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A CONTRIBUTION TO THE REINHART AND ROGOFF DEBATE: NOT 90 PERCENT BUT MAYBE 30 PERCENT

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Abstract. Using the Reinhart-Rogoff dataset, we find a debt threshold not around 90 percent but around 30 percent above which the median real GDP growth falls abruptly. Our work is the first to formally test for threshold effects in the relationship between public debt and median real GDP growth. The null hypothesis of no threshold effect is rejected at the 5 percent significance level for most cases. While we find no evidence of a threshold around 90 percent, our findings suggest that the debt threshold for economic growth may exist around a relatively small debt-to-GDP ratio of 30 percent. Empirical results are more robust with the postwar sample than the long sample that goes before World War II.

Key words: Government debt, growth, fiscal policy, median regression, testing.

JEL codes: H60, F34, E62.

1. Introduction

In this paper, we test whether there exists a threshold effect in the relationship between government debt-to-GDP ratio and median real GDP growth rate in advanced economies

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by applying a recently developed econometric technique to the Reinhart-Rogoff (RR hereafter) dataset.

Our paper is primarily motivated by Reinhart and Rogoff (2010) whose "main result is that whereas the link between growth and debt seems relatively weak at 'normal' debt levels, median growth rates for countries with public debt over roughly 90 percent of GDP are about one percent lower than otherwise; average (mean) growth rates are several percent lower." Herndon, Ash and Pollin (2014) pointed out their spread sheet errors and claimed that "overall evidence refutes RR's claim that public debt/GDP ratios above 90 percent consistently reduce a country's GDP growth." In the response to their critics, Reinhart and Rogoff (2013*b*) stressed among other things that their paper "gave significant weight to the median estimates" because they are less influenced by outliers.

A substantial body of literature since then has been devoted to testing for the threshold effect in the link between public debt and GDP growth but has not reached a general consensus. Kumar and Woo (2010), Cecchetti, Mohanty and Zampolli (2011), Checherita-Westphal and Rother (2012) and Baum, Checherita-Westphal and Rother (2013) obtained evidence supporting the proposed 90 percent debt threshold. On the other hand, Minea and Parent (2012) estimated a higher debt threshold, around 115 percent of GDP. However, Caner, Grennes and Koehler-Geib (2010) and Elmeskov and Sutherland (2012) found the threshold to be around 70 percent. Baglan and Yoldas (2013) and Égert (2013) suggested that the threshold may be even lower, around 20 percent. For more comprehensive literature review, refer to Panizza and Presbitero (2013) and Eberhardt and Presbitero (2013). However, aforementioned papers in the literature did not estimate or test for the threshold effect in terms of the *median* GDP growth rate, although Reinhart and Rogoff emphasized their median estimates. To the best of our knowledge, our paper is the first one that focuses on the *median* real GDP growth.

In this paper, we contribute to the Reinhart and Rogoff debate by formally testing for a threshold effect in the relationship between public debt and median real GDP growth.

The goal of this paper is to examine whether the empirical findings of Reinhart and Rogoff (2010) can be viewed as statistically significant evidence for the existence of the threshold effect of debt at 90 percent of GDP. Although the debate broadly encompasses the link between debt and growth in all economies, we restrict our attention to threshold effects in advanced economies in this paper using the updated Reinhart-Rogoff dataset (Reinhart and Rogoff (2013*a*)).

The remainder of the paper is organized as follows. Section 2 describes the setup and the methodology, Section 3 explains the data and gives estimation results of median regression, Section 4 presents the main testing results, Section 5 provides the results of robustness check, and Section 6 concludes. Appendix A describes the construction of the sample used in our empirical work and Appendix B provides details of the testing method.

2. The Setup and Methodology

We use the updated Reinhart-Rogoff dataset (Reinhart and Rogoff (2013*a*)) and apply the test for threshold effect developed in Lee, Seo and Shin (2011). This method allows us to test for threshold effect in median regression when the threshold value is unknown.

2.1. **Model Specification.**

Let $y_{c,t}$ be the real GDP of country c for year t. Let $G_{c,t-1,t}$ be the real GDP growth of the country between the years 't' and '$t-1$', that is, $100 \times (y_{c,t} - y_{c,t-1})/y_{c,t-1}$. Similarly, let $G_{c,t,t+5}$ be its five year forward average growth rate (annualized five-year growth hereafter) defined as $\frac{1}{5}\sum_{s=0}^{4} G_{c,t+s,t+s+1}$. The annualized five-year growth is defined so that the period included does not overlap with the period for the annual growth. Let $debt_{c,t}$ denote the debt-to-GDP ratio of country c in year t expressed in percentage.

Our main interest is how the debt to GDP ratio of a country affects its annual and annualized five-year growth rates. Specifically, we wish to test whether there is a debt/GDP threshold after which the median growth rates change abruptly. The conditional median

function is specified as

$$\text{Median}(g_{c,t}|debt_{c,t}) = \beta_1 + \beta_2 debt_{c,t} + [\alpha_1 + \alpha_2 debt_{c,t}] \times I(debt_{c,t} > \gamma),$$

where $g_{c,t}$ can be either $G_{c,t-1,t}$ or $G_{c,t,t+5}$, $I(\cdot)$ is an indicator function, and α_1, α_2, β_1, β_2 and γ are unknown true parameter values that belong to $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{B}_1$, $\mathcal{B}_2$ and Γ respectively, which are subsets of $\mathbb{R}$.

We consider two specifications for the regression. For the "intercept-only" model, we consider a conditional median function where only the intercept is allowed to change at the threshold value by imposing $\alpha_2 = 0$. For the "intercept-and-slope" model, changes in both the slope and the intercept are allowed at the threshold value.

Let $\alpha = [\alpha_1, \alpha_2]$. The null and alternative hypotheses in our setting are

$$H_0 : \alpha = 0 \text{ for any } \gamma \in \Gamma \quad \text{versus} \quad H_1 : \alpha \neq 0 \text{ for some } \gamma \in \Gamma. \tag{1}$$

When $\alpha = 0$, there is no threshold effect due to the debt to GDP ratio; whereas if $\alpha \neq 0$, there exists the threshold effect.

2.2. **Informal Description of the Testing Procedure.** There are several testing procedures available in the literature: a sup-likelihood-ratio-type test of Lee, Seo and Shin (2011), a sup-Wald-type test of Galvao et al. (2014), and a sup-score-type test of Zhang, Wang and Zhu (2014). In this paper, we use the sup-likelihood-ratio-type test of Lee, Seo and Shin (2011) since in many cases, likelihood ratio tests are known to have desirable properties.

In our analysis, we pool observations as if observations were independent and identically distributed over c and t. In what follows, we will use the subscript i for each country-year observation. This is a convenient assumption to start with, but it is also expected that the asymptotic null distribution of the sup-likelihood-ratio test statistic is the same

for stationary weakly dependent processes, as is the case with the sup-Wald-type test of Galvao et al. (2014).[1]

To give an informal description of our testing procedure, we start with the following objective function for the median regression:

$$Q_n(\alpha, \beta, \gamma) := -\frac{1}{n}\sum_{i=1}^{n}\Big|g_i - \{\beta_1 + \beta_2 debt_i + [\alpha_1 + \alpha_2 debt_i] \times I(debt_i > \gamma)\}\Big|.$$

For a given $\gamma \in \Gamma$, define $\hat{\alpha}(\gamma)$ and $\hat{\beta}(\gamma)$ to be the estimators that maximize the objective function $Q_n(\alpha, \beta, \gamma)$. Let $\hat{\gamma} := \underset{\gamma\in\Gamma}{\operatorname{argmax}}\, Q_n(\hat{\alpha}(\gamma), \hat{\beta}(\gamma), \gamma)$ and

$$\hat{Q}_n := Q_n(\hat{\alpha}(\hat{\gamma}), \hat{\beta}(\hat{\gamma}), \hat{\gamma}).$$

In addition, noting that $Q_n(\alpha, \beta, \gamma)$ does not depend on γ when $\alpha = 0$, let

$$\tilde{\beta} := \underset{\beta:\alpha=0}{\operatorname{argmax}}\, Q_n(\alpha, \beta, \gamma) \quad \text{and} \quad \tilde{Q}_n := Q_n(0, \tilde{\beta}, \gamma).$$

Define the quasi-likelihood ratio statistic by

$$QLR_n := n(\hat{Q}_n - \tilde{Q}_n). \tag{2}$$

That is, our test statistic is based on the distance between maximized restricted and unrestricted objective function values. Note that the test statistic defined in (2) can also be written as

$$QLR_n = \sup_{\gamma\in\Gamma} n\left[Q_n(\hat{\alpha}(\gamma), \hat{\beta}(\gamma), \gamma) - \tilde{Q}_n\right].$$

Thus, the statistic QLR_n can be viewed as a sup-likelihood-ratio-type statistic. We can simulate valid p-values of the quasi-likelihood ratio test, following Lee, Seo and Shin (2011). To implement the test, it is necessary to specify the range of the parameter space Γ of the threshold parameter γ. In all empirical results presented below, we set Γ to be

[1] Lee, Seo and Shin (2011) did not consider dependent observations; however, we expect that the asymptotically valid p-value can be obtained in the same way as described in this paper, in view of the asymptotic equivalence result obtained in Galvao et al. (2014) for the sup-Wald-type test.

an interval between 10% and 100% of GDP, which are approximately 7 and 94 percentiles of the debt-to-GDP ratio in the sample. See Appendix B for a detailed description of how to obtain the p-value.

3. Data and Median Regression Results

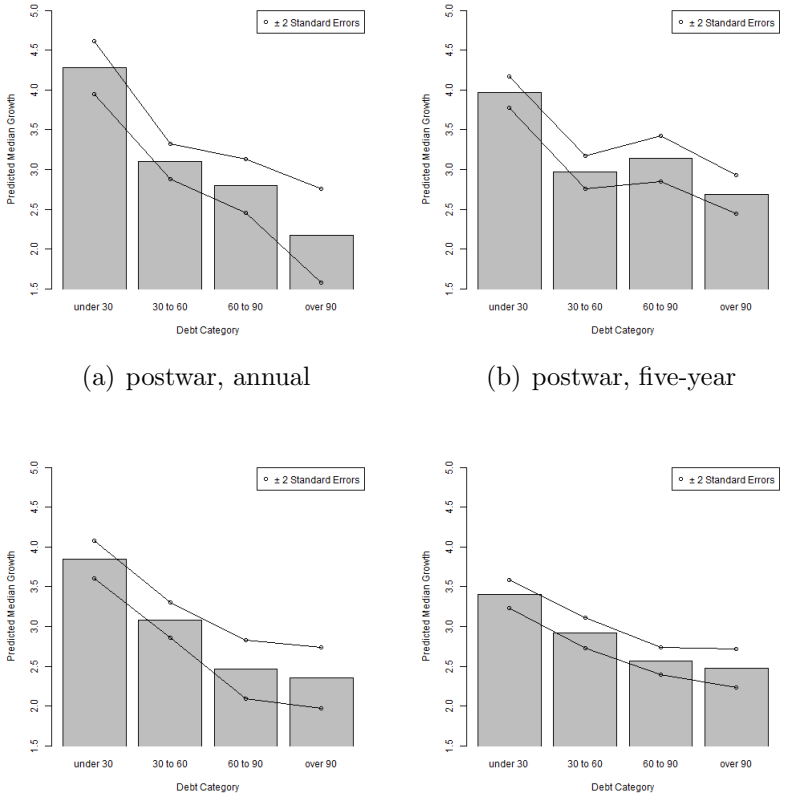
Our source of data is Reinhart and Rogoff (2013*a*) provided in Carmen Reinhart's website.[2] This is a revised and corrected version of the data used in Reinhart and Rogoff (2010). For our main analysis, we used the postwar and long samples of "advanced economies." The postwar sample covers the years 1946-2009. The long sample covers the years 1791-2009. For both samples, the countries included are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, the United Kingdom, and the United States. The specific country-years included in the sample are described in Appendix A.

For the GDP growth of New Zealand, Reinhart and Rogoff (2013) constructed two different sets of data first from Angus Maddison's Database and then from the New Zealand Historical Statistics records. We only report the results obtained using the New Zealand Historical Statistics data. We have also conducted the same tests with the Maddison data and the differences in results were minor.

Before moving to the test results, we present the predicted values from median regression of growth on dummy variables that represent debt-to-GDP ratio categories. We used the same debt categories as Reinhart and Rogoff (2010). However, we assigned an equal weight to every country-year observation for the whole sample while they assigned an equal weight to every country within a debt category (that is, the equal weight within each subsample defined by the debt level). In our test for threshold effects, since we were testing for the existence of any threshold rather than a particular threshold, we did

[2]See http://www.carmenreinhart.com/response-to-critics/.

not have ex-ante debt categories needed to construct the weights used in Reinhart and Rogoff (2010). Hence, for consistency, we used country-year equal weights in estimation of median regression.



(a) postwar, annual (b) postwar, five-year

(c) long, annual (d) long, five-year

FIGURE 1. Quantile Regression of Growth on Debt Categories

Figure 1 depicts the predicted median growth and the region within 2 standard deviations from the predicted median growth. We can observe that the difference in predicted median growth between categories "under 30%" and "30% - 60%" is larger than the difference between categories "60% - 90%" and "over 90%" for all four median regressions.

4. Test Results for Threshold Effects

Table 1 describes the result of the tests for threshold effect in median annual growth regression. The null hypothesis of no threshold effect was rejected in every test at the 5 percent significance level except for the median five-year growth regression for the long sample using the intercept-and-slope model. Other than this no-rejection case, the estimated threshold was 28 percent. In the restricted model where $\alpha = 0$ was imposed, a 10 percentage point increase in the debt was associated with a 0.22 percentage point decrease in median GDP growth for the postwar sample. For the long sample, it was associated with a 0.15 percentage point decrease in median GDP growth.

Table 2 describes the result of the tests for threshold effect in median annualized five-year growth regression. The null hypothesis of no threshold effect was rejected in every test at any conventional level. For the postwar sample, the estimated threshold was 32.5 percent in the intercept-only model and 18.1 percent in the intercept-and-slope model. For the long sample, the estimated threshold was 33.4 percent in the intercept-only model and 32.5 percent in the intercept-and-slope model. In the restricted model where $\alpha = 0$ was imposed, a 10 percentage point increase in debt was associated with a 0.12 percentage point decrease in median GDP growth for the postwar sample. For the long sample, it was associated with a 0.08 percentage point decrease in median GDP growth. The estimated coefficients suggest that the threshold level of debt/GDP is slightly higher and the negative impact of debt on growth is smaller in the medium-run compared to the short-run.

5. Robustness of Results

We check the robustness of our results by using sub-samples obtained from dividing the original sample according to the size of each observation's debt-to-GDP ratio, by omitting one or more countries from the original sample, and by adding the lagged dependent

TABLE 1. Test for threshold effect in median annual real GDP growth function

postwar	"intercept-only"	"intercept-and-slope"
n	1184	1184
p-value	0.008	0.026
$\hat{\gamma}$	28	28
$\tilde{\beta}_1$	4.331	4.331
$\tilde{\beta}_2$	-0.022	-0.022
$\hat{\beta}_1$	4.54	4.275
$\hat{\beta}_2$	-0.009	0.005
$\hat{\alpha}_1$	-1.074	-0.809
$\hat{\alpha}_2$		-0.015
long	**"intercept-only"**	**"intercept-and-slope"**
n	2313	2313
p-value	0.047	0.1
$\hat{\gamma}$	28	77.5
$\tilde{\beta}_1$	3.863	3.863
$\tilde{\beta}_2$	-0.015	-0.015
$\hat{\beta}_1$	4.022	4.073
$\hat{\beta}_2$	-0.009	-0.021
$\hat{\alpha}_1$	-0.71	-2.536
$\hat{\alpha}_2$		0.027

Note. For the "intercept-only" model, we consider a conditional median function where only the intercept is allowed to change at the threshold value by imposing $\alpha_2 = 0$. For the "intercept-and-slope" model, changes in both the slope and the intercept are allowed at the threshold value. $\tilde{\beta}_1$ and $\tilde{\beta}_2$ refer to the estimated coefficients for $\beta = (\beta_1, \beta_2)$ when the restriction $\alpha = 0$ is imposed.

variable as an additional covariate. The test results we present in this section are all obtained using the intercept-only model.

As the first robustness check, we examine whether there is a second threshold below or above the threshold estimated in the previous section. One might bring out the possibility of multiple thresholds in the link between debt and growth and justly question whether there is really no evidence of a 90 percent debt threshold. The 90 percent threshold may

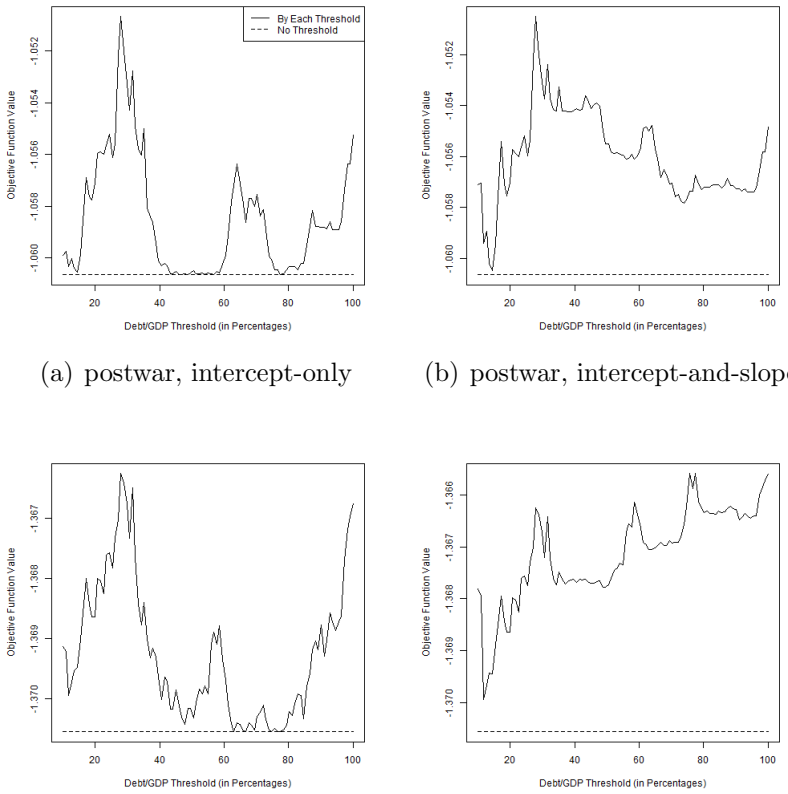
TABLE 2. Test for threshold effect in median five-year forward average real GDP growth function

postwar	"intercept-only"	"intercept-and-slope"
n	1085	1085
p-value	0	0
$\hat{\gamma}$	32.5	18.1
$\tilde{\beta}_1$	3.866	3.866
$\tilde{\beta}_2$	-0.012	-0.012
$\hat{\beta}_1$	3.987	2.101
$\hat{\beta}_2$	-0.003	0.178
$\hat{\alpha}_1$	-0.903	1.374
$\hat{\alpha}_2$		-0.184
long	**"intercept-only"**	**"intercept-and-slope"**
n	2190	2190
p-value	0	0
$\hat{\gamma}$	33.4	32.5
$\tilde{\beta}_1$	3.36	3.36
$\tilde{\beta}_2$	-0.008	-0.008
$\hat{\beta}_1$	3.461	3.276
$\hat{\beta}_2$	-0.002	0.01
$\hat{\alpha}_1$	-0.7	-0.467
$\hat{\alpha}_2$		-0.012

Note. The same note from Table 1 apply.

have existed but may not be as detectable as the 30 percent threshold. There are less observations with debt around 90 percent than those with debt around 30 percent in both the postwar sample and the long sample. Thus it may have been more difficult to detect a threshold around 90 percent compared to around 30 percent.

To examine this issue, we plot the profiled values $Q_n(\hat{\alpha}(\gamma), \hat{\beta}(\gamma), \gamma)$ of the objective function for each fixed value of γ in Figures 2 and 3. In Figure 2, there seems to be a second peak at the upper end point of the parameter space (that is, 100 percent) for the long sample; however, with relatively few observations above 100 percent, it is difficult to conclude whether a threshold really exists there. Likewise, Figure 3 does not provide evidence supporting the threshold effect at around 90 percent. To complement the eyeball



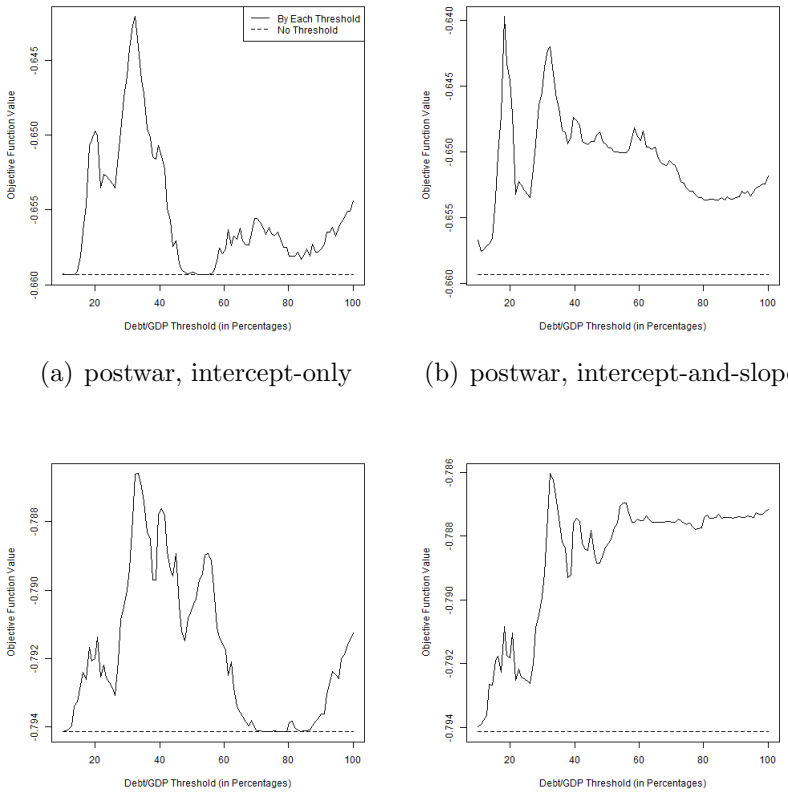
(a) postwar, intercept-only (b) postwar, intercept-and-slope

(c) long, intercept-only (d) long, intercept-and-slope

FIGURE 2. $Q_n(\hat{\alpha}(\gamma), \hat{\beta}(\gamma), \gamma)$ for Annual Growth

Note. Each panel of the figure plots the profiled value $Q_n(\hat{\alpha}(\gamma), \hat{\beta}(\gamma), \gamma)$ of the objective function for each fixed value of γ.

examination of figures, in Tables 3 and 4, we report the test results using sub-samples constructed by including only the observations with debt-to-GDP ratios below or above the estimated first thresholds. We can observe that the null hypothesis of no threshold effect is not rejected at the 5 percent significance level when annual growth is under concern. For annualized five-year growth, the null hypothesis is rejected at the 5 percent significance level only in two cases: the postwar sub-sample that includes observations with debt under 32.5 percent of GDP and the long sub-sample that includes observations with debt under 33.4 percent of GDP. The estimated second thresholds are 18.1 percent



(a) postwar, intercept-only (b) postwar, intercept-and-slope

(c) long, intercept-only (d) long, intercept-and-slope

FIGURE 3. $Q_n(\hat{\alpha}(\gamma), \hat{\beta}(\gamma), \gamma)$ for Five-year Growth

and 26.2 percent, respectively. This result further casts doubt on the claim that the 90 percent debt threshold exists.

As the second robustness check,we now check whether the results depend on the inclusion/exclusion of any particular country by carrying out the tests with country-wise sub-samples. First, we generated sub-samples by omitting one country each from the original sample. Second, we generated sub-samples by splitting the countries into two groups according to alphabetical order (10 countries in each group) or region (16 European countries and 4 non-European countries). Tables 5 to 8 describe the results. For the postwar sample, the results are scarcely affected by one-country omissions and moderately affected by splitting the countries into two groups. For the long sample, especially for the annual

TABLE 3. Test for second threshold effect in median annual real GDP growth function: Debt-to-GDP below or above the first threshold

postwar	under 28%	over 28%
n	415	769
p-value	0.509	0.285
γ	17.2	77.5
$\tilde{\beta}_1$	4.275	3.466
$\tilde{\beta}_2$	0.005	-0.009
$\hat{\beta}_1$	3.761	2.939
$\hat{\beta}_2$	0.06	0.003
$\hat{\alpha}_1$	-0.944	-1.089
long	under 28%	over 28%
n	638	1496
p-value	0.636	0.28
γ	61.3	58.6
$\tilde{\beta}_1$	3.085	3.312
$\tilde{\beta}_2$	-0.003	-0.009
$\hat{\beta}_1$	3.127	3.094
$\hat{\beta}_2$	-0.005	0
$\hat{\alpha}_1$	0.326	-0.704

growth, the results are greatly influenced. This suggests that country-wise heterogeneity in the link between debt and growth is more serious when we look at short-term growth with more historical data. In summary, we did not find any credible evidence supporting the 90 percent the debt threshold.

As the third robustness check, we consider further tests by adding a lagged dependent variable as an explanatory variable. Since we focus only on the intercept-only model in this section, the coefficient of the added lagged dependent variable does not change its value below or above the threshold.

Table 9 indicates that the threshold effect seems to disappear altogether. The test fails to reject the null hypothesis in all cases at the 5 percent significance level. For the postwar sample, a 10 percentage point increase in the debt is associated with 0.08 percentage

TABLE 4. Test for second threshold effect in median five-year forward average real GDP growth function: Debt-to-GDP below or above the first threshold

postwar	under 32.5%	over 32.5%
n	447	638
p-value	0	0.634
γ	18.1	61.3
$\tilde{\beta}_1$	3.93	3.085
$\tilde{\beta}_2$	0.003	-0.003
$\hat{\beta}_1$	2.9	3.127
$\hat{\beta}_2$	0.107	-0.005
$\hat{\alpha}_1$	-1.745	0.326
long	under 33.4%	over 33.4%
n	903	1287
p-value	0.022	0.089
γ	26.2	55.9
$\tilde{\beta}_1$	3.313	2.778
$\tilde{\beta}_2$	0.008	-0.002
$\hat{\beta}_1$	3.763	2.808
$\hat{\beta}_2$	-0.028	0.002
$\hat{\alpha}_1$	0.807	-0.416

point decrease in annual growth and 0.01 percentage point increase in annualized five-year growth. For the long sample, it is associated with a 0.11 percentage point decrease in annual growth and is neutral to the annualized five-year growth. This result further casts doubt on the existence of the proposed 90 percent debt threshold.

6. Conclusion

After testing for threshold effects in the link between public debt-to-GDP ratio and median growth, we find no evidence of a 90 percent debt threshold that is generally applicable to all countries. Instead, our findings suggest that a debt threshold, if it exists, may be around 30 percent of GDP. However, more evidence is needed to establish any credible link between such debt threshold and growth, not to mention the causal relation.

TABLE 5. Tests for threshold effects in median annual GDP growth functions : postwar sample

country omitted	p-value	$\hat{\gamma}$	$\tilde{\beta}_1$	$\tilde{\beta}_2$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\alpha}_1$
Australia	0.000	28	4.372	-0.023	4.735	-0.01	-1.291
Austria	0.029	28	4.313	-0.021	4.454	-0.01	-0.95
Belgium	0.004	28	4.341	-0.023	4.538	-0.009	-1.117
Canada	0.001	28	4.366	-0.023	4.549	-0.01	-1.134
Denmark	0.016	28	4.396	-0.022	4.587	-0.011	-0.991
Finland	0.004	28	4.396	-0.023	4.6	-0.01	-1.106
France	0.037	28	4.284	-0.021	4.38	-0.009	-0.907
Germany	0.002	28	4.396	-0.022	4.609	-0.01	-1.106
Greece	0.033	28	4.401	-0.024	4.574	-0.013	-0.899
Ireland	0.014	28	4.342	-0.023	4.518	-0.01	-1.039
Italy	0.003	28	4.244	-0.019	4.367	-0.007	-1.006
Japan	0.004	28	4.25	-0.021	4.299	-0.004	-1.146
Netherlands	0.005	28	4.353	-0.022	4.555	-0.009	-1.095
New Zealand	0.006	28	4.353	-0.022	4.557	-0.01	-1.073
Norway	0.000	28	4.395	-0.023	4.595	-0.009	-1.199
Portugal	0.021	28	4.279	-0.021	4.442	-0.01	-0.958
Spain	0.011	28	4.326	-0.022	4.504	-0.009	-1.083
Sweden	0.011	28	4.353	-0.022	4.541	-0.01	-1.057
United Kingdom	0.043	28	4.409	-0.023	4.562	-0.012	-0.927
United States	0.003	28	4.326	-0.022	4.536	-0.009	-1.145
countries included	p-value	$\hat{\gamma}$	$\tilde{\beta}_1$	$\tilde{\beta}_2$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\alpha}_1$
Australia - Ireland	0.011	24.4	4.023	-0.013	4.098	0.004	-1.322
Italy - United States	0.042	17.2	4.62	-0.029	6.351	-0.026	-1.955
European	0.001	28	4.267	-0.021	4.3	-0.001	-1.388
Non-European	0.013	12.7	5.316	-0.036	8.492	-0.016	-4.513

APPENDIX A. DATA CONSTRUCTION

Following the "Guide to Changes" in Reinhart and Rogoff (2013), we used the "Spain_new" data that includes years 1959-1980 and conducted each test twice, first using "New_Zealand_old" data constructed by Reinhart and Rogoff (RR) from Angus Maddison's Database, which has been carried over to the Total Economy Database, and then using "New_Zealand_new" data constructed by RR from The New Zealand Historical Statistics (2003).

Following the comments in the worksheet, we excluded years 1940-1945 for United Kingdom and 1941-1944 for United States. In the worksheet, debt-to-GDP ratios in

TABLE 6. Tests for threshold effects in median annual GDP growth functions : long sample

country omitted	p-value	$\hat{\gamma}$	$\tilde{\beta}_1$	$\tilde{\beta}_2$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\alpha}_1$
Australia	0.02	28	3.873	-0.016	4.026	-0.009	-0.745
Austria	0.162	100	3.828	-0.015	4.064	-0.021	0.975
Belgium	0.066	28	4.009	-0.018	4.06	-0.009	-0.712
Canada	0.013	31.6	3.883	-0.016	4.002	-0.008	-0.838
Denmark	0.04	31.6	3.974	-0.017	4.064	-0.009	-0.729
Finland	0.041	28.9	3.878	-0.016	4.027	-0.009	-0.714
France	0.071	100	3.786	-0.014	4.064	-0.022	1.196
Germany	0.072	28	3.92	-0.016	4.026	-0.009	-0.683
Greece	0.163	31.6	3.953	-0.018	4.031	-0.01	-0.633
Ireland	0.045	100	3.87	-0.016	4.101	-0.023	1.124
Italy	0.049	99.1	3.765	-0.013	4.035	-0.021	1.203
Japan	0.07	28	3.753	-0.014	3.976	-0.008	-0.712
Netherlands	0.035	28	3.854	-0.015	4.02	-0.009	-0.734
New Zealand	0.08	28	3.949	-0.017	4.029	-0.01	-0.674
Norway	0.004	28.9	3.923	-0.016	4.089	-0.008	-0.839
Portugal	0.181	28	3.809	-0.014	3.997	-0.009	-0.62
Spain	0.112	31.6	3.856	-0.015	3.997	-0.008	-0.705
Sweden	0.041	28	3.944	-0.016	4.056	-0.009	-0.744
United Kingdom	0.255	99.1	4.03	-0.018	4.147	-0.022	0.96
United States	0.04	28	3.76	-0.014	3.994	-0.008	-0.745
countries included	p-value	$\hat{\gamma}$	$\tilde{\beta}_1$	$\tilde{\beta}_2$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\alpha}_1$
Austria - Ireland	0.192	24.4	3.652	-0.01	3.745	0	-0.833
Italy - United States	0.003	58.6	4.082	-0.020	3.721	-0.005	-1.313
European	0.012	31.6	3.69	-0.014	3.878	-0.006	-0.898
Non-European	0.502	37	4.205	-0.014	4.261	-0.029	1.138

years 2008-2009 for Greece were missing while the two observations were included in the calculation of mean and median growth rate for "90 or above" debt-to-GDP ratio category. Hence, we augmented the worksheet data with debt-to-GDP ratios of Reinhart-Rogoff series provided in Carmen Reinhart's website[3] (http://www.carmenreinhart.com/data/browse-by-topic/topics/9/) to fill in the debt-to-GDP ratios for the two observations (which are 109.748642014544 for 2008 and 126.8 for 2009). In the worksheet, the post war (1946-2009) summary statistics included only years 1951-2009 for Italy although both the

[3]Last updated on November 15, 2010, downloaded on December 24, 2013.

TABLE 7. Tests for threshold effects in median five-year GDP growth functions : postwar sample

country omitted	p-value	$\hat{\gamma}$	$\tilde{\beta}_1$	$\tilde{\beta}_2$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\alpha}_1$
Australia	0	32.5	3.912	-0.012	4.101	-0.002	-1.083
Austria	0	32.5	3.822	-0.01	3.988	-0.003	-0.8
Belgium	0	32.5	3.855	-0.01	3.98	-0.002	-0.927
Canada	0	32.5	3.869	-0.012	3.989	-0.003	-0.936
Denmark	0	32.5	3.877	-0.011	3.992	-0.003	-0.804
Finland	0	32.5	3.867	-0.012	4.025	-0.003	-0.95
France	0	32.5	3.793	-0.01	3.908	-0.003	-0.822
Germany	0	32.5	3.929	-0.012	4.112	-0.003	-1.019
Greece	0	32.5	3.902	-0.012	4.028	-0.003	-0.92
Ireland	0	32.5	3.921	-0.013	3.993	-0.003	-0.945
Italy	0	32.5	3.781	-0.009	3.976	-0.002	-0.858
Japan	0	32.5	3.739	-0.009	3.915	-0.002	-0.868
Netherlands	0	32.5	3.917	-0.012	4.017	-0.002	-0.956
New Zealand	0	32.5	3.923	-0.012	4.028	-0.003	-0.92
Norway	0	32.5	3.838	-0.011	4.015	-0.002	-0.965
Portugal	0	32.5	3.793	-0.01	3.925	-0.002	-0.865
Spain	0	32.5	3.832	-0.011	3.97	-0.002	-0.95
Sweden	0	31.6	3.88	-0.011	4.067	-0.003	-0.972
United Kingdom	0	32.5	4.085	-0.016	4	-0.003	-0.831
United States	0	32.5	3.886	-0.012	4.008	-0.003	-0.938
countries included	p-value	$\hat{\gamma}$	$\tilde{\beta}_1$	$\tilde{\beta}_2$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\alpha}_1$
Australia - Ireland	0.083	20.8	3.735	-0.009	3.928	0.001	-0.981
Italy - United States	0.001	32.5	4.098	-0.017	4.24	-0.004	-1.102
European	0	32.5	3.804	-0.011	3.952	-0.001	-1.016
Non-European	0	10.9	4.342	-0.017	8.668	0	-5.399

debt-to-GDP ratio and the real GDP growth rate in 1946 for Italy were available. Hence, we also excluded this observation when constructing the post war sample in our analysis.

With the exception of years 2008-2009 for Greece, we have deleted all observations that have either the debt-to-GDP ratio or the real GDP growth rate missing. For the post war sample, 12 observations were deleted (1976-1979 Denmark; 1949, 1973-1977 France; 1951-1952 Portugal). For the long sample, 256 observations were deleted (1914-1923, 1938-1946 Austria; 1914-1920, 1940-1946 Belgium; 1880, 1914-1949, 1976-1979 Denmark; 1880, 1914-1920, 1932-1949, 1973-1977 France; 1914-1924, 1939-1950 Germany; 1914-1918, 1939-1969

TABLE 8. Tests for threshold effects in median five-year GDP growth functions : long sample

country omitted	p-value	$\hat{\gamma}$	$\tilde{\beta}_1$	$\tilde{\beta}_2$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\alpha}_1$
Australia	0	32.5	3.357	-0.008	3.542	-0.002	-0.81
Austria	0	33.4	3.343	-0.007	3.448	-0.001	-0.655
Belgium	0	41.5	3.435	-0.008	3.455	-0.001	-0.749
Canada	0	33.4	3.36	-0.008	3.46	-0.001	-0.765
Denmark	0	32.5	3.393	-0.008	3.554	-0.002	-0.722
Finland	0	33.4	3.311	-0.007	3.434	-0.001	-0.707
France	0	33.4	3.291	-0.006	3.395	-0.001	-0.639
Germany	0	33.4	3.407	-0.008	3.572	-0.002	-0.754
Greece	0	33.4	3.404	-0.009	3.48	-0.002	-0.706
Ireland	0	33.4	3.36	-0.008	3.462	-0.002	-0.735
Italy	0	33.4	3.29	-0.006	3.415	-0.001	-0.649
Japan	0	32.5	3.272	-0.006	3.441	0	-0.761
Netherlands	0	32.5	3.351	-0.007	3.516	-0.001	-0.723
New Zealand	0.001	33.4	3.414	-0.009	3.483	-0.003	-0.682
Norway	0	32.5	3.37	-0.008	3.526	-0.001	-0.811
Portugal	0	33.4	3.296	-0.006	3.425	-0.001	-0.632
Spain	0	33.4	3.282	-0.006	3.427	0	-0.673
Sweden	0	32.5	3.382	-0.008	3.544	-0.002	-0.782
United Kingdom	0.002	41.5	3.465	-0.009	3.409	0	-0.695
United States	0	32.5	3.213	-0.006	3.356	0	-0.778
countries included	p-value	$\hat{\gamma}$	$\tilde{\beta}_1$	$\tilde{\beta}_2$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\alpha}_1$
Australia - Ireland	0.028	20.8	3.123	-0.002	3.281	0.004	-0.695
Italy - United States	0	40.6	3.566	-0.012	3.6	-0.002	-1.002
European	0	32.5	3.185	-0.007	3.328	0	-0.838
Non-European	0.004	16.3	3.564	0.004	4.021	0.018	-1.352

Greece; 1947-1950 Italy; 1941-1955 Japan; 1914-1920, 1940-1955 Netherlands; 1940-1945 Norway; 1913-1949, 1951-1952 Portugal, 1936-1939 Spain). Dropping these observations with missing values let us reproduce the summary statistics reported in "Final including New Zealand (NZ Historical Statistics GDP)" and "Final including New Zealand (Maddison GDP)" in table 1 of the errata. The final data include 2313 observations for the long sample and 1184 observations for the postwar sample.

For the five-year forward average real GDP growth, we take an average of annual growth rates included in the five-year forward window. The observation was dropped from the

TABLE 9. Test for threshold effect in median real GDP growth function with lagged dependent variable

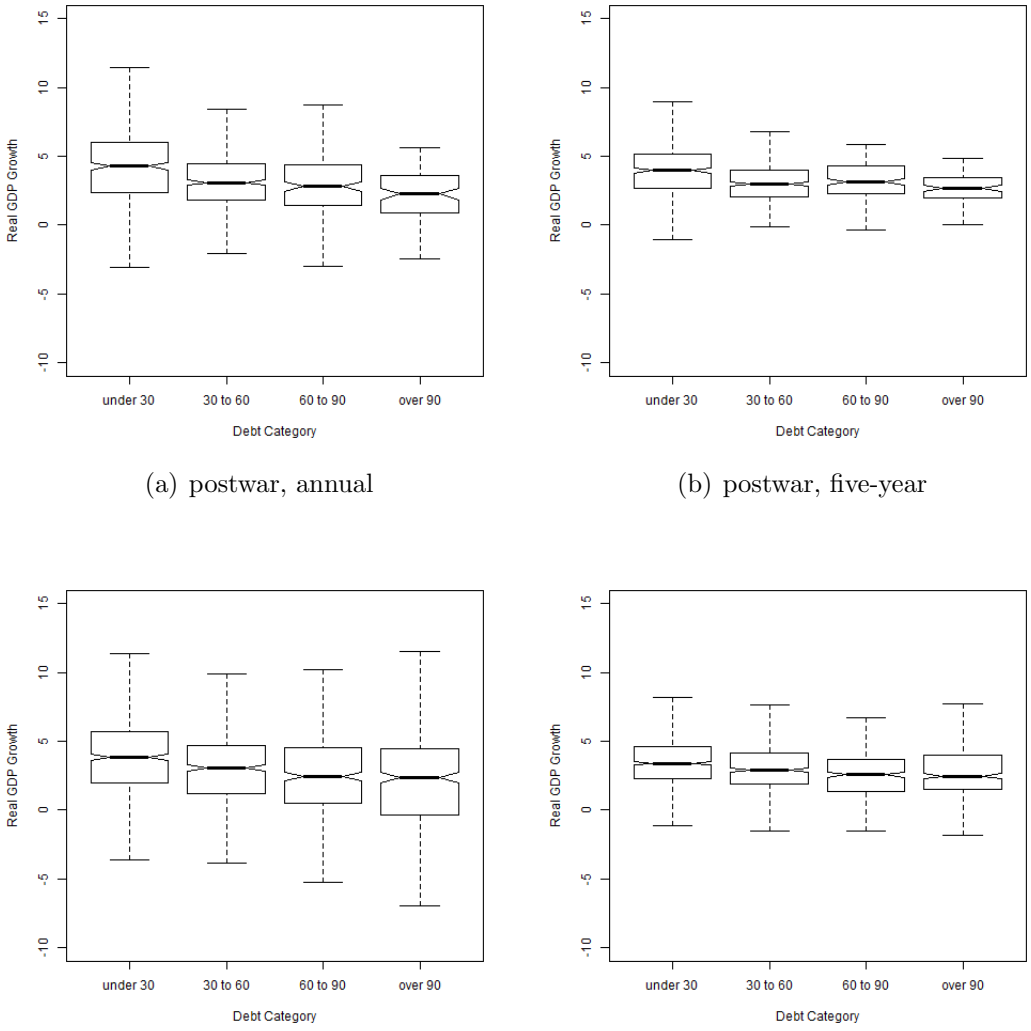
	annual		5-year	
	postwar	long	postwar	long
n	1161	2270	1062	2147
p-value	0.095	0.061	0.729	0.256
$\hat{\gamma}$	28	100	40.6	40.6
$\tilde{\beta}_1$	2.104	2.727	0.193	0.353
$\tilde{\beta}_2$	-0.008	-0.011	0.001	0
$\tilde{\beta}_3$	0.475	0.291	0.927	0.886
$\hat{\beta}_1$	2.234	2.955	0.244	0.356
$\hat{\beta}_2$	-0.002	-0.017	0.001	0.001
$\hat{\beta}_3$	0.478	0.284	0.918	0.886
$\hat{\alpha_1}$	-0.635	1.066	-0.112	-0.155

Note. $\tilde{\beta}_3$ and $\hat{\beta}_3$ are coefficients that correspond to the lagged dependent variable.

TABLE 10. Summary Statistics of GDP Growth by Debt Category

Growth Interval	Period	Debt Category[4]	n	Mean	Std. Dev.	Min	Max
Annual	Postwar	less than 30	445	4.309	2.944	-6.244	18.902
Annual	Postwar	30 to 60	442	3.076	2.930	-7.5	27.329
Annual	Postwar	60 to 90	199	2.931	2.640	-4.349	11.441
Annual	Postwar	over 90	98	2.145	3.072	-10.942	15.216
Annual	Long	less than 30	862	3.746	3.727	-15.978	19.367
Annual	Long	30 to 60	659	3.133	3.711	-13.067	31.004
Annual	Long	60 to 90	450	2.477	4.065	-21.709	25.537
Annual	Long	over 90	342	2.104	4.863	-18.750	29.058
5-Year	Postwar	less than 30	421	4.045	1.912	-1.449	10.522
5-Year	Postwar	30 to 60	399	3.103	1.527	-1.031	10.195
5-Year	Postwar	60 to 90	181	3.267	1.656	-0.386	9.652
5-Year	Postwar	over 90	84	2.751	1.065	0.047	4.880
5-Year	Long	less than 30	840	3.460	2.096	-4.713	10.773
5-Year	Long	30 to 60	609	3.061	2.070	-6.633	13.749
5-Year	Long	60 to 90	413	2.526	2.357	-5.842	13.937
5-Year	Long	over 90	328	2.699	2.331	-10.473	11.418

sample if any of $G_{c,t,t+1}$, $G_{c,t+1,t+2}$, $G_{c,t+2,t+3}$, $G_{c,t+3,t+4}$, or $G_{c,t+4,t+5}$ was missing. See Table 10 and Figure 4 for the summary of the data.



(a) postwar, annual

(b) postwar, five-year

(c) long, annual

(d) long, five-year

FIGURE 4. Box Plots of Growth by Debt Category

APPENDIX B. DETAILS OF THE TESTING PROCEDURE

In this part of the appendix, let $x_i = (1, debt_i)'$, and $w_i = debt_i$. Also, let $z_i = 1$ for the intercept-only model and $z_i = x_i$ for the intercept-and-slope model, respectively. Lee, Seo and Shin (2011) showed among other things that, under the null hypothesis (1), the

limiting distribution of QLR_n is the same as the limiting distribution of

$$QLR_j := \frac{1}{2}\sup_{\gamma}[G_n^j(\gamma)'\hat{V}(\gamma)^{-1}G_n^j(\gamma) - \tilde{G}_n^{j\prime}\tilde{V}^{-1}\tilde{G}_n^j],$$

where

$$\begin{aligned}
G_n^j(\gamma) &:= \frac{1}{\sqrt{n}}\sum_{i=1}^{n}(x_i', z_i' \times I(w_i > \gamma))'[\tau - I(u_{ij} \leq \tau)], \\
\tilde{G}_n^j &:= \frac{1}{\sqrt{n}}\sum_{i=1}^{n} x_i[\tau - I(u_{ij} \leq \tau)], \\
\hat{V}(\gamma) &:= \frac{1}{nh_n}\sum_{i=1}^{n}(x_i', z_i' \times I(w_i > \gamma))'(x_i', z_i' \times I(w_i > \gamma)) \\
&\quad \times K\left(\frac{Y_i - (x_i', z_i' \times I(w_i > \gamma)\hat{\beta}}{h_n}\right), \\
\tilde{V} &:= \frac{1}{nh_n}\sum_{i=1}^{n} x_i x_i' K\left(\frac{Y_i - x_i'\tilde{\beta}}{h_n}\right),
\end{aligned}$$

u_{ij} are iid random variables following Unif[0,1], h_n is the bandwidth, and K is a kernel function. Here, the subscript j denotes each simulation draw. We use the standard normal probability density function as the kernel function. We simulate the distribution of QLR_j and calculate the p-value for QLR_n accordingly.

In simulating the p-value, it is necessary to choose the bandwidth h_n. It was set to $\hat{\sigma} \times n^{-1/5}$, where $\hat{\sigma}$ is the sample standard deviation of $\tilde{u}_{i,t} = y_{i,t} - x_{i,t}'\tilde{\beta}$. We have also tried different bandwidths that are 0.5, 1.5, and 2 times this size. The results are not sensitive to bandwidth selection. The parameter space of the threshold is approximated by a grid such that $\Gamma = \{\gamma : \gamma = 10 + k \times 0.9\ ,\ k = 0, 1, 2, ..., 100\}$. Recall that the end points of the parameter space $[10\%, 100\%]$ are about 7 and 94 percentiles in the sample. We have also tried grids that have 51 and 201 equally-spaced points between 10% and 100%. The results are not sensitive to grid selection.

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