

# Are Two Cheap, Noisy Measures Better Than One Expensive, Accurate One?

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## Executive Summary

1. Survey responses are always subject to measurement error. In general surveys (and especially longitudinal surveys), there are severe constraints on the time that can be spent eliciting a less noisy response for any target variable. In this paper we consider when it may be better to consider multiple noisy measures of the target measure rather than improving the reliability of a single measure.
2. The Kotlarski result states that if the measurement errors in two measures of the same target variable are mutually independent and independent of the true value then we can recover the entire distribution of the quantity of interest, up to location.
3. We consider designing surveys to deliver measurement error with desirable properties. This shifts the emphasis from reliability (the signal to noise ratio for any given measure) to the joint properties of the multiple measures.
4. To illustrate our ideas, we consider a concrete example: the measurement of consumption inequality. A small simulation study suggests that the approach we propose has promise. The next step in this research agenda is experiments in survey data collection.

## 1. Introduction

Survey responses are always subject to measurement error. This is the case even for such well defined concepts as age, earnings, expenditures, net wealth and market hours; henceforth the *target variable*. In general surveys (and especially longitudinal surveys), there are severe constraints on the time that can be spent eliciting a less noisy response for any target variable. In this paper we consider when it may be better to consider multiple noisy measures of the target measure rather than improving the reliability of a single measure.

The value of multiple measures for means and regression coefficients is familiar to most empirical economists (for example, in twins studies). However the use of multiple measures is much more general and can aid in recovering the full (conditional) distribution of the target variable. Our inspiration in the analysis below is the Kotlarski result (see B.L.S Praskasa Rao, 1992). This states that if the measurement errors in two measures of the same target variable are mutually independent and independent of the true value then we can recover the entire distribution of the quantity of interest, up to location.

The Kotlarski result has been used in recent contributions for dealing with measurement error (see Tong Li and Quang Vuong, 1998, Susan M. Schennach, 2004, and Aurore Delaigle, Peter Hall and Alexander Meister, 2008). These papers follow the standard route of taking measurement errors with specific properties and then devising estimators that can take account of such measurement errors. Our interest is in survey design. Consequently we propose turning the usual procedure on its head and consider designing surveys to deliver measurement error with desirable properties. As we shall see, the emphasis then shifts from reliability (the signal to noise ratio for any given measure) to the joint properties of the multiple measures. Using an illustration of asking about total expenditure (or “consumption”), we shall show how a mixture of economic theory and analysis of auxiliary data sets can provide insights into the design of survey questions. Although we do not

consider it in this paper, this analysis also suggests complementary pre-testing and use of focus groups to further enhance the properties of multiple measures.

## 2. An Example

To make things concrete, we consider a specific measurement problem: estimating the variance of the log of consumption (total expenditure) in a population. There is a large literature that investigates consumption inequality (David Cutler and Lawrence Katz, 1992, Richard Blundell and Ian Preston, 1998, Dirk Krueger and Fabrizio Perri, 2006) and the log of the variance is common measure of inequality in this literature. Our choice of an inequality measure as our parameter of interest is intended to reinforce the point that the ideas sketched in this paper are not limited to the estimation of means.

Let  $C$  be (true) log consumption; this is the target variable. In the population  $C$  has variance  $\sigma_c^2$ ;  $\sigma_c^2$  is the parameter of interest. We suppose that we have three potential measures:  $Z$ ,  $X_1$  and  $X_2$ .  $Z$  is the ‘expensive’ measure and  $X_1$  and  $X_2$  are the two ‘noisy’ measures. Let  $c$ ,  $z$ ,  $x_1$  and  $x_2$  be deviations from the respective means. *Define* the error associated with each method as:

$$(1) \quad u = z - c$$

$$(2) \quad \varepsilon_i = x_i - c$$

These errors have variances  $\sigma_u^2$ ,  $\sigma_1^2$  and  $\sigma_2^2$  and covariances with true consumption of  $\sigma_{cu}$ ,  $\sigma_{c1}$  and  $\sigma_{c2}$ . The quantities  $\frac{\sigma_c^2}{\sigma_z^2}$  and  $\frac{\sigma_c^2}{\sigma_{x_i}^2}$  are usually referred to as the ‘reliability’ of  $z$  and  $x_i$  respectively.

The variance of the single measure  $z$  is given by:

$$(3) \quad E[z^2] = \sigma_c^2 + 2\sigma_{cu} + \sigma_u^2$$

We take the sample analogue of the variance  $E[z^2]$  as one estimate of  $\sigma_c^2$ . Improving the precision of this estimate involves reducing  $\sigma_{cu}$  and/or  $\sigma_u^2$ . For example, expenditures on individual items might be collected using diaries kept for one week. This delivers high quality data but at high cost and with substantial respondent burden. The quality of the data (as well as the cost and the respondent burden) would be increased by asking respondent to fill out the diary for longer than one week. Alternatively  $z$  might be the sum of a detailed list of recall questions. Asking more (and finer) categories of expenditure leads to better data (Joachim Winter, 2004, Menno Pradham, 2001), but this requires more interview time, and thus implies greater cost and respondent burden. The ‘cheap’ measures ( $x_1$  and  $x_2$ ) might be a ‘one shot’ recall question about total expenditure and a recall question about a single category of expenditure (food at home, for example).

When we consider using two measures to estimate the variance of  $c$  we take the covariance between the two measures:

$$(4) \quad E[x_1 x_2] = \sigma_c^2 + \sigma_{c1} + \sigma_{c2} + \sigma_{12}$$

Then the estimate of  $\sigma_c^2$  is given by the sample estimate of the covariance between the two measures:

$$(5) \quad \hat{E}[x_1 x_2] = \frac{1}{n} \sum_{i=1}^n x_{1i} x_{2i}$$

Note that the asymptotic bias  $(E[x_1 x_2] - \sigma_c^2)$  does not depend on the individual variances  $\sigma_1^2$  and  $\sigma_2^2$  and is hence not dependent on the reliability of the multiple measures. It is in this sense that we can allow them to be ‘noisy’ measures. Our interest is in designing measures that reduce the asymptotic bias,  $\sigma_{c1} + \sigma_{c2} + \sigma_{12}$ . Of course, in a finite sample, the noise will matter for the precision of the estimator.

A very common assumption for measurement errors is that they have *classical* properties. Although this term is used to denote different things for different people, in the current context the classical assumptions are  $\sigma_{cu} = 0$  and  $\sigma_{c1} = \sigma_{c2} = \sigma_{12} = 0$ . In this case the asymptotic bias of the single measure is given by:

$$(6) \quad E[z^2] - \sigma_c^2 = \sigma_u^2$$

This only reduces to zero if we eliminate measurement error altogether. On the other hand, if we assume classical measurement errors for the multiple measures then the asymptotic bias ( $E[x_1x_2] - \sigma_c^2$ ) is zero. This is an illustration of the Kotlarski result in a very specific context.

The classical assumptions are widely invoked in psychometrics and other disciplines and hence there is a widespread use in these disciplines of multiple measures.. In many contexts in economics, however, the assumption that measurement errors are independent of each other and of the true value are more expressions of hope than realistic assessments (John Bound, Charles Brown and Nancy Mathiowetz, 2001). In the next section we shall consider how to craft multiple measures which at least reduce the bias.

### **3. Multiple measures with nonclassical measurement error.**

As we have seen, if we assume classical properties for the measurement errors in our multiple measures then it would always be best to use two (or more) noisy measures rather than one less noisy one. We now consider how we might design multiple survey questions for a single target variable that induce measurement errors that come close to the classical assumptions. Any such exercise will be very specific to the target variable and will ideally involve extensive and judicious pre-testing and use of focus groups as well as analysis of other data sources and considerations from economic theory.

Consider again the basic set up:

$$(7) \quad E[z^2] = \sigma_c^2 + 2\sigma_{cu} + \sigma_u^2$$

$$(8) \quad E[x_1 x_2] = \sigma_c^2 + \sigma_{c1} + \sigma_{c2} + \sigma_{12}$$

Both cognitive theories of response behaviour and economic theory can be informative about the sources of bias ( $\sigma_{cu}$ ,  $\sigma_u^2$ ,  $\sigma_{c1}$ ,  $\sigma_{c2}$  and  $\sigma_{12}$ ). This can help us to design good measures (questions) or choose wisely from an available set of measures.

#### **4. Multiple measures for total expenditure.**

We consider again the issue of finding out about total expenditure in a given period. There is considerable evidence that well informed respondents in a household can provide reasonably accurate recall information about expenditures on specific groups of goods (such as food at home) in, say, the last month (Naeem Ahmed, Matthew Brzozowski and Thomas F. Crossley, 2006.) Suppose that we only have survey time to ask about a small number of items; which should we ask about? Martin Browning, Thomas F. Crossley and Guglielmo Weber (2003) recommended:

- *Always ask a ‘total expenditure on non-durables and services’ question.....there is a great deal of idiosyncratic behaviour in demand and sometimes households spend a good deal on sub-items that we would never think to ask about.....*
- *Always ask a ‘food at home’ and a ‘food outside the home’ question with the same time period as for total expenditure..... respondents can report food at home accurately....being a large budget item, it is very useful in imputation.....*
- *Ask about utilities such as fuel and telephones.....*

Subsequently, evidence has piled up that the first ‘one-shot’ question is very unreliable and takes a lot of survey time. Moreover, recent cognitive testing we have undertaken was particularly discouraging for this question. The idea behind the third recommendation was that these items could be measured reliably and contained variation that was orthogonal to food in/out. In making these recommendations, we very much had in mind to capture a large share of the total and/or a ‘prediction’ approach (Jonathan Skinner, 1987).

The multiple measures analysis above suggests a quite different approach. Think of log consumption of specific goods (food, clothing, telephone, recreation) as our cheap error



ridden measures  $(x_1, x_2)$  of total consumption. We then use demand theory and analysis of expenditure surveys to choose goods so that the measurement errors have desirable properties. An Engel curve relates consumption on specific items to the target variable, total expenditure. Consider a linear in logs approximate Engel curve:

$$(9) \quad x_i = \alpha_i c + \eta_i(c) + e_i$$

The parameter  $\alpha_i$  captures the income elasticity of good  $i$  if the double log form is correct; luxuries have  $\alpha_i > 1$  and necessities have  $\alpha_i < 1$ . The variable  $\eta_i(c)$  is the approximation error from using the log-log form. The variable  $e_i$  captures heterogeneity in tastes. Define the measurement error for good  $i$  as:

$$(10) \quad \varepsilon_i \equiv x_i - c = (\alpha_i - 1)c + \eta_i(c) + e_i$$

This allows us to relate economic theory concepts to the decomposition given in equation (8). Thus  $\sigma_{ci} \approx 0$  for goods with unit income elasticity and a small log-log approximation error ( $\alpha_i \approx 1$  and  $\eta_i(c) \approx 0$ ), with  $\sigma_{ci} > 0$  for luxuries and  $\sigma_{ci} < 0$  for necessities. Equation (8) then implies that it might be better to have one luxury and one necessity rather than either two luxuries or two necessities. In terms of  $\sigma_{12}$ , the best choice gives  $\sigma_{12} \square 0$ . Complementary goods (coffee and cream) will tend to have  $\sigma_{12} > 1$  and substitutes (coffee and tea) will tend to have  $\sigma_{12} < 0$ . Adding up implies  $\sigma_{12} < 0$  on average, especially for highly aggregated goods.

Based on this, we can make the following recommendations for the choice of two goods to ask about. We should choose goods that:

- Respondents can readily report,
- have close to unit income elasticities (or a luxury and a necessity), and not too much approximation error,
- are not strong complements or substitutes.

Note that large budget shares not necessary (and may be undesirable since adding up induces  $\sigma_{12} < 0$ ). Moreover, reliability (a low variance for measurement error) is helpful but not paramount, in contrast to a single measure approach.

### **5. A Simulation Study.**

To illustrate the ideas described above, we conducted a small simulation study. This experiment is based on data from the 1996 Canadian Family Expenditure Survey (FAMEX). This is an intensive, high quality budget survey based on annual recall. The nature of this survey is attractive for our purposes for two reasons. First, the responses do not suffer from infrequency. Second, recall questions are what we imagine are feasible in a general survey. From this data we selected a sample of couples without children. We treat this sample as our population of interest and take the logarithm of total nondurable consumption as the “true” value of the target variable ( $c$ ) for each household. The variance of the logarithm of total nondurable consumption in this sample is then the population parameter we wish to estimate. We consider two possibilities for  $z$  (the high cost measure). One is simply the logarithm of total nondurable consumption ( $z = c$ ) as we observe it in the data (this corresponds to observing the target variable with reliability of one). Alternatively, we take the logarithm of total nondurable consumption as observed in the data and add classical measurement so that the reliability of the measure is 0.6. For  $(x_1, x_2)$  we take pairs of goods guided by the advice summarized in the previous section. We chose to use food (the sum of food at home and food in restaurants) as one of the two “cheap” measures ( $x_1$ ) both because of the evidence (noted above) that it is well measured by recall questions and because it seems likely that questions about food will always be included in surveys that collect expenditure information. The choice of the second good was informed by subsidiary analysis of Engel curves estimated on the FAMEX budget data. On the basis of fit, income elasticity and error correlations, we concluded that recreation (or “leisure”) nondurable/semidurable goods and services would be

a good choice for the second good ( $x_2$ ). Evidence from Denmark suggests that households can and will answer recall questions on this category of expenditure (Martin Browning and Mette Gortz, 2006). Finally, we consider another estimation strategy. Following Richard Blundell, Luigi Pistaferri and Ian Preston (2004) we also impute the logarithm of total nondurable consumption by the inverse of the estimated food Engel curve:  $\left(\frac{1}{\alpha_f}\right)^{x_f}$ .

These measures of the logarithm of total nondurable consumption then imply the following estimators of the variance of the logarithm of nondurable consumption:

- i) The sample variance of  $z$
- ii) The sample variance of  $\left(\frac{1}{\alpha_f}\right)^{x_f}$
- iii) The sample covariance of  $(x_1, x_2)$

We resample repeatedly from our sub-sample of the FAMEX (making 1000 draws with replacement) and study the bias and variance of our estimators. The results are presented in Table 1 and Figure 1. The ‘true’ value of the variance of the logarithm of total nondurable expenditure in our ‘population’ (the initial sub-sample of the 1996 FAMEX) is 0.189. The first row of Table 1 indicates that if we observe the target variable with reliability 1, and simply calculate the sample variance of that variable in the sample, our estimates are, of course, centred on the true value of the parameter of interest (these estimates differ from the true value only because of sampling variability.) Note however, that when the reliability of our ‘expensive’ measure falls to 0.6 (row 2), substantial bias is introduced in our estimate of variance. This simply illustrates Equation (6): even classical measurement error biases the traditional estimator of the variance.

The next row of Table 1 (Row 3) reports the result of estimating the variance of the logarithm of total nondurable expenditure with the sample variance of an imputation of the target

variable. The imputation is from food expenditure, via an estimated Engel curve. This estimate of the variance is biased as well, for the same reason as the estimator in the row above: the imputation measures the target variable with error, and those measurement errors inflate the sample variance of the imputation. Blundell, Pistaferri and Preston (2004) provide evidence that this bias may be reasonably constant through time, so that changes in the variance can be recovered in this way, although the level of the variance cannot.

Finally, Row 4 of Table 1 reports the distribution of the estimate based on the sample covariance of two ‘cheap and noisy’ measures: in this case the logarithm of expenditure on food and the logarithm of expenditure on recreation. This estimator does very well. In our re-sampling experiment, it is centred on the true value of the parameter of interest, and not too widely dispersed. Thus our choice of goods has delivered two noisy measures whose respective measurement errors have the properties necessary to allow a good estimate of the parameter of interest.

**TABLE 1: Simulation Results**

Estimator	Mean	Std. Dev.	Mean % Bias
Direct, Reliability = 1	0.189	0.0058	-0.11
Direct, Reliability = 0.6	0.320	0.0092	64.4
Imputation from Engel curve	0.328	0.0107	73.1
2 Good (food, recreation)	0.188	0.0102	-0.87

True value = 0.189

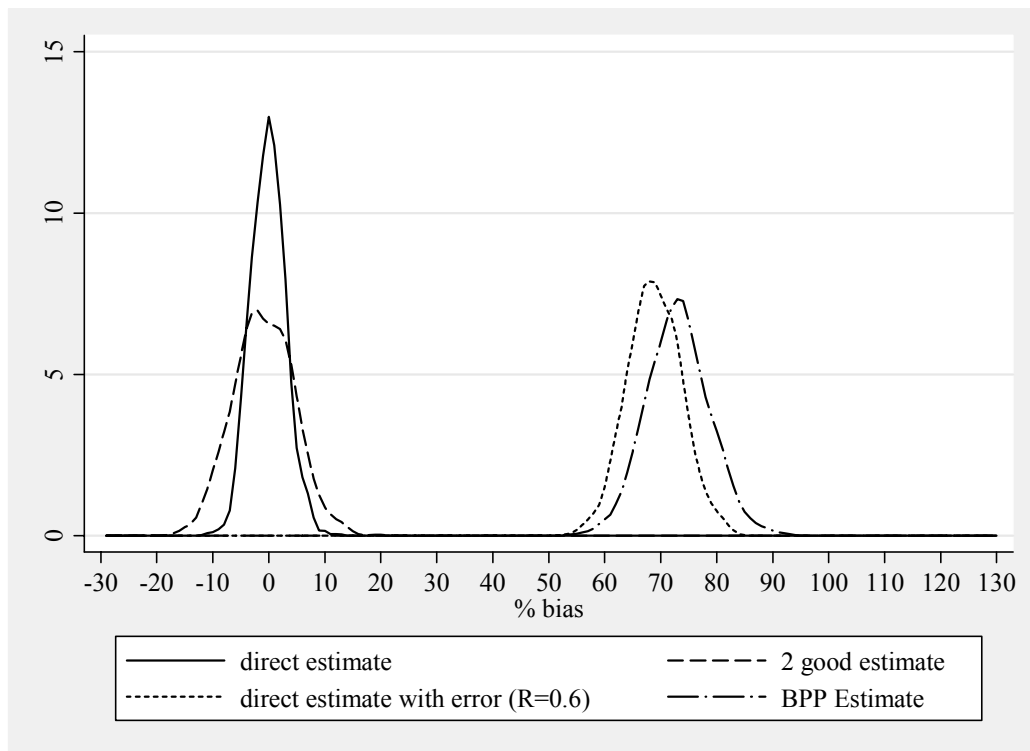
1000 replications (re-sampling with replacement)

Population size = sample size = 2379

## 6. Discussion

Designing survey questions to eliminate measurement error is very difficult – perhaps impossible. However, with the right kind of measurement errors, two error ridden measures can tell you a lot about the distribution of a quantity of interest. Our suggestion is therefore that it may be easier to design survey questions to get close (or closer) to the right kinds of measurement error.

Going forward, our research agenda is to investigate ways in we can introduce multiple measures of target variables into household surveys, and in particular, to investigate ways in which we might induce the errors in those measures to have desirable properties. Internet panels have recently been developed in both the U.S. and Europe to support social science research. These seem the natural platform on which to further explore the suggestion made in this short paper.



**Figure 1: Resampling Distribution of Alternative Estimators**

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