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Measuring wellbeing growth and convergence in multivariate ordered categorical worlds: Has there been any levelling up in the United Kingdom?

Measuring Wellbeing Growth and Convergence in Multivariate Ordered Categorical Worlds: Has there been any Levelling Up in the United Kingdom?

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Summary.

Measuring growth with ordered categorical variables is problematic due to their lack of cardinal measure and the equivocation and ambiguity inherent in the arbitrary attribution of cardinal scale to ordinal variates. Here, noting that the mean in a cardinal paradigm is the cumulation over its range of higher outcome probabilities and hence its growth is the rate of increase in those cumulated chances, application of the concept of probabilistic distance facilitates development of analogous implementable level and growth measures in ordinal paradigms that are independent of scale and unequivocal. An exemplifying analysis of the extent of “Leveling Up” growth and convergence in Income, Health and Human resources in the regions of the United Kingdom is performed over the period 2010 to 2018 prior to the Covid outbreak. The results indicate that, while there is strong evidence of growth, there is little evidence of levelling up type growth and hence little evidence of Levelling Up in that nation.

Introduction.

Perceived inequalities between a Nation's regions can be a catalyst for its deterioration as a cohesive entity (Milanovic 2011). Such imbalances are usually redressed by pursuing so-called "Levelling Up" infrastructure public policies with the intent of fostering growth and development in the disadvantaged regions. When the focus outcome variable has cardinal measure, (such as regional income and employment levels), identifying the disadvantaged regions and examining the effectiveness of such policies, is relatively straightforward. However, when the variables of concern are ordinal in nature, such as Health outcomes or Human Resource levels, identification of disadvantaged areas and measurement of their growth is more problematic. Arbitrary attribution of cardinal scale to such ordered categorical variates is not a solution since equally legitimate alternative scaling schemes often yield very different results and such equivocation impedes analysis (Bond and Lang 2019, Schroder and Yitzhaki 2017). However, this dilemma can be resolved by noting that, in the Cardinal Paradigm, the mean of a variable is the cumulation of chances of higher values over its range (rendering "growth in the mean" a "growth in the chances of higher values" interpretation) and further noting that, in the Ordered Categorical Paradigm, whilst the mean of the variate does not exist without arbitrary cardinalization, its chances of higher outcomes do exist and can be cumulated so that analogous Level and Growth measures can be computed for it. This, and employing the notion of probabilistic distance, motivates development of scale independent level and growth measures for ordinal variables which facilitate unequivocal comparison. These tools will be exemplified in an analysis of regional growth in the United Kingdom over the 2010-2018 period in the light of its "Levelling Up" policy aspirations.

The "Levelling Up" phraseology first appeared in the UK's public discourse in 1997 when Tony Blair outlined his aspirations for supporting upward social mobility in a hustings event and Theresa May described Conservatism as "Levelling Up" in a House of Commons debate. Basically, its import is the promotion of catch-up growth in those regions that are deemed to be relatively deprived. The first "official" use of the term was in the 2019 Conservative Party Manifesto written in preparation for that year's General Election, where it was argued that regional economic imbalances needed to be redressed without detriment to the more prosperous areas of the UK (essentially the South), in essence a conditional levelling up. A detailed report "Measuring Up for Levelling Up" was produced in September 2020 whereupon the Treasury Spending Review announced a £4.8 billion "Levelling Up" Fund for interim capital investment in local infrastructure. The 2021 Queen's Speech announced that the Government will "level up opportunities across all parts of the United Kingdom, supporting jobs, businesses and economic growth and addressing the impact of the pandemic on public services". In September 2021 the Ministry for Housing, Communities and Local Government was renamed the Department for Levelling Up, Housing and Communities under Secretary of State Michael Gove and a Levelling Up Task Force was formed in the Cabinet Office. Basically, Levelling Up here refers to the relative improvement of workforce quality, population health and general economic well-being within the relatively deprived areas and for policy evaluation purposes its measurement will rely upon measuring the simultaneous growth of, and convergence in, those aspects of society which are inherently ordinal in nature.

The need to redress imbalances without detriment to the more well off, is best understood in the context of Sufficientarianism¹, a doctrine emerging from political philosophy which grants special status to an outcome threshold defining “sufficient” or “good enough” and prioritizes the number of people in the population who have at least the sufficient level of the good. It is less concerned with the extent to which agents have more than enough and is more concerned with the extent to which agents do not attain a sufficient level or better. The basic idea is to designate the sufficient level, record the proportion of the population that enjoys at least a sufficient level (which is then the measure of the extent to which **sufficiency** has been achieved) and focus on that part of the population that does not achieve sufficiency. Levelling-Up growth would then concern itself with growth in the outcomes of those up to and including the sufficiency level. The standard Growth measure, the intertemporal rate of growth of the mean, does not demand growth at every outcome level, whereas Inclusive Growth measures do, Levelling-Up Growth demands growth at every outcome level up to the Sufficientarian Level, a type of Pro-Poor Growth. Here, in formulating Regional growth measures, three regional growth typologies are considered, Standard Growth, which simply requires overall growth in the nation, Inclusive Growth which requires growth in every region and, in anticipation of the announced conditional nature of levelling up policy and the special status of the more prosperous areas, Sufficientarian Growth², which requires growth in all regions that are not deemed sufficient.

In the following the growth and convergence literature is reviewed in Section 1 instruments for measuring gross and regional imbalance will be developed in Section 2 and an analysis of the United Kingdom over the pre-pandemic period 2010-2018 is reported in Section 3, conclusions are drawn in Section 4.

Section 1. Growth and Convergence.

The economic growth and convergence literature has traditionally been univariate in nature and addressed regional growth from the view-point of the lack of ‘convergence of incomes’. The approach stems from the premise that growth is expected to equalize across regions, or ‘level up’, to the regional average in the long run. The long run aspect of this assumption is quite crucial, as the early literature on the convergence of growth based on neoclassical growth models (Solow 1956, Swan 1956) required the specification of a stationary linear data generating process with a stable long run solution. Later endogenous growth theory models of (Romer 1986, Lucas 1988) required the specification of a nonlinear data generating process necessitating statistical methods that identified non-linear DGPs. Thus, levelling up has been the fundamental definition of ‘convergence’ as conceived in both neoclassical and endogenous growth theories. It should be noted that this work was always interpreted as convergence in means rather than convergence in the cumulated chances of better outcomes.

International evidence on levelling up, or regional convergence, has been elusive and disappointing, with the emergence of polarization and stratification patterns being the dominant empirics with the resulting

¹ Sufficientarian measures have recently been axiomatically developed and codified in Alcantud et.al. (2021) and Bossert et.al. (2021, 2022). The foundational work of Frankfurt (1987) only focused on those who had enough, Crisp (2003) added a refinement that drew attention to those who were not at the sufficient outcome level.

² Sentiments expressed in government documents that levelling up should not be at the expense of the most well off region i.e. the South East, suggest a Sufficientarian approach.

evolution of the literature toward the study of convergence in distribution rather than convergence in means.. The principal question for this more recent literature has thus been the following: if cross country or cross regional incomes are not converging to the mean (or, levelling up), what other distributional patterns characterize cross country incomes? The (now) vast cross country and within country literature provides significant evidence that regional growth rarely converges to the mean in the medium run, and that levelling up in the form of equalizing of all regional incomes is more likely to be preceded by other outcomes such as polarization and stratification of cross-country and within country incomes. This has thus led to the emergence of statistical methods which focus on identifying nodes or clubs and convergence to the respective means within those clubs (see Durlauf and Quah (2002) for a rich discussion on the popular methods used in the field). These methods comprise, on the one hand, non-parametric and semi-parametric approaches of identification of distribution dynamics (e.g. Quah (1997) and Pittau et al. 2010), and on the other hand, time series methods of Phillips and Sul (2007, 2009), which detect club convergence using clustering algorithms designed to identify groups of countries/regions into separate groups that are less heterogeneous than the entire set of countries/regions. The most striking finding of this regional convergence literature has been of increasing polarization and the persistence of convergence clubs over time (Bandyopadhyay 2011, Quah 1997, Pittau and Zelli 2006, Gennaioli et al. 2014).

In addition to its standard definition, further specifications of economic growth have emerged in the literature, in the form of inclusive and pro-poor growth. In its simplest form, inclusive growth implies an improvement in the outcomes of all segments of society. However, much of the inclusive growth focuses on improvement in the lot of the poor, however defined³, and seeks to identify how and whether gains from aggregate economic growth are distributed across households according to their initial incomes or expenditures. Not all growth episodes are inclusive, and the literature has defined many ways by which a growth episode can be considered inclusive. Klassen (2008) distinguishes between an absolute and a relative definition of inclusive growth: the (weak but) absolute definition refers to increased income for the poor, and the relative definition refers to growth that leads to disproportionate increases in incomes among the poor (i.e., it is accompanied by declining inequality (Klassen 2008)⁴. Ravallion and Chen (2003) propose the growth incidence curve to measure pro-poor growth, by which the contributions of growth to the respective income quantiles is recorded. Inspired by the Watts (1968)'s index of the level of poverty, they propose a measure of the rate of pro-poor growth by integration of the growth incidence curve⁵ which can be interpreted as the growth rate in the mean scaled up or down according to whether the distributional changes were pro-poor. Ali and Son (2007) also measure inclusive growth via maximization of a social opportunity function (which is similar to a social welfare function). Where opportunities created for the poor are more important than those created for the non-poor, i.e., if the opportunity enjoyed by a person is transferred to a poorer person in society, then growth may be deemed to have been pro-poor. Inclusive growth, by implication, focuses on a subset of such growth episodes. Since not all growth episodes are inclusive, it is necessary to separate those that are from those that are not.

³ There have been many alternative views, Ali & Son 2007, Foster 2013, Klassen 2010, Ravallion & Chen 2003 are examples.

⁴ By the same logic, there is no such thing as an "inclusive contraction." Kakwani and Son (2008) discuss whether pro-poor contraction can be viewed as pro-poor growth.

⁵ Their measure is the mean growth rate for the poor (as distinct from the growth rate in the mean for the poor

A further possible interpretation of pro-poor growth is ‘sufficientarianist’. Sufficientarianism, a doctrine emanating from the realms of political philosophy, focuses on an outcome threshold that is deemed to be ‘sufficient’ or ‘good enough’ and directs attention to maintaining the proportion of society who have at least the sufficient level and promoting the progress of people who possess less than that level. The key difference between Pro-Poor growth and Sufficientarian growth is that the former requires all levels of the poor to benefit, even at the expense of those who possess at least sufficient levels of income. The focus of this approach is that universal access to up to a certain level should be guaranteed, but it is less pressing to provide additional amounts above the threshold. The Sufficientarian approach avers that improvement for those below the sufficient level should not be at the expense of those who are well off. In essence, whereas for pro-poor growth, the state of the well-off is of no concern to the policy maker, for Sufficientarian growth the state of the well-off is of concern. Recent theoretical literature on Sufficientarian measurement (Alcantud et.al. 2021, Bossert et.al. 2021, 2022) addressed the subject within an axiomatic framework but, in spite of its importance, the applied framework within which we can define and implement notions of Sufficientarianism is still developing.

2. Measuring Levels and Growth Rates of Ordered Categorical Outcomes.

In the context of the United Kingdom, perceived imbalances were not confined to incomes but were also viewed in the context of Human Resource and Health outcomes, so an examination of the nature of regional growth in all of these aspects is required. The basic idea is to imagine a monotonic increasing wellbeing function $U(X, Y, Z)$ of human resources (X), health outcomes (Y) and incomes (Z)⁶ and have K joint ordinal distributions $f_k(X, Y, Z)$ for $k = 1, \dots, K$ regions and examine the comparative growth rates and increasing similarities in the chances for $U(X, Y, Z)$ of the K regions of The United Kingdom. In formulating growth measures in ordinal data environments, three generic growth typologies are considered, Basic Growth, Inclusive Growth and Sufficientarian or Pro-Poor Growth. In its simplest form, an Inclusive Growth imperative requires improvement in the outcomes of all segments of society. On the other hand, pro poor growth seeks improvement in the lot of the poor however defined. To understand the formulation of growth typologies in ordinal paradigms, the corresponding constructs in a more familiar cardinal world are first outlined.

The Cardinal Paradigm.

In cardinal paradigms, growth is usually measured in per capita terms as the rate of change over a period in a location parameter of a continuous variate (e.g. income or earnings), usually the average or median value of the variate (see for example Xu 2023). Letting $f_t(x)$ be the period t PDF of the random variable x (where $0 \leq x < Y < \infty$), with $F_t(x) (= \int_0^x f_t(z) dz)$ its period t CDF and $S_t(x) (= 1 - F_t(x))$ its corresponding survival function, integration by parts will show that $\mu_t(x)$, the period t average value of x , is the integrated survival function viz.:

⁶ This is not unlike the framework of the Human Development Index employed by the United Nations Development Program (2019) which has health, education and income as the three cornerstones of the index.

$$\mu_t(x) = E_{f_t(x)}(x) = \int_0^Y x f_t(x) dx = \int_0^Y (1 - F_t(x)) dx = \int_0^Y S_t(x) dx$$

Thus $g_{t+1}(x)$ the periodic growth in x from period t may be written as:

$$g_{t+1}(x) = (\mu_{t+1} - \mu_t) / \mu_t = \int_0^Y (S_{t+1}(x) - S_t(x)) dx / \int_0^Y S_t(x) dx \quad [1]$$

Thus, growth can also be construed as the rate of increase in cumulated chances of higher outcomes.

The Survival Function is also instrumental in defining the First Order Stochastic Dominance condition which, for any monotonic increasing Value Function $U(x)$, asserts that $E_{f_{t+1}(x)}(U(x)) - E_{f_t(x)}(U(x)) \geq 0$ when $F_{t+1}(x) \leq F_t(x) \forall x$ with $F_{t+1}(x) < F_t(x)$ somewhere. Since this can be re-written as $1 - F_{t+1}(x) \geq 1 - F_t(x) \forall x$ with $1 - F_{t+1}(x) > 1 - F_t(x)$ somewhere, the First Order Dominance condition can also be written as⁷:

$$S_{t+1}(x) \geq S_t(x) \forall x \text{ with } S_{t+1}(x) > S_t(x) \text{ somewhere} \quad [2]$$

Noting that $S_t(x)$ is the probability or chance of observing a higher level of x , [2] in essence requires that there is at least as good a chance of observing a higher level x in period $t + 1$ as there is in period t at all levels x , with a better chance at some level x . There are a plethora of tests for examining [2] (see Whang 2019 and references therein), perhaps the simplest approach is to examine $UNAMB(S_{t+1}(x), S_t(x))$ where:

$$-1 \leq UNAMB(S_{t+1}(x), S_t(x)) = \frac{\int_0^Y (S_{t+1}(x) - S_t(x)) dx}{\int_0^Y |S_{t+1}(x) - S_t(x)| dx} \leq 1 \quad [3]$$

When $UNAMB(S_{t+1}(x), S_t(x)) = 1$ [1] holds, when $UNAMB(S_{t+1}(x), S_t(x)) = -1$, $f_t(x)$ First order dominates $f_{t+1}(x)$ and when $-1 < UNAMB(S_{t+1}(x), S_t(x)) < 1$ First order dominance does not prevail (when $|UNAMB(S_{t+1}(x), S_t(x))|$ is close to 1 or -1 “Almost Dominance” (Leshno and Levy 2002) prevails).

While first order dominance is not a requirement for positive basic growth (the mean of x in period $t + 1$ can be significantly greater than that in period t without dominance prevailing) it can be invoked in order to define Inclusive Growth since, when it prevails, agents at every level of x are at least as well off in period $t + 1$ as they would be in period t ⁸. Similarly, given a Sufficentarian level M_{suf} , it can be invoked in order to define Sufficentarian growth by requiring the first order dominance condition to be satisfied up to and including the level M_{suf} but not beyond.

When a collection of K regions indexed $k = 1, \dots, K$ are under consideration with respective distribution functions $f_{t,k}(x)$ and Survival Functions $S_{t,k}(x)$, they can be ranked with respect to $U(x)$ using the Utopia-Dystopia Index (Anderson, Post and Whang 2020) and their differences examined using $UNAMB$ and the Distributional Gini and Distributional Coefficient of Variation Indices (Anderson, Linton, Pittau, Whang and Zelli 2021).

⁷ A similar condition is available in the multivariate paradigm (Atkinson and Bourguignon 1982).

⁸ When combined with a significant increase in the average, $UNAMB(S_{t+1}(x), S_t(x)) = 1$ can be seen as providing an additional level of confidence since it avers that the difference prevails at all levels of x .

The Ordered Categorical Paradigm.

Since ordinal data lacks translatable cardinal measure, analysis has frequently been facilitated by the arbitrary attribution of Cantril (1965) type scales to ordered categories. This led Bond and Lang (2019) and Schroder and Yitzhaki (2017) to reveal a “Sad Truth about Happiness Scales”, that there is inherent equivocation and ambiguity in the measurement process engendered by the scale dependency of the measures employed. Basically, equally valid alternative scaling can yield substantively different level and growth rate estimates and conclusions with respect to [1], [2] and [3]. However, by reinterpreting the mean as the cumulated chances of better outcomes across all values of a variable and entertaining the concept of Probabilistic Distance⁹ unambiguous cardinal measures of levels and growth rates of ordered categorical variates are available which facilitate analysis in ordinal paradigms without the attribution of cardinal measure.

In the ordered categorical paradigm, the variable X is reported in a sequence of ordered outcome categories. Suppose there are M ordered categories indexed $m = 1, \dots, M$, where the index reflects the ordering of the categories from the least to the most preferred, then the period t probability density function is discrete ordered categorical assigning probability $f_{m,t}$ to category m such that $\sum_{m=1}^M f_{m,t} = 1$ so that, in a “summation in the place of integration” analogy to the continuous paradigm, $F_{m,t} = \sum_{j=1}^m f_{j,t}$ and $S_{k,t} = 1 - F_{k,t}$ $k = 1, \dots, M$. $F_{m,t}$ is the chance of observing a randomly selected agent in the population at an outcome level lower than or equal to m and $S_{m,t}$ is the chance of observing a “better” outcome level than m for that agent¹⁰. Thus $\mu_{OCt} = \sum_{m=1}^M S_{m,t}$, an “Ordered Categorical Mean”, can be seen as an aggregation over outcome levels of the chances of “better” outcomes for a randomly selected agent in population t , a “Potentially Better Prospect” measure. Thus $g_{OC,t+1}$, growth in the ordered categorical paradigm, can be measured as:

$$g_{OC,t+1} = \frac{\sum_{m=1}^M (S_{m,t+1} - S_{m,t})}{\sum_{m=1}^M S_{m,t}} \quad [4]$$

[4] now has the interpretation of the proportionate increase (decrease) in cumulated better prospect probabilities. In this context First Order Dominance can be defined in terms of the survival function so that analogous to [2] for distribution $t+1$ to dominate distribution t :

$$S_{m,t+1} \geq S_{m,t} \quad \forall m \text{ with } S_{m,t+1} > S_{m,t} \text{ for some } m. \quad [5]$$

Thus, dominance of period $t+1$ over period t has the interpretation of period $t + 1$ holding at least as good a prospect probabilistically as period t at every level. Unambiguity (in order to establish inclusive growth) can be examined in a similar fashion by $UNAMB(S_{t+1}(x), S_t(x))$ where:

$$-1 \leq UNAMB(S_{t+1}(x), S_t(x)) = \frac{\sum_{m=1}^M (S_{m,t+1} - S_{m,t})}{\sum_{m=1}^M |S_{m,t+1} - S_{m,t}|} \leq 1 \quad [6]$$

Pro-poor or Sufficentarian growth at sufficient level $M^* < M$ can be examined by setting the upper summation limits in [4] and [5] to M^* .

⁹ As the basis of the Pearson (1900) family of Goodness of Fit Tests and Gini (1916) Transvariation measures, Probabilistic Distance is an old idea, Mendelson (1987) revived and adapted it to measure quantile preserving spreads. It has seen some use in ordinal outcome contexts in the Health Inequality and Bi-Polarization literatures

¹⁰ If desired $S_{k,t}^*$, the chance of observing a randomly selected agent in the population at least as high as k in the ordering, can be constructed where $S_{k,t}^* = S_{k,t} + f_{k,t}$.

To order a collection of K regional distributions indexed $k = 1, \dots, K$, with distribution functions $f_{m,k}$ and Survival Functions $S_{m,k}$ $m = 1, \dots, M$, define $S_{m,U}$ and $S_{m,D}$ $m = 1, \dots, M$, the respective Utopian and Dystopian distribution as:

$$S_{m,U} = \max_k S_{m,k} \text{ and } S_{m,D} = \min_k S_{m,k} \text{ for } m = 1, \dots, M$$

Then $UD1(k)$, region k 's First Order Utopia-Dystopia index is given by:

$$UD1(k) = \frac{\sum_{m=1}^M (S_{m,k} - S_{m,D})}{\sum_{m=1}^M (S_{m,U} - S_{m,D})}$$

Distributional differences between regions can be examined using the ordered categorical equivalent of $DGIN$, the Distributional Gini Coefficient and $DCOV$, the Distributional Coefficient of Variation where:

$$DGIN = \frac{K}{K-1} \sum_{j=1}^K \sum_{k=j+1}^K \sum_{m=1}^M |f_{m,j} - f_{m,k}| \text{ and } DCOV = \frac{K}{K-1} \sum_{k=1}^K \sum_{m=1}^M 0.5 |f_{m,o} - f_{m,k}|$$

Where $f_{m,o}$ is an object distribution like an average or target distribution.

Inference.

Assume t are independently observed populations with n_t observations respectively and let the true m 'th outcome level probability $f_{t,m} = p_{t,m}$ for $m = 1, \dots, M$ be stacked in the $m \times 1$ vector \underline{p}_t and let $\hat{\underline{p}}_t$ be the corresponding relative frequency estimates of those probabilities based upon a simple random sample. Then, following Rao (2009), $\sqrt{n_t}(\hat{\underline{p}}_t - \underline{p}_t) \sim_a N(0, V(\underline{p}_t))$ where:

$$V(\underline{p}_t) = \begin{pmatrix} p_1 & 0 & \cdot & 0 \\ 0 & p_2 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & p_m \end{pmatrix} - \begin{pmatrix} p_1^2 & p_1 p_2 & \cdot & p_1 p_m \\ p_2 p_1 & p_2^2 & \cdot & p_2 p_m \\ \cdot & \cdot & \cdot & \cdot \\ p_m p_1 & p_m p_2 & \cdot & p_m^2 \end{pmatrix}$$

Given the $m \times m$ dimensioned integrating matrix D , where:

$$D = \begin{pmatrix} 1 & 0 & \cdot & 0 \\ 1 & 1 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot \\ 1 & 1 & \cdot & 1 \end{pmatrix}$$

\underline{F}_t , the t 'th period vector of CDF values is such that, given \underline{I} is an m dimensioned vector of ones:

$$\underline{F}_t = D\underline{p}_t \text{ and } \underline{S}_t = \underline{I} - D\underline{p}_t \quad [A1]$$

Each will have variance $DV(\underline{p}_t)D'$, so that $\sqrt{n_t}(\hat{\underline{F}}_t - \underline{F}_t) \sim_a N(0, DV(\underline{p}_t)D')$ and $\sqrt{n_t}(\hat{\underline{S}}_t - \underline{S}_t) \sim_a N(0, DV(\underline{p}_t)D')$. Since $\mu_t = \underline{I}'\underline{S}_t$, $\sqrt{n_t}(\hat{\mu}_t - \mu_t) \sim_a N(0, \underline{I}'DV(\underline{p}_t)D'\underline{I})$.

Multidimensionality.

It should be noted that, unlike the cardinal paradigm, these growth, level, distributional variation, ambiguity measures and inference procedures are viable in multidimensional frameworks. Thus, when a variable (such as human resources) is based on more than one ordered categorical variate (such as embodied human capital and experience), multidimensional versions of [4], [5] and [6] are readily

available. For example, suppose 4 jointly distributed ordered categorical variates with respectively M, N, P and Q categories, then the object of interest becomes $\mu_{OC,t}$:

$$\mu_{OC,t} = \sum_{m=1}^M \sum_{n=1}^N \sum_{p=1}^P \sum_{q=1}^Q S_{m,n,p,q,t} \quad [7]$$

with [4], [5] and [6] redefined accordingly, extending the inference procedures is similarly straight forward. The maximum value that [7] could take on is $M * N * P * Q - 1$ (when all mass is concentrated in the highest category so that $p_{M,N,P,Q} = 1$) and its minimum value is 0 (when all mass is concentrated in the lowest category $p_{1,1,1,1} = 1$) so that $\mu_{OC,t}/(M * N * P * Q - 1)$ would be a standardized mean confined to the unit interval which would facilitate comparison of means with different dimensions and numbers of categories.

Section 3. Regional Growth in the UK 2010-2018: Levelling up, or Not?

In order to understand the challenges of leveling up the regions in the United Kingdom, and much in the spirit of the UNDP's Human Development Index (UNDP 2019), individual regional outcomes with respect to income, health and human resources are examined and compared over the period 2010 to 2018 just prior to the COVID outbreak. As with the HDI it is the combined improvement of incomes, health and human resources that are of interest, to this end data from the UK survey for 12 regions was employed which included responses to questions about household income, health status and educational status and age.

Data.

The data was sourced from the Understanding Society¹¹ dataset, a large-scale longitudinal study conducted in the United Kingdom. Understanding Society: the UK Household Longitudinal Study (UKHLS) is a survey, collecting data from participants, and it is one of the largest household panel studies in the world, by surveying the members of approximately 40,000 households (at Wave 1). Households recruited at the first round of data collection are visited each year to collect information on changes to their household and individual circumstances. It began in 2009 but carries on from the earlier British Household Panel survey, which ran from 1991 to 2008 (University of Essex, Institute for Social and Economic Research, 2022). After incomplete records were excluded a 2-period sample of 75487 subjects remained with recorded levels of self-reported Education, Health, Income and Age group.

The five Education Categories were <certificate of secondary education or less, "O" level or equivalent, "A" level or equivalent, Post secondary training, 1st Degree and above. The seven Health Status Categories were 1 Completely, 2 Mostly, 3 Somewhat dissatisfied, 4 neither dissatisfied nor satisfied, 5 Somewhat, 6 Mostly, 7 Completely Satisfied. Monthly gross nominal incomes before taxes were reported with the 2018 incomes deflated by 1.1816 based upon the Cost-of-Living index change over the period. Deciles for the pooled income data were then determined to establish the ten categories of an

¹¹ The overall purpose of Understanding Society is to provide high-quality longitudinal data on subjects such as health, education, income, regional location, and well-being to help understand the long-term effects of social and economic change, as well as policy interventions designed to impact upon the general wellbeing of the UK population. To this end, the study collects both objective and subjective indicators, which makes it an ideal for the purpose of the project. The samples are constructed by taking yearly cross-sections of the data by region with all the variables of interest mentioned above to compute the different measures described in the paper.

income level categorical variate¹². Finally, Six Age Group Categories were determined as <26, 26-35, 36-45, 46-55, 56-65 and >65. Details of the Survival Functions for Incomes, Health Outcomes, Age Groups and Human Resources are provided in the appendix.

For information purposes, based upon category identifying indices, Table 1 reports the Means, Medians, Maximums, Minimums and Standard Deviations of the variables employed in the analysis though it should be stressed that these indices were not employed in any way in the subsequent analysis.

Table 1. Summary Statistics.

		Education	Health	Income	Age Group	Income Category
Pooled Sample N=75487	Mean	3.1102	4.8674	3459.0305	3.6781	5.5000
	Median	3.0000	6.0000	2833.0230	4.0000	6.0000
	Maximum	5.0000	7.0000	1165802.9790	6.0000	10.0000
	Minimum	1.0000	1.0000	0.0677	1.0000	1.0000
	Std Deviation	1.6775	1.6613	6489.4301	1.4893	2.8723
2010 Sample N=43411	Mean	3.0562	4.9006	3335.9242	3.7135	5.3670
	Median	3.0000	6.0000	2738.4900	4.0000	5.0000
	Maximum	5.0000	7.0000	47410.7110	6.0000	10.0000
	Minimum	1.0000	1.0000	0.1700	1.0000	1.0000
	Std Deviation	1.6920	1.6727	2478.0630	1.4918	2.8828
2018 Sample N=32076	Mean	3.1833	4.8226	3625.6400	3.6302	5.6801
	Median	3.0000	5.0000	2960.2192	4.0000	6.0000
	Maximum	5.0000	7.0000	1165802.9790	6.0000	10.0000
	Minimum	1.0000	1.0000	0.0677	1.0000	1.0000
	Std Deviation	1.6548	1.6447	9526.2680	1.4846	2.8482

Over the observation period there appears to have been an improvement in the overall Education and Income levels with a deterioration in the self-reported health level. Preliminary data analysis indicated strong outcome similarities between contiguous Regions so the original 12 Regions were amalgamated into 6 distinctive North-Eastern, North-Western, Midlands, Southern, Wales and a Scotland + Northern Ireland combined regions with the resultant sample sizes reported in Table 2.

Table 2. Sample sizes.

	North-Eastern	North-Western	Midlands	Southern	Wales	Scotland + Northern Ireland
2010	4840	4557	6440	17087	3336	7151
2018	3929	3385	4946	12972	2047	4797

What follows is separate analyses of Age, Income, Health and Human Resources in those regions followed by a joint overall analysis.

The Age Distribution.

Since changes in the age distribution in a population have consequences for its income levels, health outcomes and human resources it will be explored first. Earnings tend to increase with age, Health

¹² The pooled income deciles were: 1033.0000, 1457.1600, 1878.8084, 2337.7878, 2833.0000, 3393.2718, 4093.3225, 5004.0117, 6572.4199, 1165802.9790.

outcomes tend to deteriorate with age and human resources, a combination of embodied human capital and experience, are affected by age because of the lifetime experience accumulation process.

In the context of an Age Group Distribution, the Survival Function reports the chance of being in a higher age group and an increase in the Summed Survival Function measure is evidence of an ageing population, the consequence of a low replication rate combined with increasing Life Expectancy. Table 3 reports the ordered categorical means, growth rates and significance tests for the age distribution over the 2010-2018 period.

Table 3. Age Distribution.

	Summed Higher Age Prob		8 Year Growth Rate	Unambiguity	P(diff > 0)
	2010	2018			
North East, Yorks and Humberside	2.5275 {5} (0.0018)	2.7539 {6} (0.0023)	0.0896	1.0000	0.0002
North West and Merseyside	2.5556 {4} (0.0019)	2.7421 {5} (0.0026)	0.0730	1.0000	0.0029
East and West Midlands	2.5652 {3} (0.0014)	2.7950 {4} (0.0018)	0.0896	1.0000	0.0000
London, South East and West and East	2.5183 {6} (0.0005)	2.8092 {3} (0.0007)	0.1155	1.0000	0.0000
Wales	2.7440 {1} (0.0027)	2.9946 {1} (0.0044)	0.0913	1.0000	0.0014
Scotland and Northern Ireland	2.6198 {2} (0.0012)	2.9402 {2} (0.0019)	0.1223	1.0000	0.0000

* {Rank} (estimate variance)

	2010	2018
Distributional Gini's	0.0777	0.0855
Distributional Difference from the average	0.0309	0.0328
Distributional Difference from the South	0.0370	0.0331

In both observation years the oldest populations were in Wales and Scotland + Northern Ireland, the youngest was in the South in 2010 and the North East in 2018. Population aging was ubiquitous, significant and unambiguous with 8 year growth rates of the order of 9% with the greatest growth in Scotland and Northern Ireland and London and the South and the slowest growth in the North-West. Whilst aging inequality increased overall there was some convergence towards the Souths Age Distribution.

Table 4. The Income Distribution

	Summed Better Chances		8 Year	Unambiguity	P(diff <= 0)
	2010	2018	Growth Rate		
North East, Yorks and Humberside	3.9560 {6} (0.0049)	4.2291 {6} (0.0062)	0.0690	1.0000	0.0049
North West and Merseyside	4.1914 {3} (0.0053)	4.4886 {3} (0.0073)	0.0709	0.9831	0.0041
East and West Midlands	4.1328 {4} (0.0038)	4.4968 {2} (0.0050)	0.0881	1.0000	0.0001
London, South East and West and East	4.7204 {1} (0.0015)	5.0708 {1} (0.0019)	0.0742	1.0000	0.0000
Wales	4.0204 {5} (0.0072)	4.3522 {5} (0.0120)	0.0825	1.0000	0.0083
Scotland and Northern Ireland	4.2850 {2} (0.0034)	4.4570 {4} (0.0052)	0.0401	1.0000	0.0317

* {Rank} (estimate variance)

	2010	2018
Distributional Gini's	0.1213	0.1401
Distributional Difference from the average	0.0451	0.0533
Distributional Difference from the South	0.0939	0.1098

Real Income growth was significant, ubiquitous and unambiguous across most of the regions and, with the exception of Scotland and Northern Ireland, of a similar order of magnitude (roughly 1% per annum). London and the South had the highest income levels, whereas the Northeast, Yorkshire and Humberside had the lowest incomes in both periods. Notably the growth rate in the lowest ranked region was lower than the growth rates in all regions except for Scotland which had the lowest growth rate. Overall distributional inequality, whether it be between all groups or differences from the average or differences from the Souths distribution, grew throughout the period, suggesting an absence of levelling up with respect to incomes.

Table 5. The Health Distribution.

	Summed Better Chances 2010	2018	8 Year Growth Rate	Unambiguity	P(diff > 0)
North East, Yorks and Humberside	3.8560 {4} (0.0025)	3.7956 {5} (0.0031)	-0.0157	-0.8186	0.2099
North West and Merseyside	3.8431 {5} (0.0027)	3.7959 {4} (0.0036)	-0.0123	-0.5261	0.2752
East and West Midlands	3.8731 {3} (0.0019)	3.8039 {3} (0.0025)	-0.0179	-0.8595	0.1465
London, South East and West and East	3.9582 {1} (0.0007)	3.8558 {2} (0.0009)	-0.0259	-0.9747	0.0057
Wales	3.8168 {6} (0.0036)	3.6365 {6} (0.0060)	-0.0472	-1.0000	0.0330
Scotland and Northern Ireland	3.8933 {2} (0.0017)	3.8724 {1} (0.0025)	-0.0054	-0.2760	0.3739

* {Rank} (estimated variance)

	2010	2018
Distributional Gini's	0.0728	0.0755
Distributional Difference from the average	0.0289	0.0302
Distributional Difference from the South	0.0373	0.0395

A function of the aging population whose health outcomes deteriorate with age, Health deteriorated over the period universally though, with the exception of Wales, it was never unambiguous or significant. The South and Scotland and Northern Ireland enjoyed the best health whereas Wales enjoyed the worst health. Wales aside, Health deteriorated fastest in the South. Health inequality grew throughout the period both overall and in terms of differences from the average and differences from the South.

Table 6. Human Resources

	Summed Better Chances 2010	2018	8 Year Growth Rate	Unambiguity	P(diff > 0)
North East, Yorks and Humberside	19.3409 {6} (0.0242)	20.5167 {6} (0.0301)	0.0608	1.0000	0.0000
North West and Merseyside	19.6880 {4} (0.0251)	20.8313 {4} (0.0339)	0.0581	1.0000	0.0000
East and West Midlands	19.4337 {5} (0.0177)	20.6846 {5} (0.0230)	0.0644	1.0000	0.0000
London, South East and West and East	20.2449 {3} (0.0063)	21.6023 {1} (0.0082)	0.0670	1.0000	0.0000
Wales	21.6760 {1} (0.0389)	21.5423 {2} (0.0594)	-0.0062	-0.1718	0.6651
Scotland and Northern Ireland	21.1217 {2} (0.0169)	21.3634 {3} (0.0247)	0.0114	0.3707	0.1179

* {Rank} (estimate variance)

	2010	2018
Distributional Gini's	0.2927	0.2462
Distributional Difference from the average	0.1227	0.0988
Distributional Difference from the South	0.1273	0.1231

Human resources, a combination of embodied human capital and experience, grew throughout the period with the exception of Wales, which in the first period was the best placed. With the exception of Wales, Scotland and Northern Ireland the growth was unambiguous. Inequality in the distribution of human resources diminished over the period in all 3 measures signaling convergence in resource distributions and some equalization or Levelling Up.

Table 7. Income, Health and Human Resources Joint Distribution.

	Summed Better Chances 2010	2018	8 Year Growth Rate	Unambiguity	P(diff < 0)
North East, Yorks and Humberside	282.7238 {6} (0.0931)	288.2736 {6} (0.1000)	0.0196	0.8765	0.0000
North West and Merseyside	283.9638 {4} (0.0966)	292.1315 {4} (0.1054)	0.0288	0.9283	0.0000
East and West Midlands	283.2868 {5} (0.0809)	291.1559 {5} (0.0877)	0.0278	0.9618	0.0000
London, South East and West and East	295.3521 {3} (0.0463)	302.2663 {1} (0.0502)	0.0234	0.9517	0.0000
Wales	296.8450 {2} (0.1036)	295.8935 {3} (0.1320)	-0.0032	-0.1546	1.0000
Scotland and Northern Ireland	298.7237 {1} (0.0699)	297.9760 {2} (0.0856)	-0.0025	-0.2510	1.0000

* {Rank} (Standard Error)

	2010	2018
Distributional Gini's	0.4099	0.4039
Distributional Difference from the average	0.1611	0.1574
Distributional Difference from the South	0.1907	0.1964

In terms of an Integrated Income, Health and Human Resource Wellbeing Index, North-Eastern, North-Western and Midland regions consistently ranked 6, 4 and 5 respectively over the period based upon the health, income, health and human resource outcome Joint Distribution. London and The South, Wales and Scotland and Northern Ireland were closely aligned at the beginning of the period, ranking 3, 2 and 1 respectively in 2010, in 2018 their respective ranks were 1, 3 and 2. Growth was always significant in all regions though it was negative for Wales and Scotland and Northern Ireland it was never unambiguous in any region. While there appeared to be convergence in terms of overall differences and differences from the average, there was divergence and an absence of levelling up convergence with regard to closer proximity to the South.

Conclusions.

By noting that the mean of a cardinally measurable variable is the cumulated probabilities of higher values over its range and applying the notion of probabilistic distance, analogous scale independent tools for measuring growth and convergence in multivariate ordered categorical variates have been developed together with their sampling distributions. These tools obviate the need for arbitrary cardinalisation of ordered variables with all its attendant vagueness and facilitate assessment of inclusive, pro-poor and Sufficentarian growth. They were exemplified in an analysis of the extent of Levelling Up in Incomes, Health outcomes and Human Resources, the three pillars of the United Nations Development Programs Human Development Index, in six regions of the United Kingdom over the period 2010-2018.

While there was evidence of growth in the Income and Human Resources dimensions of wellbeing, Health outcomes exhibited negative growth in all regions, the result of an ageing population. Within the multidimensional framework the most advantaged areas in the initial period were The South, Wales, and Scotland + Northern Ireland (largely a consequence of their superior Human Resource stocks), however their multidimensional growth rates were low, indeed negative in the case of Wales, and Scotland + Northern Ireland. The three relatively disadvantaged areas in the initial period all had positive growth with the North-West and the Midlands having the highest growth rates, which outstripped that of the South. Though the changes were all statistically significant, none of the regional growth was unambiguous in a three dimensional sense. Whilst there was some evidence of convergence, with Distributional Ginis and Distributional Variation diminishing over time, there was no evidence of Levelling Up type convergence with distributional differences from the South actually increasing over time.

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