The Regressivity of a Value Added Tax: Tax Credit Method and Subtraction Method — A Japanese Case

MASAYUKI TAMAOKA

1. INTRODUCTION

Value added tax (VAT) is often described as a regressive tax because it taxes consumption, and the propensity to consume tends to decrease as income rises. Countries that maintain progressive tax systems take several measures to remove the regressivity of VAT. These measures include (i) exempting food and social necessities and (ii) taxing luxuries at high rates and necessities at low rates.

VAT of the subtraction type has two main variants, namely the tax credit method and the subtraction method. These two types of VAT have theoretically the same effects on a firm’s liability for VAT and a firm’s decision on price in cases where there is only one rate and there are no exemptions. However, in cases where multiple rates and exemptions are implemented, these two types of

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2 VAT using the tax credit method is often called invoice method or EU-type VAT and VAT using the subtraction method is often called accounts method VAT. According to McLure (1987), VAT using the subtraction method also has two variants, namely the naïve type and the sophisticated type. In this paper we only deal with the naïve-type VAT — this characterises the subtraction method VAT well. See McLure (1987, Ch. 6).

3 In order to compare the two types of VAT we consider exemptions, in what follows, as a means of exclusion from VAT. Another type of exclusion is zero rating under the tax credit method. Under the subtraction method the distinction between exemption and zero rating is meaningless.

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VAT no longer have the same effects on a firm’s liability for VAT and a firm’s decision on price.

In the European Union, multiple rates are used to mitigate the regressivity of VAT in several countries. In a recent EU Directive, the EU proposed a dual-rate structure, namely a standard rate of at least 15 per cent and a minimum rate of at least 5 per cent. Also proposed was the abolition of internal border controls by switching from the destination principle to the origin principle. This means that each Member State can set its own VAT rates to mitigate regressivity and to secure its revenue.

In Japan, VAT using the subtraction method under the name ‘Consumption Tax’ was introduced in 1989 at a rate of 3 per cent. The reason Japan adopted the subtraction method instead of the tax credit method at a rate of 3 per cent is that there was strong opposition to introducing a new kind of indirect tax from several groups — especially small traders, retailers and consumers — and the Ministry of Finance therefore wanted to keep compliance costs for small businesses down to a minimum to win support from them and to keep the tax rate as low as possible to get support from consumers. As a result, the EU-type VAT method was not suitable, and a subtraction method, which had previously been a textbook option but which had never been used in any other country, was adopted.

VAT under the subtraction method, unlike the familiar EU-type VAT, is simple in its mechanism because it does not require the use of invoices in calculating the tax due — the tax due is calculated by multiplying the tax rate directly by the difference between a firm’s sales and inputs. Japanese Consumption Tax covers almost all goods and services, taxes them at a basic rate of 3 per cent (with the exception of 6 per cent on the car industry) and exempts financial and insurance companies and many small businesses. It has been argued since the introduction of Consumption Tax that the regressivity of VAT itself was a serious problem and extra problems were caused by using the subtraction method rather than the tax credit method. The use of the tax credit method instead of the subtraction method and an increase in tax rate coupled with exemption of foods have been under consideration. It is important, therefore, to estimate the different burdens imposed on households by each type of VAT when considering which type of VAT to adopt, particularly if the policy aim is to mitigate the regressivity of VAT.

If we assume one tax rate and no exemption, it is well known that the two types of VAT give very similar results. The distributional effects of these two types of VAT differ, however, when we consider multiple tax rates and

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4 For the rate structure of the EC VAT, see Cnossen (1989).
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exemption, because then the two types of VAT use different methods to calculate the tax due and so the prices of goods and services are different.

The purpose of this paper is to highlight the differences between the two types of VAT, with particular emphasis on the distributional effects for different income groups when exemptions or multiple rates are used as a means of removing the regressivity of VAT. In the next section, we discuss the method of analysis. Then, in Section III, we explore several measures aimed at relieving regressivity. Section IV provides a summary and conclusion.

II. METHOD OF ANALYSIS

There are two main tools for analysing the effects of VAT. One is an input–output table which can be used to examine the effects of VAT on prices, and the other is a survey of family incomes and expenditures which can be used to look at the burden of VAT for different income groups. By using the input–output table, we can see the effects of VAT on prices in different industries when multiple rates and measures of exemption are used. By using the household survey, we can look at the burden of VAT for different income groups while considering the pattern of consumption of those groups.

First, we look at the household survey. The household survey consists of revenue items and expenditure items. On the revenue side, there are wages and salaries, property income such as interest receipts and dividends, social security benefits and so on. On the expenditure side, there are consumption expenditures, taxes such as personal income tax, residential tax, social security contributions, interest payments and so on. Consumption expenditures consist of 16 items including food and we calculate personal income tax and residential tax liabilities for different income groups by calculating personal deductions.

As for the calculation of the burden of VAT for different income groups, we assume each household consumes each consumption item at a fixed proportion of the basic year’s (1986) disposable income (equation (1)) and each household pays VAT per yen equivalent to the ratio of the tax to the tax-exclusive price of each consumption item (equation (2)).

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6 We use 1980’s 72-sector input–output table and 1986’s survey of family income and expenditure.
7 For the mathematical treatment of input–output tables, see Appendix A.
8 Residential tax is a kind of income tax levied by the local government.
9 In calculating the liability to personal income tax and residential tax, we consider a household with one income and assume that the number of children is the number of persons in the household in the household survey minus two (i.e. the couple themselves). We consider the following income groups, where annual income is in 10,000 yen — 1: under 100, 2: 100–150, 3: 150–200, 4: 200–250, 5: 250–300, 6: 300–350, 7: 350–400, 8: 400–450, 9: 450–500, 10: 500–550, 11: 550–600, 12: 600–650, 13: 650–700, 14: 700–750, 15: 750–800, 16: 800–900, 17: 900–1,000, 18: over 1,000.
Fiscal Studies

(1) \[ \text{CHN}_t^j = \text{CHN}_{1986}^j \frac{YHD_t^j}{YHD_{1986}^j} \quad (i=1,\ldots,16, j=1,\ldots,18) \]

(2) \[ \text{VAT}_t^j = \frac{\text{PA}_t^j}{\text{PB}_t^j} - 1 \cdot \text{CHN}_t^j \]

(3) \[ \text{VAT}_t^j = \sum_{i=1}^{16} \text{VAT}_{t}^i \]

where CHN^j_t is the jth income group’s nominal consumption expenditure on the ith good in period t, YHD^j_t is the jth income group’s disposable income in period t, VAT^j_t is the jth income group’s burden of VAT on the ith good in period t, PA^j_t is the tax-inclusive price of the ith good in period t, PB^j_t is the tax-exclusive price of the ith good in period t and VAT^j_t is the jth income group’s total VAT burden in period t.

We substitute the tax-exclusive price and tax-inclusive price of the ith good that we get from the input–output model into equation (2). Tax-cum price in the input–output model is given by equations (A.3) and (A.5) and the tax-exclusive price in the input–output model is given by equation (A.1’) in Appendix A. Equation (A.1’) means that each industry’s price depends on other industries’ prices and the tax-exclusive price is the same as the price if there were no VAT.

III. MEASURES FOR REDUCING REGRESSIVITY AND THEIR RESULTS

It is often said that the burden of VAT is distributed regressively with respect to income because consumption per income falls as income rises. However, the burden of VAT becomes proportional once we calculate it as a share of consumption or lifetime income in the case of a uniform tax rate. Nevertheless, annual income is often used in calculating the burden of VAT, for reasons that are partly political. We then need to consider the consequences of using multiple rates and exemption to mitigate the regressivity of VAT against income. It is also worth while examining empirically whether the distributional effects of VAT with respect to income, disposable income and consumption are progressive,

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10 As there are 16 consumption items in the household survey and 72 sectors in the input–output table, we apply the latter’s sectors to the former’s items. See Appendix B.

11 This means tax-inclusive price.
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proportional or regressive, because only a few studies have so far attempted to clarify the distributional impacts of VAT in relation to various measures comprehensively.

According to the US Department of the Treasury (1984), there are four ways to reduce the regressivity of VAT:

- adjustment of government transfer payments;
- zero rating of food and other necessities;
- provision of a refundable credit;
- personal exemption value added tax.

There are also non-VAT measures to offset the regressivity of VAT, such as other tax and expenditure measures. However, the only measures to reduce the regressivity of VAT we look at in this paper are altering the tax base and changing tax rates, because we consider these to be practicable in a system of VAT as an indirect tax. Therefore, we consider (i) exemption of necessities such as food and exemption of social needs such as health and medical services and education and (ii) setting multiple tax rates on goods that were previously taxed at higher rates than other goods and services in the commodity tax system (e.g. electrical appliances, cars).

**TABLE 1**

**Alternative VAT Structures**

<table>
<thead>
<tr>
<th></th>
<th>3% standard rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bench-mark</strong></td>
<td>6% for car industry</td>
</tr>
<tr>
<td></td>
<td>Financial and insurance sectors exempt</td>
</tr>
<tr>
<td><strong>Exemptions</strong></td>
<td>3% standard rate</td>
</tr>
<tr>
<td></td>
<td>3% for car industry</td>
</tr>
<tr>
<td></td>
<td>Financial and insurance sectors, medical service and education exempt</td>
</tr>
<tr>
<td><strong>Multiple rates</strong></td>
<td>3% standard rate</td>
</tr>
<tr>
<td></td>
<td>10% for car industry and electrical appliances</td>
</tr>
<tr>
<td></td>
<td>Financial and insurance sectors, medical service and education exempt</td>
</tr>
</tbody>
</table>

We measure the regressivity of VAT against gross income, against disposable income and against consumption, as noted above (see also OECD (1988)). Income consists of wages and salaries, property income such as interest receipts and dividends, and social security benefits. Disposable income is obtained by subtracting taxes (e.g. personal income tax, residential tax, property tax, car weight tax, stamp tax, inheritance tax), social security contributions and interest payments from gross income. In calculating the burden of income tax and residential tax, we use 1989’s table of deductions and tax rates since it is the
year of tax reform. As for consumption, we use tax-cum consumption, which is equal to free-of-VAT consumption plus VAT, as a denominator for calculating the burden of VAT.

We take the tax base and VAT rates that existed in 1990 — and which are based on the Consumption Tax Law of 1989 — as our bench-mark figures from which comparisons can be made. Table 1 sets out the alternative structures from which comparisons can be made. We examine the impact on households’ incomes of these alternatives when implemented through the tax credit method and the subtraction method.

1. Tax Credit Method

First we mention the results for the tax credit method VAT — EU-style VAT. Figure 1 highlights the impact on households of the three alternative tax structures using the tax credit method. AT payments represented 2.04 per cent of gross income for the lowest income group and 1.18 per cent for the highest. VAT under all structures is regressive.

![FIGURE 1](image)

VAT Burden with respect to Gross Income

It can be seen that in the exemptions case (Exempt) the burden of the highest income group is reduced as much as that of the lowest because the ratio of food and medical expenditures to disposable income falls as income rises but the ratio of education expenditure rises as income rises.

In the multiple rates case (Multi) where we add high tax rates on electrical appliances and cars to the Exempt case, the burden ratios of all income groups
### TABLE 2

Rates of Price Increase of Consumption Items and Tax Liabilities$^a$

<table>
<thead>
<tr>
<th></th>
<th>TAX CREDIT METHOD</th>
<th>SUBTRACTION METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Consumption Tax</td>
<td>Exempt</td>
</tr>
<tr>
<td>Food$^b$</td>
<td>2.613 (1,450)</td>
<td>1.313 (0)</td>
</tr>
<tr>
<td>Beverages</td>
<td>2.659 (1,157)</td>
<td>0.815 (0)</td>
</tr>
<tr>
<td>Housing</td>
<td>2.940 (2,193)</td>
<td>2.947 (2,192)</td>
</tr>
<tr>
<td>Fuel, light and water</td>
<td>2.622 (1,053)</td>
<td>2.636 (288)</td>
</tr>
<tr>
<td>charges$^c$</td>
<td>0.707 (−974)</td>
<td>0.712 (−978)</td>
</tr>
<tr>
<td>Electricity</td>
<td>2.601 (285)</td>
<td>2.636 (288)</td>
</tr>
<tr>
<td>Gas</td>
<td>2.539 (598)</td>
<td>2.564 (599)</td>
</tr>
<tr>
<td>Furniture and household</td>
<td>1.138 (167)</td>
<td>1.168 (182)</td>
</tr>
<tr>
<td>utensils</td>
<td>2.238 (1,081)</td>
<td>2.265 (1,088)</td>
</tr>
<tr>
<td>Household durables</td>
<td>2.144 (3,018)</td>
<td>0.915 (0)</td>
</tr>
<tr>
<td>Medical care</td>
<td>1.739 (2,189)</td>
<td>1.803 (4,148)</td>
</tr>
<tr>
<td>Public transportation</td>
<td>0.506 (5,100)</td>
<td>0.506 (−1,972)</td>
</tr>
<tr>
<td>Automobiles</td>
<td>2.458 (3,897)</td>
<td>0.324 (0)</td>
</tr>
<tr>
<td>Education</td>
<td>1.422 (3,080)</td>
<td>1.713 (5,333)</td>
</tr>
<tr>
<td>Reading and recreation</td>
<td>1.422 (3,080)</td>
<td>1.713 (5,333)</td>
</tr>
<tr>
<td>Other living expenditures</td>
<td>2.871 (904)</td>
<td>2.880 (905)</td>
</tr>
</tbody>
</table>

$^a$Rate of price increase represents rate of increase of tax-inclusive price to tax-exclusive price. Tax liabilities are given in parentheses.

$^b$Excludes expenditure on beverages.

$^c$Excludes expenditure on electricity and gas.
except the lowest decrease. This happens because the purchase ratio of electrical appliances is highest for the lowest income group in the benchmark year, 1986. Table 2 shows the rate of price increase of each consumption item and the tax payments of industry corresponding to each consumption item. The numbers in Table 2 are not all 3 per cent, reflecting the fact that some industries are exempt and others are taxed at a higher rate. It can be seen that the rate of price increase of consumer durables, including electrical appliances, is 7.9 per cent, and that of cars is 7.2 per cent. These increases are higher than those in other industries, reflecting the high tax rate applied to the electrical appliance and car industries. This leads to an increasing burden on the consumer. On the other hand, industries that purchase inputs from these two industries can now deduct 10 per cent of tax-exclusive purchases as a tax credit instead of 3 per cent. For example, a food company buying a refrigerator worth £10,000 from an electrician’s shop could deduct £300 as a tax credit previously, but it can now deduct £1,000 as a tax credit. As a result, the rate of price increases in such industries is lower than in the Exempt case, which reduces the burden on the consumer when we consider the rate of price increases of inputs from these two industries.

FIGURE 2

VAT Burden with respect to Disposable Income

12 The newest edition of the household survey of 1988 shows that this ratio is zero in the lowest income group.
The pattern of the tax burden with respect to disposable income is almost the same as for gross income (see Figure 2). The degree of regressivity is slightly less in this case than in the gross income case because disposable income reflects the progressivity of income and residential taxes. We can say that the regressivity against disposable income is another expression of the progressivity of direct taxes.

Lastly, we can see a mild regressivity against consumption in the benchmark case, ranging from 2.15 per cent for the lowest income group to 1.94 per cent for the highest, although the tax rate is basically 3 per cent for almost all industries (see Figure 3). This result contrasts with the ordinary view\footnote{See OECD (1988, Ch. 8). Mild progressivity is found in the Netherlands in 1983 and mild regressivity is found in Denmark in 1981.} that the burden of VAT is proportional to consumption across all income groups. The reason for this phenomenon is the following. Let $T_{ct}$ be the burden ratio of VAT against consumption for income group $j$. As $T_{ct}$ is the burden ratio of VAT against tax-cum-consumption, using equations (2) and (3) we get:

![Figure 3: VAT Burden with respect to Consumption](image)

\[ T_{ct} = \text{burden ratio of VAT against tax-cum-consumption} \]
As we can see from equation (5), $W_{jt}$ is the weighted average of the rate of increase of the tax-inclusive price to tax-exclusive price for each industry, where the weights are the consumption rates of each income group for each industry’s product. As the rate of price increase is given, the $W_{jt}$ of an income group that spends much of its income on a good that has a high rate of price increase is higher than that of an income group that spends a smaller percentage of its income on that good. Therefore, we can say $T_{jt}$ is also higher. We can see from Table 2 that the items with a high rate of price increase are cars, housing, tobacco, drink, water, lighting, heating and food, and the items with a low rate of price increase are electricity charges, durable goods, culture, amusement and other expenditures. We confirm the condition that $W_{jt}$ decreases as income rises in the consumption pattern of the bench-mark year. If there is a uniform rate, each industry may have a different rate of price increase for its own product so that the pattern of the burden of VAT can be proportional, progressive or regressive.

In the cases of Exempt and Multi, essentially the same thing can be said as in the bench-mark case and they can reduce the overall burden of VAT but cannot reduce the regressivity of VAT so much.

2. Subtraction Method

Next, we mention the results for the subtraction method VAT — Japanese Consumption Tax. VAT payments represented 1.96 per cent of gross income for

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14 See Appendix C.
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the lowest income group and 1.08 per cent for the highest. Figure 1 shows that VAT under the subtraction method is regressive too. We can also see that the burden of VAT under the subtraction method for each income group is lower than that under the tax credit method even though the same tax base and the same tax rates as under the Consumption Tax Law are used. In other words, the rate of price increase for each industry, except education, under the subtraction method is lower than that under the tax credit method. We can confirm this by looking at Table 2. This is because the treatment of a taxable industry which purchases inputs from a tax-exempt industry at an intermediate stage of transaction is different under the two methods of VAT. Under the tax credit method, the taxable industry has no tax on inputs from the tax-exempt industry for which credit can be claimed; however, under the subtraction method, the taxable industry can claim credit on those inputs. Since the pyramiding effect of tax under the tax credit method cannot happen under the subtraction method, prices under the subtraction method are in many cases lower than those under the tax credit method when the same tax base and tax rates are used.

The patterns of tax burden for the Exempt and Multi cases are similar under both methods but the effect of the price increase in the industries to which a high tax rate is applied is fundamentally different. The typical case is the price increase in the car industry. It is 7.2 per cent under the tax credit method but –0.9 per cent under the subtraction method even though the same tax rate of 10 per cent is applied. As a result, the price increases in other industries are a factor in reducing the tax burden of households under the tax credit method but the price increase in the car industry itself is a factor in reducing the tax burden of households under the subtraction method. The following is the reason.

Now, we let \( S_j \) be sales excluding taxes in industry \( j \), \( S_{ij} \) be the value of inputs excluding taxes in industry \( j \) from industry \( i \), and \( t_j \) be the tax-exclusive tax rate for industry \( j \). Then the tax liability of industry \( j \) under the tax credit method, \( T_{TC} \), and that under the subtraction method, \( T_S \), in the no exemption case are given by the following equations:

\[
T_{TC}^j = t_j S_j - \sum_{i=1}^{n} t_i S_{ij}, \quad T_S^j = t_j (S_j - \sum_{i=1}^{n} S_{ij}) \quad (i, j = 1, 2, \ldots, n)
\]

There are four cases to consider, namely,

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15 The difference in treatment of tax-exempt industries under each method of calculating VAT depends on whether an invoice is used. Under the tax credit method, the criterion for whether an industry is in the VAT system is whether it uses invoices, but under the subtraction method, invoices are not used so one cannot judge whether an industry is taxable or tax-exempt. In both methods, the neutrality of the tax, which is claimed to be an advantage of VAT, is distorted.
(i) \( T^r_{TC} > 0, \ T^r_S > 0, \)
(ii) \( T^r_{TC} > 0, \ T^r_S < 0, \)
(iii) \( T^r_{TC} < 0, \ T^r_S > 0, \)
(iv) \( T^r_{TC} < 0, \ T^r_S < 0. \)

Case (ii) holds for the car industry. That is, when sales exclusive of exports are smaller than purchases, the tax liability under the subtraction method is necessarily negative but that under the tax credit method can be positive when the industry concerned is taxed at a higher rate. This phenomenon is based on the fundamental feature that each method of VAT has. Namely, an industry’s ‘value added of consumption type’ is taxed directly under the subtraction method but under the tax credit method an industry’s ‘sales’ are taxed and tax credits on purchases are subtracted at its own rate.\(^{16}\)

Therefore, when one wishes to increase the tax burden of the consumer by using higher rates, it may be that one cannot attain that objective under the subtraction method of the naïve type because the rate of price increase depends upon sales and purchases of industries that are taxed at higher rates.

Lastly, the patterns of the tax burden in relation to disposable income and consumption are as regressive as those under the tax credit method, and almost the same reasoning can be applied here as that of the tax credit method. Reducing the regressivity under the Exempt and Multi cases is not as successful as with the tax credit method. The pattern of the tax burden is, however, different from that under the tax credit method in the Multi case because the price increases of the car industry and consumer durable goods that are taxed at a higher rate are different from those under the tax credit method.

**IV. CONCLUDING REMARKS**

In this paper, we have considered the extent to which the regressivity of VAT is reduced by (i) the exemption of necessities and (ii) a multiple-rate system, both under the tax credit method of VAT and under the subtraction method of VAT. We came to the following conclusions:

(1) VAT based on the tax base and tax rates of the Japanese Consumption Tax Law is regressive against income, disposable income and consumption.

(2) In the case of the exemption of necessities, we saw a reduction in the tax burden across all income groups under both methods, although there was no reduction in regressivity.

(3) In the case of multiple rates, the tax burdens under both methods are milder than those in the exemption case. The difference between each

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\(^{16}\) The Japanese Consumption Tax lies between the tax credit and the subtraction methods as far as the manner of tax calculation is concerned in this respect.
method in the rate of price increase for an industry that is taxed at a higher rate affects the pattern of tax burden for households.

We assumed in this paper that each industry can set its price by fully shifting the tax liability. However, it is a well-known fact that it is very difficult to set prices under the subtraction method when multiple rates are used. Even if price setting is successful, there can be unintended results, as noted in this paper. Therefore, one needs careful consideration when setting multiple rates because unintended results may occur if the structure of industries and the consumption patterns of households are not known. In my view, these conclusions have much relevance to the criteria which should be used when introducing a VAT as a new source of revenue in Central and Eastern European countries.

The measures of exemption and the system of multiple rates not only distort the proper neutrality of VAT but increase rapidly the compliance cost for the taxpayer and the administrative cost for the tax authorities. The problem remains of whether one reduces the regressivity of VAT by the system of VAT itself, or through the system of income tax, or through the social security system.

APPENDIX A. INPUT–OUTPUT TABLES AND VALUE ADDED TAX

In this appendix, we look at the model that explains the effect of VAT on prices for each industry. The basic model is a price determination model using input–output analysis.

Now, the following relationship holds for each industry in the economy where there is no VAT:

\[
\begin{align*}
\text{\text{P}}_i = \sum_{j=1}^{n} a_{ij} \text{P}_j + v_i \quad (i, j = 1, \ldots, n)
\end{align*}
\]

where \( p_i \) is the unit price of the product of the \( i \)th industry, \( a_{ij} \) is the input coefficient per unit of output and \( v_i \) is the rate of value added per unit of product. In what follows, we assume each industry sets its product’s price by fully shifting VAT liability.

Solving equation (A.1) for \( p \) and showing it in matrix form we have

\[
\begin{align*}
\text{\text{P}} = [\text{I} - A^T]^{-1} \text{V}
\end{align*}
\]

\[\text{For a literature on VAT in Central and Eastern European countries, see Cnossen (1992). For an analysis of criteria in choosing among types of VAT, see Shoup (1990).}\]

\[\text{For a critical view on the use of multiple rates, see Cnossen (1989).}\]
where \( P=(p_1, p_2, \ldots, p_n)' \), \( I \) is the unit matrix of order \( n \), \( A \) is the input coefficient matrix, \( ' \) denotes the transpose of a matrix, the superscript \(-1\) denotes the inverse of a matrix and \( V=(v_1, v_2, \ldots, v_n)' \).

In the case of VAT under the tax credit method, the tax liability of each industry is the tax on sales excluding exports minus the tax on purchases including capital goods. Then we get the following expression instead of equation (A.1):

\[
(A.2) \quad p_i = \sum_{j=1}^{n} a_{ij} p_j + v_i + \tau_i p_i (1-e_i) - \sum_{j=1}^{n} k_{ji} \quad (i, j = 1, \ldots, n)
\]

where \( \tau_i \) is the tax-inclusive tax rate for the \( i \)th industry, \( e_i \) is export per unit of output and \( k_{ji} \) is the investment coefficient per unit of output. For exemption under the tax credit method, equation (A.1) still holds for a tax-exempt industry because it has no VAT liability. For a VAT-liable industry, however, since it can obtain no credits on inputs purchased from a tax-exempt industry, the last term on the right-hand side of equation (A.2) for a corresponding tax-exempt industry becomes zero. Solving equation (A.2) for \( p \) and showing it in matrix form we have

\[
(A.3) \quad P = \left[ I - A' - (I - A' - E - K')_0 \left[ \tau \right] \right]^{-1} V
\]

where \( E \) is the diagonal matrix whose diagonal elements are \( e_i \) (the export coefficient matrix), \( K \) is the investment coefficient matrix per unit of output, \( [\tau] \) is a diagonal matrix whose diagonal elements are \( \tau_i \), and \( (I - A' - E - K')_0 \) is the matrix \( (I - A' - E - K') \) but with the elements of the \( i \)th row and \( i \)th column replaced by zero when the \( i \)th industry is exempt.

In the case of VAT under the subtraction method, the tax liability of each industry is its tax rate multiplied by the tax base, which is sales excluding exports minus purchases including capital goods. Then equation (A.1) becomes

\[
(A.4) \quad p_i = \sum_{j=1}^{n} a_{ij} p_j + v_i + \tau_i (1-e_i) - \sum_{j=1}^{n} a_{ij} + \sum_{j=1}^{n} k_{ji} \quad (i, j = 1, \ldots, n)
\]

For exemption under the subtraction method, as under the tax credit method, equation (A.1) still holds for a tax-exempt industry because it has no VAT liability. But, for a VAT-liable industry, since it can deduct purchases from tax-
exempt industries, price determination remains as in equation (A.4). We solve equation (A.4) for $p$ and show it in matrix form to get

$$ P = \left[ I - A' - [c](I - A' - E - K')^{(i)} \right]^{-1} V $$

where $(I - A' - E - K')^{(i)}$ is the matrix $(I - A' - E - K')$ but with the elements of the $i$th row replaced by zero when the $i$th industry is exempt.

We use equations (A.1'), (A.3) and (A.5) in the analysis.

### APPENDIX B. CORRESPONDENCE BETWEEN HOUSEHOLD SURVEY AND INPUT–OUTPUT TABLE

<table>
<thead>
<tr>
<th>Household survey</th>
<th>Input–output table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>Other food</td>
</tr>
<tr>
<td>Beverages</td>
<td>Beverages</td>
</tr>
<tr>
<td>Housing</td>
<td>Real estate rent</td>
</tr>
<tr>
<td>Fuel, light and water charges</td>
<td>Water service</td>
</tr>
<tr>
<td>Electricity</td>
<td>Electric power</td>
</tr>
<tr>
<td>Gas</td>
<td>City gas</td>
</tr>
<tr>
<td>Furniture and household utensils</td>
<td>Furniture</td>
</tr>
<tr>
<td>Household durables</td>
<td>Electric appliances</td>
</tr>
<tr>
<td>Clothes and footwear</td>
<td>Knit wears, other textiles and personal effects</td>
</tr>
<tr>
<td>Medical care</td>
<td>Health and social security institution</td>
</tr>
<tr>
<td>Public transportation</td>
<td>Transportation,, private transportation and communication</td>
</tr>
<tr>
<td>Automobiles</td>
<td>Car</td>
</tr>
<tr>
<td>Education</td>
<td>Education</td>
</tr>
<tr>
<td>Reading and recreation</td>
<td>Other services</td>
</tr>
<tr>
<td>Other living expenditures</td>
<td>Other services</td>
</tr>
<tr>
<td>Tobacco</td>
<td>Tobacco</td>
</tr>
</tbody>
</table>

### APPENDIX C. PRICE DETERMINATION FOR ORIGIN PRINCIPLE AND GNP-TYPE VAT

As we assume in this paper that VAT is calculated according to the destination principle and consumption type, price is determined by equations (A.3) and (A.5) in Appendix A. In this case, the rate of change of price for each industry is different although there is a uniform tax rate.

When we assume VAT of origin principle and gross-national-product-type, equation (A.3) in the absence of exemption becomes

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19 One cannot distinguish between a VAT-liable firm and a VAT-exempt firm under the subtraction method because of the absence of an invoice.
\[ P = \left[ I - A' - (I - A'[t]) \right] V \]
\[ = (I - [t])^{-1}(I - A')^{-1}V \]
\[ = (I + [t])P_0 \]

where \([t]\) is the tax-exclusive tax rate matrix with diagonal elements \(t_i\) and off-diagonal elements zero, and \(P_0\) is the vector of tax-exclusive prices. The rate of price increase for each industry is equal to the tax-exclusive tax rate. Therefore, the rate of price increase is the same across all industries in the case of a uniform tax rate and under the tax credit method.

Under the subtraction method, on the other hand, when we assume VAT of origin principle and gross-national-product-type, equation (A.5) in the absence of exemption becomes

\[ P = \left[ I - A' - [t]I - A' \right] V \]
\[ = (I - A')^{-1}(I - [t])^{-1}V \]
\[ = (I - A')^{-1}(I + [t])V. \]

The rate of price increase for each industry depends not only on its own tax rate but on other industries' tax rates as well. Only when there is a uniform tax rate does the rate of price increase for each industry equal the tax-exclusive tax rate and also equal the rate of price increase under the tax credit method.

See also Appendix A.

REFERENCES

Regressivity of VAT