Data**

Figure A9: Autocorrelations Employment: Simulations versus Data
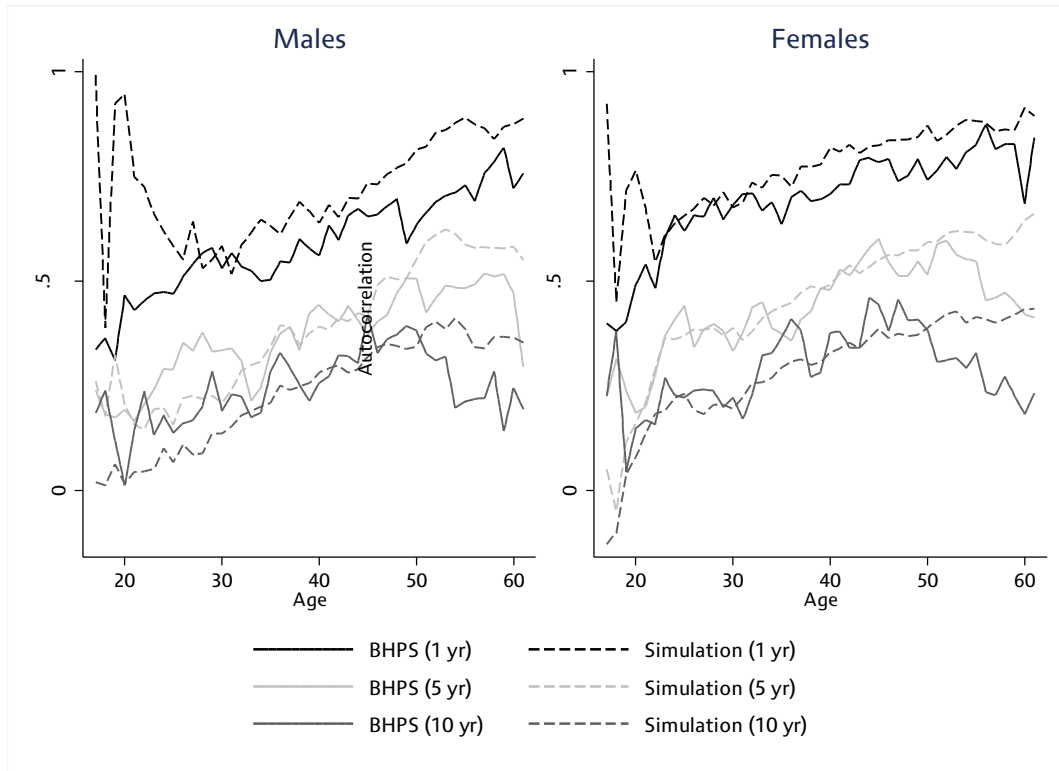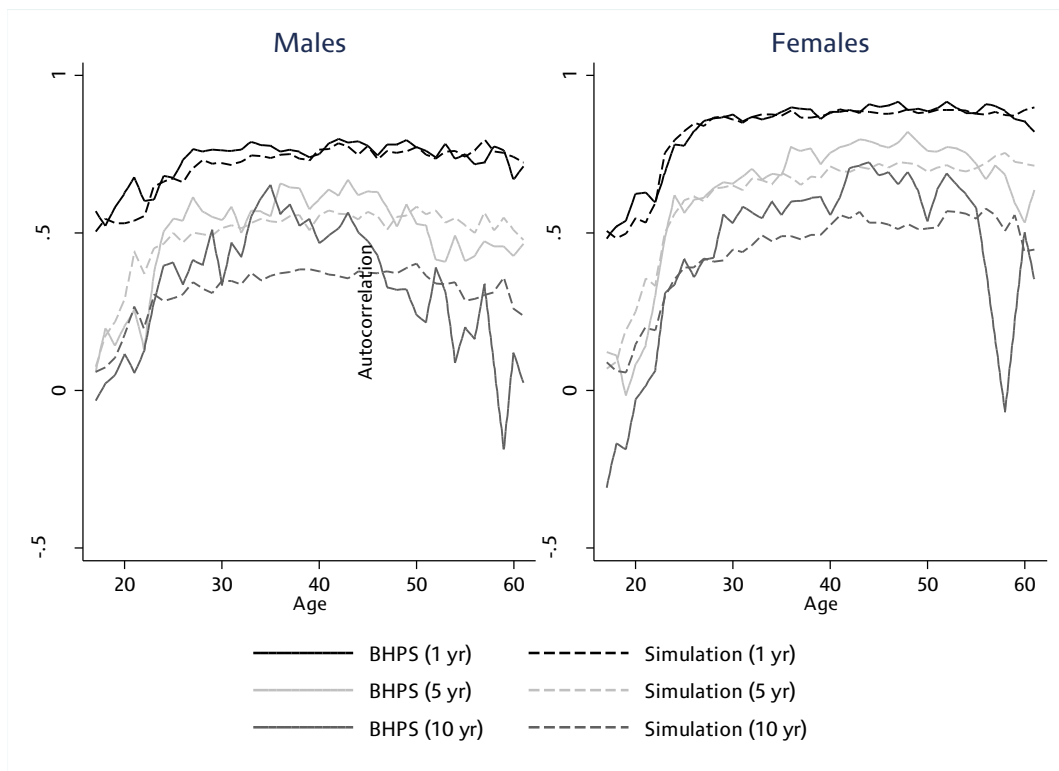

Figure A10: Autocorrelations Earnings Ranks: Simulations versus Data

As a final check on the performance of our simulations, we can compare our simulated individuals to those from the same cohort in the waves they are

observed in the BHPS. Table A3 shows the proportions always employed, always unemployed, always in a couple and always single over 10 years from 1995 to 2004 (inclusive) in the BHPS and in our simulations. Our simulations come very close to matching these proportions.

Table A3: Persistence of Employment and Couple Status for 1945-54 Cohort in BHPS and Simulations

	BHPS	*Simulations*
Always employed	55.9%	56.3%
Always unemployed	12.4%	11.4%
Always couple	73.7%	73.6%
Always single	15.6%	12.9%
N	676	4666

Appendix B: Intrapersonal Redistribution

In this appendix, we derive a formula for the share of redistribution that is intrapersonal. Let B denote benefits received, T taxes paid and $R = B - T - K$ redistribution, where K is the no-redistribution baseline (either lump-sum or proportional to gross earnings), all defined in PV 2015 terms. We use i to index individuals and a to index age, and the absence of a subscript from one of these variables indicates summation, e.g. $R_i = \sum_a R_{ia}$. We can write:

$$\underbrace{\sum_a |R_{ia}|}_{\text{Total redistribution}} = \underbrace{\sum_a [1(R_{ia} > 0)R_{ia}]}_{\text{Total redistribution towards individual}} - \underbrace{\sum_a [1(R_{ia} \leq 0)R_{ia}]}_{\text{Total redistribution against individual}} .$$

We can decompose the two terms on the right-hand side separately. For the redistribution towards term, we write:

$$\underbrace{\sum_a [1(R_{ia} > 0)R_{ia}]}_{\text{Total redistribution towards}} = 1[R_i > 0]\left[\sum_a [1(R_{ia} > 0)R_{ia}]\right] + 1[R_i \le 0]\left[\sum_a [1(R_{ia} > 0)R_{ia}]\right]$$

$$= 1[R_i > 0]\left[\sum_a [1(R_{ia} > 0)R_{ia}] + \sum_a [1(R_{ia} \le 0)R_{ia}]\right]$$

$$- 1[R_i > 0]\left[\sum_a [1(R_{ia} \le 0)R_{ia}]\right] + 1[R_i \le 0]\left[\sum_a [1(R_{ia} > 0)R_{ia}]\right]$$

$$= 1[R_i > 0]R_i - 1[R_i > 0]\left[\sum_a [1(R_{ia} \le 0)R_{ia}]\right]$$

$$+ 1[R_i \le 0]\left[\sum_a [1(R_{ia} > 0)R_{ia}]\right]$$

$$= \underbrace{1[R_i > 0]R_i}_{\text{Interpersonal towards}} + \underbrace{\min\left\{\sum_a [1(R_{ia} > 0)R_{ia}], -\sum_a [1(R_{ia} \le 0)R_{ia}]\right\}}_{\text{Intrapersonal towards}}.$$

Likewise, for the redistribution against term, we can write:

$$\underbrace{\sum_a [1(R_{ia} \le 0)R_{ia}]}_{\text{Total redistribution against}} = 1[R_i > 0]\left[\sum_a [1(R_{ia} \le 0)R_{ia}]\right] + 1[R_i \le 0]\left[\sum_a [1(R_{ia} \le 0)R_{ia}]\right]$$

$$= 1[R_i \le 0]\left[\sum_a [1(R_{ia} \le 0)R_{ia}] + \sum_a [1(R_{ia} > 0)R_{ia}]\right]$$

$$- 1[R_i \le 0]\left[\sum_a [1(R_{ia} > 0)R_{ia}]\right] + 1[R_i > 0]\left[\sum_a [1(R_{ia} \le 0)R_{ia}]\right]$$

$$= 1[R_i \le 0]R_i - 1[R_i \le 0]\left[\sum_a [1(R_{ia} > 0)R_{ia}]\right]$$

$$+ 1[R_i > 0]\left[\sum_a [1(R_{ia} \le 0)R_{ia}]\right]$$

$$= \underbrace{1[R_i \le 0]R_i}_{\text{Interpersonal against}} - \underbrace{\min\left\{\sum_a [1(R_{ia} > 0)R_{ia}], -\sum_a [1(R_{ia} \le 0)R_{ia}]\right\}}_{\text{Intrapersonal against}}.$$

Summing these two together, we find:

$$\underbrace{\sum_a |R_{ia}|}_{\text{Total redistribution}} = \underbrace{|R_i|}_{\substack{\text{Total} \\ \text{interpersonal}}} + \underbrace{2\min\left\{\sum_a [1(R_{ia} > 0)R_{ia}], -\sum_a [1(R_{ia} \le 0)R_{ia}]\right\}}_{\text{Total intrapersonal}}.$$

Summing across individuals, we obtain:

$$\underbrace{\sum_i \sum_a |R_{ia}|}_{\text{Total redistribution}} = \underbrace{\sum_i |R_i|}_{\substack{\text{Total} \\ \text{interpersonal}}}$$

$$\underbrace{+ 2\sum_i \min\left\{\sum_a [1(R_{ia} > 0)R_{ia}], -\sum_a [1(R_{ia} \leq 0)R_{ia}]\right\}}_{\text{Total intrapersonal}}.$$

We then define the intrapersonal share as:

$$\alpha = 1 - \frac{\sum_i |R_i|}{\sum_i \sum_a |R_{ia}|}.$$

We ask how this compares with the intrapersonal shares calculated from the towards and against equations, defined as follows:

$$\alpha_T = 1 - \frac{\sum_i 1[R_i > 0]R_i}{\sum_i \sum_a [1(R_{ia} > 0)R_{ia}]};$$

$$\alpha_A = 1 - \frac{\sum_i 1[R_i \leq 0]R_i}{\sum_i \sum_a [1(R_{ia} \leq 0)R_{ia}]}.$$

Note that, by definition:

$$\sum_i \sum_a R_{ia} = 0.$$

This implies that all interpersonal redistribution must offset across individuals:

$$\sum_i 1[R_i > 0]R_i = -\sum_i 1[R_i \leq 0]R_i.$$

It also implies that total towards and total against offset across individuals:

$$\sum_i \sum_a [1(R_{ia} > 0)R_{ia}] = -\sum_i \sum_a [1(R_{ia} \leq 0)R_{ia}].$$

Substituting these into the equations for α_T and α_A, we find that $\alpha_T = \alpha_A = \alpha$.