Consumption Smoothing, Assets and Family Labor Supply

Richard Blundell  University College London & IFS

ECB October 2013
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- Based on recent work with Luigi Pistaferri and Itay Eksten at Stanford: "Consumption Inequality and Family Labor Supply".

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  - Taxes and welfare: (earnings $\rightarrow$ income)
  - Assets: Saving and borrowing (income $\rightarrow$ consumption)
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  - Informal contracts, gifts, etc.
- But how important are each of these mechanisms?
Overview

- Focus on how families deal with labor market shocks.
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- Point to the importance of constructing panel/administrative linked data that allow all three measures.

With Luigi and Itay we make use of the new PSID data on consumption, earnings and assets. We show that family labor supply, credit market and the tax/welfare system all have key roles to play in the ‘insurance’ of shocks. Credit and family labor supply act together to insure shocks.

Finding: Once assets, family labor supply and taxes (and welfare) are properly accounted for, we can explain the link between these series and there is less evidence for additional insurance.

Some consumption inequality descriptives...
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- Some consumption inequality descriptives....
CONSUMPTION INEQUALITY IN THE UK
By age and birth cohort

Variance of log nondurable consumption

Age
20 30 40 50 60 70

Variance
.1 .15 .2 .25 .3 .35 .4 .45 .5

1940 1950 1960 1970

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Income Inequality in the UK
By age and birth cohort

Variance of log net income

Age
20 30 40 50 60 70

Variance
.25 .3 .35 .4 .45 .5

1940 1950 1960 1970

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Consumption and Family Labor Supply
ECB October 2013
CONSUMPTION INEQUALITY IN THE US

By age and birth cohort

![Graph showing consumption inequality by age and birth cohort.]
The existing literature (references in paper) usually relates movements in consumption to **predictable and unpredictable income changes** as well as **persistent and non-persistent shocks** to economic resources.
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A little background on the empirical strategy for income and consumption dynamics behind these results...
INCOME DYNAMICS

Consider consumer $i$ (of age $a$) in time period $t$, has log income $y_{it}(\equiv \ln Y_{i,a,t})$ written

$$y_{it} = Z_{it} \varphi + f_{0i} + pt f_{1i} + y_{it}^P + y_{it}^T$$

A key consideration is to allow variances (or factor loadings) of $y_{it}^P$ and $y_{it}^T$ to vary with age/time for each birth cohort.

Find $p_t f_{1i}$ to be less important and $\rho$ closer to unity, especially in the 30-59 age selection.

Detailed work on Norwegian population register panel data...
**Income Dynamics**

Consider consumer \( i \) (of age \( a \)) in time period \( t \), has log income \( y_{it}(\equiv \ln Y_{i,a,t}) \) written

\[
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where \( y_{it}^{P} \) is a persistent process of income shocks, say

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which adds to the individual-specific trend $ptf_i$ and where $y_{it}^T$ is a transitory shock represented by some low order MA process, say

$$y_{it}^T = \epsilon_{it} + \theta_1\epsilon_{i,t-1}$$
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Consider consumer \( i \) (of age \( a \)) in time period \( t \), has log income \( y_{it}(\equiv \ln Y_{i,a,t}) \) written

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y_{it} = Z_{it}' \varphi + f_{0i} + p_{tf_{1i}} + y_{it}^P + y_{it}^T
\]

where \( y_{it}^P \) is a persistent process of income shocks, say

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y_{it}^P = \rho y_{it-1}^P + \nu_{it}
\]

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- Find \( p_{tf_{1i}} \) to be less important and \( \rho \) closer to unity, especially in the 30-59 age selection.
- Detailed work on Norwegian population register panel data....
Variance of permanent shocks over the life-cycle

Source: Blundell, Graber and Mogstad (2013), Norwegian Population Panel.
LIFE-CYCLE INCOME DYNAMICS
Norwegian population panel (low skilled)

Source: Blundell, Graber and Mogstad (2013).
To account for the impact of income shocks on consumption we introduce transmission parameters: $\kappa_{cvt}$ and $\kappa_{c\epsilon t}$, writing consumption growth as:

$$\Delta \ln C_{it} \approx \Gamma_{it} + \Delta Z_{it}' \varphi^c + \kappa_{cvt} \nu_{it} + \kappa_{c\epsilon t} \epsilon_{it} + \zeta_{it}$$
CONSUMPTION GROWTH AND INCOME "SHOCKS"

To account for the impact of income shocks on consumption we introduce *transmission parameters*: $\kappa_{cvt}$ and $\kappa_{c\epsilon t}$, writing consumption growth as:

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which provides the link between the consumption and income distributions.
**Consumption Growth and Income "Shocks"**

To account for the impact of income shocks on consumption we introduce *transmission parameters*: $\kappa_{cvt}$ and $\kappa_{c\epsilon t}$, writing consumption growth as:

$$\Delta \ln C_{it} \cong \Gamma_{it} + \Delta Z'_{it} \varphi^c + \kappa_{cvt} v_{it} + \kappa_{c\epsilon t} \epsilon_{it} + \zeta_{it}$$

which provides the link between the consumption and income distributions.

For example, in Blundell, Low and Preston (QE, 2013) show, for any birth-cohort,

$$\Delta \ln C_{it} \cong \Gamma_{it} + \Delta Z'_{it} \varphi^c + (1 - \pi_{it}) v_{it} + (1 - \pi_{it}) \gamma_{Lt} \epsilon_{it} + \zeta_{it}$$

where

$$\pi_{it} \approx \frac{\text{Assets}_{it}}{\text{Assets}_{it} + \text{Human Wealth}_{it}}$$

and $\gamma_{Lt}$ is the annuity value of a transitory shock for an individual aged $t$ retiring at age $L$. 
In this paper we look closer at four key mechanisms:

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3. Non-linear taxes and welfare
4. Other (un-modeled) mechanisms.
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Distinctive features of this paper:
- Allow for
  - non-separability,
  - heterogeneous assets,
  - correlated shocks to individual wages.
- Use new data from the PSID 1999-2009
  - More comprehensive consumption measure.
  - Asset data collected in every wave.
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>PSID Total</td>
<td>3,276</td>
<td>3,769</td>
<td>4,285</td>
<td>5,058</td>
<td>5,926</td>
<td>5,736</td>
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<tr>
<td>NIPA Total</td>
<td>5,139</td>
<td>5,915</td>
<td>6,447</td>
<td>7,224</td>
<td>8,190</td>
<td>9,021</td>
</tr>
<tr>
<td>ratio</td>
<td>0.64</td>
<td>0.64</td>
<td>0.66</td>
<td>0.7</td>
<td>0.72</td>
<td>0.64</td>
</tr>
<tr>
<td>PSID Nondurables</td>
<td>746</td>
<td>855</td>
<td>887</td>
<td>1,015</td>
<td>1,188</td>
<td>1,146</td>
</tr>
<tr>
<td>NIPA Nondurables</td>
<td>1,330</td>
<td>1,543</td>
<td>1,618</td>
<td>1,831</td>
<td>2,089</td>
<td>2,296</td>
</tr>
<tr>
<td>ratio</td>
<td>0.56</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
<td>0.57</td>
<td>0.5</td>
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<tr>
<td>PSID Services</td>
<td>2,530</td>
<td>2,914</td>
<td>3,398</td>
<td>4,043</td>
<td>4,738</td>
<td>4,590</td>
</tr>
<tr>
<td>NIPA Services</td>
<td>3,809</td>
<td>4,371</td>
<td>4,829</td>
<td>5,393</td>
<td>6,101</td>
<td>6,725</td>
</tr>
<tr>
<td>ratio</td>
<td>0.66</td>
<td>0.67</td>
<td>0.7</td>
<td>0.75</td>
<td>0.78</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Note: PSID weights are applied for the non-sampled PSID data (47,206 observations for these years). Total consumption is defined as Nondurables + Services. PSID consumption categories include food, gasoline, utilities, health, rent (or rent equivalent), transportation, child care, education and other insurance. NIPA numbers are from NIPA table 2.3.5. All numbers are nonminal.
## Descriptive Statistics for Consumption

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td><strong>Consumption</strong></td>
<td>27,290</td>
<td>31,973</td>
<td>35,277</td>
<td>41,555</td>
<td>45,863</td>
<td>44,006</td>
</tr>
<tr>
<td><strong>Nondurable Consumption</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food (at home)</td>
<td>6,859</td>
<td>7,827</td>
<td>7,827</td>
<td>8,873</td>
<td>9,889</td>
<td>9,246</td>
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<tr>
<td>Gasoline</td>
<td>1,387</td>
<td>2,041</td>
<td>1,916</td>
<td>2,601</td>
<td>3,301</td>
<td>2,611</td>
</tr>
<tr>
<td><strong>Services</strong></td>
<td>21,319</td>
<td>25,150</td>
<td>28,419</td>
<td>33,755</td>
<td>36,949</td>
<td>35,575</td>
</tr>
<tr>
<td>Food (out)</td>
<td>2,029</td>
<td>2,279</td>
<td>2,382</td>
<td>2,582</td>
<td>2,693</td>
<td>2,492</td>
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<tr>
<td>Health Insurance</td>
<td>1,056</td>
<td>1,268</td>
<td>1,461</td>
<td>1,750</td>
<td>1,916</td>
<td>2,188</td>
</tr>
<tr>
<td>Health Services</td>
<td>902</td>
<td>1,134</td>
<td>1,334</td>
<td>1,447</td>
<td>1,615</td>
<td>1,844</td>
</tr>
<tr>
<td>Utilities</td>
<td>2,282</td>
<td>2,651</td>
<td>2,702</td>
<td>4,655</td>
<td>5,038</td>
<td>5,600</td>
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<tr>
<td>Transportation</td>
<td>3,122</td>
<td>3,758</td>
<td>4,474</td>
<td>3,797</td>
<td>3,970</td>
<td>3,759</td>
</tr>
<tr>
<td>Education</td>
<td>1,946</td>
<td>2,283</td>
<td>2,390</td>
<td>2,557</td>
<td>2,728</td>
<td>2,584</td>
</tr>
<tr>
<td>Child Care</td>
<td>601</td>
<td>653</td>
<td>660</td>
<td>689</td>
<td>648</td>
<td>783</td>
</tr>
<tr>
<td>Home Insurance</td>
<td>430</td>
<td>480</td>
<td>552</td>
<td>629</td>
<td>717</td>
<td>729</td>
</tr>
<tr>
<td>Rent (or rent equivalent)</td>
<td>8,950</td>
<td>10,645</td>
<td>12,464</td>
<td>15,650</td>
<td>17,623</td>
<td>15,595</td>
</tr>
<tr>
<td>Observations</td>
<td>1,872</td>
<td>1,951</td>
<td>1,984</td>
<td>2,011</td>
<td>2,115</td>
<td>2,221</td>
</tr>
</tbody>
</table>

Notes: PSID data from 1999-2009 PSID waves. PSID means are given for the main sample of estimation: married couples with working males aged 30 to 65. SEO sample excluded. PSID rent is imputed as 6% of reported house value for homeowners. Missing values in consumption and assets sub-categories were treated as zeros.
## Descriptive Statistics for Assets and Earnings

<table>
<thead>
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</thead>
<tbody>
<tr>
<td>Total assets</td>
<td>332,625</td>
<td>352,247</td>
<td>382,600</td>
<td>476,626</td>
<td>555,951</td>
<td>506,823</td>
</tr>
<tr>
<td>Housing and RE assets</td>
<td>159,856</td>
<td>187,969</td>
<td>227,224</td>
<td>283,913</td>
<td>327,719</td>
<td>292,910</td>
</tr>
<tr>
<td>Financial assets</td>
<td>173,026</td>
<td>164,567</td>
<td>155,605</td>
<td>192,995</td>
<td>228,805</td>
<td>214,441</td>
</tr>
<tr>
<td>Total debt</td>
<td>72,718</td>
<td>82,806</td>
<td>98,580</td>
<td>115,873</td>
<td>131,316</td>
<td>137,348</td>
</tr>
<tr>
<td>Mortgage</td>
<td>65,876</td>
<td>74,288</td>
<td>89,583</td>
<td>106,423</td>
<td>120,333</td>
<td>123,324</td>
</tr>
<tr>
<td>Other debt</td>
<td>7,021</td>
<td>8,687</td>
<td>9,217</td>
<td>9,744</td>
<td>11,584</td>
<td>14,561</td>
</tr>
</tbody>
</table>

First earner (head)
- Earnings                      | 54,220 | 61,251 | 63,674 | 68,500 | 72,794 | 75,588 |
- Hours worked                  | 2,357  | 2,317  | 2,309  | 2,309  | 2,284  | 2,140  |

Second earner (wife)
- Participation rate            | 0.81   | 0.8    | 0.81   | 0.81   | 0.81   | 0.8    |
- Earnings (conditional on participation) | 26,035 | 28,611 | 31,693 | 33,987 | 36,185 | 39,973 |
- Hours worked (conditional on participation) | 1,666  | 1,691  | 1,697  | 1,707  | 1,659  | 1,648  |

Observations                   | 1,872  | 1,951  | 1,984  | 2,011  | 2,115  | 2,221  |

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Household Decisions in a Unitary Framework

Household chooses \( \{C_{i,t+j}, H_{i,1,t+j}, H_{i,2,t+j}\}_{j=0}^{T-t} \) to maximize

\[
\mathbb{E}_t \sum_{\tau=0}^{T-t} (1 + \delta)^{-\tau} \nu (C_{i,t+\tau}, H_{i,1,t+\tau}, H_{i,2,t+\tau}; Z_{i,t+\tau})
\]

subject to

\[
C_{i,t} + \frac{A_{i,t+1}}{1 + r} = A_{i,t} + H_{i,1,t}W_{i,1,t} + H_{i,1,t}W_{i,2,t}
\]
Household Decisions in a Unitary Framework

Household chooses \( \{C_{i,t+j}, H_{i,1,t+j}, H_{i,2,t+j}\}_{j=0}^{T-t} \) to maximize

\[
\mathbb{E}_t \sum_{\tau=0}^{T-t} (1 + \delta)^{-\tau} \nu (C_{i,t+\tau}, H_{i,1,t+\tau}, H_{i,2,t+\tau}; Z_{i,t+\tau})
\]

subject to

\[
C_{i,t} + \frac{A_{i,t+1}}{1 + r} = A_{i,t} + H_{i,1,t} W_{i,1,t} + H_{i,1,t} W_{i,2,t}
\]

Our approach

- Extend previous work and express the distributional dynamics of consumption and earnings growth as functions of Frisch elasticities, ‘insurance parameters’ and wage shocks
Wage Process

For earner $j = \{1, 2\}$ in household $i$, period $t$, wage growth is:

$$\Delta \log W_{i,j,t} = \Delta X'_{i,j,t} \beta_j + \Delta u_{i,j,t} + v_{i,j,t}$$
WAGE PROCESS

For earner $j = \{1, 2\}$ in household $i$, period $t$, wage growth is:

$$\Delta \log W_{i,j,t} = \Delta X'_{i,j,t} \beta_j + \Delta u_{i,j,t} + \nu_{i,j,t}$$

\[
\begin{pmatrix}
    u_{i,1,t} \\
    u_{i,2,t} \\
    \nu_{i,1,t} \\
    \nu_{i,2,t}
\end{pmatrix}
\sim i.i.d. \quad \begin{pmatrix}
    0 \\
    0 \\
    0 \\
    0
\end{pmatrix}, \quad \begin{pmatrix}
    \sigma_{u,1}^2 & \sigma_{u,1,u_2} & 0 & 0 \\
    \sigma_{u,1,u_2} & \sigma_{u,2}^2 & 0 & 0 \\
    0 & 0 & \sigma_{v,1}^2 & \sigma_{v,1,v_2} \\
    0 & 0 & \sigma_{v,1,v_2} & \sigma_{v,2}^2
\end{pmatrix}
\]
For earner \( j = \{1, 2\} \) in household \( i \), period \( t \), wage growth is:

\[
\Delta \log W_{i,j,t} = \Delta X'_{i,j,t} \beta_j + \Delta u_{i,j,t} + v_{i,j,t}
\]

\[
\begin{pmatrix}
    u_{i,1,t} \\
    u_{i,2,t} \\
    v_{i,1,t} \\
    v_{i,2,t}
\end{pmatrix}
\sim i.i.d.
\begin{pmatrix}
    0 \\
    0 \\
    0 \\
    0
\end{pmatrix},
\begin{pmatrix}
    \sigma_{u,1}^2 & \sigma_{u,1,u_2} & 0 & 0 \\
    \sigma_{u,1,u_2} & \sigma_{u,2}^2 & 0 & 0 \\
    0 & 0 & \sigma_{v,1}^2 & \sigma_{v,1,v_2} \\
    0 & 0 & \sigma_{v,1,v_2} & \sigma_{v,2}^2
\end{pmatrix}
\]

- Allow the variances to differ by gender and across the life-cycle.
## Wage Parameters Estimates

### Baseline

<table>
<thead>
<tr>
<th>Sample</th>
<th></th>
<th></th>
<th>All</th>
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<tbody>
<tr>
<td></td>
<td>Trans.</td>
<td>Perm.</td>
<td></td>
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<tr>
<td>Males</td>
<td>$\sigma^2_{u_1}$</td>
<td>$\sigma^2_{v_1}$</td>
<td>0.033 (0.007)</td>
</tr>
<tr>
<td>Females</td>
<td>$\sigma^2_{u_2}$</td>
<td>$\sigma^2_{v_2}$</td>
<td>0.012 (0.006)</td>
</tr>
<tr>
<td>Correlation</td>
<td>Trans.</td>
<td>Perm.</td>
<td></td>
</tr>
<tr>
<td>of shocks</td>
<td>$\rho_{u_1,u_2}$</td>
<td>$\rho_{v_1,v_2}$</td>
<td>0.244 (0.22)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.113 (0.07)</td>
</tr>
</tbody>
</table>
CONSUMPTION AND EARNINGS GROWTH

The ‘Simple’ Separable Case

\[
\begin{pmatrix}
\Delta c_t \\
\Delta y_{1,t} \\
\Delta y_{2,t}
\end{pmatrix}
\sim
\begin{pmatrix}
0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \\
\kappa_{y_1,u_1} & 0 & \kappa_{y_1,v_1} & \kappa_{y_1,v_2} \\
0 & \kappa_{y_2,u_2} & \kappa_{y_2,v_1} & \kappa_{y_2,v_2}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{1,t} \\
\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]
The ‘Simple’ Separable Case

\[
\begin{pmatrix}
\Delta c_t \\
\Delta y_{1,t} \\
\Delta y_{2,t}
\end{pmatrix}
\approx
\begin{pmatrix}
0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \\
\kappa_{y_1,u_1} & 0 & \kappa_{y_1,v_1} & \kappa_{y_1,v_2} \\
0 & \kappa_{y_2,u_2} & \kappa_{y_2,v_1} & \kappa_{y_2,v_2}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{1,t} \\
\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]

where the key transmission parameters

\[
\kappa_{y_j,u_j} = \left(1 + \eta_{h_j,w_j}\right) \to \text{[Frisch]}
\]
CONSUMPTION AND EARNINGS GROWTH

The ’Simple’ Separable Case

\[
\begin{pmatrix}
\Delta c_t \\
\Delta y_{1,t} \\
\Delta y_{2,t}
\end{pmatrix} \approx
\begin{pmatrix}
0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \\
\kappa_{y_1,u_1} & 0 & \kappa_{y_1,v_1} & \kappa_{y_1,v_2} \\
0 & \kappa_{y_2,u_2} & \kappa_{y_2,v_1} & \kappa_{y_2,v_2}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{1,t} \\
\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]

where the key transmission parameters

\[
\kappa_{y_j,u_j} = \left( 1 + \eta_{h_j,w_j} \right) \rightarrow \text{[Frisch]} \quad \kappa_{y_j,v_j} \rightarrow \text{[Marshall]}
\]
CONSUMPTION AND EARNINGS GROWTH

The ‘Simple’ Separable Case

\[
\begin{pmatrix}
\Delta c_t \\
\Delta y_{1,t} \\
\Delta y_{2,t}
\end{pmatrix}
\approx
\begin{pmatrix}
0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \\
\kappa_{y_1,u_1} & 0 & \kappa_{y_1,v_1} & \kappa_{y_1,v_2} \\
0 & \kappa_{y_2,u_2} & \kappa_{y_2,v_1} & \kappa_{y_2,v_2}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{1,t} \\
\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]

where the key transmission parameters

\[\kappa_{y_j,u_j} = \left(1 + \eta_{h_j,w_j}\right) \rightarrow \text{[Frisch]} \quad \kappa_{y_j,v_j} \rightarrow \text{[Marshall]}\]

\[\kappa_{c,v_j} = (1 - \pi_{i,t}) s_{i,j,t} \frac{\eta_{c,p} \left(1 + \eta_{h_j,w_j}\right)}{\eta_{c,p} + \left(1 - \pi_{i,t}\right) \eta_{h,w}}\]
CONSUMPTION AND EARNINGS GROWTH

The ‘Simple’ Separable Case

\[
\begin{pmatrix}
\Delta c_t \\
\Delta y_{1,t} \\
\Delta y_{2,t}
\end{pmatrix}
\approx
\begin{pmatrix}
0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \\
\kappa_{y_{1,u_1}} & 0 & \kappa_{y_{1,v_1}} & \kappa_{y_{1,v_2}} \\
0 & \kappa_{y_{2,u_2}} & \kappa_{y_{2,v_1}} & \kappa_{y_{2,v_2}}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{1,t} \\
\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]

where the key transmission parameters

\[
\kappa_{y_{j,u_j}} = \left(1 + \eta_{h_{j,w_j}}\right) \rightarrow [\text{Frisch}] \quad \kappa_{y_{j,v_j}} \rightarrow [\text{Marshall}]
\]

\[
\kappa_{c,v_j} = \frac{\eta_{c,p} \left(1 + \eta_{h_{j,w_j}}\right)}{\eta_{c,p} + \left(1 - \pi_{i,t}\right) \eta_{h,w}}
\]

\[
\pi_{i,t} \approx \frac{\text{Assets}_{i,t}}{\text{Assets}_{i,t} + \text{Human Wealth}_{i,t}}
\]
CONSUMPTION AND EARNINGS GROWTH

The 'Simple' Separable Case

\[
\begin{pmatrix}
\Delta c_t \\
\Delta y_{1,t} \\
\Delta y_{2,t}
\end{pmatrix}
\approx
\begin{pmatrix}
0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \\
\kappa_{y_{1,u_1}} & 0 & \kappa_{y_{1,v_1}} & \kappa_{y_{1,v_2}} \\
0 & \kappa_{y_{2,u_2}} & \kappa_{y_{2,v_1}} & \kappa_{y_{2,v_2}}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{1,t} \\
\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]

where the key transmission parameters

\[
\kappa_{y_{j,u_j}} = \left(1 + \eta_{h_{j,w_j}}\right) \rightarrow [Frisch] \quad \kappa_{y_{j,v_j}} \rightarrow [Marshall]
\]

\[
\kappa_{c,v_j} = \left(1 - \pi_{i,t}\right) s_{i,j,t} \frac{\eta_{c,p}}{\eta_{c,p} + \eta_{c,p} \left(1 + \eta_{h_{j,w_j}}\right)}
\]

\[
s_{i,j,t} \approx \frac{\text{Human Wealth}_{i,j,t}}{\text{Human Wealth}_{i,t}}
\]
CONSUMPTION AND EARNINGS GROWTH

The 'Simple' Separable Case

\[
\begin{pmatrix}
\Delta c_t \\
\Delta y_{1,t} \\
\Delta y_{2,t}
\end{pmatrix}
\sim
\begin{pmatrix}
0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \\
\kappa_{y_1,u_1} & 0 & \kappa_{y_1,v_1} & \kappa_{y_1,v_2} \\
0 & \kappa_{y_2,u_2} & \kappa_{y_2,v_1} & \kappa_{y_2,v_2}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{1,t} \\
\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]

where the key transmission parameters

\[
\kappa_{y_j,u_j} = \left(1 + \eta_{h_j,w_j}\right) \rightarrow \text{[Frisch]} \quad \kappa_{y_j,v_j} \rightarrow \text{[Marshall]}
\]

\[
\kappa_{c,v_j} = \frac{\eta_{c,p} \left(1 + \eta_{h_j,w_j}\right)}{(1 - \pi_{i,t}) \left(s_{i,j,t} + \eta_{c,p} + \eta_{c,p} \left(1 - \pi_{i,t}\right) \overline{\eta}_{h,w}\right)}
\]

\[
\overline{\eta}_{h,w} = s_{i,j,t} \eta_{h_j,w_j} + s_{i,-j,t} \eta_{h_{-j},w_{-j}}
\]
The 'Simple' Separable Case

\[
\begin{pmatrix}
\Delta c_t \\
\Delta y_{1,t} \\
\Delta y_{2,t}
\end{pmatrix}
\approx
\begin{pmatrix}
0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \\
\kappa_{y_1,u_1} & 0 & \kappa_{y_1,v_1} & \kappa_{y_1,v_2} \\
0 & \kappa_{y_2,u_2} & \kappa_{y_2,v_1} & \kappa_{y_2,v_2}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{1,t} \\
\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]

Introduce now \( \beta \), representing insurance over and above savings, taxes and labour supply \( \rightarrow \) networks, etc.

Key transmission parameter becomes:

\[
\kappa_{c,v_j} = (1 - \beta) (1 - \pi_{i,t}) s_{i,j,t} \frac{\eta_{c,p} (1 + \eta_{h,j,w_j})}{\eta_{c,p} + (1 - \beta) (1 - \pi_{i,t}) \eta_{h,w}}
\]
Identification with Asset Data

- Note that $\beta$ is not identified separately from $\pi$
- Back out $\pi$ from the data and estimate $\beta$

$$\pi_{i,t} \approx \frac{Observed\ in\ PSID}{\text{Human Wealth}_{i,t} + \text{Assets}_{i,t}}$$

- Human wealth is projected using observables that evolve deterministically (e.g. age).
When preferences are non-separable, we have:

$$\begin{pmatrix} \Delta c_t \\ \Delta y_{1,t} \\ \Delta y_{2,t} \end{pmatrix} \sim \begin{pmatrix} \kappa_{c,u_1} & \kappa_{c,u_2} & \kappa_{c,v_1} & \kappa_{c,v_2} \\ \kappa_{y_{1,u_1}} & \kappa_{y_{1,u_2}} & \kappa_{y_{1,v_1}} & \kappa_{y_{1,v_2}} \\ \kappa_{y_{2,u_1}} & \kappa_{y_{2,u_2}} & \kappa_{y_{2,v_1}} & \kappa_{y_{2,v_2}} \end{pmatrix} \begin{pmatrix} \Delta u_{1,t} \\ \Delta u_{2,t} \\ \nu_{1,t} \\ \nu_{2,t} \end{pmatrix}$$

- $\kappa_{c,u_j}$ → non-separability between consumption and leisure $j$
- $\kappa_{y_j,u_k}$ → non-separability between spouses’ leisures
Distribution of $s$ by Age

\[ s_{i,t} \approx \frac{\text{Human Wealth}_{\text{male},i,t}}{\text{Human Wealth}_{i,t}} \]
DISTRIBUTION OF $s$ BY AGE

$$s_{i,t} \approx \frac{\text{Human Wealth}_{male,i,t}}{\text{Human Wealth}_{i,t}}.$$
**Distribution of \( \pi \) by Age**

\[ \pi_{i,t} \approx \frac{\text{Assets}_{i,t}}{\text{Assets}_{i,t} + \text{Human Wealth}_{i,t}} : \]

Graph showing the distribution of \( \pi \) by age, with the y-axis representing \( \pi \) values ranging from 0 to 0.5, and the x-axis representing age groups (30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-65). The graph includes a line for Total Assets (Thousands of Dollars) and another line for \( \pi \).
Distribution of $\pi$ by Age

$\pi_{i,t} \approx \frac{\text{Assets}_{i,t}}{\text{Assets}_{i,t} + \text{Human Wealth}_{i,t}}$:
## Results: With and Without Separability

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<tbody>
<tr>
<td></td>
<td>$(1)$</td>
<td>$(2)$</td>
<td>$(3)$</td>
</tr>
<tr>
<td>$E(\pi)$</td>
<td>0.181</td>
<td>0.181</td>
<td>0.181</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.741</td>
<td>-0.120</td>
<td>0</td>
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<tr>
<td></td>
<td>(0.085)</td>
<td>(0.098)</td>
<td></td>
</tr>
<tr>
<td>$\eta_{c,p}$</td>
<td>0.201</td>
<td>0.437</td>
<td>0.448</td>
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<tr>
<td></td>
<td>(0.077)</td>
<td>(0.124)</td>
<td>(0.126)</td>
</tr>
<tr>
<td>$\eta_{h_1,w_1}$</td>
<td>0.431</td>
<td>0.514</td>
<td>0.497</td>
</tr>
<tr>
<td></td>
<td>(0.097)</td>
<td>(0.150)</td>
<td>(0.150)</td>
</tr>
<tr>
<td>$\eta_{h_2,w_2}$</td>
<td>0.831</td>
<td>1.032</td>
<td>1.041</td>
</tr>
<tr>
<td></td>
<td>(0.133)</td>
<td>(0.265)</td>
<td>(0.275)</td>
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<tr>
<td>$\eta_{c,w_1}$</td>
<td>--.</td>
<td>-0.141</td>
<td>-0.141</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.051)</td>
<td>(0.053)</td>
</tr>
<tr>
<td>$\eta_{h_1,p}$</td>
<td>--.</td>
<td>0.082</td>
<td>0.082</td>
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<tr>
<td></td>
<td></td>
<td>(0.030)</td>
<td>(0.031)</td>
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<tr>
<td>$\eta_{c,w_2}$</td>
<td>--.</td>
<td>-0.138</td>
<td>-0.158</td>
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<tr>
<td></td>
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<td>(0.139)</td>
<td>(0.121)</td>
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<tr>
<td>$\eta_{h_2,p}$</td>
<td>--.</td>
<td>0.162</td>
<td>0.185</td>
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<td>(0.145)</td>
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<tr>
<td>$\eta_{h_1,w_2}$</td>
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<td>0.128</td>
<td>0.120</td>
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<tr>
<td></td>
<td></td>
<td>(0.052)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>$\eta_{h_2,w_1}$</td>
<td>--.</td>
<td>0.258</td>
<td>0.242</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.103)</td>
<td>(0.119)</td>
</tr>
</tbody>
</table>
Marshallian Elasticities: By Age

Wife's Marshallian Elasticity ($\kappa_{12}$)

Age of household head

30-34  35-39  40-44  45-49  50-54  55-59  60-65
Marshallian Elasticities: By Age

Head's Marshallian Elasticity ($\kappa_7$)

Age of household head

95th perc., 90th perc., 75th perc., 50th perc., 25th perc., 10th perc., 5th perc.
**INTERPRETATION: INSURANCE VIA LABOR SUPPLY (SHOCK TO MALE WAGES)**

The average response of total earnings \( (y = y_1 + y_2) \) to a permanent shock to the male’s wages:

\[
\frac{\partial \Delta y}{\partial v_1} = s \cdot \frac{\partial \Delta y_1}{\partial v_1} + (1-s) \cdot \frac{\partial \Delta y_2}{\partial v_1} = 0.44
\]

\[
\hat{s} = 0.69, \quad \hat{\kappa}_{y_1,v_1} = 0.98, \quad \hat{\kappa}_{y_2,v_1} = -0.81
\]

Response of consumption to a 10% permanent decrease in the male’s wage rate \((v_1 = 0.1)\):

- one earner, fixed labor supply and no insurance -10%
- two earners, fixed labor supply and no insurance -6.9%
- with husband labor supply adjustment -6.8%
- with family labor supply adjustment -4.4%
- with family labor supply adjustment and other insurance -3.8%

---

*Blundell, UCL & IFS ( ) Consumption and Family Labor Supply ECB October 2013 31 / 56*
**INTERPRETATION: INSURANCE VIA LABOR SUPPLY (SHOCK TO MALE WAGES)**

The average response of total earnings \( (y = y_1 + y_2) \) to a permanent shock to the male’s wages:

\[
\frac{\partial \Delta y}{\partial v_1} = \hat{s} \cdot \frac{\partial \Delta y_1}{\partial v_1} + (1 - \hat{s}) \cdot \frac{\partial \Delta y_2}{\partial v_1} = 0.44
\]

\( \hat{s} = 0.69 \)
\( \hat{\kappa}_{y_1,v_1} = 0.98 \)
\( \hat{\kappa}_{y_2,v_1} = 0.81 \)

Response of consumption to a 10% permanent decrease in the male’s wage rate \( (v_1 = -0.1) \):

one earner, fixed labor supply and no insurance -10%
**INTERPRETATION: INSURANCE VIA LABOR SUPPLY (SHOCK TO MALE WAGES)**

The average response of total earnings \( y = y_1 + y_2 \) to a permanent shock to the male’s wages:

\[
\frac{\partial \Delta y}{\partial v_1} = \hat{s} \cdot \frac{\partial \Delta y_1}{\partial v_1} + (1 - \hat{s}) \cdot \frac{\partial \Delta y_2}{\partial v_1} = 0.44
\]

\[
\hat{s} = 0.69, \quad \hat{\kappa}_{y_1,v_1} = 0.98, \quad \hat{\kappa}_{y_2,v_1} = -0.81
\]

Response of consumption to a 10% permanent decrease in the male’s wage rate \( v_1 = -0.1 \):

- one earner, fixed labor supply and no insurance: -10%
- two earners, fixed labor supply and no insurance: -6.9%
**INTERPRETATION: INSURANCE VIA LABOR SUPPLY (SHOCK TO MALE WAGES)**

The average response of total earnings \( (y = y_1 + y_2) \) to a permanent shock to the male’s wages:

\[
\frac{\partial \Delta y}{\partial v_1} = s \left( \frac{\partial \Delta y_1}{\partial v_1} \right) + (1-s) \left( \frac{\partial \Delta y_2}{\partial v_1} \right) = 0.44
\]

\[
\hat{s} = 0.69, \quad \hat{k}_{y_1,v_1} = 0.98, \quad \hat{k}_{y_2,v_1} = -0.81
\]

Response of *consumption* to a 10% permanent decrease in the male’s wage rate \( (v_1 = -0.1) \):

- one earner, fixed labor supply and no insurance: -10%
- two earners, fixed labor supply and no insurance: -6.9%
- with husband labor supply adjustment: -6.8%
**INTERPRETATION: INSURANCE VIA LABOR SUPPLY (SHOCK TO MALE WAGES)**

The average response of total earnings \((y = y_1 + y_2)\) to a permanent shock to the male’s wages:

\[
\frac{\partial \Delta y}{\partial v_1} = s \left( \frac{\partial \Delta y_1}{\partial v_1} \right) + (1 - s) \left( \frac{\partial \Delta y_2}{\partial v_1} \right) = 0.44
\]

\(s = 0.69\), \(\hat{\kappa}_{y_1, v_1} = 0.98\), \(1 - s = 0.31\), \(\hat{\kappa}_{y_2, v_1} = -0.81\)

Response of consumption to a 10% permanent decrease in the male’s wage rate \((v_1 = -0.1)\):

- one earner, fixed labor supply and no insurance: -10%
- two earners, fixed labor supply and no insurance: -6.9%
- with husband labor supply adjustment: -6.8%
- with family labor supply adjustment: -4.4%
INTERPRETATION: INSURANCE VIA LABOR SUPPLY (SHOCK TO MALE WAGES)

The average response of total earnings \((y = y_1 + y_2)\) to a permanent shock to the male’s wages:

\[
\frac{\partial \Delta y}{\partial v_1} = s * \frac{\partial \Delta y_1}{\partial v_1} + (1 - s) * \frac{\partial \Delta y_2}{\partial v_1} = 0.44
\]

\(s = 0.69\) \(\hat{\kappa}_{y_1,v_1} = 0.98\) \(1 - s = 0.31\) \(\hat{\kappa}_{y_2,v_1} = -0.81\)

Response of consumption to a 10% permanent decrease in the male’s wage rate \((v_1 = -0.1)\):

- one earner, fixed labor supply and no insurance: -10%
- two earners, fixed labor supply and no insurance: -6.9%
- with husband labor supply adjustment: -6.8%
- with family labor supply adjustment: -4.4%
- with family labor supply adjustment and other insurance: -3.8%
Response of Consumption to a 10% Permanent Decrease in the Male’s Wage Rate

- fixed labor supply and no insurance
- with family labor supply adjustment
- with family labor supply adjustment and other insurance
INSURANCE VIA LABOR SUPPLY (SHOCK TO MALE WAGES): BY AGE

Consumption Response to a -10% Permanent Shock to Head's Wages ($\kappa_3$)
The average response of total earnings to a permanent shock to the female’s wages:

\[
\frac{\partial \Delta y}{\partial v_2} = s \cdot \frac{\partial \Delta y_1}{\partial v_2} + (1 - s) \cdot \frac{\partial \Delta y_2}{\partial v_2} = 0.25
\]

Response of consumption to a 10% permanent decrease in the female’s wage rate ($v_2 = -0.1$):

- two earners, fixed labor supply and no insurance: -3.1%
**INTERPRETATION: INSURANCE VIA LABOR SUPPLY (SHOCK TO FEMALE WAGES)**

The average response of total earnings to a permanent shock to the female’s wages:

\[
\frac{\partial \Delta y}{\partial v_2} = s \left( \frac{\partial \Delta y_1}{\partial v_2} \right) + (1 - s) \left( \frac{\partial \Delta y_2}{\partial v_2} \right) = 0.25
\]

\[\kappa_{y_1,v_2}=-0.23\quad \kappa_{y_2,v_2}=1.32\]

Response of consumption to a 10% permanent decrease in the female’s wage rate \((v_2 = -0.1)\):

- two earners, fixed labor supply and no insurance \(-3.1\%\)
- with family labor supply adjustment \(-2.5\%\)
**INTERPRETATION: INSURANCE VIA LABOR SUPPLY (SHOCK TO FEMALE WAGES)**

The average response of total earnings to a permanent shock to the female’s wages:

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\frac{\partial \Delta y}{\partial v_2} = s \frac{\partial \Delta y_1}{\partial v_2} + (1 - s) \frac{\partial \Delta y_2}{\partial v_2} = 0.25
\]

Response of consumption to a 10% permanent decrease in the female’s wage rate \((v_2 = -0.1)\):

- two earners, fixed labor supply and no insurance: -3.1%
- with family labor supply adjustment: -2.5%
- with family labor supply adjustment and other insurance: -2.1%
SUMMARY AND CONCLUSIONS...

- Focus on understanding the transmission of inequality over the working life.
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Summary and Conclusions...

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- Once family labor supply, assets and taxes (and benefits) are properly accounted for, there is less evidence for additional insurance,
  - lots to be done to dig deeper into these, and other, mechanisms.
  - consider detailed consumption components and form of non-separability.
## Results by Age, Education and Asset Selections

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Age 30-55</th>
<th>Some college+</th>
<th>Top 2 asset terc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\pi)$</td>
<td>0.181</td>
<td>0.142</td>
<td>0.202</td>
<td>0.245</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.120</td>
<td>-0.177</td>
<td>0.117</td>
<td>-0.046</td>
</tr>
<tr>
<td></td>
<td>(0.098)</td>
<td>(0.089)</td>
<td>(0.072)</td>
<td>(0.084)</td>
</tr>
<tr>
<td>$\eta_{c,p}$</td>
<td>0.437</td>
<td>0.465</td>
<td>0.368</td>
<td>0.343</td>
</tr>
<tr>
<td></td>
<td>(0.124)</td>
<td>(0.044)</td>
<td>(0.05)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>$\eta_{h_1,w_1}$</td>
<td>0.514</td>
<td>0.467</td>
<td>0.542</td>
<td>0.388</td>
</tr>
<tr>
<td></td>
<td>(0.150)</td>
<td>(0.036)</td>
<td>(0.045)</td>
<td>(0.037)</td>
</tr>
<tr>
<td>$\eta_{h_2,w_2}$</td>
<td>1.032</td>
<td>1.039</td>
<td>0.858</td>
<td>0.986</td>
</tr>
<tr>
<td></td>
<td>(0.265)</td>
<td>(0.099)</td>
<td>(0.097)</td>
<td>(0.105)</td>
</tr>
<tr>
<td>$\eta_{c,w_1}$</td>
<td>-0.141</td>
<td>-0.113</td>
<td>-0.162</td>
<td>-0.127</td>
</tr>
<tr>
<td></td>
<td>(0.051)</td>
<td>(0.018)</td>
<td>(0.022)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>$\eta_{h_1,p}$</td>
<td>0.082</td>
<td>0.065</td>
<td>0.087</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.01)</td>
<td>(0.012)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>$\eta_{c,w_2}$</td>
<td>-0.138</td>
<td>-0.083</td>
<td>-0.142</td>
<td>-0.129</td>
</tr>
<tr>
<td></td>
<td>(0.139)</td>
<td>(0.029)</td>
<td>(0.032)</td>
<td>(0.154)</td>
</tr>
<tr>
<td>$\eta_{h_2,p}$</td>
<td>0.162</td>
<td>0.097</td>
<td>0.169</td>
<td>0.154</td>
</tr>
<tr>
<td></td>
<td>(0.166)</td>
<td>(0.034)</td>
<td>(0.038)</td>
<td>(0.038)</td>
</tr>
<tr>
<td>$\eta_{h_1,w_2}$</td>
<td>0.128</td>
<td>0.101</td>
<td>0.115</td>
<td>0.079</td>
</tr>
<tr>
<td></td>
<td>(0.052)</td>
<td>(0.011)</td>
<td>(0.012)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>$\eta_{h_2,w_1}$</td>
<td>0.258</td>
<td>0.205</td>
<td>0.255</td>
<td>0.172</td>
</tr>
<tr>
<td></td>
<td>(0.103)</td>
<td>(0.022)</td>
<td>(0.027)</td>
<td>(0.021)</td>
</tr>
</tbody>
</table>

Note: Specifications (2) to (4) - Non-bootstrap s.e.’s
Concavity and Advance Information

- **Concavity of preferences.** Use the fact that:

\[
\begin{pmatrix}
\eta_{cp} \frac{c}{p} & \eta_{cw_1} \frac{c}{w_1} & \eta_{cw_2} \frac{c}{w_2} \\
-\eta_{h_1} \frac{h_1}{p} & -\eta_{h_1} \frac{h_1}{w_1} & -\eta_{h_1} \frac{h_1}{w_2} \\
-\eta_{h_2} \frac{h_2}{p} & -\eta_{h_2} \frac{h_2}{w_1} & -\eta_{h_2} \frac{h_2}{w_2}
\end{pmatrix}
= \lambda
\begin{pmatrix}
\frac{d^2u}{dc^2} & \frac{d^2u}{dcdl_1} & \frac{d^2u}{dcdl_2} \\
\frac{d^2u}{dl_1dc} & \frac{d^2u}{dl_1^2} & \frac{d^2u}{dl_1dl_2} \\
\frac{d^2u}{dl_2dc} & \frac{d^2u}{dl_2dl_1} & \frac{d^2u}{dl_2^2}
\end{pmatrix}^{-1}
\]

- Appendix shows concavity cannot rejected, and is numerically satisfied at average values of wages, hours, consumption.
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\end{pmatrix}
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\begin{pmatrix}
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\end{pmatrix}^{-1}
\]

- Appendix shows concavity cannot be rejected, and is numerically satisfied at average values of wages, hours, consumption.

- **Advance Information.** Consumption growth should be correlated with future wage growth (Cunha et al., 2008, and BPP 2008).
  - Test has p-value 13%
**RESULTS: EXTENSIVE MARGIN**

- Estimate a "conditional" Euler equation, controlling for changes in hours (intensive margin) and changes in participation (extensive margin)
**Results: Extensive Margin**

Estimate a "conditional" Euler equation, controlling for changes in hours (intensive margin) and changes in participation (extensive margin)

<table>
<thead>
<tr>
<th></th>
<th>Regression results</th>
<th>First stage F-stats</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td><strong>$\Delta EMP_t (Male)$</strong></td>
<td>0.144 (0.269)</td>
<td>23.4</td>
</tr>
<tr>
<td><strong>$\Delta h_t (Male)$</strong></td>
<td>$-0.073 \ (0.075)$</td>
<td>$-0.013 \ (0.021)$</td>
</tr>
<tr>
<td><strong>$\Delta EMP_t (Female)$</strong></td>
<td>0.356 (0.169)</td>
<td>98.4</td>
</tr>
<tr>
<td><strong>$\Delta h_t (Female)$</strong></td>
<td>$-0.220 \ (0.100)$</td>
<td>$-0.171 \ (0.094)$</td>
</tr>
</tbody>
</table>

Sample All

Instruments $2^{nd}, 4^{th}$ lags

Note: $\Delta x_t$ is defined as $(x_t - x_{t-1}) / [0.5 \ (x_t + x_{t-1})]$
## Wage Parameters by Assets and Age

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sample</strong></td>
<td>All</td>
<td>$1^{st}$ asset tercile</td>
<td>$2^{nd}, 3^{rd}$ asset terciles</td>
<td>age&lt;40</td>
<td>age&gt;=40</td>
</tr>
<tr>
<td><strong>Males</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trans. $\sigma^2_{u1}$</td>
<td>0.033 (0.007)</td>
<td>0.03 (0.009)</td>
<td>0.042 (0.009)</td>
<td>0.042 (0.013)</td>
<td>0.028 (0.008)</td>
</tr>
<tr>
<td>Perm. $\sigma^2_{v1}$</td>
<td>0.035 (0.005)</td>
<td>0.027 (0.006)</td>
<td>0.039 (0.007)</td>
<td>0.025 (0.009)</td>
<td>0.039 (0.007)</td>
</tr>
<tr>
<td><strong>Females</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trans. $\sigma^2_{u2}$</td>
<td>0.012 (0.005)</td>
<td>0.023 (0.009)</td>
<td>0.011 (0.007)</td>
<td>0.02 (0.015)</td>
<td>0.01 (0.005)</td>
</tr>
<tr>
<td>Perm. $\sigma^2_{v2}$</td>
<td>0.046 (0.004)</td>
<td>0.036 (0.007)</td>
<td>0.05 (0.006)</td>
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<td>0.042 (0.005)</td>
</tr>
<tr>
<td><strong>Correlations of Shocks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trans. $\sigma_{u1,u2}$</td>
<td>0.202 (0.159)</td>
<td>-0.264 (0.181)</td>
<td>0.39 (0.197)</td>
<td>0.459 (0.28)</td>
<td>0.115 (0.201)</td>
</tr>
<tr>
<td>Perm. $\sigma_{v1,v2}$</td>
<td>0.153 (0.06)</td>
<td>0.366 (0.142)</td>
<td>0.096 (0.066)</td>
<td>0.041 (0.174)</td>
<td>0.162 (0.063)</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>8,191</td>
<td>2,626</td>
<td>5,565</td>
<td>2,172</td>
<td>6,019</td>
</tr>
</tbody>
</table>

---

**Sources:**

- **Blundell, UCL & IFS ( ):**
- **Consumption and Family Labor Supply:**
- **ECB October 2013:**
**Transmission Parameters:**
Consumption response to $j$’s permanent wage shock:

$$\kappa_{c,vj} = (1 - \beta) (1 - \pi_{i,t}) s_{i,j,t} \frac{\eta_{c,p} (1 + \eta_{h,j,w})}{\eta_{c,p} + (1 - \beta) (1 - \pi_{i,t}) \eta_{h,w}}$$
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$$

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- declines with $\eta_{h_j,w_j}$ only if $j$’s labor supply responds negatively to own permanent shock. In one-earner case, true if

$$(1 - \beta) (1 - \pi_{i,t}) - \eta_{c,p} > 0$$
Data and Sample Selection

- **PSID biennial 1999-2009:**
  - PSID consumption went through a major revision in 1999
    - ~70% of consumption expenditures. Good match with NIPA
    - The sum of food at home, food away from home, gasoline, health, transportation, utilities, etc.
    - Main items that are missing: clothing (now included), recreation, alcohol and tobacco
  - Earning and hours for each earner
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  - Stable household composition
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- Methodology: Use structural restrictions that ‘theory’ imposes on the variance covariance structure of $\Delta c_{i,t}$, $\Delta y_{i,1,t}$ and $\Delta y_{i,2,t}$
Some Econometrics Issues

- Measurement error
  - For consumption, use martingale assumption and mean-reversion
  - For wages, use external estimates from Bound et al. (1994)
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  - Selection adjusted second moments
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- Inference
  - Multi-step procedure
  - Block bootstrap standard errors
**Inference**

- Multi-step estimation procedure:
  - Regress $c_{i,t}, y_{i,j,t}, w_{i,j,t}$ on observable characteristics, and construct the residuals $\Delta c_{i,t}, \Delta y_{i,j,t}$ and $\Delta w_{i,j,t}$
  - Estimate the wage parameters using the conditional second order moments for $\Delta w_{i,1,t}$ and $\Delta w_{i,2,t}$
  - Estimate $\pi_{i,t}$ and $s_{i,t}$ using asset and (current and projected) earnings data
  - Estimate preference parameters using restrictions on the joint behavior of $\Delta c_{i,t}, \Delta y_{i,j,t}$ and $\Delta w_{i,j,t}$

- GMM with standard errors corrected by the block bootstrap.
where $\xi_{i,j,t}, \xi_{i,t}^c$ and $\xi_{i,j,t}^y$ are measurement errors in log wages of earner $j$, log consumption, and log earnings of earner $j$. 
INCOME DYNAMICS - MORE DETAILS

- Our focus here is on non-stationarity, heterogenous profiles, and shocks of varying persistence.
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Individual $i$ of age $a$ in time period $t$, has log income $y_{i,a} (\equiv \ln Y_{i,a,t})$:

$$y_{i,a} = Z_{i,a}^{T} \varphi_{a} + f_{0i} + f_{1i}p_{a} + y_{i,a}^{P} + \varepsilon_{i,a}$$

where $\beta_{i}p_{a}$ is an individual-specific trend, allow non-zero covariance between $f_{0}$ and $f_{1}$. 
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where $\beta_ip_a$ is an individual-specific trend, allow non-zero covariance between $f_0$ and $f_1$.

$y_{i,a}^T$ is the persistent process with variance $\sigma_a^2$

\[ y_{i,a}^T = \rho y_{i,a-1}^T + v_{i,a} \]

and $\varepsilon_{i,a}$ is a transitory process (can be low order MA) with variance $\omega_a^2$ (can be low order MA).
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\[
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\]

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\[
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\]

and \( \varepsilon_{i,a} \) is a transitory process (can be low order MA) with variance \( \omega_a^2 \) (can be low order MA).

- Allow variances (or factor loadings) of \( \nu_{i,a} \) and \( \varepsilon_{i,a} \) to vary with age/time for each birth cohort and education group.
The idiosyncratic trend term $p_i f_{1i}$ could take a number of forms. Two alternatives are worth highlighting:

- (a) Deterministic idiosyncratic trend:
  
  $p_i f_{1i} = r(t) f_{1i}$

  where $r$ is a known function of $t$, e.g. $r(t) = t$, 

- (b) Stochastic trend in "ability prices":
  
  $p_i f_{1i} = p_i f_{1i} + \xi_t$ with $E_t \xi_t = 0$. 

Evidence points to some periods of time where each of these is of key importance. Deterministic trends as in (a), appear most prominently early in the working life. Formally, this is a life-cycle effect. Alternatively, stochastic trends (b) are most likely to occur during periods of technical change when skill prices are changing across the unobserved ability distribution. Formally, this is a calendar time effect.
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**Idiosyncratic Trends**

- For each cohort we consider several alternative models for the heterogenous profile $\beta_i p_a$:
Idiosyncratic Trends

- For each cohort we consider several alternative models for the heterogenous profile $\beta_i p_a$:

1. Baseline Specification: $f_{1i} = 0$

2. Linear Specification: $p_a = \gamma_1 a + \gamma_0$, so that

$$\Delta p_a = (\rho_1 + \gamma_0 \xi) + \gamma_1 \xi_0$$

3. Quadratic Specification: $p_a = \gamma_0 + \gamma_1 a + \gamma_2 a^2$

4. Piecewise-Linear Specification: $p_a = \kappa_1 a + 35 (1 - \kappa_1) a \kappa_2 + 52 (1 - \kappa_2)$ if $a < 35$ or otherwise

5. Polynomials up to degree 4.
Idiosyncratic Trends

For each cohort we consider several alternative models for the heterogenous profile $\beta_i p_a$:

1. Baseline Specification: $f_{1i} = 0$
2. Linear Specification: $p_a = \gamma_1 a + \gamma_0$, so that
   \[
   \Delta^\rho p_a = (1 - \rho) \gamma_0 i + \gamma_1 \xi_0
   \]
   where $\xi_0 \equiv [a - \rho (a - 1)]$.

3. Quadratic Specification: $p_a = \gamma_0 + \gamma_1 a + \gamma_2 a^2$
4. Piecewise-Linear Specification:
   
   \[
   p_a = \begin{cases} 
   \kappa_1 a + \gamma_0 \kappa_1 & \text{if } a < 35 \\
   \kappa_2 a + \gamma_0 \kappa_2 & \text{otherwise}
   \end{cases}
   \]
   with knots at age 35 and age 52.
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Idiosyncratic Trends

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     \[
p_a = \begin{cases} 
      \kappa_1 a + 35 (1 - \kappa_1) & \text{if } a \leq 35 \\
      a & \text{otherwise} \\
      \kappa_2 a + 52 (1 - \kappa_2) & \text{if } a \geq 52 
     \end{cases}
\]
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Idiosyncratic Trends

For each cohort we consider several alternative models for the heterogenous profile $\beta_i p_a$:

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   \end{cases}$$

   with knots at age 35 and age 52.
5. **Polynomials up to degree 4**.
Covariance structure

Suppose we observe individual \( i \) at age \( a = 1, \ldots, T \), we then have \( T - 1 \) equations \( \Delta^\rho y_{ia} (\equiv y_{i,a} - \rho y_{i,a-1}) \). In vector form

\[
\Delta^\rho y_i = ((1 - \rho) \nu, \Delta^\rho p_a) \begin{pmatrix} f_{0i} \\ f_{1i} \end{pmatrix} + v_i + \Delta^\rho \epsilon_i.
\]
### Covariance Structure

Suppose we observe individual $i$ at age $a = 1, ..., T$, we then have $T - 1$ equations $\Delta^\rho y_{ia} (\equiv y_{i,a} - \rho y_{i,a-1})$. In vector form

$$\Delta^\rho y_i = ((1 - \rho) \iota, \Delta^\rho p_a) \begin{pmatrix} f_{0i} \\ f_{1i} \end{pmatrix} + \nu_i + \Delta^\rho \varepsilon_i.$$ 

The Variance-Covariance matrix in general has the form:

$$\text{Var}(\Delta^\rho y_i) = \Omega + W$$

where $W =$

$$\begin{pmatrix}
\sigma_2^2 + \omega_2^2 + \rho^2 \omega_1^2 & -\rho \omega_2^2 & 0 & 0 \\
-\rho \omega_2^2 & \sigma_3^2 + \omega_3^2 + \rho^2 \omega_2^2 & -\rho \omega_3^2 & 0 \\
0 & -\rho \omega_3^2 & \ddots & -\rho \omega_{T-1}^2 \\
0 & 0 & -\rho \omega_{T-1}^2 & \sigma_T^2 + \omega_T^2 + \rho^2 \omega_{T-1}^2
\end{pmatrix}.$$
Suppose we observe individual \( i \) at age \( a = 1, ..., T \), we then have \( T - 1 \) equations \( \Delta^\rho y_{ia} (= y_{i,a} - \rho y_{i,a-1}) \). In vector form

\[
\Delta^\rho y_i = ((1 - \rho) \iota, \Delta^\rho p_a) \begin{pmatrix} f_{0i} \\ f_{1i} \end{pmatrix} + \nu_i + \Delta^\rho \varepsilon_i.
\]

The Variance-Covariance matrix in general has the form:

\[
\text{Var}(\Delta^\rho y_i) = \Omega + W
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where \( W = \)

\[
\begin{pmatrix}
\sigma^2_2 + \omega_2^2 + \rho^2 \omega_1^2 & -\rho \omega_2^2 & 0 & 0 \\
-\rho \omega_2^2 & \sigma^2_3 + \omega_3^2 + \rho^2 \omega_2^2 & -\rho \omega_3^2 & 0 \\
0 & -\rho \omega_3^2 & \ddots & -\rho \omega_{T-1}^2 \\
0 & 0 & \cdots & \sigma^2_T + \omega_T^2 + \rho^2 \omega_{T-1}^2
\end{pmatrix}
\]

For the linear heterogeneous profiles case:

\[
\Omega = [(1 - \rho) \iota, \xi_0] \begin{pmatrix}
\sigma^2_0 \\
\rho_{01}^2 \sigma_0 \sigma_1 \\
\rho_{01} \sigma_0 \sigma_1 \\
\sigma^2_1
\end{pmatrix} [(1 - \rho) \iota, \xi_0]^T.
\]
Removing Additive Separability: Theory

Approximating the first order conditions (intensive margin):

\[
\Delta c_{i,t} \approx \left( \eta_{c,w_1} + \eta_{c,w_2} - \eta_{c,p} \right) \Delta \ln \lambda_{i,t} \\
+ \eta_{c,w_1} \Delta w_{i,1t+1} + \eta_{c,w_2} \Delta w_{i,2t+1}
\]
REMOVING ADDITIVE SEPARABILITY: THEORY

- Approximating the first order conditions (intensive margin):
  \[
  \Delta c_{i,t} \approx \left( \eta_{c,w_1} + \eta_{c,w_2} - \eta_{c,p} \right) \Delta \ln \lambda_{i,t} \\
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  \]

- Interpretation:
  - C and H substitutes ($\eta_{c,w_j} < 0$) ⇒ Excess smoothing
  - C and H complements ($\eta_{c,w_j} > 0$) ⇒ Excess sensitivity
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\]

- Interpretation:
  - C and H substitutes \((\eta_{c,w_j} < 0) \Rightarrow\) Excess smoothing
  - C and H complements \((\eta_{c,w_j} > 0) \Rightarrow\) Excess sensitivity

- Moments

\[
\begin{pmatrix}
\Delta c_{i,t} \\
\Delta y_{i,1,t} \\
\Delta y_{i,2,t}
\end{pmatrix} \simeq
\begin{pmatrix}
\kappa_{i,c,u_1} & \kappa_{i,c,u_2} & \kappa_{i,c,v_1} & \kappa_{i,c,v_2} \\
\kappa_{i,y_{1,u_1}} & \kappa_{i,y_{1,u_2}} & \kappa_{i,y_{1,v_1}} & \kappa_{i,y_{1,v_2}} \\
\kappa_{i,y_{2,u_1}} & \kappa_{i,y_{2,u_2}} & \kappa_{i,y_{2,v_1}} & \kappa_{i,y_{2,v_2}}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{i,1,t} \\
\Delta u_{i,2,t} \\
v_{i,1,t} \\
v_{i,2,t}
\end{pmatrix}
\]

where (for \(j = 1, 2\))

\[
\kappa_{i,c,u_j} = \eta_{c,w_j}; \quad \kappa_{i,y_{j,u_j}} = 1 + \eta_{h_j,w_j}; \quad \kappa_{i,y_{j,u_{-j}}} = \eta_{h_j,w_{-j}}
\]
Non-linear Taxes

\[
\tilde{Y}_{it} = (1 - \chi_t) (H_{1,t} W_{1,t} + H_{2,t} W_{2,t})^{1-\mu_t}
\]
Non-linear Taxes

\[ \tilde{Y}_{it} = (1 - \chi_t) (H_{1,t}W_{1,t} + H_{2,t}W_{2,t})^{1-\mu_t} \]

- Implications for underlying structural preference parameters, e.g.

\[ \tilde{\eta}_{h_j, w_j} = \frac{\eta_{h_j, w_j} (1 - \mu)}{1 + \mu \eta_{h_j, w_j}} \text{(with } \tilde{\eta}_{h_j, w_j} \leq \eta_{h_j, w_j} \text{ for } 0 \leq \mu \leq 1) \]
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- Labor supply elasticities (w.r.t. W) are dampened: Return to work decreases as people cross tax brackets
## Loading Factor Matrix: Estimates

<table>
<thead>
<tr>
<th>Response to</th>
<th>Separable case</th>
<th></th>
<th>Non-separable case</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Consump.</td>
<td>Husband’s</td>
<td>Wife’s</td>
<td>Consump.</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>earnings</td>
<td>earnings</td>
<td>(4)</td>
</tr>
<tr>
<td>$v_1$</td>
<td>0.13</td>
<td>1.15</td>
<td>-0.54</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
<td>(0.067)</td>
<td>(0.206)</td>
<td>(0.057)</td>
</tr>
<tr>
<td>$v_2$</td>
<td>0.07</td>
<td>-0.16</td>
<td>1.53</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.057)</td>
<td>(0.101)</td>
<td>(0.037)</td>
</tr>
<tr>
<td>$\Delta u_1$</td>
<td>0</td>
<td>1.43</td>
<td>0</td>
<td>-0.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.097)</td>
<td></td>
<td>(0.051)</td>
</tr>
<tr>
<td>$\Delta u_2$</td>
<td>0</td>
<td>0</td>
<td>1.83</td>
<td>-0.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.133)</td>
<td>(0.139)</td>
</tr>
</tbody>
</table>
**Heterogeneity:**

<table>
<thead>
<tr>
<th></th>
<th>(1) Baseline</th>
<th>(2) Age 30-55</th>
<th>(3) Some college+</th>
<th>(4) Top 2 asset terc.</th>
<th>(5) Age variance</th>
<th>(6) Sel.correct.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\pi)$</td>
<td>0.181</td>
<td>0.142</td>
<td>0.202</td>
<td>0.245</td>
<td>0.181</td>
<td>0.176</td>
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<tr>
<td>$\beta$</td>
<td>-0.120</td>
<td>-0.177</td>
<td>0.117</td>
<td>-0.046</td>
<td>-0.109</td>
<td>-0.129</td>
</tr>
<tr>
<td></td>
<td>(0.198)</td>
<td>(0.089)</td>
<td>(0.072)</td>
<td>(0.084)</td>
<td>(0.077)</td>
<td>(0.076)</td>
</tr>
<tr>
<td>$\eta_{c,p}$</td>
<td>0.437</td>
<td>0.465</td>
<td>0.368</td>
<td>0.343</td>
<td>0.42</td>
<td>0.473</td>
</tr>
<tr>
<td></td>
<td>(0.124)</td>
<td>(0.044)</td>
<td>(0.05)</td>
<td>(0.04)</td>
<td>(0.037)</td>
<td>(0.041)</td>
</tr>
<tr>
<td>$\eta_{h1,w1}$</td>
<td>0.514</td>
<td>0.467</td>
<td>0.542</td>
<td>0.388</td>
<td>0.575</td>
<td>0.509</td>
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<td>(0.045)</td>
<td>(0.037)</td>
<td>(0.04)</td>
<td>(0.038)</td>
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<tr>
<td>$\eta_{h2,w2}$</td>
<td>1.032</td>
<td>1.039</td>
<td>0.858</td>
<td>0.986</td>
<td>1.005</td>
<td>1.095</td>
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<tr>
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<td>(0.099)</td>
<td>(0.097)</td>
<td>(0.105)</td>
<td>(0.086)</td>
<td>(0.092)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
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<th>(5) Age variance</th>
<th>(6) Sel.correct.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{c,w1}$</td>
<td>-0.141</td>
<td>-0.113</td>
<td>-0.162</td>
<td>-0.127</td>
<td>-0.15</td>
<td>-0.150</td>
</tr>
<tr>
<td></td>
<td>(0.051)</td>
<td>(0.018)</td>
<td>(0.022)</td>
<td>(0.016)</td>
<td>(0.018)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>$\eta_{h1,p}$</td>
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<td>0.065</td>
<td>0.087</td>
<td>0.07</td>
<td>0.087</td>
<td>0.088</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.01)</td>
<td>(0.012)</td>
<td>(0.009)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>$\eta_{c,w2}$</td>
<td>-0.138</td>
<td>-0.083</td>
<td>-0.142</td>
<td>-0.129</td>
<td>-0.11</td>
<td>-0.122</td>
</tr>
<tr>
<td></td>
<td>(0.139)</td>
<td>(0.029)</td>
<td>(0.032)</td>
<td>(0.154)</td>
<td>(0.026)</td>
<td>(0.028)</td>
</tr>
<tr>
<td>$\eta_{h2,p}$</td>
<td>0.162</td>
<td>0.097</td>
<td>0.169</td>
<td>0.154</td>
<td>0.129</td>
<td>0.143</td>
</tr>
<tr>
<td></td>
<td>(0.166)</td>
<td>(0.034)</td>
<td>(0.038)</td>
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<td>(0.033)</td>
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<tr>
<td>$\eta_{h1,w2}$</td>
<td>0.128</td>
<td>0.101</td>
<td>0.115</td>
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<td>0.141</td>
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<td>(0.012)</td>
<td>(0.01)</td>
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<td>(0.01)</td>
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<tr>
<td>$\eta_{h2,w1}$</td>
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<td>0.205</td>
<td>0.255</td>
<td>0.172</td>
<td>0.285</td>
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<td></td>
<td>(0.103)</td>
<td>(0.022)</td>
<td>(0.027)</td>
<td>(0.021)</td>
<td>(0.022)</td>
<td>(0.021)</td>
</tr>
</tbody>
</table>

Note: Specifications (2) to (6) - Non-bootstrap s.e.’s
Approximation of the Euler Equation (1)

- From $\lambda_{i,t} = \frac{1+\delta}{1+r} E_t \lambda_{i,t+1}$, use a second order Taylor approximation (with $r = \delta$) to yield:

$$\Delta \ln \lambda_{i,t+1} \approx \omega_t + \varepsilon_{i,t+1}$$

- where

$$\omega_t = -\frac{1}{2} E_t (\Delta \ln \lambda_{i,t+1})^2$$

$$\varepsilon_{i,t+1} = \Delta \ln \lambda_{i,t+1} - E_t (\Delta \ln \lambda_{i,t+1})$$

- Then use the fact that

$$\Delta \ln U_{C_{i,t+1}} = \Delta \ln \lambda_{i,t+1}$$

$$\Delta \ln U_{H_{i,j,t+1}} = -\Delta \ln \lambda_{i,t+1} - \Delta \ln W_{i,j,t+1}$$
Approximation of the Euler Equation (2)

- Consider now Taylor expansion of $U_{C_{i,t+1}}(\lambda_{i,t+1})$:

\[
\frac{U_{C_{i,t+1}} - U_{C_{i,t}}}{U_{C_{i,t}}} \approx \frac{C_{i,t+1} - C_{i,t}}{C_{i,t}} \frac{U_{C_{i,t}C_{i,t}}}{U_{C_{i,t}}}
\]
\[
\Delta \ln U_{C_{i,t+1}} \approx -\frac{1}{\eta_{c,p}} \Delta \ln C_{i,t+1}
\]

- and therefore, from

\[
\Delta \ln \lambda_{i,t+1} \approx \omega_{t+1} + \varepsilon_{i,t+1}
\]

- get

\[
\Delta \ln C_{i,t+1} = -\eta_{c,p} (\omega_{t+1} + \varepsilon_{i,t+1})
\]
APPROXIMATION OF THE LIFE TIME BUDGET CONSTRAINT

- Use the fact that

\[
\mathbb{E}_I \left[ \ln \sum_{i=0}^{T-t} X_{t+i} \right] = \ln \sum_{i=0}^{T-t} \exp \mathbb{E}_{t-1} \ln X_{t+i} + \sum_{i=0}^{T-t} \frac{\exp \mathbb{E}_{t-1} \ln X_{t+i}}{\sum_{j=0}^{T-t} \exp \mathbb{E}_{t-1} \ln X_{t+j}} (\mathbb{E}_I - \mathbb{E}_{t-1}) \ln X_{t+i} + O \left( \mathbb{E}_I \left\| \xi_t^T \right\|^2 \right)
\]

for \( X = C, WH \) and appropriate choice of \( \mathbb{E}_I \).

- Goal: obtain a mapping from wage innovations to innovations in consumption (marginal utility of wealth)