# The Macro-dynamics of Sorting between Workers and Firms 

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## Contribution

- We develop an equilibrium random on-the-job search model of the Labor market, with ex-ante heterogeneous workers and firms, and aggregate productivity shocks
- We calibrate the model to US time-series data 1951-2007 and assess the model predictions for patterns during 2008-12 recession
- We use the model to asses the cyclicality of sorting/mismatch between workers and jobs, both for those hired from unemployment and those who were employed the period before


## Contribution

- The model delivers rich dynamics in terms of the cyclical composition of
- unemployed workers
- vacancies
- productive matches
- transition rates
- measured labor productivity
- The model has a recursive structure that implies that:
- knowledge of the current aggregate shock (and the stochastic process) is a sufficient statistic for decisions regarding which worker-firm matches to form or dissolve, and who change jobs
- the decision of which types of vacancies to create depends on the current distribution of worker-types among the unemployed and the current distribution of worker-types across job-types


## Related Literature

Models of aggregate shocks with heterogeneity

- Directed search: Menzio \& Shi (2010a,b, 2011), Kaas \& Kircher (2011), Schaal (2011); Wage posting: Moscarini \& Postel-Vinay (2011a,b), Coles \& Mortensen (2011);
Cyclical behavior of labor productivity and labor market variables
- Shimer (2005), Hall (2005), Hagedorn \& Manovskii (2008), Gertler \& Trigari (2009), Hagedorn \& Manovskii (2010), ...
Sorting between workers and firms (or unemployed and vacancies)
- Shimer \& Smith (2001), Eekhout \& Kircher (2011), Lise, Meghir, Robin (2012), Melo (2009), Bagger \& Lentz (2012), Barlevey (2002), Sahin, Song, Topa \& Violante (2012), Hagedorn, Law \& Manovskii (2012), Mueller (2012), ...
As far as we know, there is still very little work with double-sided worker-firm heterogeneity. Yet there is a lot of interest in understanding the evolution of match quality in recessions and booms.


## Agents and Technology

- Time is discrete and indexed by $t$.
- The planning horizon for workers and firms is infinite
- All agents are risk neutral and discount the future at rate $r$
- Let $x, y$, and $z$ index worker type, firm type and the aggregate productivity level


## Agents and Technology

- Time is discrete and indexed by $t$.
- The planning horizon for workers and firms is infinite
- All agents are risk neutral and discount the future at rate $r$
- Let $x, y$, and $z$ index worker type, firm type and the aggregate productivity level
- There is a continuum of workers indexed by type $x \in[0,1]$
- with distribution $\ell(x)$ and home production $b(x, z)$
- workers search both when unemployed and employed
- There is a continuum of profit maximizing firms $y \in[0,1]$
- type is defined by their technology $p(x, y, z)$
- recruit by posting vacancies $v(y)$ at increasing convex cost $c[v(y)]$
- retain workers by responding to outside offers


## Aggregate States

- $u_{t}(x)$ : the distribution of unemployed workers at the beginning of period $t$ (prior to realization of $z_{t}$ )
- $h_{t}(x, y)$ : the distribution of worker-firm matches at the beginning of period $t$ (prior to realization of $z_{t}$ )
- $z_{t}$ is updated from $z_{t-1}$ according to $\pi\left(z, z^{\prime}\right)$
- The state at the beginning of period $t$ is defined by $\left\{u_{t}(x), h_{t}(x, y), z_{t}\right\}$


## Three Key Modeling Assumptions

(1) Transferable Utility

- Workers and firms value a wage change the same way.
(2) Firms make state-contingent offers and counter-offers to workers
- When firms contact unemployed workers, they offer them their reservation value.
- When firms contact employed workers, they engage in Bertrand competition with current employer.
(3) Firms operate constant returns to scale production and pay flow costs to recruit new workers
- Hiring a new worker does not affect the productivity of existing matches, or the ability to hire more workers in the future.


## Values and Match Surplus

- Let $W_{t}(w, x, y)$ be the present value to a worker of type $x$ of receiving a wage $w$ when employed by a firm of type $y$.
- The subscript $t$ indicates that the function depends, in general, on the aggregate state at time $t:\left\{u_{t}(x), h_{t}(x, y), z_{t}\right\}$


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- The subscript $t$ indicates that the function depends, in general, on the aggregate state at time $t:\left\{u_{t}(x), h_{t}(x, y), z_{t}\right\}$
- Let $B_{t}(x)$ be the value of unemployment
- Let $\Pi_{t}(w, x, y)$ be the present value to a firm of type $y$ employing a worker of type $x$, paying a wage $w$
- The match surplus is given by

$$
W_{t}(w, x, y)-B_{t}(x)+\Pi_{t}(w, x, y)=S_{t}(x, y)
$$

## Timing

- Within a period
(1) The aggregate shock $z_{t}$ is realized, endogenous and exogenous separations occur
(2) Firms post vacancies and new meetings occur
(3) Production takes place


## Separations (Layoffs)

- The aggregate state changes from $z_{t-1}=z$ to $z_{t}=z^{\prime}$.
- All jobs such that $S_{t}(x, y) \leq 0$ are immediately destroyed,
- A fraction $\delta$ of the viable ones are also destroyed.
- Hence the stock of unemployed workers of type $x$ immediately after the realization of $z_{t}$ (at time $t+$ ) is

$$
u_{t+}(x)=u_{t}(x)+\int\left[\mathbf{1}\left\{S_{t}(x, y) \leq 0\right\}+\delta \mathbf{1}\left\{S_{t}(x, y)>0\right\}\right] h_{t}(x, y) \mathrm{d} y
$$

- The stock of matches of type $(x, y)$ is

$$
h_{t+}(x, y)=(1-\delta) \mathbf{1}\left\{S_{t}(x, y)>0\right\} h_{t}(x, y) .
$$

## Meeting Function

- The total measure of meeting at time $t$ is given by

$$
M_{t}=M\left(L_{t}, V_{t}\right)=\min \left\{\alpha \sqrt{L_{t} V_{t}}, L_{t}, V_{t}\right\}
$$

where $M\left(L_{t}, V_{t}\right)$ in strictly increasing in $L_{t}$ and $V_{t}$ and constant returns to scale.

- For the purposes of new meetings, the Labor force is defined by:

$$
L_{t}=f\left(u_{t+}, h_{t+}\right)=s_{0} \int u_{t+}(x) \mathrm{d} x+s_{1} \iint h_{t+}(x, y) \mathrm{d} x \mathrm{~d} y
$$

- Firms observe the new aggregate state and choose visibility $v_{t}(y)$, with aggregator:

$$
V_{t}=g\left(v_{t}\right)=\int v_{t}(y) \mathrm{d} y
$$

## Laws of Motion

For unemployment:

$$
u_{t+1}(x)=u_{t+}(x)\left[1-\int \lambda_{0, t} \frac{q_{t} v_{t}(y)}{M_{t}} \mathbf{1}\left\{S_{t}(x, y)>0\right\} \mathrm{d} y\right]
$$

For employment:

$$
\begin{aligned}
h_{t+1}(x, y)= & h_{t+}(x, y)+u_{t+}(x) \lambda_{0, t} \frac{q_{t} v_{t}(y)}{M_{t}} \mathbf{1}\left\{S_{t}(x, y)>0\right\} \\
+ & \int h_{t+}\left(x, y^{\prime}\right) \lambda_{1, t} \frac{q_{t} v_{t}(y)}{M_{t}} \mathbf{1}\left\{S_{t}(x, y)>S_{t}\left(x, y^{\prime}\right)\right\} \mathrm{d} y^{\prime} \\
& -h_{t+}(x, y) \int \lambda_{1, t} \frac{q_{t}\left(y^{\prime}\right) v_{t}\left(y^{\prime}\right)}{M_{t}} \mathbf{1}\left\{S_{t}\left(x, y^{\prime}\right)>S_{t}(x, y)\right\} \mathrm{d} y^{\prime}
\end{aligned}
$$

where $\lambda_{0, t}, \lambda_{1, t}$ and $q_{t}$ are the equilibrium meeting probabilities for unemployed workers, employed workers and vacancies

## Contracting and Re-contracting

## Postel-Vinay \& Robin (2001) and Postel-Vinay \& Turon (2010)

- An unemployed worker is offered her reservation wage:

$$
W_{t}\left(\phi_{0, t}(x, y), x, y\right)-B_{t}(x)=0
$$

- An employed worker is offered the minimum to outbid current (or poaching) firm,

$$
W_{t}\left(\phi_{1, t}\left(x, y, y^{\prime}\right), x, y\right)-B_{t}(x)=S_{t}\left(x, y^{\prime}\right)
$$

where $S_{t}(x, y)>S_{t}\left(x, y^{\prime}\right)$

- After an aggregate shock the current wage $w$ may not be viable. We assume that $w^{\prime}=\phi_{2, t}(w, x, y)$ with
- $\phi_{2, t}(w, x, y)=\phi_{0, t}(x, y)$ if $W_{t}(w, x, y)-B_{t}(x)<0$ (Worker PC binds)
- $\phi_{2, t}(w, x, y)=\phi_{1, t}(x, y, y)$ if $\Pi_{t}(w, x, y)<0$ (Firm PC binds)
- $\phi_{2, t}(w, x, y)=w$ otherwise (status quo)


## The Match Surplus and the Aggregate State

- The value to the worker and the value to the firm depend on $x, y$, aggregate productivity $z_{t}$, and on the distributions $v_{t}(y), u_{t}(x)$, and $h_{t}(x, y)$ (they affect the expectations of outside offers available to the worker)


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- However, the match surplus depends on time only through $z$
- Outside offers trigger a change to the transfer between firm and worker (the wage) but leave the size of the surplus unchanged
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- Outside offers trigger a change to the transfer between firm and worker (the wage) but leave the size of the surplus unchanged
- If the worker leaves to another firm she receives all of the current surplus
- We can write the surplus as

$$
S(x, y, z)=s(x, y, z)+\frac{1-\delta}{1+r} \int \max \left\{S\left(x, y, z^{\prime}\right), 0\right\} \pi\left(z, z^{\prime}\right) \mathrm{d} z^{\prime}
$$

with $s(x, y, z)=p(x, y, z)-b(x, z)$.

## Vacancy Creation and the Aggregate State

Firms choose $v_{t}(y)$ to maximize the return to recruiting:

$$
\max _{v_{t}(y)}\left\{-c\left[v_{t}(y)\right]+q_{t} v_{t}(y) J_{t}(y)\right\}
$$

where $J_{t}(y)$ is the expected value of a new match

$$
J_{t}(y)=\int \frac{s_{0} u_{t+}(x)}{L_{t}} S(x, y, z)^{+} \mathrm{d} x+\iint \frac{s_{1} h_{t+}\left(x, y^{\prime}\right)}{L_{t}}\left[S(x, y, z)-S\left(x, y^{\prime}, z\right)\right]^{+} \mathrm{d} x \mathrm{~d} y^{\prime}
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$$

For cost function $c_{0}[v(y)]=\frac{c_{0}}{1+c_{1}} v_{t}(y)^{1+c_{1}}$ and CD meeting technology: $q_{t}=\alpha \theta_{t}^{-\omega}$ we have a closed form for vacancy creation:

$$
\begin{gathered}
\theta_{t} \equiv \frac{V_{t}}{L_{t}}=\left(\frac{\alpha}{c_{0}}\right)^{\frac{1}{c_{1}+\omega}}\left(\frac{J_{t}}{L_{t}}\right)^{\frac{c_{1}}{c_{1}+\omega}}, \\
v_{t}(y)=\left(\frac{q_{t} J_{t}(y)}{c_{0}}\right)^{\frac{1}{c_{1}}} .
\end{gathered}
$$

## Computation of the Stochastic Search Equilibrium

(1) Solve for the fixed point in $S(x, y, z)$ independently of the actual realization of aggregate productivity shocks
(2) Given an initial distribution of workers across jobs and employment states, $u_{0}(x), h_{0}(x, y)$ and a realized sequence of aggregate productivity shocks $\left\{z_{0}, z_{1}, \ldots\right\}$ we can solve for the sequence of distributions of unemployed worker types, worker-firm matches, and vacancies $\left\{u_{t+1}(x), h_{t+1}(x, y), v_{t}(y)\right\}_{t=0}^{T}$.

## Parametric Specification

- Meeting function

$$
M_{t}=M\left(L_{t}, V_{t}\right)=\min \left\{\alpha \sqrt{L_{t} V_{t}}, L_{t}, V_{t}\right\}, \quad \alpha>0
$$

- Vacancy costs

$$
c\left[v_{t}(y)\right]=\frac{c_{0} v_{t}(y)^{1+c_{1}}}{1+c_{1}}, \quad c_{0}>0, \quad c_{1}>0
$$

- Value added

$$
p(x, y, z)=z \times\left(p_{1}+p_{2} x+p_{3} y+p_{4} x^{2}+p_{5} y^{2}+p_{6} x y\right)
$$

- Home production

$$
b(x, z)=b_{0}+z \times\left(b_{1} x+b_{2} x^{2}\right)
$$

- Worker type distribution

$$
x \sim \operatorname{Beta}\left(\beta_{1}, \beta_{2}\right)
$$

## Calibration

- We calibrate the model parameters by method of simulated moments
- The model is solved at a weekly frequency and the simulated data is then aggregated (exactly as the BLS and BEA data) to form quarterly moments
- From the data we remove a quadratic trend from log transformed data (1951-2007)


## Some Comments on Identification

- $\alpha, s_{1}$, and $\delta$ (mobility) identified by the average transition rates between unemployment and employment, between jobs, and from employment to unemployment
- $\sigma$ and $\rho$ (process for $z$ ) identified by standard deviation and auto-correlation of output
- $c_{0}$ and $c_{1}$ (vacancy costs) identified by the standard deviation of vacancies and the correlation of vacancies with output
- $\beta_{i}, b_{i}$, and $p_{i}$ (heterogeneity and match production)
- The distribution of worker types is identified by the pattern in the number of workers unemployed 5,15 and 27 or more weeks
- The contribution of firm type to value added is identified by the cross-sectional variation in value added per job, and its correlation with output


## Model Fit to Moments

| Moments | Data | Model | Moments | Data | Model |
| :--- | ---: | ---: | :--- | ---: | ---: |
| $\mathbb{E}[U]$ | 0.0562 | 0.0568 | $\operatorname{sd}[U]$ | 0.2140 | 0.2063 |
| $\mathbb{E}\left[U^{5 p}\right]$ | 0.0324 | 0.0339 | $\operatorname{sd}\left[U^{5 p}\right]$ | 0.3138 | 0.2670 |
| $\mathbb{E}\left[U^{15 p}\right]$ | 0.0153 | 0.0148 | $\operatorname{sd}\left[U^{15 p}\right]$ | 0.4435 | 0.3699 |
| $\mathbb{E}\left[U^{27 p}\right]$ | 0.0078 | 0.0064 | $\operatorname{sd}\left[U^{27 p}\right]$ | 0.5388 | 0.4740 |
| $\mathbb{E}[U 2 E]$ | 0.4376 | 0.4188 | $\operatorname{sd}[U 2 E]$ | 0.1257 | 0.1509 |
| $\mathbb{E}[E 2 U]$ | 0.0254 | 0.0244 | $\operatorname{sd}[E 2 U]$ | 0.1291 | 0.1267 |
| $\mathbb{E}[J 2 J]$ | 0.0273 | 0.0260 | $\operatorname{sd}[J 2 J]$ | 0.0924 | 0.1069 |
| $\mathbb{E}[$ prod. disp. $]$ | 0.7478 | 0.6623 | $\operatorname{sd}[\operatorname{prod} . \operatorname{disp}]$. | 0.0166 | 0.0082 |
| $\operatorname{sd}[V]$ | 0.2291 | 0.1860 | $\operatorname{corr}[U, V A]$ | -0.7742 | -0.9406 |
| $\operatorname{sd}[V / U]$ | 0.4162 | 0.3722 | $\operatorname{corr}[V, V A]$ | 0.6372 | 0.9159 |
| $\operatorname{sd}[V A]$ | 0.0363 | 0.0379 | $\operatorname{corr}[U 2 E, V A]$ | 0.8143 | 0.9010 |
| autocorr $[V A]$ | 0.9427 | 0.9553 | $\operatorname{corr}[E 2 U, V A]$ | -0.5984 | -0.5169 |
| $\operatorname{corr}[V, U]$ | -0.7642 | -0.8005 | $\operatorname{corr}[\operatorname{prod} . \operatorname{disp}, V A]$ | -0.3902 | -0.4552 |
| $\operatorname{corr}[U 2 E, J 2 J]$ | 0.6333 | 0.5526 |  |  |  |
|  |  |  |  |  |  |

## Parameter Estimates



Distribution of worker types


Production function

| $M(L, V)=1.89 \sqrt{L V}$ |  | $c[v(y)]=0.03 v(y)^{2.12}$ |  |  |  |
| :--- | :---: | :---: | :--- | :---: | :---: |
| Search intensity | $s_{1} / s_{0}$ | 0.022 | $b(x, z)=0.5+e^{z}\left(-0.1 x+4.7 x^{2}\right)$ |  |  |
| Exogenous separation | $\delta$ | 0.007 | Productivity shocks | $\sigma$ | 0.049 |
|  |  |  | Gaussian copula $(\sigma, \rho)$ | $\rho$ | 0.999 |

Feasible matches with aggregate shock at median


Feasible matches with aggregate shock at 90th percentile


Feasible matches with aggregate shock at 10th percentile


## Feasible matches



## Recovering the realized shock process $z_{t}$



We filter out the series for $z_{t}$ that best matches the output series 1951q1 to 2012q4.

std ratio $=1.018$, corr $=0.831$


std ratio $=0.275$, corr $=0.197$

std ratio $=1.362$, corr $=0.580$


std ratio $=1.192$, corr $=0.845$


## Labor Productivity and Output



Data - blue; Model prediction - green

## Cyclical composition of unemployed workers



Cyclicality: low skilled 0.84 , high skilled 1.23 (from regression of log unemployment rate by skill on log unemployment rate)

## Relative productivity, sorting and Firms' surplus share

|  |  | Baseline | constant $b$ | No heterogeneity |  |
| :--- | :--- | :--- | :--- | :--- | :---: |
| $\frac{b(x, \bar{z})}{p(x, y(x), \bar{z})}$ | mean | 0.9564 | 0.8350 | 0.9631 |  |
|  | min | 0.9040 | 0.1780 | 0.9631 |  |
|  | max | 0.9803 | 0.9585 | 0.9631 |  |
| corr $(x, y)$ |  | 0.736 | 0.709 | na |  |
| Firm share of |  | 0.274 | 0.372 | 0.558 |  |
| surplus at matching |  |  |  |  |  |

## Mismatch (Sorting)

- Let $y(x)=\arg \max _{y} S_{t}(x, y)$
absolute $\operatorname{mismatch}_{t}=\frac{1}{H_{t}^{j}} \int\left[S_{t}(x, y(x))-S_{t}(x, y)\right] h_{t}^{j}(x, y) \mathrm{d} x \mathrm{~d} y$ relative mismatch $_{t}=\frac{1}{H_{t}^{j}} \int\left[\frac{S_{t}(x, y(x))-S_{t}(x, y)}{S_{t}(x, y(x))}\right] h_{t}^{j}(x, y) \mathrm{d} x \mathrm{~d} y$
- Distribution of matches with workers hired out of unemployment

$$
h_{t}^{0}(x, y)=u_{t+}(x) \lambda_{0, t} \frac{q_{t} v_{t}(y)}{M_{t}} \mathbf{1}\left\{S_{t}(x, y) \geq 0\right\}
$$

- Distribution of matches where the worker was employed last period

$$
\begin{aligned}
h_{t}^{1}(x, y)=h_{t+}(x, y) & {\left[1-\int \lambda_{1, t} \frac{q_{t} v_{t}\left(y^{\prime}\right)}{M_{t}} \mathbf{1}\left\{S_{t}\left(x, y^{\prime}\right)>S_{t}(x, y)\right\} \mathrm{d} y^{\prime}\right] } \\
& +\int h_{t+}\left(x, y^{\prime}\right) \lambda_{1, t} \frac{q_{t} v_{t}(y)}{M_{t}} \mathbf{1}\left\{S_{t}(x, y)>S_{t}\left(x, y^{\prime}\right)\right\} \mathrm{d} y^{\prime}
\end{aligned}
$$

## Cyclical Mismatch



Absolute Mismatch


Relative Mismatch

- $\times$ - worker-job pairs where the worker was hired out of unemployment.
- ○- worker-job pairs in which the worker was employed in the previous period.


## Cyclical Mismatch



- $\times$ - worker-job pairs where the worker was hired out of unemployment.
- ○- worker-job pairs in which the worker was employed in the previous period.


## Summary

- We develop an equilibrium random on-the-job search model of the Labor market, with ex-ante heterogeneous workers and firms, and aggregate productivity shocks
- The model fits the US time-series data 1951-2007 and does quite well predicting the patterns over 2008-12
- In booms, workers initially accept worse matches on average than in recessions. At the same time, once employed they move more quickly to better matches in booms than in recessions


## The Value of Unemployment

Consider a worker of type $x$ who is unemployed for the whole period $t$.

$$
\begin{aligned}
& B_{t}(x)=b(x, z)+\frac{1}{1+r} \mathbb{E}_{t}\left[\left(1-\lambda_{0, t+1}\right) B_{t+1}(x)\right. \\
+ & \left.\lambda_{0, t+1} \int \max \left\{W_{t+1}\left(\phi_{0, t+1}(x, y), x, y\right), B_{t+1}(x)\right\} \frac{q_{t+1}(y) v_{t+1}(y)}{M_{t+1}} \mathrm{~d} y\right]
\end{aligned}
$$

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\end{aligned}
$$

Since any firm the worker contacts will offer her reservation value this simplifies to

$$
B_{t}(x)=b(x, z)+\frac{1}{1+r} \mathbb{E}_{t} B_{t+1}(x)
$$

## The Value of Employment

$$
\left.\begin{array}{rl}
W_{t}(w, x, y)=w+\frac{1}{1+r} \mathbb{E}_{t}[ & {\left[\mathbf{1}\left\{S_{t+1}(x, y)<0\right\}+\delta \mathbf{1}\left\{S_{t+1}(x, y) \geq 0\right\}\right] B_{t+1}(x)} \\
& +(1-\delta) \mathbf{1}\left\{S_{t+1}(x, y) \geq 0\right\} \\
\times\left[\lambda_{1, t+1}\right. & \int_{y^{\prime} \in \mathcal{M}_{1, t+1}(x, y)} W_{t+1}\left(\phi_{1, t+1}\left(x, y^{\prime}, y\right), x, y^{\prime}\right) \frac{q_{t+1}\left(y^{\prime}\right) v_{t+1}\left(y^{\prime}\right)}{M_{t+1}} \mathrm{~d} y^{\prime} \\
+ & \lambda_{1, t+1} \int_{y^{\prime} \in \mathcal{M}_{2, t+1}(w, x, y)} W_{t+1}\left(\phi_{1, t+1}\left(x, y, y^{\prime}\right), x, y\right) \frac{q_{t+1}\left(y^{\prime}\right) v_{t+1}\left(y^{\prime}\right)}{M_{t+1}} \mathrm{~d} y^{\prime} \\
& +\left[1-\lambda_{1, t+1} \int_{y^{\prime} \in \mathcal{M}_{3, t+1}(w, x, y)} \frac{q_{t+1}\left(y^{\prime}\right) v_{t+1}\left(y^{\prime}\right)}{M_{t+1}} \mathrm{~d} y^{\prime}\right]
\end{array}\right] \quad \begin{aligned}
& \\
&\left.\times \min \left\{W_{t+1}(w, x, y), \max \left\{S_{t+1}(x, y)+B_{t+1}(x), B_{t+1}(x)\right\}\right]\right] \\
& \mathcal{M}_{1, t}(x, y) \equiv\left\{y^{\prime} \mid S_{t}\left(x, y^{\prime}\right)>S_{t}(x, y)\right\}, \\
& \mathcal{M}_{2, t}(w, x, y) \equiv\left\{y^{\prime} \mid W_{t}(w, x, y)-B_{t}(x)<S_{t}\left(x, y^{\prime}\right)<S_{t}(x, y),\right. \\
& \mathcal{M}_{3, t}(w, x, y) \equiv\left\{y^{\prime} \mid S_{t}\left(x, y^{\prime}\right)<W_{t}(w, x, y)-B_{t}(x)\right\} .
\end{aligned}
$$

## Firm Value

$$
\begin{aligned}
& \Pi_{t}(w, x, y)=p(x, y, z)-w+\frac{1}{1+r} \mathbb{E}_{t}\left[(1-\delta) \mathbf{1}\left\{S_{t+1}(x, y) \geq 0\right\}\right. \\
& \times\left[\lambda_{1, t+1} \int_{y^{\prime} \in \mathcal{M}_{2, t+1}(w, x, y)} \Pi_{t+1}\left(\phi_{1, t+1}\left(x, y, y^{\prime}\right), x, y\right) \frac{q_{t+1}\left(y^{\prime}\right) v_{t+1}\left(y^{\prime}\right)}{M_{t+1}} \mathrm{~d} y^{\prime}\right. \\
& \quad+\left[1-\lambda_{1, t+1} \int_{y^{\prime} \in \mathcal{M}_{3, t+1}(w, x, y)} \frac{q_{t+1}\left(y^{\prime}\right) v_{t+1}\left(y^{\prime}\right)}{M_{t+1}} \mathrm{~d} y^{\prime}\right] \\
& \left.\left.\quad \times \min \left\{\Pi_{t+1}(w, x, y), S_{t+1}(x, y)^{+}\right\}\right]\right]
\end{aligned}
$$

## Match Surplus

## Estimated Parameters

| Matching $M=\alpha \sqrt{L V}$ | $\alpha$ | 1.894 | Home production | $b_{0}$ | 0.553 |
| :--- | :---: | :--- | :---: | ---: | ---: |
| Interest rate | $r$ | 0.05 | $b(x, z)=b_{0}+e^{z}$ | $b_{1}$ | -0.095 |
| Search intensity | $s_{1} / s_{0}$ | 0.022 | $\times\left(b_{1} x+b_{2} x^{2}\right)$ | $b_{2}$ | 4.688 |
| Vacancy posting costs | $c_{0}$ | 0.055 | Value added | $p_{1}$ | 0.612 |
| $\quad c[v(y)]=\frac{c_{0}}{1+c_{1}} v(y)^{1+c_{1}}$ | $c_{1}$ | 1.120 | $p(x, y, z)=e^{z}$ | $p_{2}$ | -0.171 |
| Exogenous separation | $\delta$ | 0.007 | $\times\left(p_{1}+p_{2} x\right.$ | $p_{3}$ | -1.024 |
| Productivity shocks | $\sigma$ | 0.049 | $+p_{3} y+p_{4} x^{2}$ | $p_{4}$ | 4.650 |
| $\quad$ Gaussian copula $(\sigma, \rho)$ | $\rho$ | 0.999 | $\left.+p_{5} y^{2}+p_{6} x y\right)$ | $p_{5}$ | -2.995 |
| Worker heterogeneity | $\beta_{1}$ | 1.105 |  | $p_{6}$ | 3.093 |
| $\quad \operatorname{Beta}\left(\beta_{1}, \beta_{2}\right)$ | $\beta_{2}$ | 1.407 |  |  |  |

Note: $r$ is fixed at 0.05 annually. Moments

| Fitted Moments | Data | I | II | III | IV | V | VI |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\mathbb{E}[U]$ | 0.0562 | 0.0568 | 0.0573 | 0.0541 | 0.0549 | 0.0614 | 0.0615 |
| $\mathbb{E}\left[U^{5 p}\right]$ | 0.0324 | 0.0339 | 0.0348 | 0.0294 | 0.0309 | 0.0320 | 0.0312 |
| $\mathbb{E}\left[U^{15 p}\right]$ | 0.0153 | 0.0148 | 0.0155 | 0.0090 | 0.0103 | 0.0091 | 0.0089 |
| $\mathbb{E}\left[U^{27 p}\right]$ | 0.0078 | 0.0064 | 0.0067 | 0.0023 | 0.0032 | 0.0024 | 0.0029 |
| $\mathbb{E}[U 2 E]$ | 0.4376 | 0.4188 | 0.4090 | 0.4680 | 0.4465 | 0.4881 | 0.5109 |
| $\mathbb{E}[E 2 U]$ | 0.0254 | 0.0244 | 0.0240 | 0.0262 | 0.0254 | 0.0314 | 0.0323 |
| $\mathbb{E}[J 2 J]$ | 0.0273 | 0.0260 | 0.0311 | 0.0277 | 0.0276 | 0.0382 | 0.0231 |
| $\mathbb{E}[$ sd labor prod $]$ | 0.7478 | 0.6623 | 0.3537 | na | 0.0683 | 0.1856 | 0.0953 |
| $\operatorname{sd}[U]$ | 0.2140 | 0.2063 | 0.2126 | 0.1731 | 0.1633 | 0.1678 | 0.2098 |
| $\operatorname{sd}\left[U^{5 p}\right]$ | 0.3138 | 0.2670 | 0.2791 | 0.2728 | 0.2197 | 0.2238 | 0.2898 |
| $\operatorname{sd}\left[U^{15 p}\right]$ | 0.4435 | 0.3699 | 0.3979 | 0.4647 | 0.3615 | 0.3344 | 0.4435 |
| $\operatorname{sd}\left[U^{27 p}\right]$ | 0.5388 | 0.4740 | 0.5332 | 0.6823 | 0.5429 | 0.4601 | 0.6356 |
| $\operatorname{sd}[U 2 E]$ | 0.1257 | 0.1509 | 0.1599 | 0.1400 | 0.1228 | 0.1130 | 0.1655 |
| $\operatorname{sd}[E 2 U]$ | 0.1291 | 0.1267 | 0.1300 | 0.0573 | 0.1033 | 0.1335 | 0.1374 |
| $\operatorname{sd}[J 2 J]$ | 0.0924 | 0.1069 | 0.1037 | 0.1899 | 0.1285 | 0.1984 | 0.1288 |
| $\operatorname{sd}[\operatorname{sd}$ labor prod $]$ | 0.0166 | 0.0082 | 0.0063 | na | 0.0042 | 0.0009 | 0.0087 |
| $\operatorname{sd}[V]$ | 0.2291 | 0.1860 | 0.1163 | 0.2349 | 0.2384 | 0.2260 | 0.1777 |
| $\operatorname{sd}[V / U]$ | 0.4162 | 0.3722 | 0.3157 | 0.3964 | 0.3223 | 0.3147 | 0.3185 |
| $\operatorname{sd}[\mathrm{VA}]$ | 0.0363 | 0.0379 | 0.0389 | 0.0384 | 0.0379 | 0.0344 | 0.0354 |
| autocorr $[\mathrm{VA}]$ | 0.9427 | 0.9553 | 0.9557 | 0.8804 | 0.9254 | 0.7976 | 0.8754 |
| $\operatorname{corr}[V, U]$ | -0.7642 | -0.8005 | -0.8272 | -0.8846 | -0.2614 | -0.2608 | -0.3463 |
| $\operatorname{corr}[U, \mathrm{VA}]$ | -0.7742 | -0.9406 | -0.9528 | -0.9778 | -0.3586 | -0.7664 | -0.7380 |
| $\operatorname{corr}[V, \mathrm{VA}]$ | 0.6372 | 0.9159 | 0.8881 | 0.9477 | 0.9315 | 0.7690 | 0.8604 |
| $\operatorname{corr}[U 2 E, \mathrm{VA}]$ | 0.8143 | 0.9010 | 0.9360 | 0.9416 | 0.2102 | 0.4501 | 0.6420 |
| $\operatorname{corr}[E 2 U, \mathrm{VA}]$ | -0.5984 | -0.5169 | -0.4455 | -0.9226 | -0.2932 | -0.3915 | -0.3132 |
| $\operatorname{corr}[U 2 E, J 2 J]$ | 0.6333 | 0.5526 | 0.5494 | 0.9974 | 0.2857 | 0.5842 | 0.4270 |
| $\operatorname{corr}[\mathrm{sd}$ labor prod, VA $]$ | -0.3902 | -0.4552 | -0.3910 | na | 0.7465 | -0.2184 | -0.2690 |

Model (I) baseline model; (II) home production is independent of worker type and aggregate state $b(x, z)=b$; (III) no worker or firm heterogeneity; (IV) only worker heterogeneity; (V) has no production complementarities: $p_{x y}=0$; (VI) has production of the form $p(x, y, z)=x y z$.

