Abstract

The growth in labor market participation among women with young children has raised concerns about the potential negative impact of the mother’s absence from home on child outcomes. Recent data show that mother’s time spent with children has in fact declined in the last decade, while the indicators of children’s cognitive and noncognitive outcomes have worsened. The objective of our research is to estimate a model of the cognitive development process of children nested within and otherwise standard model of household life cycle behavior. The model is able to capture a number of possible feedbacks between the child quality and employment processes in the household, which we believe sheds some light on the intertemporal relationships observed in the data and the possible impacts of labor market shocks on the welfare of children.
1 Introduction

Economic theory does not provide unambiguous predictions regarding the impact of parental employment on the welfare of children. While family income is necessary to provide for the public and private consumption of the household and its members, as well as for investments undertaken collectively and individually, there are opportunity costs associated with time supplied to the labor market beyond the foregone leisure of the parents. As discussed below, numerous studies have found a positive relationship between the time spent with parents and the quality of a child’s outcomes, measured on any of one of several dimensions. The nature of the tradeoff facing household decision-makers is apparent if we think of the household as an enterprise with a complicated set of preferences, outputs, and constraints, as in the seminal work of Becker (1981). From a societal perspective, perhaps the most important output of the household is the number and the “quality” of children it produces. It is helpful to think of there being a production technology for child quality, in which some initial endowment of child quality, present at birth, say, is augmented over time by inputs of time contributed by parents, siblings, other relatives, paid child-care workers, teachers, etc., and various types of goods purchased as inputs to the development process, such as formal schooling, toys, books, sporting goods, etc. Given the household’s objectives, its mode of decision-making, and the constraints it is confronted with over time, it makes a sequence of time allocation, consumption, and investment decisions at each stage of the child development process. Ignoring the possibility of borrowing and saving for the moment, the more income the household has at a point in time, the more can be spent on child investment goods, among other things. By the same token, decreasing time with the child, holding other inputs fixed, leads to worse child quality outcomes. How does the household properly balance these trade-offs?

It may be useful to compare and contrast the situation that prevails in the standard profit-maximizing firm. In this case, firm behavior is decomposed into a (1) cost-minimization problem, which yields a function that provides the minimum cost required to produce any level of output, and (2) a profit-maximization problem in which the cost function is considered as being predetermined. The solution of these two sequential problems yields the profit-maximizing output quantity and the demand for the factors of production. This is all reasonably straightforward.

In the case of child quality, we face a number of additional challenges, some of them nicely described in Todd and Wolpin (2004). The production process is truly dynamic in the case of child quality, with input decisions from past years typically influencing current investment choices and child quality levels. Unlike the case of the firm, whose objective is to choose output levels so as to maximize profits, the objectives of the parents are not defined only with respect to the quality, or the quantity, of their children. Their utility levels also are determined by their leisure, their consumption of adult private goods, and public goods that affect the welfare of all members of the household. In other words, the household’s objective is not to maximize child quality given the resources available to them.
Our focus is on the estimation of child outcome technologies, or production functions, that include as arguments a limited number of (potentially observable) factors of production, as well as functions characterizing the dynamic evolution of the budget constraint of the household. Given the arguments we have just presented, it appears necessary to model the household decision-making process in a relatively complete way if we are to disentangle the impact of household preferences, technologies, and constraints on child quality outcomes. Our model utilizes a reasonably standard life cycle framework, in which parents face partially endogenous constraint sets that evolve over time. The Child Development Supplements of the Panel Study of Income Dynamics, hereafter referred to as the PSID-CDS, gives us a substantial amount of useful information, but only at one or two points in time. The keys to being able to use such limited (in a dynamic sense) information to estimate a growth model are (1) assuming time-invariance of the functions and processes describing the households’ objectives and constraints and (2) the use of simulation-based estimation techniques that allow us to “fill in” the huge numbers of gaps we face in our data on the development process at the household level.

Even under the restrictive assumptions made for purposes of tractability and for parameter identification, we find that we are able to fit many of the patterns observed in the empirical household income, labor supply, child investment, and child outcome processes reasonably well. Using the parameter estimates, we are able to analyze the impact of changes in the time inputs of mothers and fathers on the child development process. Of course, both of these processes are endogenous within the model, so that any changes in the relationship between them must be produced by changes in the parental wage and nonlabor income processes and the prices of the consumption and investment goods. The model is able to generate a number of behavioral links between the child quality and employment processes in the household, which we believe shed some light on dynamic relationships observed in the raw data and the possible impacts of labor market shocks to the parents on the welfare of children.

There exists a large empirical literature on the relationship between household characteristics, parental employment patterns, and child outcomes, and in section 2 we provide a brief survey. In section 3 we present the model, and section 4 contains a discussion of estimation issues, the data utilized in the empirical work, and some descriptive empirical results. Section 5 contains the model estimates, and some “empirical” comparative statics exercises. In section 6 we conduct a policy experiment meant to mimic some features of the Head Start program. Section 7 concludes.

2 Parental Time Inputs and Child Outcomes

A large number of studies have assessed the effect of parental time on children’s cognitive development. Most studies have used parents’ employment as a proxy for time with the children. These studies report evidence that, on the negative side, the loss of parental
time with the children has a negative impact on certain measures of the child's well-being (e.g. socio-emotional adjustment and cognitive outcomes), while, on the positive side, the additional labor income has positive implications for expenditures on goods consumed by the child (Brooks-Gunn, Han and Waldfogel, 2001; Ermisch and Francesconi, 2005; Bernal, 2008). It remains unclear which effect is predominant, since the existing literature provides conflicting conclusions. There is wide variation in reported empirical estimates, even for studies based on the same data set. Estimates range from parental employment being detrimental (Baydar and Brooks-Gunn, 1991; Desai et al., 1989), to its having no effect (Blau and Grossberg, 1992), to its being beneficial (Vandell and Ramanan, 1992). Reasons for the diversity of these results may include the wide range of specifications that are estimated, as well as the common limitation of failing to control for potential biases that may arise due to the endogeneity of parental time and other child quality inputs included in the analyses.

Most studies limit their attention to mother’s inputs. The literature on the effects of maternal time on child cognitive outcomes is extensive. Some studies have focused on the timing of the process showing that there are deleterious impacts of maternal employment during the child’s first year, while the influence of maternal employment on child outcomes after the first year is ambiguous (Baydar and Brooks-Gunn, 1991; Ruhm, 2004). Many studies have demonstrated that the influence of maternal employment on child outcomes differs by the economic and demographic characteristics of mothers and their families, suggesting that maternal employment may be more harmful for children from advantaged backgrounds: children from wealthier families, non-Hispanic white children, and children from intact families. Desai, Chase-Lansdale and Michael (1989) find that maternal employment negatively influences children from higher income families, but not children from middle or low income families (1989), while Waldfogel, Han and Brooks-Gunn (2002) find a persistent negative effect of mother’s employment on cognitive test scores for non-Hispanic white children, but not for African-American or Hispanic children.

Very few studies have used direct measures of the time parents spend with children to examine the relationship between parental investments and children’s cognitive development. Time diary data suggest that women’s entry into the labor force is associated with changes in time use that make employment status a poor proxy for maternal involvement. Booth et al (2002) analyze time diaries administered to mothers from the NICHD Study of Early Child Care and find that the amount of time mothers spend with their young children is not significantly correlated with measures of cognitive skills. Huston and Aronson (2005) find that a measure of the mother’s time with children even relates negatively to child language skills, though there are obvious reverse causality issues that may be important. Their studies are limited to the investigation of behaviors and outcomes in the first two years of life and do not take into account differences in the initial endowments of children. More recently, Hsin (2008), using the Child Development Supplement of the PSID, investigated the effect of maternal time with children during pre-school years on the child’s cognitive outcomes, while conditioning on characteristics of children that may...
to bias the estimated impact of maternal time allocations. For example, children clearly differ in their initial endowments such as innate cognitive ability, health status, and their physical development. Mothers may respond to observed difficulties faced by children by spending more time with them. She finds a positive and persistent effect of the time mothers spend with children on children’s language development, but only among children who spend time with verbally-skilled mothers.

While mother’s time is a crucial input in the production process of child outcomes, father’s time may be equally productive, especially in some stages in the child’s development process; time spent with children by fathers has increased over time, partly offsetting the decline in mother’s time spent with the child. Studies considering father’s time show that fathers’ care for infants is no better or worse than other types of arrangements (Averett et al 2005), and the amount of time a father spends with children is affected by the gender composition of the children (Lundberg et al 2006, Mammen 2005). Several studies suggest that there is no long term benefit of paternal investments on children’s achievement and behavior (Yeung, Hill, and Duncan, 1999; Haveman and Wolfe, 1995).

Other reasons for the diversity of the results are methodological and are associated with the variety of model specifications employed. Bernal (2008) estimates a dynamic model of mothers’ choices to control for potential biases that may arise as a result of the fact that women who work and use child care may be systematically different from women who do not, as well as allowing for the child’s initial cognitive ability to influence the mother’s work decisions. Todd and Wolpin (2003) estimate a dynamic child quality production function that views child development as a cumulative process, with the final child quality level being determined by the sequence of family and school inputs supplied during the developmental process and on heritable endowments (i.e., initial conditions). Their estimating framework allows for unobserved endowment effects, potentially endogenous input choices, and for the cumulative effects of child investments. Their results show that both contemporaneous and lagged inputs matter in the production of current achievement, and that it is important to allow for unobserved child-specific endowment effects and the endogeneity of inputs. Cunha and Heckman (2007) estimate a dynamic factor model of child cognitive and non-cognitive outcomes in a model of skill formation, taking into account the problem of endogeneity of inputs and the unobserved nature of both the inputs and outputs. They find that early environments play a large role in shaping later outcomes, and children’s cognitive and non-cognitive outcomes are largely determined early in life.

3 Model

This section develops the model that is the basis of our empirical estimation. As noted in the Introduction, the model is based on a set of assumptions that allow us to derive closed-form solutions to the household’s dynamic optimization problem; it is the simple forms of the life-cycle demand functions that allows us to determine a relatively large number of
endogenous variables in a straightforward way. The special characteristics of the decision rules also allow us to sort out identification issues when we discuss estimation issues below.

3.1 Timing and Preferences

The model begins with the birth of a child. The household makes decisions in each period of a child’s life, where the child’s age is indexed by $t$. For simplicity, we assume that the family has only one child.\\footnote{We only consider investments in one child, and ignore the presence of siblings. In the data used below, detailed information on no more than two children in the household is collected. While there may be very important externalities present when more than two child qualities are being determined simultaneously, the nature of these spillovers is likely to be a complicated function of the age, gender, and native endowment differences among the children. This is a subject worthy of further research, but is ignored in this paper. In terms of our model, heterogeneity in the utility from income not spent on the one child, and time not spent with the child or in market work, what we call parental “consumption” and “leisure”, can be thought of as in part reflecting the value of these resources being invested in the other children in the household.}

Parents make investments in child quality from the first period of the child’s life, $t = 1$, through the last developmental period, $T$. At this “terminal” point (from the perspective of the parents’ investment in the child), the child has reached adulthood and adult outcomes depend (in part) on the level of child quality obtained at this point.\\footnote{The terminal date $T$ need not correspond to the end of the investment period in the child. In a more elaborate model of child development, it may correspond to the end of a particular developmental stage, with the final value of child quality in the current stage of development serving as an initial condition into the next stage of development, which may be characterized by very different production technologies. While we have not pursued such an approach in this paper, it is a subject of our on-going research.}

In each period, the household makes five choices: hours of work for each parent: $h_{1t}$ (mother) and $h_{2t}$ (father); time spent in child care for each parent: $\tau_{1t}$ (mother) and $\tau_{2t}$ (father); and expenditures on “child” goods, $e_t$. Household utility in period $t$ is a function of each parent’s hours of leisure, $l_{1t}$ for the mother and $l_{2t}$ for the father, joint private consumption of the parents and child, $c_t$, and the level of their child’s quality, $k_t$. We assume a Cobb-Douglas form for preferences and restrict the preference parameters to be stable over time:

$$u(l_{1t}, l_{2t}, c_t, k_t) = \alpha_1 \ln l_{1t} + \alpha_2 \ln l_{2t} + \alpha_3 \ln c_t + \alpha_4 \ln k_t, \quad (1)$$

where $\sum_j \alpha_j = 1$.

3.2 Child Quality Production

Next period child quality is produced by the current level of child quality $k_t$, parental time investments in the child, and expenditures on the child. We assume a Cobb-Douglas form for the child quality technology:

$$k_{t+1} = f(k_t, \tau_{1t}, \tau_{2t}, e_t) = R_t^{\delta_{1t}} \tau_{1t}^{\delta_{2t}} \tau_{2t}^{\delta_{3t}} e_t^{\delta_{4t}} k_t^{\delta_{4t}} \quad (2)$$
where $R_t > 0$ is a scaling factor.

While the Cobb-Douglas form restricts the substitution possibilities, we allow the productivities of the various inputs to vary over the age of the child. This allows us to capture the important insights in the economics and child development literatures that the marginal productivity of inputs varies over the stages of child development (for a useful survey, see Heckman and Masterov (2007)). As written in (2), the production technology is deterministic. An extension of the model is to assume there are stochastic shocks to child quality production, which may be attractive for purposes of estimation, though under our functional form assumptions the presence of such a shock has no substantive impact on the household’s decision rules. We will return to this topic below.

### 3.3 Dynamic Problem

Given wage offers and the current level of child quality, parents optimally choose their labor supply and child inputs to maximize expected lifetime discounted utility. The value function for the household at period $t$ is then

$$V_t(S_t) = \max_{l_{1t}, \tau_{1t}, l_{2t}, \tau_{2t}, e_t} u(l_{1t}, l_{2t}, c_t, k_t) + \beta V_{t+1}(S_{t+1}),$$

where the vector of state variables $S_t$ consist of the current level of child quality, the wage offers to the parents, and nonlabor income,

$$S_t = (k_t w_{1t} w_{2t} I_t),$$

and $\beta (\in [0,1))$ is the discount rate. The state variable vector at the birth of the child are the initial conditions of the problem, $S_1 = (k_1 w_{11} w_{21} I_1)$.

The constraint set faced by the household in period $t$ consists of time and market good expenditures restrictions. We assume that each parent has a time endowment of $TT$ each period, so that

$$TT = l_{jt} + h_{jt} + \tau_{jt}, \ j = 1,2.$$

With no borrowing or saving, the household expenditure constraint in period $t$ is

$$c_t + e_t = w_{1t} h_{1t} + w_{2t} h_{2t} + I_t.$$

### 3.4 Terminal Value

Parental investments in child quality are limited to the first $T$ period’s of the child’s life. The terminal level of child quality is then $k_{T+1}$. Beyond $T$, the parents make only labor supply and consumption decisions, since $k_t = k_{T+1}$ for $t = T + 1, T + 2, \ldots$. Then we can write the households optimization problem at time $T + 1$ as

$$V_{T+1}(w_{1,T+1} w_{2,T+1} I_{T+1}; k_{T+1}) = \tilde{V}_{T+1}(w_{1,T+1}, w_{2,T+1}, I_{T+1}) + \psi \alpha_4 \ln k_{T+1},$$

7
where

$$\hat{V}_{T+1}(w_{1,T+1}, w_{2,T+1}, I_{T+1}) = \max_{l_{1,T+1}, l_{2,T+1}} \alpha_1 \ln l_{1,T+1} + \alpha_2 \ln l_{2,T+1} + \alpha_3 \ln (w_{1,T+1}(TT - l_{1,T+1}) + w_{2,T+1}(TT - l_{2,T+1}) + I_{T+1})$$

$$+ \beta \hat{V}_{T+2}(w_{1,T+2}, w_{2,T+2}, I_{T+2}).$$

A few things to note about this expression are the following. First, assuming that the household is infinitely-lived, \(\pi\), the household problem after period \(T\) becomes stationary, so that we can write

$$\hat{V}_{T+s}(\tilde{S}_{T+s}) = \hat{V}(\tilde{S}_{T+s}), \ s = 1, 2, \ldots,$$

where \(\tilde{S}_t = (w_{1t}, w_{2t}, I_t)\). Second, we have assumed that the “terminal” value of child quality in period \(T + 1\) is given by \(\psi \alpha_4 \ln k_{T+1}\). If we made the stringent assumption that child quality yielded the same value in all of the remaining periods of the household’s life, it would be the case that

$$\psi = (1 - \beta)^{-1}.$$  

However, we felt that it was more reasonable to freely estimate the parameter \(\psi\) instead of imposing this potentially over-identifying and somewhat controversial restriction.

Third, it is apparent that with no savings or borrowing possibilities and with no state dependence in wages and/or nonlabor income (i.e., the wage and nonlabor income processes are strictly exogenous), once the child investment process is finished, the household solves a sequence of static optimization problems. Thus the period \(s\) optimization problem, \(s = T + 1, T + 2, \ldots\), reduces to

$$\max_{l_{1,s}, l_{2,s}} \alpha_1 \ln l_{1,s} + \alpha_2 \ln l_{2,s} + \alpha_3 \ln (w_{1,s}(TT - l_{1,s}) + w_{2,s}(TT - l_{2,s}) + I_s).$$

All of this implies that we can write the period \(T\) optimization problem as

$$V_T(w_{1T}, w_{2T}, I_T, k_T) = \max_{l_{1T}, r_{1T}, l_{2T}, r_{2T}, c_T} \alpha_1 \ln l_{1T} + \alpha_2 \ln l_{2T} + \alpha_3 \ln c_T + \alpha_4 \ln k_T$$

$$+ \psi \alpha_4 \{\delta_1 T \ln r_{1T} + \delta_2 T \ln r_{2T} + \delta_3 T \ln c_T + \delta_4 T \ln k_T\}$$

$$+ \hat{V}(\tilde{S}_{T+1})$$

From the point of view of the choice problem, we can write this expression as

$$V_T(w_{1T}, w_{2T}, I_T, k_T) = \max_{l_{1T}, r_{1T}, l_{2T}, r_{2T}, c_T} \alpha_1 \ln l_{1T} + \alpha_2 \ln l_{2T} + \alpha_3 \ln c_T$$

$$+ \psi \alpha_4 \{\delta_1 T \ln r_{1T} + \delta_2 T \ln r_{2T} + \delta_3 T \ln c_T\}$$

$$+ Q_T(k_T, \tilde{S}_{T+1}). \quad (4)$$

\(^3\)It is not strictly necessary that the household be infinitely-lived. All of the properties discussed follow if the has a constant probability of death, say \(\pi\), the value of which is subsumed within the discount factor \(\beta\).
where \( Q_T(k_T, \tilde{S}_{T+1}) \) is not a function of any current period choices. Thus the time allocation and consumption decisions are determined solely from the expression on the first two lines of the right hand side of (4).

3.5 Model Solution

We devote some time to describing the solution to the model, which will be important in understanding the ability of the model to fit the data and, more formally, in evaluating the ability of our proposed estimator to recover the primitive parameters that characterize the model.

As is clear from the nature of the production technology, there are never any corner solutions to the household input choice problem during the investment period.\(^4\) We can write the conditional factor demands for child inputs, where we are conditioning on labor supply choices and nonlabor income, as

\[
\tau_{1t} = (T_T - h_{1t}) \frac{\varphi_{1t}}{\alpha_1 + \varphi_{1t}}
\]

\[
\tau_{2t} = (T_T - h_{2t}) \frac{\varphi_{2t}}{\alpha_2 + \varphi_{2t}}
\]

\[
e_t = (w_{1t}h_{1t} + w_{2t}h_{2t} + I_t) \frac{\varphi_{3t}}{\alpha_3 + \varphi_{3t}},
\]

where

\[
\varphi_{jt} = \beta \delta_{jt} \eta_{t+1}, \quad j = 1, 2, 3.
\]

The sequence \( \{\eta_t\}_{t=1}^{T+1} \) is defined (backwards-) recursively as

\[
\eta_{T+1} = \psi \alpha_4
\]

\[
\eta_T = \alpha_4 + \beta \delta_{4,T} \eta_{T+1}
\]

\[
\vdots
\]

\[
\eta_t = \alpha_4 + \beta \delta_{4,t} \eta_{t+1}
\]

\[
\vdots
\]

\[
\eta_1 = \alpha_4 + \beta \delta_{4,2} \eta_2.
\]

\( \eta_t \) is the period \( t \) marginal utility of (log) child quality to the household: \( \eta_t = \partial V(S_t) / \partial \ln k_t \). \( \eta_t \) reflects both the present period flow marginal utility of (log) child quality to the household, given by \( \alpha_4 \), and the discounted value of child quality to future utility. The latter

\(^4\)If any factor is set at 0, then child quality will be 0 in all subsequent periods, and household utility diverges to \(-\infty\) as \( k \to 0 \).
value of current child quality depends on the discount rate and the technologically determined productivity of the current stock of child quality in producing future child quality, given by the time varying parameter $\delta_{t,t}$.

The solution to the spousal labor supplies problem in period $t$ also has a simple form. Define two “latent” labor supply variables in period $t$ by

$$
\hat{h}_{1t} = \frac{A_{1t} - A_{2t}B_{1t}}{1 - A_{2t}B_{2t}},
$$

$$
\hat{h}_{2t} = \frac{B_{1t} - B_{2t}A_{1t}}{1 - A_{2t}B_{2t}},
$$

(8)

where

$$
A_{1t} = \frac{w_{1t}TT\xi_{t}^{12} - I_{t}\xi_{t}^{11}}{w_{1t}(\xi_{t}^{11} + \xi_{t}^{12})},
$$

$$
A_{2t} = \frac{w_{2t}\xi_{t}^{11}}{w_{1t}(\xi_{t}^{11} + \xi_{t}^{12})},
$$

$$
B_{1t} = \frac{w_{2t}TT\xi_{t}^{12} - I_{t}\xi_{t}^{22}}{w_{2t}(\xi_{t}^{22} + \xi_{t}^{12})},
$$

$$
B_{2t} = \frac{w_{1t}\xi_{t}^{22}}{w_{2t}(\xi_{t}^{22} + \xi_{t}^{12})},
$$

and where

$$
\xi_{t}^{11} = \alpha_{1} + \beta \eta_{t+1}\delta_{1t},
$$

$$
\xi_{t}^{12} = \alpha_{3} + \beta \eta_{t+1}\delta_{3t},
$$

$$
\xi_{t}^{22} = \alpha_{2} + \beta \eta_{t+1}\delta_{2t}.
$$

Given these latent labor supplies, we can define the actual optimal hour choices that satisfy the rationing constraint on the time allocations of the parents. The solution to the optimization problem in terms of the “latent” labor supply choices is derived from functions similar to reaction functions, which express the optimal latent labor supply choice of parent $i$ as a linear function of the latent labor supply choice of parent $i'$, $i' \neq i$. These functions are

$$
\hat{h}_{1t} = A_{1t} - A_{2t}\hat{h}_{2t},
$$

$$
\hat{h}_{2t} = B_{1t} - B_{2t}\hat{h}_{1t}.
$$

If the latent labor supplies on the right hand sides are set to zero, it is apparent that the condition required for the conditional latent labor supplies to both be 0 is

$$(h_{1t}^{*} = 0, h_{2t}^{*} = 0) \iff A_{1t} \leq 0 \text{ and } B_{1t} \leq 0.$$
If both of these intercept terms are nonpositive, then the household supplies no time to the market. For this to be the case, it is necessary that the household’s nonlabor income be strictly positive.

Going back to the “full” solutions to the model given in (8), if both of the solutions are positive, then both satisfy the time allocation constraints, and these are the solutions to the household optimization problem. If the latent labor supply of parent 1 is positive and that of parent 2 is negative, then \((h^*_1 t, h^*_2 t) = (A_1 t, 0)\), while if the situation is reversed, the solution is \((h^*_1 t = 0, h^*_2 t = B_1 t)\). In summary, optimal labor supplies are

\[
(h^*_1 t, h^*_2 t) = \begin{cases} 
(0, 0) & \text{if } A_1 t \leq 0 \text{ and } B_1 t \leq 0 \\
(A_1 t, 0) & \text{if } A_1 t - A_2 t B_1 t > 0 \text{ and } B_1 t - B_2 t A_1 t < 0 \\
(0, B_1 t) & \text{if } A_1 t - A_2 t B_1 t < 0 \text{ and } B_1 t - B_2 t A_1 t > 0 \\
(h^*_1 t, h^*_2 t) & \text{if } A_1 t - A_2 t B_1 t \geq 0 \text{ and } B_1 t - B_2 t A_1 t \geq 0 
\end{cases}
\]

Using these optimal labor supply choices, the investment decisions are determined using (5), (6), and (7) after substituting \(h^*_1 t\) and \(h^*_2 t\) into the functions.

### 3.6 Characteristics of Decision Rules

We conclude this section by emphasizing a few characteristics of the decision rules derived under our modeling assumptions. Most notable is the fact that our functional form assumptions result in decision rules that are independent of the current child quality state. Child quality remains a state variable in the problem since it enters the utility function of the household in every period. The lack of dependence of investment and labor supply decisions on child quality levels greatly simplifies the computational burden of solving the model, enabling us to find closed-form solutions for all five endogenous variables. Even though the functional form assumptions are restrictive, it is not necessary to assume temporal invariance of either the child quality production function or of household preferences. For purposes of estimation, we have assumed time-invariant household preferences, but in principle there is no restriction on the manner in which the Cobb-Douglas can vary over time. This makes the specification sufficiently flexible to fit most patterns in the data while preserving its very attractive computational properties.

The model as it has been developed so far does not include “shocks” or uncertainty in any form. Once again, due to the properties of the decision rules, such “realistic” changes to the model setup can be easily accommodated. For example, say that the child quality production function is altered to include a stochastic total factor productivity, that is,

\[
k_{t+1} = f(k_t, \tau_{1t}, \tau_{2t}, e_t, A_t) = R_t \tau_{1t}^{\delta_{1t}} \tau_{2t}^{\delta_{2t}} e_t^{\delta_{3t}} k_t^{\delta_{4t}},
\]

where \(R_t\) has a nondegenerate distribution \(H_t\). Then

\[
\ln k_{t+1} = \ln R_t + \delta_{1t} \ln \tau_{1t} + \delta_{2t} \ln \tau_{2t} + \delta_{3t} \ln e_t + \delta_{4t} \ln k_t + \varepsilon_t,
\]

(9)
where \( \varepsilon_t \equiv \ln R_t - \ln R_t \) and \( E_t = \ln R_t \), assuming that this expectation exists. No other restrictions are necessary on the distribution of the sequence \( \{\ln R_t\}_t=1^T \), since the decision rules are independent of the \( \{\ln k_t\}_t=1^T \).

It is also not necessary to assume that the household knows future values of the wage and nonlabor income process when making period \( t \) under our assumption of no borrowing or saving. Because of this assumption, labor supply decisions were made as if the model was static in nature. Due to this, future wages or nonlabor income receipts cannot influence current period decisions, and all decisions are consistent with various assumptions about how households form expectations, including perfect foresight, rational expectations, and adaptive expectations. Given the ability to save and borrow, current period labor supply decisions will be affected by future wage expectations, and the model would have to take a stand on how these were formed. Moreover, given the limited amount of data to which we have access, incorporation of a complicated expectation process in the model can become a daunting task.

### 4 Econometric Issues

We now discuss the manner in which we have taken the model developed above to the data, and we provide some details concerning identification issues. A key element of taking any model to data is the manner in which heterogeneity is included, if allowed to be present at all, and the nature of the randomness in the model that is a requirement for there to be a well-posed estimation problem.

#### 4.1 Econometric Specification

We begin by discussing some of our assumptions regarding the model specification. As noted above, we allow the production function parameters to vary with the age of the child, but do not allow any further heterogeneity in the function. That is, we assume that all families possess the same child production technology.\(^5\) In order to economize on parameters, we assume that the parameters \( \{\delta_{1t}, \delta_{2t}, \delta_{3t}, \delta_{4t}\}_t=1^T \) vary as follows:

\[
\delta_{jt} = \exp(\gamma_{j0} + \gamma_{j1}t), \quad j = 1, ..., 4; \quad t = 1, ..., T.
\]

Similarly, we assume that TFP, \( R_t \), is given by

\[
R_t = \exp(\gamma_{00} + \gamma_{01}t + \varepsilon_t),
\]

where \( \varepsilon_t \) is i.i.d. \( N(0, \sigma_R^2) \). Thus the production process is characterized by 11 parameters, \( \{\gamma_{j0}, \gamma_{j1}\}_j=1^5, \sigma_R \).

\(^5\)Below, we do consider another version of the model where the productivity of each parent’s time in producing child quality is affected by the parent’s level of education.
Household preferences are assumed to be fixed over time, however, we do allow heterogeneity in utility function preferences across households. We specify the distribution of preferences parameters as $G(\alpha; \theta)$, where $G$ is a parametric distribution function characterized by the finite-dimensional parameter vector $\theta$, and with the three dimensional vector $\alpha = (\alpha_1 \alpha_2 \alpha_3)'$ defined such that $0 < \alpha_1 + \alpha_2 + \alpha_3 < 1$, $\alpha_j > 0$, $j = 1, 2, 3$. A household objective is determined by a draw from the distribution $G$, with $\alpha_4 = 1 - \alpha_1 - \alpha_2 - \alpha_3$.

In terms of the specific functional form for $G$, we assume that it is generated as follows. The $3 \times 1$ vector $\nu$ is normally distributed with

$$\nu \sim N(\mu_\alpha, \Sigma_\alpha),$$

where $\mu_\alpha$ is a $3 \times 1$ vector and $\Sigma_\alpha$ is a $3 \times 3$ covariance matrix of full rank. Define $D = 1 + \sum_{j=1}^{3} \exp(\nu_j)$. Then a draw $\nu$ from the trivariate normal is mapped into $\alpha$ as

$$\begin{align*}
\alpha_1 &= D^{-1} \exp(\nu_1) \\
\alpha_2 &= D^{-1} \exp(\nu_2) \\
\alpha_3 &= D^{-1} \exp(\nu_3) \\
\alpha_4 &= D^{-1}
\end{align*}$$

Then the c.d.f. of $\alpha$ is given by

$$G(\alpha) = \int \int \int \chi[D^{-1} \exp(\nu_1) \leq \alpha_1] \chi[D^{-1} \exp(\nu_2) \leq \alpha_2] \chi[D^{-1} \exp(\nu_3) \leq \alpha_3] dF(\nu|\mu_\alpha, \Sigma_\alpha).$$

Thus the population distribution of $\alpha$ is characterized in terms of the parameter vectors $\mu_\alpha$ and $\text{vec}(\Sigma_\alpha)$, the vectorization of the nonredundant elements in $\Sigma_\alpha$.\footnote{When estimating $\Sigma_\alpha$, it is necessary to choose a parameterization that ensures that any estimate $\hat{\Sigma}_\alpha$ is symmetric, positive definite. The most straightforward way of doing so is to use the Cholesky decomposition of $\Sigma_\alpha$. There are 10 parameters to estimate, with $\mu_\alpha = [\mu_{\alpha,1}, \mu_{\alpha,2}, \mu_{\alpha,3}]$ and

$$\Sigma_\alpha = \begin{bmatrix}
    c_1^2 & c_{12} & c_{13} \\
    c_{12} & c_{22} & c_{23} \\
    c_{13} & c_{23} & c_3^2
\end{bmatrix},$$

where $c_1, c_2, c_3$ are constrained to be positive using the exponential function, and $c_{12}, c_{13}, c_{23}$ are constrained to be between $-1$ and 1 using the hyperbolic tangent function. These constraints ensure that the diagonal elements are strictly positive and the matrix has proper scaling for the numerical implementation of the Cholesky decomposition. (see Pinheiro and Bates, 1996).}

The final piece of the specification of the model involves the evolution of the wage and nonlabor income processes. We assume that stochastic terms associated with these processes have an $AR(1)$ structure. In particular, we specify

$$\begin{bmatrix}
    \ln w_{1,t} \\
    \ln w_{2,t}
\end{bmatrix} = \begin{bmatrix}
    \mu_{1,t} \\
    \mu_{2,t}
\end{bmatrix} + \begin{bmatrix}
    \omega_{1,t} \\
    \omega_{2,t}
\end{bmatrix},$$

where $\omega_{1,t}$ and $\omega_{2,t}$ are $N(0, \Sigma_\omega)$.
where
\[
\begin{bmatrix}
\omega_{1,t} \\
\omega_{2,t}
\end{bmatrix} = \frac{\rho_1}{\rho_2} \begin{bmatrix} \omega_{1,t-1} \\ \omega_{2,t-1} \end{bmatrix} + \frac{\epsilon_{1,t}}{\epsilon_{2,t}},
\]
and where
\[
\begin{bmatrix}
\epsilon_{1,t} \\
\epsilon_{2,t}
\end{bmatrix} \sim i.i.d. N\left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} \right), t = 2, 3, ...
\]
The terms \(\mu_{1,t}\) and \(\mu_{2,t}\) are the means of the log wage draws of spouse 1 and spouse 2 at time \(t\). We can think of these processes as beginning at \(t = 1\), when the child is born. The initial draw of \((\omega_{1,1}, \omega_{2,1})'\) taken from the steady state marginal distribution, so that
\[
\begin{bmatrix}
\omega_{1,1} \\
\omega_{2,1}
\end{bmatrix} \sim i.i.d. N\left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 - \rho_1 \sigma_{11} & (1 - \rho_1)(1 - \rho_2)^{-1}\sigma_{12} \\ (1 - \rho_1)(1 - \rho_2)^{-1}\sigma_{12} & 1 - \rho_2 \sigma_{22} \end{bmatrix} \right). \tag{10}
\]
In terms of the nonlabor income process, there are a a large number of households with no nonlabor income in a given period, so we posit this process to be a truncated version of an AR(1) latent variable process in levels (instead of logs). In particular, let
\[
I^*_t = \mu_{3,t} + \omega_{3,t},
\]
be the latent nonlabor income in period \(t\), with a mean given by \(\mu_{3,t}\) and with \(\omega_{3,t} = \rho_3 \omega_{3,t-1} + \epsilon_{3,t}\), where \(\epsilon_{3,t} \sim i.i.d. N(0, \sigma_{33})\). This process begins when the child is one year of age as well, with \(\omega_{3,1} \sim i.i.d. N(0, (1 - \rho_3)^{-2}\sigma_{33})\). The actual nonlabor income process is given by
\[
I_t = \max(0, I^*_t), \text{ for all } t.
\]

### 4.2 Identification

In this discussion we indicate the manner in which the behavioral parameters characterizing the model can be recovered from the data at our disposal in a reasonably straightforward manner given the requisite data. The estimator we actually implement has several advantages (both theoretical and practical), to be detailed below, over those discussed in this section. But in a reasonably complex dynamic model it is useful to develop some intuition as to the key sources of identifying information under our modeling assumptions.

We first consider the estimation of production technology parameters. As noted above, there are 11 parameters to be determined in our parameterization. Estimation of all parameters is hampered by the fact that the PSID-CDS data are not collected on a yearly basis. In our case, data were collected in 1997 and 2002. The timing convention we use is that the child quality measures observed in year \(t\) are contemporaneous with the child quality measure(s) that are available in the same survey year. Thus, in 2002 for example, for a child of age \(t\) in that year, the measured child quality and inputs in 2002 map into the child quality measure of 2003. Unfortunately, that information is unavailable to us.
We will carry out our discussion assuming that it is, and then we will consider how our estimation strategy can be adjusted to deal with the sizable amount of missing data.

If we did have information available in successive periods, then recovery of the $\delta$ parameters would be completely straightforward. Say that we had child test score data for both the years 2003 and 2002, as well as the investment information concurrent with the 2002 child quality measure. Using the specification given in (9) and substituting in the functional form of the age-dependent production function parameters, we have

\[
\ln k_{i,t,t+1} = \ln R_{i,t} + \delta_1 t_i \ln \tau_{1,i,t} + \delta_2 t_i \ln \tau_{2,i,t} + \delta_3 t_i \ln \varepsilon_{i,t} + \delta_4 t_i \ln k_{i,t} = \gamma_0 + \gamma_0 t_i + \exp(\gamma_1 t_i) \ln \tau_{1,i,t} + \exp(\gamma_2 t_i) \ln \tau_{2,i,t} + \exp(\gamma_3 t_i) \ln \varepsilon_{i,t} + \exp(\gamma_4 t_i) \ln k_{i,t} + \varepsilon_{i,t} \equiv X(t_i, \tau_{1,i,t}, \tau_{2,i,t}, \varepsilon_{i,t}, k_{i,t}; \gamma) + \varepsilon_{i,t} \quad i = 1, \ldots, N.
\]

where $i$ denotes the household, $t_i$ is the age of the child in household $i$ at the time of the survey, and $\varepsilon_{i,t} \overset{i.i.d.}{\sim} N(0, \sigma^2_R)$. Then subject to the usual full rank conditions, the nonlinear least squares estimator

\[
\hat{\gamma}_{NLS} = \arg\min_\gamma \sum_{i=1}^N (\ln k_{i,t,t+1} - K(t_i, \tau_{1,i,t}, \tau_{2,i,t}, \varepsilon_{i,t}, k_{i,t}; \gamma))^2.
\]

is a consistent estimator of $\gamma$, that is, $\text{plim} \hat{\gamma}_{NLS} = \gamma$. The “full rank conditions” in this case primarily mean that not all households choose the same values of investments, which is trivially satisfied in the data, and that not all households have a child of the same age. Since the parameters characterizing the production function are a linear function of age, it is enough that the sample contain children of two different ages for the full rank condition to be satisfied. A consistent estimator for the final production process parameter is given by

\[
\hat{\delta}_R = N^{-1} \sum_{i=1}^N (\ln k_{i,t,t+1} - K(t_i, \tau_{1,i,t}, \tau_{2,i,t}, \varepsilon_{i,t}, k_{i,t}; \hat{\gamma}_{NLS}))^2.
\]

In terms of the identification of the wage and income processes, the PSID contains multiple observations of wages (for those parents who work) and non-labor income over several periods across the child development process, including periods for which we do not have child quality data from the CDS. In estimating the nonlabor income process, which is assumed to be independent of the wage processes, there are no issues. For illustration, say that we have access to only two observations (in consecutive periods) on the nonlabor income process for each household in a random sample of $N$ households. Given the stationarity of the stochastic process, the first time that household $i$ is sampled (when the child

\footnote{If the production parameters were a quadratic function of child age, then the full rank condition would require that the sample contain children of at least three different ages, and so on.}

\footnote{Note that if we have observations of child quality and inputs for children of every age, we could consistently estimate the production function parameters $\delta_{1z}, \delta_{2z}, \delta_{3z}, \delta_{qt}$ directly using an OLS regression of child quality on inputs for each age. The absence of this information for every child age necessitates some functional form restriction on how these parameters vary by child age.}
is of age $t_i$, say), a drawn is taken from the marginal distribution of the disturbance term, which is $N(0, (1 - \rho_3)^{-2}\sigma_{33})$. We parameterize the time-varying average of the (latent) nonlabor income process as

$$
\mu_{3, t_i, i} = X_{3, t_i, i}\zeta_3,
$$

where $X_{3, t_i, i}$ is a $1 \times K_3$ vector of characteristics of household $i$ when the child is $t_i$ years of age, which includes a ‘1’ as the first element, and $\zeta_3$ is a $K_3 \times 1$ parameter vector. Then given $\{I_{t_i, i}, I_{t_i+1, i}, X_{3, t_i, i}, X_{3, t_i+1, i}\}_{i=1}^{N}$, using the bivariate normality assumptions it is straightforward to construct a maximum likelihood estimator of $\zeta_3, \sigma_{33},$ and $\rho_3$. Conditions for identification are full column rank of the $X_3$ matrix in the population and not all observations censored at 0. Under our model specification, the limiting probability of this event is 0.

While we have access to wage observations for multiple periods, wage observations are nonrandomly missing due to the significant number of corner solutions associated with labor supply choice. When one or both parents is not in the labor market, we do not observe the wage. Under our model specification, we can “correct” our estimator of model parameters for the nonrandomly missing data using our modeling assumptions. Then both the wage processes and the parameters characterizing preferences must be simultaneously estimated.

Each household, at every point in time, makes five decisions. The time-invariant household utility function is characterized in terms of 3 free parameters, since $\sum_{j=1}^{4} \alpha_j = 1$. Other parameters characterizing preferences that remain to be considered are the discount rate, $\beta$, and the parameter $\psi$ that, in conjunction with $\alpha_4$, determines the terminal valuation of child quality at this stage of the development process. Both the $\beta$ and $\psi$ parameters are assumed to be homogenous in the population.

If we condition on values of $\beta$ and $\psi$, then the marginal distribution of $\alpha$ is nonparametrically identified given only one period of observed input demands, labor supplies, and total household income per household. To see this, simply note that by the structure of the production and utility functions, input demands are positive in every period of the production process. The conditional (on labor supply and household income) demand functions are given by (5), (6), and (7). Given $\beta$ and $\psi$, this system of equations can be inverted to yield unique values of $\alpha_1, \alpha_2,$ and $\alpha_3$ for each household in the sample. Then the empirical distribution of these values is the nonparametric maximum likelihood estimator of $G(\alpha)$, with the estimator conditional on estimates of the production function parameters and values of $\beta$ and $\psi$.

What remains is the determination of the wage process parameters and $\beta$ and $\psi$. Under our assumption of homogeneity of these parameters, we could use any household in which both parents work, in conjunction with our estimation of $\alpha_1, \alpha_2,$ and $\alpha_3$ for that household to determine values of $\beta$ and $\psi$. This is accomplished by using the labor supply decisions evaluated at the actual hours choices and wage offers to back these two values out. By the structure of the model, if we observed the wages for all parents in the sample, estimation
of the wage processes would not be necessary, rather, we could simply condition on wage realizations. Wages would be observed for all parents only in the case in which they are not at corner solutions, and, as noted above, a substantial number of parents are (particularly mothers). It is for this reason alone that we must estimate the wage processes.

While our identification argument regarding the distribution of preferences is appealing at some level, it does present conceptual problems. Most obvious is the multiplicity of estimates of $G$, $\beta$, and $\psi$ that are available. We have access to two observations on input demands, household labor supplies, and total income for all households in the subsample that we utilize. Conditional on $\beta$ and $\psi$, this means that two separate estimates of the household $\alpha$ vector are available. There is no possibility that these “backed out” values of the $\alpha$ vector will be the same at both CDS dates. Of course, one could rationalize input choices at several points in time by adding classical measurement error to the conditional demand equations, but by doing so, there is no possibility of using (11) to consistently estimate the production function parameters (due to the errors in variables problem).

The main objective of our discussion of identification has been to illustrate that there is a substantial amount of information regarding preferences and technology available in the data available to us, even if we have ignored some of the pressing issues of missing data in carrying out our discussion. There are two problems with missing data in our sample. One is the gaps in the data that make it impossible to use successive observations on child quality along with input demand to estimate the production parameters directly, as in (11). We observe what we consider to be an accurate measurement of child quality in 1997, along with input utilization in that year, but don’t observe the outcome of these choices until 2002, five years later. In between these dates, input decisions have been made and levels of child quality have been determined; these input decisions depend on wage and nonlabor income draws in intervening years, possible shocks to productivity, etc. The only way to fill in these holes is to simulate the path of all of the state variables over this period using the model structure.

The other type of missing data problem faced involves nonrandom missing data on wages. As mentioned above, wages have to be generated for the years between the observed child quality levels in any event. But in addition, when or both parents supply no time to the market, the wage offer is not observed for the period. This type of selection is particularly troublesome when preferences are treated as random in the population. In this case, seeing a parent not supply time to the market is consistent with (1) that parent’s wage offer being low, (2) the household utility function weight on that parent’s leisure being high, or (3) both (1) and (2). In order to “extrapolate” preferences and wages when a large number of households have at least one parent out of the labor force requires parametric

---

9While wage and nonlabor income is gathered at every interview date, PSID interviews are conducted every 2 years at this point, and our decision periods correspond to single years. For this reason, even the wage and nonlabor income process has to be simulated between the times when the child quality measures are available.
assumptions on both processes.

4.3 Estimator

The family data we have available consist of a sample households with observed characteristics $X$ and includes children interviewed at various ages, where child age is indexed $t$. The observed household characteristics include parental variables, such as education and ages of parents at the birth of the child. We observe for each mother and father in the household, hours worked, hours spent with children, and repeated measures of child quality: $h_{1t}, \tau_{1t}, h_{2t}, \tau_{2t}, k_t$. In addition, we observe, (accepted) wages for both parents and total household income, as described in the Data Section. Although data on some child specific expenditures is available, we assume expenditures on children are unobserved.

The family data we have can utilize to estimate primitive parameters is primarily taken from the CDS administered in 1997 and 2002 and from the standard questions concerning household income, wages, and labor supply asked of all adult household members in the PSID. In particular, in each household $i$ we observe

$$\{t^j_{1i}, X^j_i, h^j_{1,i,t_i}, w^j_{1,i,t_i}, h^j_{2,i,t_i}, w^j_{2,i,t_i}, h^j_{1,i,t_i}, w^j_{1,i,t_i}, h^j_{2,i,t_i}, w^j_{2,i,t_i}, \tau^j_{1,i,t_i}, \tau^j_{2,i,t_i}, k^j_{it_i}, e^j_{it_i}, \}i=1,...,N; j=1,2.$$  

The indexing convention is that the superscript $j = 1$ indicates the value of the characteristic in the first wave of the CDS, 1997, and $j = 2$ indicates the value in the second wave, 2002. For example, $t^1_{15}$ is the age of the child in the 45th sample household in 1997, $w^2_{1,98,14}$ is the wage of parent 1 (the mother) in sample household 98 observed in the year 2002 when the child is age 14, etc. $X^j_i$ is the vector of household characteristics observed in period $j$, including education levels and age.

The estimator is based on simulation. We first define a set of sample moments, which summarize the relationships in the sample at each survey date and across survey dates. Let the vector of sample characteristics in our sample of size $N$ be denoted by $M_N$. For each household $i$, we generate a set of $NS$ sample paths over the development period in the following manner. The empirical process begins in 1997 when the child is $t^1_{1i}$ years of age in household $i$ and his or her “quality” level, as measured by the test score we use, is $k^1_{it_i}$. Given the parent’s characteristics at the sample date, $X^1_i$, we draw from the stationary distribution of shocks to wages and nonlabor incomes, and in conjunction with the “mean shifters” in $X^1_i$, we determine the initial wage and income draws. We also draw from the distribution of household preferences, $G(\alpha)$, and this draw stays with the household over the entire sample path. Since the production technology is assumed to be deterministic, up to an additive shock that does not appear in the decision rules for labor supply and child investment, we can then solve the household’s decision problem in 1997, yielding values of labor supply and investment decisions.

In 1998 we have no observations on child quality or investments, yet using the model structure we can simulate these values. Using last period’s wage and nonlabor income draws, we draw new shocks to these processes and generate wage and nonlabor income
realizations for 1998. Child quality in 1998 is determined from child quality in 1997, the inputs used in 1997, and a TFP shock. Household decisions for 1998 are then determined, and we repeat this process through the year 2002. Thus in 2002 we will have generated a sequence of wage and nonlabor income draws from 1997 through 2002, child quality realizations in all years, and sequences of all of the dependent variables in the model.

For the same household \(i\), this process is repeated \(NS\) times (for each of the replications for household \(i\) a new draw from the preference distribution \(G\) is made), so that in the end we have \(NS \times N\) sample paths. Using the simulated data set, we then compute the analogous simulated moments to those determined from the actual data sample. The simulated moments generated are determined by \(\Omega\), the vector of all primitive parameters that characterize the model, and the actual (pseudo) random number draws made in generating the sample paths. Denote the Simulated sample characteristics generated under the parameter vector \(\Omega\) by \(\hat{M}_{NS}(\Omega)\). The Method of Simulated Moments (MSM) estimator of \(\Omega\) is then given by

\[
\hat{\Omega}_{NS,N,W} = \arg \min_{\Omega} (M_N - \hat{M}_{NS,N}(\Omega))'W(M_N - \hat{M}_{NS,N}(\Omega)),
\]

where \(W\) is a symmetric, positive-definite weighting matrix. Now by the Law of Large Numbers (LLN), \(\text{plim}_{N \to \infty} M_N = M\), the population vector of values of the characteristics comprising \(M_N\). Since we do not have access to unbiased simulators, for consistency of the MSM estimator, we require that \(NS\) also grow indefinitely large. Let the true value of the parameter vector characterizing the model be denoted by \(\Omega_0\). Then

\[
\text{plim}_{NS \to \infty} \hat{M}_{NS,N}(\Omega_0) = M_N(\Omega_0).
\]

Given identification and these conditions,

\[
\text{plim}_{N \to \infty, NS \to \infty} \hat{\Omega}_{NS,N,W} = \Omega \text{ for all } W.
\]

The moments we use include the average and standard deviation of child quality at each child age,\(^{10}\) the average and standard deviation of hours of work for mothers and fathers at each child age, and the average and standard deviation of child care hours for mothers and fathers at each child age. In addition, we use the average and standard deviation of accepted wages, auto-correlation in wages over time, and correlation in wages across parents. We also compute a number of contemporaneous and lagged correlations between the observed

---

\(^{10}\)We assume child quality is measured without error using a specific measure, described in the data section. Extending this model to allow for multiple measures of child quality and measurement error would not change the fundamental nature of the latent child quality process. For each child, we have two observations of child quality at different ages. In order to match the ceiling effect present in the actual measured test score, we impose this ceiling on our latent child quality process. We assume for any period after the initial period, the observed measure \(k_{t+1}^*\) is constructed as \(k_{t+1}^* = \max\{k_{t+1}, \bar{k}\}\), where \(\bar{k}\) is the particular test score ceiling for our measure. Since this ceiling only binds for older children, we do not adjust our initial observed value of child quality, which is measured only for younger children, and assume that the initial observed value in the data is the latent value. For the second observation of child quality, when the children are 5 years older, children may have a latent child quality in excess of the test score ceiling. In practice, only a handful of children achieved the maximum test score.
labor supply, time with children, child quality, wages, and income. It is important to note that while we do not observe child inputs, labor supply, wages, and income in the same periods, our Method of Moments framework allows us to tractably combine moments from various points in the child development process into a single estimator.

4.4 Data

We utilize data from the Panel Study of Income Dynamics (PSID) and the first two waves of the Child Development Supplements (CDS-I and CD-II). The PSID is a longitudinal study that began in 1968 with a nationally representative sample of about 5,000 American families, with an oversample of black and low-income families. In 1997, the PSID began collecting data on a random sample of the PSID families that have children under the age of 13 in a Child Development Supplement (CDS-I). Data were collected for up to two children per family. The CDS collects information on child development and family dynamics, including parent-child relationships, home environment, indicators of children’s health, cognitive achievements, social-emotional development and time use, among other variables. The entire CDS sample size in 1997 is approximately 3,500 children residing in 2,400 households. A follow-up study with these children and families was conducted in 2002-03 (CDS-II). These children were between the ages of 5-18 in 2003. No new children were added to the study.

Starting in 1997, children’s time diaries were collected along with detailed assessments of children’s cognitive development. Children’s cognitive skills are conceived broadly to include language skills, literacy and problem-solving skills and are measured with the Woodcock Johnson Achievement Test-Revised (Woodcock and Johnson, 1989). In 1997, children aged 3-5 received the Letter-Word Identification and Applied Problem sub-tests. Children aged 6 and above received Letter-Word and Passage Comprehension sub-tests as well as Applied Problems and Calculation sub-tests. In the 2002-03 (second) wave, these tests were re-administered, with the exception of the the Calculation sub-test. Given the wide range of ages to which the Letter-Word tests was administered, we use this test as our measure of child development. We use the raw scores on this exam rather than the age standardized exam. The test contains 57 items, and the range in the raw scores is 0 to 57.

For each household, we observe hours, hourly wages, and non-labor income during the 1997, 1999, and 2000 surveys reported for the previous year (1996, 1998 and 2000). The monetary values have been deflated and are all in 2001 USD. All of this wage and income

---

11 An important addition in CDS-II is a set of detailed questions about childhood consumption. These questions concern the amount of money the family and others outside of the family unit paid for various types of the consumption of and the investment in the target child over the previous 12 months. These items include tuition, tutoring programs, lessons, and sports. There is also a set of questions concerning the expenditures the family or others outside the family unit make for all the children within the household. These include toys or presents, vacations, school supplies, food and clothes or shoes. There is also information on health care and childcare and related care expenses.
information is used in estimating the model, even though child investment and achievement information is only available in 1997 and 2002-3.

We are interested in households in which both biological parents were present in both waves. Most of the variables we use in the model are collected from the primary caregiver of a child and for the head and wife of the household. Therefore, our sample consists of children who (1) have valid test scores in both waves of the Child Development Supplements, (2) have the mother as the primary caregiver of the target child (95% of the entire CDS sample), and (3) are sons or daughters of the head of the household (94 percent of the original CDS sample). In addition we drop observations with missing information on mother’s or father’s time with the child, those reporting (real) hourly wages above $150 per hour in any period, and those with weekly non-labor income above $1,000 in any period. Our total sample consists of 491 children.

Table 1 reports descriptive statistics of our selected sample. At the initial 1997 wave of the sample, the children are aged 3-12, with the average child age of 7.5 years. In 1997, the parent’s average age is 38 for fathers and 36 for mothers. Average years of schooling is similar across parents at about 13 years.

Father’s time is 5.6 hours a week and in 1997 and 4.2 in 2002. Mothers time is about four times as great in 1997 and three times as great in 2002. The total amount of parents spend with the child did not change much between the two waves. Figure 1 shows the pattern of mothers and fathers time with the child and their hours of work. On average, mothers spend more time with the child than do father’s, though mother’s time investment shows a steep decline with child age while the time input of the father is essentially constant.

Table 2 provides descriptive statistics on labor supply, wages, and income. Average hours worked by fathers is over 50 percent larger than for mothers. Average father’s wages are about twice as large as average mother’s wages. Average non-labor income is about $67 in 1996 and $99 in 2000. About 30 percent of households have no nonlabor income in any given year.

Figure 3 presents average letter word score by the child’s age. The average score is about 5 for 3 year old children and rises to about 50 for 14 year old children. Since the scores are not age-normed, there is a substantial degree of growth, particularly at early ages. The function flattens out due to ceiling effects at the upper ages in the development period we examine.

5 Estimates

This section begins with a discussion of the estimates of the behavioral model. We then provide an assessment of within-sample model fit. We conclude the section with a number of “local” comparitive statics exercises performed using the point estimates we obtained.
5.1 Household Preference Parameters

Periods are in years and the assumed planning horizon is age 16, $T = 16$. As discussed above, parents may continue to make child investments after this point but we do not explicitly model these investments and rely on our terminal period specification to capture the utility value of them. The annualized discount rate for the household is fixed at $\beta = 0.95$.

Table 3 presents the parameter estimates of the behavioral model in which it is assumed that parental preferences are time (i.e., child age) invariant, though we allow for population in preferences. We estimate two versions of the model. The first version restricts the productivity of mother’s and father’s time to be independent of parental education. The second version allows mother’s productivity time to depend on mother’s years of schooling and father’s time productivity to depend on father’s years of schooling.

The transformed parameters of the distribution are difficult to interpret, so instead we present their mean values, standard deviations, and the three correlation coefficients which taken together describe the (estimated) first two moments of the population distribution of preferences under our parametric assumption on this multivariate distribution. In terms of the average household preference weights, under the second specification, there is a substantially higher weight placed on father’s leisure than the mother’s (0.22 versus 0.10). The average weight applied to household consumption is 0.34, and the weight attached to child “quality” is 0.34. The parameter estimates are mainly attributable to the fact that women spend “too little” time in leisure given the wage differences between fathers and mothers and the declining gender differences in the productivity of time investments with the age of the child to be consistent with equal average household preference weights.

Turning next to what the estimates imply about the dispersion of preferences, the parameter estimates for Model 2 yield standard deviations for the four preference parameters of between 0.024 and 0.055. These estimates imply a relatively concentrated distribution of preferences with the mean of the marginal distribution about 4-6 times larger than the standard deviation. We also estimate a strong correlation in leisure preferences across spouses, with the correlation in $\alpha_1$ and $\alpha_2$ estimated at 0.95. This may reflect the extent of assortative matching in the marriage market with regard to preferences in leisure. In contrast, we estimate a negative correlation between preferences for parents’ leisure and household consumption, with the correlation between mother’s leisure $\alpha_1$ and consumption $\alpha_3$ estimated at $-0.69$ and the correlation between father’s leisure $\alpha_2$ and consumption $\alpha_3$ at $-0.88$.

The scaling factor applied to the terminal valuation of child quality ($\psi$) is estimated to be approximately 8. These estimates imply that the household highly values child quality both in terms of flows and its terminal value, where the termination date here is considered the end of this particular (sub) development process. The high value of $\psi$ could indicate that $k_T$ serves as an “important” initial condition for developmental processes that begin in the later teen years.
5.2 Child Quality Technology Parameters

We next discuss the estimated production process for child quality, which we allow to change in a “smooth” way with the age of the child, as described above. Table 4 provides the parameter estimates, which differ in the restrictions placed on the production technology. Our baseline specification was given in section 4.1. Our second, slightly more general, specification allows parental education to directly affect the productivity of each parent’s time inputs, so that

\[ \delta_{jt} = \exp(\gamma_{j0} + \gamma_{j1}t + \gamma_{j2}s_j), \ j = 1, 2, \]

where \( s_j \) is parent \( j \)'s level of education (years of schooling). The parameter \( \gamma_{k2} \) reflects the “return to schooling” in producing child quality through parental time.

The TFP profile is quite flat with an estimated age slope of 0.0003 or 0.0012, depending on the specification. We estimate that there is little productivity change as the child ages that is not “explained” by our four input model. The standard deviation of the child shock is also quite low at 0.0031 and 0.0008, depending on the specification.

The estimates of the impacts of parental education on the productivity of their time inputs, reported in column 2 of Table 4, indicates that there is no significant impact. We estimate a small, positive “return” to the mother’s educational attainment on the productivity of her time inputs, but not for father’s time. Of course, though we have introduced these parental characteristics into the production function, the manner in which we have done so is largely arbitrary. It may also be the case that these effects are nonlinear in a more substantive manner, in that the time inputs of parents with extremely low levels of formal schooling or cognitive ability may have no, or even negative, impacts on child quality improvements, while the time inputs of parents with moderate or high levels of cognitive abilities may have roughly similar positive impacts on child quality. Only further empirical exploration, including incorporating better measures of parents’ cognitive skills, can help clarify this.

Figures 4 and Figures 5 graph the estimated technology parameters from the unrestricted model 2 from Table 4. Figure 4 shows that child quality is quite persistent given the estimated high productivity of the previous level of child quality in producing the next period’s level (\( \delta_{4t} \)). There is more inertia in the child quality process as the child reaches the upper age limit of our analysis. While we believe that this may reflect a real characteristic of the development process, there is no doubt that it also reflects the ceiling effect produced by our fixed interval measure of child quality that is not age-normed.

We see that for the two time “flow” inputs into the dynamic production process (mother’s time and father’s time), the marginal productivity of the input is declining in child age. Interestingly, we estimate that father’s time is more productive than mother’s time at every child age. Based on these estimates, the model leads us to the conclusion that mothers spend more time with the child than does the father not due to their absolute advantage in producing child quality, but rather because their opportunity costs of spend-
ing time with the child are lower than the father’s, due to lower wage offers, and because the household values the leisure of the mother less than that of the father.

The declining productivity of parental time makes some intuitive sense given our model specification. Once children attain the age of 5 or 6, they typically leave the home for significant periods of time each day for formal schooling activities. This amounts to a large, probably discontinuous, shift in the child quality process. Even after the onset of formal schooling activities, the child may increasingly be subject to inputs, both good and bad, from teachers and other students, that supplant the interactions the child had previously with the parents. From the point of view of parental inputs, their input decisions have increasingly small effects on child outcomes as they are “crowded out” by these others.\textsuperscript{12}

While one could argue about the form of the dependence of the production process on the age of the child, it is reasonable to think that the impact of parental inputs is, in general, declining.\textsuperscript{13}

5.3 Wage and Non-Labor Income Process Parameters

Wage offers are unobserved with a distribution that depends on observed household characteristics, including mother’s and father’s age and mother’s and father’s education. We parameterize the mean of the distribution of log wage offers as

\[
\ln \mu_{jt} = \mu_{0j} + \mu_{1j}s_j + \mu_{2j}age_{jt} + \mu_{3j}age_{jt}^2 + \epsilon_{jt}
\]

where \(s_j\) is the years of schooling for parent \(j\), which is assumed to be constant over the child’s lifetime, and \(age_{jt}\) is parent \(j\)’s at the age of the child \(t\). Variation in parental age across children of the same age is produced by variation in the timing of births.

Table 5 displays the estimated wage and income parameter estimates. The labor market return to parental schooling is estimated to be quite high. Each year of schooling for mother’s increases her mean log wage offer by around 15 percent, and for fathers, by around 16 percent. There is a less pronounced age earnings profile, although we do estimate that wage offers have the standard concave increasing relationship with age. The relatively slow increase in earnings with age that we estimate is likely due to the limited age range (only over the 13 year child development process) at which we estimate the wage offer distribution.

We estimate that the innovation process for wage offers for mothers and fathers have substantial negative auto-correlation, around nearly \(-0.5\) to \(-0.6\). We estimate a high

\textsuperscript{12}Of course, the parents continue to have a major impact on the factor inputs through their choice of the child’s schooling environment. Liu et al. (2010) focus on this important aspect of child investment decisions.

\textsuperscript{13}Several researchers have pointed to the importance of the phenomenon of self-investment as the child ages (e.g., Monfardini (2008)). The persistence we note in child quality process as the child ages may be due to the child, and others, supplying inputs that are unobserved and persistent.
correlation in mother’s and father’s wage offer innovations of around 0.75, which probably reflects both common labor market effects and assortative mating.

Non-labor income for the household is partially observed for some periods but unobserved for others. Unlike the wage offer distribution, we do not face the problem of endogenous non-labor income since we have an observation of non-labor income for each household. The mean of the latent non-labor income is given $\mu_0^0$ and does not depend on parental characteristics. For the latent non-labor income process, we estimate a high standard deviation in the innovation and a high degree of (positive) autocorrelation.

5.4 Within Sample Fit

Table 7 displays the sample fit of our simulated model to some features of the wage and income data. In general, we fit the mean and standard deviation of accepted wages and non-labor income well.

The estimated model is able to fit basic patterns in extensive labor supply (employment) and intensive labor supply for both patterns. Figure 6 presents sample fit to the observed employment probabilities. Employment is defined as working any hours during the week of the survey (note any hours during the year). The model is able to replicate the high employment rates for fathers relative to mothers. In addition, the model reproduces the increasing labor market participation of women as their children age. Figure 7 provides evidence on the sample fit of average hours worked across child ages. As with the employment probabilities, the estimated model fits the higher average labor supply of fathers and the increasing average number of hours mothers worked.

Figures 8 provides evidence of the sample fit of the estimated model to observed time the parents spend with children. Generally the model fits the lower average time fathers spend with their child than mothers. In addition, the model replicates the declining time mothers spend with children as their children age. Figure 9 provides evidence on the sample fit of the estimated child quality process to the observed child quality measure. The estimated model fits the concave increasing average level of child quality as the child ages.

5.5 Comparative Statics Exercise: Wage Changes

Next, we probe the predictions of the model at the estimated parameters by performing a number of comparative statics exercises. A potentially interesting question is how changes in wages earned by fathers or mothers impact household decisions and outcomes. The elasticities we compute are “finite” ones, with the change in each respective parent’s wage offer equal to 10 percent. For each period $t$, we increase the wage draw for all mothers or all fathers by 10 percent. We then re-simulate the model for each sample household from the

\[14\] We found that non-labor income is in general unrelated to household characteristics of parents, education and age. Hence we model the latent distribution not to depend on these characteristics.
initial child interview in 1997 to the terminal period at age 16, and re-calculate simulated moments over the data sample and our simulation draws from the estimated distributions of preferences, wages, non-labor income, and child quality shocks.

Table 8 computes elasticities for mean child quality (at the terminal age of 16), mean hours worked by both parents, mean time with the children of both parents, child expenditures, and parental consumption. The first column provides the baseline levels at the original wage draws. The mean level of child quality at age 16 at baseline is 53.7. Age 16 child quality is the terminal value of child quality produced by parental inputs at the last period of development, age 16, and the current level of child quality at that age.\(^{15}\) We focus on this statistic since the terminal value of child quality seems to be the most policy relevant from the perspective of producing adult outcomes. This terminal level of child quality is the initial condition for the child as she transitions to adulthood. The remaining statistics are summary measures over all child ages. Mean hours that the mother works under the baseline of 30.96 hours is the simulation estimate of the average labor supply for all sample mothers who have children that range in age from 3 to 16. The elasticity estimates then indicate that a 10 percent increase in the women’s wage offer would increase mean labor supply for mothers by 13.79 percent from baseline.

Our main interest is in the difference in the responsiveness of each of the variables to wage changes by gender. Since our main concern is child quality, we begin with the first row of Table 8. We note that increasing the mother’s wage has a positive impact on child quality. A 10 percent increase in the mother’s wage increases child quality by 0.08 percent. The net effect of the change depends on the preference valuations of consumption, the parents’ leisure, and child quality by the household. As we see from the fourth row, the mother reduces her time with the child by almost 5.3 percent as a result of a wage increase. This effect alone would reduce child quality. However, the higher wage for mothers induces higher labor supply, thus labor income for the household increases. With this added labor income, the household optimally increases its child expenditures by almost 2.89 percent. This effect increases child quality. In addition, the increase in the mother’s wage offer changes the relative prices of mothers and fathers time and the father works less and spends more time with his child. Given that fathers have similar time productivity with their children, fathers are an effective substitute for the lost mothers time.

In the next policy experiment (column 3 of Table 8, we increase the father’s wage by 10 percent. An increase in the wage of the father has a stronger effect on final child quality: a 10 percent increase in father’s time increases mean child quality by 0.96 percent. Notice that the effects of the wage increase for fathers has a smaller own effect on father’s labor supply and a stronger cross-effect on mother’s labor supply through an income effect. An increase in the father’s wage results in the mother spending less time in the market, and this reduction is spread between her increased leisure and increased time with her child,

\(^{15}\)Given our timing convention, where inputs in period \(t\) produce child quality in period \(t+1\), the terminal level of child quality produced from age 16 inputs is \(k_{17}\)
which results in an improvement in child quality. Mothers increase their time with the child on average by 5.11 percent and expenditures on children increase by 6.95 percent. It should be noted however that given that fathers have higher average wages, a 10 percent increase in the father’s wage offer is a larger wage effect than is that for mother’s.

5.6 Input Allocations

We investigate the importance of modeling the child development process within a household framework by considering a number of special cases of our more general model.

5.6.1 Child Quality Maximizing Preferences

In the first special case, we examine the optimal level of child inputs if we were to assume the household only has preferences over child quality. The optimal allocation of inputs under these “child quality maximizing” preferences is given by solving the dynamic household problem setting the weight on parental leisure and consumption to zero: \( \alpha_1 = \alpha_2 = \alpha_3 = 0 \). This is equivalent to assuming the parents solve the following maximization process:

\[
\max_{\tau_1, \tau_2} f(k_t, \tau_{1,t}, \tau_{2,t}, e_t),
\]

subject to the constraints: \( TT = h_{jt} + \tau_{jt} \) for \( j = 1, 2 \) and \( e_t = w_1 h_{1t} + w_2 h_{2t} + I_t \). Note that the under these “child quality maximizing” preferences, the household problem is strictly a static problem as the optimal allocation of inputs to maximize child quality in each period also maximizes the terminal period \( T \) level of child quality.

Table 9 presents the mean level of endogenous choices and final period (age 16) child quality under the baseline unrestricted model, with heterogeneous preferences discussed above, and the restricted child quality maximizing preferences. The first row indicates that the mean level of final period child quality more than doubles under the child quality maximizing preferences. Recall that while our child quality test score measure has a ceiling at 57, which is imposed in our estimation in order to fit the observed distribution of child quality, our production technology has no defined ceiling (see Estimator sub-section). Hence, under our estimated production technology parameters, we can examine what the latent level of child quality would be for each child, ignoring the ceiling effect of actual test score measures.

We see that under the child quality maximizing preferences, the parents choose to work in the labor market substantially less than before, with mother’s working 6.2 hours on average relative to 31 hours in the unrestricted model, and father’s working 18.5 rather than 45 hours. The time spent with their children increases substantially from 38.7 hours to 105.8 hours for mothers and 26.6 hours to 93.4 hours. All of these figures are summary averages over the simulated sample and the age range of children. Note that both mothers and fathers still work under the child quality maximizing preferences and do not spend their entire time endowment on child rearing. This is because market work funds child
good expenditures, and it is optimal for the parents to continue working some hours. Note also that although mother’s time is less productive than father’s time, mothers still spend more time on average with their children than fathers due to the relative productivity of fathers in the labor market. Even under the child quality maximizing preferences, the relative specialization of mothers to child rearing and fathers to market work still occurs. Even with the lower labor income, under the child maximizing preferences, expenditures on children still increases over three fold over the baseline. By definition all of the household’s income is spent on child goods.

5.6.2 Selfish Parents Preferences

The next special case we consider is the opposite of the previous case. Here we set household preferences so that there is no weight on child quality: $\alpha_4 = 0$. With these “selfish parents preferences,” the parents solve the following problem:

$$\max_{h_{1t}, h_{2t}} U(l_{1t}, l_{2t}, c_t),$$

subject to the constraints: $TT = h_{jt} + l_{jt}$ for $j = 1, 2$ and $c_t = w_{1t}h_{1t} + w_{2t}h_{2t} + I_t$. We maintain the same distribution of preferences over leisure and consumption as estimated. These selfish parents preferences imply that parents spend no time with their children $\tau_{1t} = \tau_{2t} = 0$ and there are no expenditures on child goods $e_t = 0$. As with the child quality maximizing preferences, the household problem under the selfish parent preferences is a strictly static problem. These preferences are essentially the preferences that would prevail for couples without children.

The third column of Table 9 presents the results for the model estimated under these selfish parent preferences. By definition, child quality, time with children, and expenditures are zero. The interesting aspect of this special case is the contrast of the labor supply decision with that for the baseline case. If the parents place no weight on child quality, the mother and father would both work substantially more hours than under the baseline where they derive utility from child quality. Average mother’s labor supply increases from 31.1 hours under the baseline to 50 hours under the selfish preferences. Average father’s labor supply increases from 45 hours to 54 hours under the selfish preferences. As a result of the additional labor income and no child expenditures, household consumption increases from $1,270 to $1,809 per week. These results suggest that parents optimally reduce their labor supply in order to invest in child quality through time spent with their children. With no weight on child quality, parents would spend considerably more time in the labor market in order to finance their own consumption. The most substantial change is for women, as their average labor supply when the household puts no weight on child quality is much closer to that of their husbands (50 vs. 54) than under the baseline preferences (31 vs 45). Our estimates are in line then with the view that gender specialization of time between market work and childcare is due to household preferences over child quality.
5.6.3 Technology Optimal Allocations

In the third special case we consider, the optimal level of child inputs is determined by the technology alone. This special case is intended to replicate one of the main approaches to child investment taken in the current literature, in which the production technology alone is used to draw inferences about the optimal allocation of child inputs across stages in the child’s development and across different types of inputs. Our estimate of the production technology provides a “selection corrected” estimate of the production technology because we jointly estimate the production technology with our model of the endogenous parental inputs. We can then use the estimated technology to indicate what the optimal allocation of inputs would be if a social planner were to use this technology to allocate resources but ignore household preferences and differential wage offers. The degree to which this technologically-optimal allocation would differ from the actual allocation parents make demonstrates the importance of considering the child quality technology jointly with the household decision model, including preferences, wages, and the non-labor income processes.

With no resource constraint, we cannot consider the level of inputs using the estimated production technology alone. Instead we define the “technology optimal” ratio of inputs as the ratio of marginal productivities of the inputs in each period. The technology optimal ratio of mother’s time to father’s time is

\[
\frac{\tau_{1t}}{\tau_{2t}} = \frac{\delta_{1t}}{\delta_{2t}},
\]

and the technology optimal ratio mother’s time to child good expenditures is given by

\[
\frac{\tau_{1t}}{e_t} = \frac{\delta_{1t}}{\delta_{3t}}.
\]

The difference between the optimal ratio of inputs chosen by the household in our unrestricted baseline model and the technology optimal ratio is that the household optimal allocation takes into account the cost of child investments from foregone parental leisure and consumption (given by \(\alpha_1, \alpha_2, \alpha_3\)) and the differential opportunity cost of mother’s and father’s time due to them having different wage offers.

Figure 10 displays the optimal ratio of mother’s time to father’s time with the child under the baseline unrestricted household-optimal model and the technology-optimal model. This figure uses the technology estimates from Model 2. For the technology optimal allocation, we use the estimates for high-school educated mother’s and fathers, although the patterns are similar for all parents. For the household optimal model, we plot the mean input choices in our sample at each child age.

Figure 10 shows that the technology optimal allocation reflects the change in productivity of mothers and fathers as the child ages (Figure 5) and allocates relatively less mother’s time to child development than the household optimal solution. The household optimal
allocation takes into account that mothers are less productive in the labor market than fathers, and therefore the optimal allocation of time is for mother’s to spend relatively more time with the child than fathers. Both allocations are downward sloping given that the main time invariant feature of the model is the technology. Mother’s time is becoming increasingly less productive relative to father’s time as the child ages.

Figure 11 repeats the analysis of optimal allocations focusing on another ratio of inputs, the ratio of mother’s time to child good expenditures. The technology optimal allocation of mother’s time to expenditures is several times higher than the baseline household optimal ratio. This reflects the fact that the household optimally allocates a much higher relative level of expenditures to the child than would be indicated solely by technological considerations. The household optimally substitutes child goods for mother’s time. The technology optimal solution ignores the fact that mother’s time with the child has both an opportunity cost in terms of foregone leisure for the mother and foregone labor income from mother’s labor supply, and therefore foregone parental consumption and child expenditures.

6 Social Policy and Child Development

In this section we consider, in an admittedly highly stylized manner, the impact of two extant social policies on child development and household welfare. Our entire analysis has been conducted outside of an equilibrium framework, so the exercises reported here are not to be considered part of a serious policy analysis. However, given that one could reasonably argue that general equilibrium effects would tend to attenuate the impacts found here, our exercise might at least establish some bounds of the impacts of these policies on child development.

6.1 Public Schooling

Given that the developmental period we examine covers the age range 3-16, school attendance occurs over most of the period. We have not accounted for this in the model explicitly, which may arguably have resulted in model misspecification regarding the choice sets of parents and inputs in the child development process. In the United States, households with children 5 years of age or less receive minimal, child-related, contributions from governmental and nongovernmental organizations. At the age of 5 or 6, most children enter public schooling; within this institutional environment, children and their parents receive large transfers from state and local governments, primarily, in the form of teachers, administrators, school buildings, books, subsidized meals, etc. Data from the National Center for Educational Statistics suggests that the average of state expenditures per pupil (across

---

16Expanding the set of inputs to include time spent with teachers and others encountered in institutional settings, along with the monetary contributions made by such actors, is an objective of our current research.
all grade levels) was approximately 10,000 dollars in 2005. We use this amount in our exercise, and assume that this translates into a weekly amount of approximately $192.\(^{17}\)

Of course, there are many potential benefits of public schooling aside from indirect transfers to households with school-age children. The most obvious ones are efficiency gains, since there would seem to be scale economies in the provision of basic education, and some studies have pointed to the positive impacts of group interactions when young on noncognitive skills later in life. While there is a healthy debate concerning the deleterious effects of classroom overcrowding, few researchers or parents call for class sizes of less than 10 students. We ignore all of these important arguments for the benefits of public schooling, and here examine only the implications of these indirect transfers for the welfare of children and households within our behavioral framework.

In terms of the household’s problem, in our model public schooling simply amounts to a contribution to \(e\), expenditures on the child. The difference between this form of subsidy and a direct transfer of $192 to the household with no restrictions on how it is to be spent are obvious. Household welfare can never be increased by restricting the manner in which the transfer is used, whereas child quality could be. Given our functional form assumptions regarding child quality production, every household spends some money on child quality production. If without the public schooling subsidy, the household spent $10 per week on the child, say, then the subsidy takes the household far beyond this level. In terms of household welfare, the constraint that at least $192 of total household income be spent on child quality would be a binding one. The household will be better off than it would be without the transfer, but not as well off as if the transfer were unrestricted. Child quality may raise appreciably in such a case, however. In contrast, when the household was already spending more than $192 before receiving the public school subsidy, the constraint is non-binding. The household will be better off, and a portion of its higher income will be spent on child quality investment. In this case, the percentage gain in child quality may not be as large as when the constraint is binding, however.

Table 10 provides the results from this policy experiment. We compute the results using the same technique as described above for the wage offer comparative statistics exercises. Column 2 reports the results from a transfer of income of $192 to each household for all periods and imposing a constraint that at least $192 must be spent on child goods. We see that this policy increases average terminal child quality over the baseline by about 8 percent (from an average level of 53.7 to 58). Mothers and fathers react to this policy by reducing their labor supply. This occurs because of the income effect of the increase in household income. On average, mothers and fathers increase their time spent with their children, but the average increase is less than the average reduction in labor supply. We can see then that the reduction in labor supply caused by the income transfer is being split between greater leisure for the parents and greater time investment in their children. Child

\(^{17}\)We have simply expressed the total school year amount as a weekly flow value. Our model with its conventions regarding decision periods (the year) and measurement units (weeks) does not allow us to allocate the transfers only to weeks in which school is in session.
good expenditures increase quite substantially, by about 41 percent (from $167 per week to $236), as a result of the policy. Household consumption also increases because of the income transfer but far less than the increase in child good expenditures. For unconstrained households, those households already spending in excess of the required $192 on child goods, this policy is a pure income transfer.

The third column of Table 10 presents a second policy experiment in which we simply give the household the income transfer of $192 but place no restrictions on how this transfer is spent. In this experiment, the household could simply consume all of the transfer and not increase child good expenditures at all. Thus these two experiments present a contrast between two general approaches to child quality welfare policies: a “targeted” child expenditure subsidy and an “untargeted” subsidy in the form of a pure income transfer.

We see that mean child quality under the untargeted income transfer increases over the baseline but is less than under the targeted subsidy. As expected, child good expenditures increases over baseline but far less than under the restricted targeted subsidy because the household is not restricted in how income is allocated. Labor supply also falls more under the untargeted transfer. This is because the income effect is stronger with the pure income transfer because there are no “strings” or restrictions attached to the extra household income. Under the untargeted subsidy, labor income is allowed to be optimally allocated across consumption and child goods expenditures according to household preferences. Under the targeted subsidy, constrained households are forced to sub-optimally allocate a greater share of labor income to child goods expenditures. The marginal utility of labor income is then less under the restricted targeted subsidy than the untargeted subsidy. Because of the greater decline in labor supply, time with children by the parents is actually higher under the untargeted subsidy than the targeted subsidy. This makes the relative effect of the targeted subsidy versus the untargeted subsidy ambiguous: the untargeted subsidy encourages the parents to spend more time their children through a stronger income effect on labor supply, while the targeted subsidy forces the household to allocate a greater share of income to child goods expenditures.

6.2 Limitations on Work Hours

In 2000, the left-wing government of Lionel Jospin passed a law establishing 35 hours as the new legal work week, with strict limits on and penalties for work over this limit. While the rationale for decreasing the length of the work week was not framed explicitly in terms of child development, such a law could possibly be expected to have important impacts on the child development process, since it directly impacts the household’s ability to freely allocate time and financial resources across competing uses. Needless to say, if such a law were to be imposed and seriously enforced in the U.S. context, we would expect to see potentially major changes in wage distributions, the demand for capital, etc. These

---

18The limitations on overtime hours were substantially relaxed by amendments introduced over the period 2004 through 2007.
are all neglected in our exercise, where we simply impose a binding 35 hour work limitation on each of the parents, and we make no other changes to the choice problem the household faces. We assume that the law is perfectly enforced and permanent.

Table 10 provides the results from this policy experiment in the last column. Overall under this policy, mean child quality increases by about 4.3 percent (from 53.7 under baseline to 56 under the hours restriction). Under this policy, mean hours of work for the mother and father fall, where the greatest change is for fathers whose average labor supply falls from 45 hours on average for the baseline to 33.8 hours with the restriction. Average mother’s labor supply changes relatively modestly from 31.1 hours to 29.8 hours. Because of these hours restrictions, both parents spend more time with their children on average. However, mirroring the labor supply changes, the most substantial increase is for fathers. Child good expenditures falls from the baseline because of the lower labor supply and reduction in labor income. This alone would cause child quality to be reduced. However, it appears that the positive effect on child quality due to the increase in time with children swamps the lower level of expenditures on children. The net effect of this policy is therefore estimated to be positive on child quality. Although, as suggested by the fall in household consumption, this restricted hours policy reduces overall household utility.

7 Conclusion

Using a simple dynamic production technology for child quality and a Cobb-Douglas specification of a household utility function, we employ unique data from the PSID-CDS on investments in children to recover estimates of the primitive parameters of the problem. One of the main points of the paper has been that household time and money investments in children can only be properly understood when preferences and technologies are simultaneously estimated. We found that, while the average household attaches a substantial weight to child quality in its utility function, it is by no means only concerned with child quality. This was made abundantly clear in the comparative statics exercises and policy experiments. For example, we provided quantitative evidence regarding the extent of “crowding out” of household investment in children by public investment, in the form of schooling transfers. Though we have not allowed, at present, for time inputs of teachers and other agents, it is clear that these factors of production belong in the production function as well, and we can expend the time inputs of parents to be supplanted by those of other agents when the child begins formal schooling. Policy-makers considering changes in the school environment must be aware of parental responses to changes in schooling inputs or their attempts at enhancing child welfare are likely to come to naught.

We believe that we have uncovered a number of interesting empirical results and interpretations, though obviously much more needs to be done on both the preferences and technology sides of the model. In the paragraph above, we mentioned that the set of inputs should be expanded to include the time and money contributions of teachers, tutors,
care-givers, and other relatives. Our analysis is also limited in the sense that we specify a unitary model to represent household preferences. We used a household utility function approach to ease the computational burden and to focus attention on the child quality production function. For both conceptual and empirical reasons, the household utility function set up is suspect. In future research we will be introducing individualistic preferences of the parents along with a dynamic contracting approach to decision-making. In this case, child investments and outcomes will also reflect their parents preferences and each parent’s “power” in determining household decisions.

Perhaps the biggest limitation of the analysis is that only households in which both biological parents are present are modeled. It is well-known that children who spent a significant amount of time living in households without both biological parents have significantly worse outcomes in terms of scores on standardized ability or achievement tests, educational attainment, unemployment, etc. Even in cases where to divorced parents continue to invest in the child, the production technology is likely to differ from the one that would be available if the parents were not divorced and still cohabiting. The PSID-CDS data includes many households in which only one parent is present, so that, solely on a descriptive level, it is possible to determine how the pattern of investments differs between intact and non-intact households. In order to actually determine the characteristics of production processes in the two types of settings, as well as the preferences of divorced and married parents, necessitates endogenizing the divorce decision. Brown and Flinn (2006) attempt something along these lines, but they do not have access to the rich investment data that are found in the PSID-CDS, which considerably complicates their ability to model the child’s developmental process.
References


Table 1: Descriptive Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>1997 Mean (Std.)</th>
<th>2002-03 Mean (Std.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child’s age</td>
<td>7.45045 (2.839872)</td>
<td></td>
</tr>
<tr>
<td>Father’s age</td>
<td>38.43372 (6.305459)</td>
<td></td>
</tr>
<tr>
<td>Mother’s age</td>
<td>36.12484 (5.606386)</td>
<td></td>
</tr>
<tr>
<td>Father’s education</td>
<td>13.25997 (2.776811)</td>
<td></td>
</tr>
<tr>
<td>Mother’s education</td>
<td>13.30116 (2.600934)</td>
<td></td>
</tr>
<tr>
<td>Father’s time with child</td>
<td>5.577536 (8.48059)</td>
<td>4.255899 (7.304067)</td>
</tr>
<tr>
<td>Mother’s time with child</td>
<td>22.33422 (16.94523)</td>
<td>12.95109 (12.64746)</td>
</tr>
<tr>
<td>Letter Word raw score</td>
<td>29.38867 (16.35427)</td>
<td>47.19691 (6.304849)</td>
</tr>
</tbody>
</table>

Notes: Sample of
Source: PSID.
Table 2: Descriptive Statistics for Labor Supply, Wages, and Income

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>(Std.)</td>
<td>Mean</td>
</tr>
<tr>
<td>Mother’s work hours per week</td>
<td>25.75</td>
<td>(18.59)</td>
<td>25.89</td>
</tr>
<tr>
<td>Father’s work hours per week</td>
<td>44.43</td>
<td>(10.01)</td>
<td>45.49</td>
</tr>
<tr>
<td>Father’s hourly wage</td>
<td>20.71</td>
<td>(13.40)</td>
<td>22.03</td>
</tr>
<tr>
<td>Mother’s hourly wage</td>
<td>10.05</td>
<td>(10.03)</td>
<td>11.13</td>
</tr>
<tr>
<td>Weekly non-labor income</td>
<td>66.90</td>
<td>(135.08)</td>
<td>–</td>
</tr>
</tbody>
</table>

Notes: Sample of
Source: PSID.
Figure 1: Average Hours with Children for Mothers and Fathers by Child Age

Notes:
Figure 2: Average Hours Working for Mothers and Fathers by Child Age

Notes:
Figure 3: Average Letter Work Score by Child Age

Notes:
Table 3: **Preference Parameter Estimates**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean of $\alpha_1$</td>
<td>0.1058</td>
<td>0.1028</td>
</tr>
<tr>
<td></td>
<td>()</td>
<td>()</td>
</tr>
<tr>
<td>Mean of $\alpha_2$</td>
<td>0.2181</td>
<td>0.2201</td>
</tr>
<tr>
<td></td>
<td>()</td>
<td>()</td>
</tr>
<tr>
<td>Mean of $\alpha_3$</td>
<td>0.3394</td>
<td>0.3374</td>
</tr>
<tr>
<td></td>
<td>()</td>
<td>()</td>
</tr>
<tr>
<td>Mean of $\alpha_4$</td>
<td>0.3367</td>
<td>0.339</td>
</tr>
<tr>
<td></td>
<td>()</td>
<td>()</td>
</tr>
<tr>
<td>Std. of $\alpha_1$</td>
<td>0.0050</td>
<td>0.0242</td>
</tr>
<tr>
<td></td>
<td>()</td>
<td>()</td>
</tr>
<tr>
<td>Std. of $\alpha_2$</td>
<td>0.0417</td>
<td>0.0445</td>
</tr>
<tr>
<td></td>
<td>()</td>
<td>()</td>
</tr>
<tr>
<td>Std. of $\alpha_3$</td>
<td>0.0509</td>
<td>0.0553</td>
</tr>
<tr>
<td></td>
<td>()</td>
<td>()</td>
</tr>
<tr>
<td>Std. of $\alpha_4$</td>
<td>0.0309</td>
<td>0.0380</td>
</tr>
<tr>
<td></td>
<td>()</td>
<td>()</td>
</tr>
<tr>
<td>Correlation of $\alpha_1$ and $\alpha_2$</td>
<td>-0.0466</td>
<td>0.9484</td>
</tr>
<tr>
<td></td>
<td>()</td>
<td>()</td>
</tr>
<tr>
<td>Correlation of $\alpha_1$ and $\alpha_3$</td>
<td>-0.6241</td>
<td>-0.7063</td>
</tr>
<tr>
<td></td>
<td>()</td>
<td>()</td>
</tr>
<tr>
<td>Correlation of $\alpha_2$ and $\alpha_3$</td>
<td>-0.7215</td>
<td>-0.8794</td>
</tr>
<tr>
<td></td>
<td>()</td>
<td>()</td>
</tr>
<tr>
<td>$\psi$ (Terminal Payoff to Child Quality)</td>
<td>8.3595</td>
<td>7.7165</td>
</tr>
<tr>
<td></td>
<td>()</td>
<td>()</td>
</tr>
</tbody>
</table>

Notes: Model 1 does not include parental education in the production function. Model 2 includes parental education.
Table 4: **Technology Parameter Estimates**

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_{00} ) (TFP R intercept)</td>
<td>0.2106</td>
<td>0.2113</td>
</tr>
<tr>
<td>( \gamma_{01} ) (TFP R slope)</td>
<td>0.0003</td>
<td>0.0012</td>
</tr>
<tr>
<td>( \sigma_R ) (Std. of Child Quality Shock)</td>
<td>0.0031</td>
<td>0.0008</td>
</tr>
<tr>
<td>( \gamma_{10} ) (Mother’s Time intercept)</td>
<td>-1.9948</td>
<td>-2.0115</td>
</tr>
<tr>
<td>( \gamma_{11} ) (Mother’s Time slope)</td>
<td>-0.0981</td>
<td>-0.0990</td>
</tr>
<tr>
<td>( \gamma_{12} ) (Mother’s Time education slope)</td>
<td>-0.0026</td>
<td>(NA)</td>
</tr>
<tr>
<td>( \gamma_{20} ) (Father’s Time intercept)</td>
<td>-1.7948</td>
<td>-1.8143</td>
</tr>
<tr>
<td>( \gamma_{21} ) (Father’s Time slope)</td>
<td>-0.0704</td>
<td>-0.0707</td>
</tr>
<tr>
<td>( \gamma_{22} ) (Father’s Time education slope)</td>
<td>(NA)</td>
<td>(NA)</td>
</tr>
<tr>
<td>( \gamma_{30} ) (Child Expenditures intercept)</td>
<td>-3.6741</td>
<td>-3.7037</td>
</tr>
<tr>
<td>( \gamma_{31} ) (Child Expenditures slope)</td>
<td>-0.0011</td>
<td>-0.0003</td>
</tr>
<tr>
<td>( \gamma_{40} ) (Last Period’s Child Quality intercept)</td>
<td>-0.3221</td>
<td>-0.3238</td>
</tr>
<tr>
<td>( \gamma_{41} ) (Last Period’s Child Quality slope)</td>
<td>0.0101</td>
<td>0.0103</td>
</tr>
</tbody>
</table>

Notes: Model 1 does not include parental education in the production function. Model 2 includes parental education.
Figure 4: Estimated Child Development Parameters by Child Age

Notes:
Figure 5: Estimated Child Development Parameters by Child Age

Notes:
Table 5: Wage and Income Parameter Estimates

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mother’s Log Wage Offer</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_0^1$ (Intercept)</td>
<td>-0.4465</td>
<td>-0.3656</td>
</tr>
<tr>
<td>$\mu_1^1$ (Mother’s Schooling)</td>
<td>0.1555</td>
<td>0.1510</td>
</tr>
<tr>
<td>$\mu_2^1$ (Mother’s Age)</td>
<td>0.0107</td>
<td>0.0102</td>
</tr>
<tr>
<td>$\mu_3^1$ (Mother’s Age Sq x 1000)</td>
<td>-0.0601</td>
<td>-0.0648</td>
</tr>
<tr>
<td>$\sigma_1$ (Standard Deviation of Innovation)</td>
<td>0.2153</td>
<td>0.2179</td>
</tr>
<tr>
<td>$\rho_1$ (Auto-Correlation)</td>
<td>-0.5072</td>
<td>-0.4976</td>
</tr>
<tr>
<td>$\rho_{12}$ (Correlation with Father’s Wage Shock)</td>
<td>0.7595</td>
<td>0.7519</td>
</tr>
</tbody>
</table>

|                          |         |         |
| **Father’s Log Wage Offer** |         |         |
| $\mu_2^2$ (Intercept)    | 0.6560  | 0.6417  |
| $\mu_3^2$ (Father’s Schooling) | 0.1643  | 0.1627  |
| $\mu_2^2$ (Father’s Age)  | -0.0009 | -0.0002 |
| $\mu_3^2$ (Father’s Age Sq x 1000) | -0.0059 | -0.0114 |
| $\sigma_2$ (Standard Deviation of Innovation) | 0.7018  | 0.7312  |
| $\rho_2$ (Auto-Correlation) | -0.6221 | -0.5875 |

Notes: Model 1 does not include parental education in the production function. Model 2 includes parental education.
Table 6: **Wage and Income Parameter Estimates (con’t)**

<table>
<thead>
<tr>
<th>Latent Non-Labor Income</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_3^0$ (Intercept)</td>
<td>-92.5490</td>
<td>-92.5490</td>
</tr>
<tr>
<td>$\sigma_3$ (Standard Deviation of Innovation)</td>
<td>289.2813</td>
<td>289.2813</td>
</tr>
<tr>
<td>$\rho_3$ (Auto-Correlation)</td>
<td>0.3591</td>
<td>0.3591</td>
</tr>
</tbody>
</table>

Notes: Model 1 does not include parental education in the production function. Model 2 includes parental education.
Table 7: **Sample Fit for Wages and Income**

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Simulated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Mother’s Wage</td>
<td>10.8278</td>
<td>10.4955</td>
</tr>
<tr>
<td>Std. Mother’s Wage</td>
<td>10.3097</td>
<td>9.1187</td>
</tr>
<tr>
<td>Avg. Father’s Wage</td>
<td>22.2421</td>
<td>21.5814</td>
</tr>
<tr>
<td>Std. Father’s Wage</td>
<td>14.7498</td>
<td>17.3501</td>
</tr>
<tr>
<td>Avg. Non-Labor Income</td>
<td>82.7660</td>
<td>82.7801</td>
</tr>
<tr>
<td>Std. Non-Labor Income</td>
<td>150.6585</td>
<td>150.1142</td>
</tr>
</tbody>
</table>

Notes: Sample of
Figure 6: Employment Probabilities for Mothers and Fathers by Child Age

Notes:
Figure 7: Average Hours Working for Mothers and Fathers by Child Age

Notes:
Figure 8: Average Hours with Children for Mothers and Fathers by Child Age

Notes:
Figure 9: Average Letter Work Score by Child Age

Notes:
Table 8: **Comparative Statics**

<table>
<thead>
<tr>
<th></th>
<th>Level at Baseline</th>
<th>10 % Increase in Mother’s Wage</th>
<th>10 % Increase in Father’s Wage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Child Quality at Age 16</td>
<td>53.7174</td>
<td>0.0008</td>
<td>0.0096</td>
</tr>
<tr>
<td>Mean Hours Work (Mother)</td>
<td>30.9604</td>
<td>0.1379</td>
<td>-0.1329</td>
</tr>
<tr>
<td>Mean Hours Work (Father)</td>
<td>45.0087</td>
<td>-0.0423</td>
<td>0.0425</td>
</tr>
<tr>
<td>Mean Time w/ Child (Mother)</td>
<td>38.8619</td>
<td>-0.0530</td>
<td>0.0511</td>
</tr>
<tr>
<td>Mean Time w/ Child (Father)</td>
<td>26.6258</td>
<td>0.0286</td>
<td>-0.0287</td>
</tr>
<tr>
<td>Mean Child Expenditures</td>
<td>166.9252</td>
<td>0.0289</td>
<td>0.0695</td>
</tr>
<tr>
<td>Mean Household Consumption</td>
<td>1,273.30</td>
<td>0.0288</td>
<td>0.0696</td>
</tr>
</tbody>
</table>

Notes: Results computed using a 10 percent increase in wages for mother and father, respectively. Estimates reported are elasticities (1 percent equivalent). A value of 0.1379 indicates that mean hours worked by the mother increased by 13.79 percent after a 10 percent increase in the Mother’s wage.
Table 9: Optimal Decisions with Alternative Preferences

<table>
<thead>
<tr>
<th></th>
<th>Level at Baseline</th>
<th>Child Quality Maximizing Preferences</th>
<th>Selfish Parent Preferences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Child Quality at Age 16</td>
<td>53.6935</td>
<td>121.9042</td>
<td>0</td>
</tr>
<tr>
<td>Mean Hours Work (Mother)</td>
<td>31.1472</td>
<td>6.2011</td>
<td>49.9173</td>
</tr>
<tr>
<td>Mean Hours Work (Father)</td>
<td>45.0536</td>
<td>18.5512</td>
<td>54.1575</td>
</tr>
<tr>
<td>Mean Time w/ Child (Mother)</td>
<td>38.7207</td>
<td>105.7989</td>
<td>0</td>
</tr>
<tr>
<td>Mean Time w/ Child (Father)</td>
<td>26.5843</td>
<td>93.4488</td>
<td>0</td>
</tr>
<tr>
<td>Mean Child Expenditures</td>
<td>167.0617</td>
<td>600.1822</td>
<td>0</td>
</tr>
<tr>
<td>Mean Household Consumption</td>
<td>1,270.60</td>
<td>0</td>
<td>1,809.40</td>
</tr>
</tbody>
</table>

Notes: Child Quality Maximizing Preferences set preference weight on parental leisure and consumption to 0: $\alpha_1 = \alpha_2 = \alpha_3 = 0$. Under these preferences, the household then maximizes the level child quality, and consumption $c_t = 0$ for all $t$. Selfish Parent Preferences set $\alpha_4 = 0$, and the household puts no weight on child quality. With these preferences, $\tau_{1t} = \tau_{2t} = e_t = 0$ for all $t$. Mean Child Quality at Age 16 is the terminal latent child quality level (i.e. the level at the start of period $t = 17, k_{17}$). Note that with the test score ceiling of $\bar{k} = 57$, the measured score would be truncated. We impose this test score ceiling in the estimation to match the data generating process. Here we report the latent, non-truncated scores in the policy simulations. Latent child quality reported here can therefore exceed $\bar{k}$. 
Figure 10: Optimal Ratio of Mother’s and Father’s Time with Child under Different Modeling Assumptions

Notes:
Figure 11: Optimal Ratio of Mother’s Time with Child and Child Expenditures under Different Modeling Assumptions

Notes:
Table 10: **Policy Simulations**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level at Baseline</td>
<td>Child Expend. Floor of $e \geq 192$ and Inc. Transf. of $192$</td>
<td>Income Transfer of $192$ Only</td>
<td>Hours Ceiling $h_{1t} \leq 35$ and $h_{2t} \leq 35$</td>
</tr>
<tr>
<td>Mean Child Quality at Age 16</td>
<td>53.6935</td>
<td>58.0148</td>
<td>56.6422</td>
</tr>
<tr>
<td>Mean Hours Work (Mother)</td>
<td>31.1472</td>
<td>28.6156</td>
<td>26.1458</td>
</tr>
<tr>
<td>Mean Hours Work (Father)</td>
<td>45.0536</td>
<td>41.7178</td>
<td>38.7838</td>
</tr>
<tr>
<td>Mean Time w/ Child (Mother)</td>
<td>38.7207</td>
<td>39.4077</td>
<td>40.5746</td>
</tr>
<tr>
<td>Mean Time w/ Child (Father)</td>
<td>26.5843</td>
<td>27.7023</td>
<td>28.8464</td>
</tr>
<tr>
<td>Mean Child Expenditures</td>
<td>167.0617</td>
<td>236.1468</td>
<td>180.2980</td>
</tr>
<tr>
<td>Mean Household Consumption</td>
<td>1,270.60</td>
<td>1,318.70</td>
<td>1,341.40</td>
</tr>
</tbody>
</table>

Notes: Experiment 1 gives each household an income transfer of $192 each week and forces the household to spend at least $192 on child goods. Experiment 2 gives each household the same $192 income transfer each week but leaves the household unrestricted in how the household spends the extra income. Experiment 3 imposes a labor supply ceiling on the parents and forces mothers and fathers to work 35 or fewer hours each week. Mean Child Quality at Age 16 is the terminal latent child quality level (i.e. the level at the start of period $t = 17, k_{17}$). Note that with the test score ceiling of $\bar{k} = 57$, the measured score would be truncated. We impose this test score ceiling in the estimation to match the data generating process. Here we report the latent, non-truncated scores in the policy simulations.