Fiscal Fan Charts: A Tool for Assessing Member States’ (Likely?) Compliance with EU Fiscal Rules

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Appendix A. Modelling and projecting Irish real GDP growth

This appendix explains the method used to project Irish real GDP growth rates over the period 2011–15. We wish to do so using Penn World Tables data for 1951–2009 with the 2010 observation coming from the Department of Finance (2011) Update.

A natural starting point is to fit a simple parametric model such as an AR(1) and use that to project growth rates forwards. This gives us an equation of the form

\[ g_t = \alpha + \beta g_{t-1} + \epsilon_t \]

where \( \epsilon_t \) is some well-behaved (for example, Gaussian) noise process. This equation fits the Irish data well.

Unfortunately, this process also has the property that the projected prediction intervals always widen with the horizon – in fact, they widen in accordance with the well-known square-root-of-time rule – so becoming implausibly wide at some point. Our projections suggest that this approach gives implausibly wide intervals before the end of the five-year horizon.

Hence, even though we can fit an AR(1) to the data and get a reasonable fit, we cannot use that fitted process to produce plausible longer-term projections.

A related problem is that this process involves no mean-reversion feature that would mitigate the tendency of the prediction bounds to widen excessively as the horizon increases.

This suggests the alternative of a mean-reverting process such as

\[ g_t = \gamma + \delta (\bar{g} - g_{t-1}) + \epsilon_t \]

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where we require $\delta > 0$ for mean reversion to occur and $\bar{g}$ is the $g$ value to which the process reverts. For any given value of $\bar{g}$, this process is observationally equivalent to (A1) where the parameters of the two processes are related as follows:

$$
\gamma = \alpha + \beta \bar{g},
$$
$$
\delta = -\beta.
$$

Unfortunately, estimates of (A1) indicate that $\beta$ is significantly positive, and this implies that $\delta < 0$ and not positive as required. This indicates that we cannot fit a mean-reversion process that actually reverts to the mean.

Another approach is simple historical simulation (HS): we project from the historical outcomes using the VaR (value at risk) or quantile forecasting methods familiar from financial risk management (see, for example, Dowd (2005, ch. 4)). Vanilla HS has the attraction that the prediction bounds can never exceed the range of the historical data, but it also suffers from some limitations:

- It can be inaccurate, especially with smaller sample sizes.
- It does not take account of the autocorrelation of growth rates.
- It can be difficult obtaining multi-step-ahead projections using vanilla HS.

The first drawback can be ameliorated by using a kernel or smoothing procedure, which also has the advantage of making it easier to obtain quantile estimates: we can then bootstrap from the kernel density.\(^1\) The second and third can be solved by introducing autocorrelation in the bootstrapped $g$ projections. More specifically, the procedure goes as follows:

- We fit a kernel to the historical data: we actually use a box kernel but the choice of kernel does not appear to make much difference.
- For each of $t$ from 1 to 5, we obtain a bootstrapped sample of 20,000 simulations from the kernel density. We call this sample `boot_kernel_g`, and denote the $j^{th}$ individual value in this sample as `boot_kernel_g(j)`.
- Note that each such distribution of bootstrapped values can be regarded as the long-run economic growth rate density because this bootstrap implicitly ignores any temporal dependence between growth rates.
- For each of $t$ from 0 to 5, denote the to-be-simulated value of $g$ at time $t$ as $g(t)$.
- For $t = 0$, set $g(j) = g_0$ for all $j$.

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\(^1\)An additional benefit of bootstrapping from the kernel density rather than from the original data is that we can more easily obtain estimates of the 1 per cent, 2 per cent, 3 per cent etc. percentiles. The estimation of such percentiles from a sample of 60 observations is otherwise non-trivial.
• For $t = 1$, set $g_t(j) = \rho g_0 + (1-\rho)\text{boot}_\text{kernel}_g(j)$, where $\rho$ is our assumed autocorrelation coefficient, which is taken to be equal to our best estimate, 0.568.
• For $t = 2$, set $g_t(j) = \rho^2 g_0 + (1-\rho^2)\text{boot}_\text{kernel}_g(j)$.
• For $t = 3$, set $g_t(j) = \rho^3 g_0 + (1-\rho^3)\text{boot}_\text{kernel}_g(j)$.
• For $t = 4$, set $g_t(j) = \rho^4 g_0 + (1-\rho^4)\text{boot}_\text{kernel}_g(j)$.
• For $t = 5$, set $g_t(j) = \rho^5 g_0 + (1-\rho^5)\text{boot}_\text{kernel}_g(j)$.

This procedure gives us simulated $g$ values whilst imposing minimal structure on the data. These give us prediction intervals that increase with the horizon and gradually level out, being effectively bounded by the range of the historical data. The rate at which they widen depends on $\rho$, which

**FIGURE A1**

*Projections of real GDP growth rate*
functions as a persistence parameter: the greater is $\rho$, the greater the persistence and the smaller the rate at which prediction intervals widen with the horizon.

To illustrate the sensitivity of the projections to the value of $\rho$, the panels of Figure A1 give alternative projections of the real GDP growth rate for values of $\rho$ equal to 0.8, 0.6 and 0.4 respectively.

The plausibility of these alternative projections can assessed by the speed at which they fan out, which itself reflects their temporal dependence, and by how they compare against the historical data and/or kernel density. Note that the range of the historical data is $[-8.44 \text{ per cent}, 12.57 \text{ per cent}]$, whilst the 5 per cent lower bound and 95 per cent upper bound of the kernel density are $-3.31 \text{ per cent}$ and $10.42 \text{ per cent}$ respectively.

The middle chart would appear to give a plausible set of projections by these criteria, and is also based on a $\rho$ value close to our best estimate.

Finally, note that the above procedure gives us a set of fan chart projections whose only calibration comes from the sample of Irish growth rates given in the Penn World Tables. If we wish to re-centre the projections on an alternative central path – as is done in the paper – then for each $t$, we add to $g_t$ the assumed central growth rate for $t$ minus the mean of $g_t$.

**Appendix B. Impact of a fiscal tightening in 2013–15 on the fiscal and growth rate variables**

We assume a tightening of the structural primary budget balance beyond that already planned in the Update of 1 per cent of nominal GDP in 2013, 2014 and 2015. This has the effect of reducing real GDP by 0.317 per cent in 2013, by 0.615 per cent in 2014 and by 0.516 per cent in 2015.

**1. 2013**

The new baseline deficit ratio, taking account of the fiscal tightening, in 2013 is now calculated as follows:

$$d_{2013} = (s_{2013}^{U} + 0.01)R_{2013} + c_{2013} - \sigma_{2013}^{U} R_{2013}$$

where, in this case, $R_{2013} = (1 - 0.00317)^{-1}$ and $c_{2013}$ is calculated by noting that the output gap is now

$$\frac{Y_{2013} - P_{2013}^{U}}{P_{2013}^{U}} = \frac{Y_{2013}^{U}(1 - 0.00317)}{P_{2013}^{U}} - 1$$

and therefore
The growth rate, $g_{2013}$, used below, is $(1 + g_{2013})(1 - 0.00317) - 1$.

The new baseline debt ratio in 2013 is

$$b_{2013} = (b_{2013}^U - 0.01)R_{2013} + (c_{2013}^U - c_{2013}).$$

2. 2014

$$d_{2014} = (S_{2014} + 0.01)R_{2014} + c_{2014} - \{a_{2014}^UR_{2014} + i_{2014}b_{2013} - \hat{b}_{2013}^U\}$$

where $x_{2014} = (1 + g_{2014})(1 + n_{2014}^U)$; $n_{2014}^U$ is the GDP deflator value for 2014 given in the Update (the 2014 and 2015 values for the deflator are included in Table 1); $c_{2014}$ is calculated as $\alpha\{Y_{2014}^U(1 - 0.00615) / R_{2014}^U - 1\}$; $R_{2014}$ equals $(1 - 0.00615)^{-1}$; and $\hat{b}_{2013}^U$ is, in this case, the original Update debt ratio for 2013, less the 1 per cent fiscal consolidation in that year, rebased, i.e. $(b_{2013}^U - 0.01)R_{2013}$.

The growth rate, $g_{2014}$, is $(1 + g_{2013}^U)(1 + g_{2014}^U)(1 + 0.00615)/(1 + g_{2013}) - 1$.

The interest rate variable, $i$, has a value of 0.0582 in both 2014 and 2015, as in the exercise in Section IV.

The debt ratio in 2014 is

$$b_{2014} = \{(b_{2014}^U - 0.01) - 0.01\}R_{2014} + (c_{2014}^U - c_{2014}) + (1 + i_{2014})b_{2013} - \hat{b}_{2013}^U$$

where $x_{2014}^U = (1 + g_{2014}^U)(1 + n_{2014}^U)$.

The entry 0.01/$x_{2014}^U$ in this equation reflects the need to reduce the Update debt ratio value for 2014 by the 1 per cent reduction in the debt ratio arising directly from consolidation already undertaken in 2013, prior to rebasing for the lower level of GDP in 2014 arising from the fiscal consolidation. Similar adjustments for the policy-induced change in the structural budget balance ratio in 2013 and 2014 have to be accounted for in adjusting the Update debt ratio in 2015 for the effects of consolidation in all three years.

3. 2015

The deficit ratio and debt ratio in 2015 are, respectively,
\[ d_{2015} = (s_{2015}^{U} + 0.01)R_{2015} + c_{2015} - \{o_{2015}^{U}R_{2015} + i_{2015} \frac{b_{2014} - \tilde{b}_{2014}}{x_{2015}} \} \]

and

\[ b_{2015} = \{[(b_{2015}^{U} - 0.01) - \frac{0.01}{x_{2015}^{U}} - \frac{0.01}{x_{2014}^{U}x_{2015}^{U}}]R_{2015} \} + \{c_{2015}^{U} - c_{2015} \} \]

\[ + (1 + i_{2015}) \frac{b_{2014} - \tilde{b}_{2014}}{x_{2015}} \]

where \( c_{2015} = \alpha \{Y_{2015}^{U}(1-0.00516) / P_{2015}^{U} - 1 \}; \ R_{2015} = (1-0.00516)^{-1}; \)

\( g_{2015} = (1 + g_{2013}^{U})(1 + g_{2014}^{U})(1 + g_{2015}^{U})(1-0.00516)/(1 + g_{2013}^{U})(1 + g_{2014}^{U}) - 1; \)

\( x_{2015} = (1 + g_{2015}^{U})(1 + n_{2015}^{U}); \ x_{2015}^{U} = (1 + g_{2015}^{U})(1 + n_{2015}^{U}); \)

\( \tilde{b}_{2014} = \{(b_{2014}^{U} - 0.01) - (0.01/x_{2014}^{U})\}R_{2014}. \)

**References**
