B. Online technical appendix

In this appendix, we set out how we calculate our estimates of changes in value added due to trade barriers.

B.1 Estimating each industry's use of EU inputs from different industries

The UK input–output tables from the Office for National Statistics (ONS) record supply–use relationships between different UK industries. We group these into 102 industries.[1] The tables also tell us how much each industry imports from the rest of the world. However, they do not record the source of these inputs. Since we are considering a situation in which trade barriers are raised specifically on EU imports, we need to understand what proportion of each industry's imported inputs come from EU countries.

To do this, we combine the information in the input–output tables with data from the World Input–Output Database (WIOD).[2] This database records supply–use relationships between industries across countries, showing for example the importance of UK imports of French machinery for the UK transport equipment industry. The WIOD contains data on all EU member countries (including the UK) and for 56 different industries.

Our aim is to use the data on the 56 industry groupings in the WIOD tables in combination with data from the ONS input–output tables to estimate the importance of EU inputs for all the 102 industries in our analysis.

To do this, we make use of the following approach. Let i denote one of the 102 industries we take from the UK input–output tables and g one of the 56 industry groupings used in the WIOD. Let m_i denote the share of each industry's inputs that are imported. Let the share of intermediate imports that are imported by the EU in the WIOD tables by industry group g be denoted by m_g^{EU}. Then denote purchases of imported intermediate inputs by industry i from industry j (where industry j is in WIOD industry g) by x_{ijg}. We want to estimate the share of UK inputs purchased from EU industry i that are purchased from j, which we call $\widehat{m}^{EU}{}_{ij}$.

We calculate this as

(B1) $$\widehat{m}^{EU}{}_{ij} = m_i \times m_g^{EU} \times \frac{x_{ijg}}{\sum_{k \in g} x_{ikg}},$$

which is the share of industry i's inputs that are imported multiplied by the share of intermediate inputs that WIOD industry g purchases from the EU multiplied by the proportion of imported inputs within WIOD group g that UK industry i purchases from industry j.

1 We aggregate some industries so as to be able to map these industries to external trade and tariff data.

2 For details, see M. Timmer, P. Dietzenbacher, R. Stehrer and G. de Vries, 'An illustrated user guide to the World Input–Output Database: the case of global automotive production', *Review of International Economics*, 2014, 23, 575–605.

B.2 Calculating industry-level exposure measures

Our exposure measures can be thought of as capturing the change in each industry's *value added* (the value of output less the value of inputs) under the assumption that UK industries do not change their output levels. To obtain these, we calculate: the price change needed to ensure that demand for each industry's output remains constant after trade barriers are introduced; the change in input costs per unit of output each industry experiences; and, from those, the proportional change in each industry's value added.

Change in demand for output

Demand for output produced by industry i is given by

(B2) $$X_i = X_i^{EU} + X_i^{NEU} + X_i^{Final} + \sum_j X_{ij}^{Inter} \ ,$$

which is the sum of exports to the EU X_i^{EU}, exports to non-EU destinations X_i^{NEU}, UK final demand X_i^{Final} and intermediate demand for inputs from other UK industries X_{ij}^{Inter}.

This means we can write the proportional change in output as

(B3) $$\hat{X}_i = \sigma_i^{EU}\hat{X}_i^{EU} + \sigma_i^{NEU}\hat{X}_i^{NEU} + \sigma_i^{Final}\hat{X}_i^{Final} + \sum_i \frac{\gamma_{ij}^{UK} X_j}{X_i}\hat{X}_{ij}^{Inter} \ ,$$

where $\hat{x}$ denotes a proportional change in some variable x. σ_i^{EU}, σ_i^{NEU} and σ_i^{Final} refer to the shares of output for each industry sold to the EU, to non-EU countries and for final consumption in the UK and γ_{ij}^{UK} is the value of inputs from industry j needed to produce a unit of industry i's output X_i.

It is convenient to write (B3) in terms of vectors and matrices, with each row denoting an industry. Letting bold terms denote vectors/matrices, we can write the following system of equations:

(B4) $$\hat{\boldsymbol{X}} = \boldsymbol{\Sigma}^{EU}\hat{\boldsymbol{X}}^{EU} + \boldsymbol{\Sigma}^{NEU}\hat{\boldsymbol{X}}^{NEU} + \boldsymbol{\Sigma}^{Final}\hat{\boldsymbol{X}}^{Final} + \boldsymbol{S}\hat{\boldsymbol{X}}^{Inter} \ ,$$

where $\boldsymbol{\Sigma}^{EU}$, for example, is a diagonal matrix with $\sigma_1^{EU} \ldots \sigma_N^{EU}$ on the diagonal. $\boldsymbol{S}$ is a matrix with $\frac{\gamma_{ij}^{UK} X_j}{X_i}$ in element ij.

Let us now assume that UK firms view inputs from the EU, non-EU and UK as imperfect substitutes. In particular, let us assume that they are combined with a constant elasticity of substitution (CES) production function. This implies that the change in intermediate demand for UK output given a marginal change in prices of EU imports is given by

(B5) $$\hat{X}_{ij}^{Inter} = \hat{X}_j + \xi_j(\theta_{ij}^{UK} - 1)\hat{P}_i + \xi_j\theta_{ij}^{EU}\hat{\tau}_i^{EU,M} \ ,$$

where ξ_j is the elasticity of substitution between UK, non-EU and EU inputs, and θ_{ij}^{UK} and θ_{ij}^{EU} are the shares of inputs used by industry j from industry i that are drawn from the UK and the EU respectively. $\hat{\tau}_i^{EU,M}$ is the change in the *ad valorem* trade cost applying to imports of goods or services produced by industry i from the EU following Brexit, and $\hat{P}_i$ is the output price change for UK output produced by industry i. Substituting (B5) into (B4) and rearranging gives

(B6) $\quad \widehat{\boldsymbol{X}} \quad = (\boldsymbol{I} - \boldsymbol{S})^{-1}(\boldsymbol{\Sigma}^{EU}\widehat{\boldsymbol{X}}^{EU} + \boldsymbol{\Sigma}^{NEU}\widehat{\boldsymbol{X}}^{NEU} + \boldsymbol{\Sigma}^{Final}\widehat{\boldsymbol{X}}^{Final} + \boldsymbol{B}^{EU}\widehat{\boldsymbol{\tau}}^{EU,M} + \boldsymbol{B}^{UK}\widehat{\boldsymbol{P}})$,

where $\boldsymbol{B}^{EU}$ has diagonal entries $\sum_j \frac{\gamma_{ij}^{UK} X_j}{X_i} \xi_j \theta_{ij}^{EU}$ and $\boldsymbol{B}^{UK}$ has diagonal entries $\sum_j \frac{\gamma_{ij}^{UK} X_j}{X_i} \xi_j (\theta_{ij}^{UK} - 1)$. Finally, we can also write

(B7) $\quad \widehat{\boldsymbol{X}} \quad = (\boldsymbol{I} - \boldsymbol{S})^{-1}(\boldsymbol{\Sigma}^{EU}\boldsymbol{E}^{EU}(\widehat{\boldsymbol{P}} + \widehat{\boldsymbol{\tau}}^{EU,X}) + \boldsymbol{\Sigma}^{Final}(\boldsymbol{E}^{UK,EU}\widehat{\boldsymbol{\tau}}^{EU,M} + \boldsymbol{E}^{UK}\widehat{\boldsymbol{P}})$

$$+\boldsymbol{\Sigma}^{NEU}\boldsymbol{E}^{NEU}\widehat{\boldsymbol{P}} + \boldsymbol{B}^{EU}\widehat{\boldsymbol{\tau}}^{EU,M} + \boldsymbol{B}^{UK}\widehat{\boldsymbol{P}}) ,$$

where $\boldsymbol{E}^{UK}$, $\boldsymbol{E}^{EU}$ and $\boldsymbol{E}^{NEU}$ are matrices of elasticities of demand for UK output in the UK, EU and non-EU countries respectively, $\widehat{\boldsymbol{\tau}}^{EU,X}$ is a vector of changes in the *ad valorem* trade costs that apply to exports to the EU after Brexit and $\boldsymbol{E}^{UK,EU}$ is a matrix of cross-price elasticities giving the proportional change in demand for UK output by final consumers given a proportional change in the price of EU imports.

To pin down these elasticities for a given set of estimated trade elasticities, we need to impose some structure on consumers' preferences. We assume Cobb–Douglas preferences for demand across industries, but CES demands for different national varieties within each industry. That is, we assume that consumers in both the EU and the UK maximise utility U of the following form:

(B8) $\quad U = \sum_i \alpha_i \ln c_i$,

where

(B9) $\quad c_i = \left[\pi_i{}^{UK}(c_i^{UK})^{\frac{\rho_i - 1}{\rho_i}} + \pi_i{}^{EU}(c_i^{EU})^{\frac{\rho_i - 1}{\rho_i}} + \pi_i{}^{NEU}(c_i{}^{NEU})^{\frac{\rho_i - 1}{\rho_i}}\right]^{\frac{\rho_i}{\rho_i - 1}}$.

where ρ_i is the elasticity of substitution for industy i and c_i^{UK}, c_i^{EU} and c_i^{NEU} are goods or services produced by industry i in the UK, EU and non-EU respectively. $\pi_i{}^{UK}$, $\pi_i{}^{EU}$ and $\pi_i{}^{NEU}$ are parameters that determine consumers' preferences over these national varieties. Given these preferences, it follows that the demand elasticity for UK output given a change in the price of EU inputs is

(B10) $\quad \epsilon_i^{UK,EU} \quad = -s_i^{EU}(1 - \rho_i)$,

where s_i^{EU} is the market share of EU producers in final demand for the output of industry i.

Assuming that the market share of UK exporters in the EU-wide market is 'small', the elasticity of demand from EU consumers for UK output is $-\rho_i$.

We take the values of ρ_i and ξ_i for each industry using (1 plus) the trade elasticity estimates in Caliendo and Parro (2015).[3] For service industries, for which Caliendo and Parro do not provide estimates, we follow Costinot and Rodríguez-Clare (2014) in choosing

[3] L. Caliendo and F. Parro, 'Estimates of the trade and welfare effects of NAFTA', *Review of Economic Studies*, 2015, 82, 1–44.

an elasticity that is roughly the median of trade elasticities estimated in other studies (that is, a value of 5).[4] Results shown below for which we employ a constant elasticity across sectors make use of an elasticity of substitution of 5 for all sectors.

These elasticities do not account for changes in UK demand as a result of changes in incomes following Brexit; they represent substitution responses to price changes only.

Changes in costs of inputs

As well as reducing the value of UK output, changes in trade barriers will also affect the costs of firms' inputs. We calculate these as

(B11) $$\mathit{Change\ in\ input\ costs}_j = \gamma_{ij}^{UK}\hat{P}_i + \gamma_{ij}^{EU}\hat{\tau}_i^{EU,M} ,$$

where $\gamma_{ij}^{EU} = \gamma_{ij}\theta_{ij}^{EU}$, where γ_{ij} are the total inputs purchased from industry i by industry j.

Calculating value added changes

Output price changes are defined implicitly by setting $\widehat{\boldsymbol{X}} = \boldsymbol{0}$ in (B6) and solving.

Proportional value added changes for industry i are then given by

(B12) $$\hat{p}_i = \frac{\hat{P}_i - \mathit{Change\ in\ input\ costs}}{(1-\sum_j \gamma_{ji})} .$$

[4] A. Costinot and A. Rodríguez-Clare, 'Trade theory with numbers: quantifying the consequences of globalization', in *Handbook of International Economics*, 2014, 4, 197–261.