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Alternatives to Current Regression Methods in the Calculation of Standard Spending Assessments

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Preface

This is the final report of a research project undertaken for the Department of the Environment at the Institute for Fiscal Studies. The authors have benefitted from help and advice from staff at the Department of the Environment, and, at IFS, from Najma Rajah, Michael Ridge and Frank Windmeijer. The views expressed, and any errors, are the reponsibility of the authors alone, and not of the IFS, which has no corporate views.

Executive Summary.

Central government estimates the costs which local authorities would incur if they provided a similar level of service to their local populations. These costs are used as the basis for the distribution of revenue support grant to local authorities. At present, central government estimates the *relative needs* of each local authority by calculating a series of Standard Spending Assessments (SSAs). SSAs attempt to control for local variations in the costs of providing a similar, or *standard* service, reflecting differences in the number and needs of potential clients for a particular service in each local area and differences between areas in the costs of providing the service to each potential client.

This report has explored the validity of the methods which are currently used in the calculation of SSAs. It identifies a number of conceptual and technical issues which may need to be addressed in any future recalibration of the SSA system, and discusses the problems which the present capping arrangements pose for the identification of spending needs.

Standard Spending Assessments are currently estimated using a statistical tool known as regression analysis. In principle, the aim of the analysis is to assess how differences between local authorities in the *needs* for local authority services affect the level of *expenditure* which they must incur in meeting these needs. Local authority expenditures may differ for a range of reasons - including differences in efficiency and in political preferences regarding local public spending, as well as differences in the characteristics of the population and other factors which give rise to different needs. Regression analysis is, in principle, a tool which makes it possible to assess the separate contribution to expenditure of needs differences from the effects of the other influences on spending. However, its ability to assess the effects of needs on spending depends on some key requirements:

First, there are technical requirements about the methods employed. The aim of the exercise is to uncover the relationship between needs and spending, uncontaminated by other possible effects on spending arising from sources such as efficiency differences and political preferences. How far any particular regression technique will be able to estimate the "pure" effect of needs differences will depend on the particular ways in which efficiency and political factors affect spending. This will determine whether relatively straightforward techniques can get close to the needs/spending relationship, or whether more complex techniques are required.

It is a matter for judgement in particular cases, how much technical sophistication is needed in the estimation of the relationships underlying SSAs. A considerable part of our report has been devoted to assessing this issue.

Second, for the approach to be in any way meaningful, a much more fundamental condition must be met. The method requires that local authorities should have the ability to vary their spending levels to reflect differences in need; if they are not able to do this, then we cannot discover anything about needs by looking at their spending levels. More generally, the restrictions placed on the spending decisions of some authorities by the capping system can invalidate the assumptions which implicitly underlie the methods which have been used until now to estimate the relationship between needs indicators and spending. The SSA calculations currently in operation are based on an analysis of local authority spending data for a period (1990/91) when only a small number (20) of local authorities were capped. Capping now affects many more local authorities, and there is an important issue about its implications for any future updating of the SSA analysis. The second main element in our report has been an assessment of the implications of capping for the estimation of local authority needs through regression analysis.

We set out our main conclusions below. In assessing their implications it is important to bear in mind the relatively limited number of real alternatives which exist to the current approach. Whilst there may be scope for methodological refinement, there are, in our view, few plausible radical alternatives to statistical analysis of past spending patterns as the basis for assessing the relationship between needs indicators and spending. If it remains the intention to have a system of revenue support grants which aims to compensate local authorities for differences in spending needs, something like the present arrangements may continue to be needed.

Many of the important issues for policy will then involve assessing the balance of advantage between the current, admittedly imperfect, arrangements, and other, also imperfect, alternatives. The system of transferring resources from one tier of government to another should be clearly understood, transparent to local electors and seen to be fair. Any modifications to the present system intended to improve the technical merit of the system should be understood by those who then apply them, and should produce results which are, as far as possible, robust and stable from year to year. It is important that any methods used are, as far as possible, non-contentious and defensible, and should avoid the use of data which is unreliable or which may not be available at the time when the methods have to be employed.

Increases in the technical merit of the methods currently employed have to be balanced against arguments concerning the practicality of implementing the suggested techniques.

Likewise, when assessing the desirability of updating the analysis to use more recent data than the 1990/91 spending data currently employed, it will be necessary to weigh up the disadvantages of an analysis based on old data against the difficulties of employing the regression approach to analyse a period when capping has been more widespread.

The methods currently employed

In Sections 3 and 4 of the report we review a range of issues concerning the methods currently used to estimate the relationship between needs indicators and local authority spending. These fall into three groups:

- The underlying validity of the approach as a way of discovering the true relationship between needs and spending (Section 3).

- Some technical issues concerning possible "short-term" refinements to the current methodology, not involving major new data, or a new approach (Sections 4.1 - 4.7).

- Some more fundamental changes to methodology which could be considered, involving new data, or wholesale changes to the estimation techniques (Sections 4.8 - 4.9).

Regarding the first question, our analysis concludes that the validity of the models currently used in the SSA calculations depends on a number of fairly restrictive assumptions. It is far from obvious that these conditions are met, either conceptually or technically. In particular, we highlight the role played in the analysis by *variations in service quality* across authorities. The validity of the approach currently used to estimate SSAs requires *either* that local authorities should all be providing a common standard of service, *or* that variations in service standards between authorities should be unrelated to the pattern of needs across authorities.

There is, however, good reason to expect variation in service levels. The costs of providing a given standard of some services in sparsely-populated rural areas may, for example, be much higher than in urban areas, and rural areas may not, therefore, be able to afford the same standard of provision as in urban areas. Political preferences with regard to local spending may

also give rise to differences in service levels between areas. Some correlation between needs and service standards is also likely. For example, if the political control of an authority carries with it any implications for service quality and if political control is partly a function of some of the needs variables (such as socio-economic conditions) then service standards will be correlated with the needs variables. If this is the case, then the present method of calculating SSAs would tend to generate biased estimates of the importance of each of the needs indicators on the level of local spending.

- The obvious response to rectifying the omission of information on the standard of local services provided from the present SSA regressions is simply to include some measure of service quality in the estimation of the regressions, not with a view to basing grant in any way on service level but rather to prevent service level differences contaminating the estimates of how the various needs indicators affect the expenditure needed to provide local services at a standard level. Measuring service quality is, however, fraught with difficulties. First, there may be fundamental disagreements over the objectives of particular local services and therefore how performance would ideally be measured. Second, whilst peripheral measures of service standards such as the time taken to answer telephone enquiries are relatively straightforward to identify, more fundamental aspects of service delivery may often be more difficult to measure. In the longer term, however, we think there would be considerable gains from developing a set of good indicators of service quality for local authorities for use in the analysis, although we accept that the choice of quality indicators is unlikely to avoid controversy.

- An alternative method of taking account of variations in the quality of services provided by different local authorities is to include those factors which might be expected to influence a local authority's choice of service standard within the SSA regressions. Such factors might include the relative costs of providing each service in each area, local incomes, the level of grant received by the local authority and political and cultural influences. The inclusion of these characteristics of a local area would not be used to allocate grant to local authorities but rather to allow for such factors in the estimates of the impact of the various needs indicators on local spending. We illustrate the use of this approach by applying it to some of the regression equations, and show the difference that it makes to the results. Further research would be needed to assess whether more general application of these methods would result in major changes to the SSA results.

We assess secondly the scope for "short-term" technical refinements to the methodology. Regardless of whether SSAs continue to be estimated along the lines used at present or not, we describe a number of technical improvements which could be made to calculation of SSAs. These would allow a consideration of whether estimated SSAs can explain variations in spending across all authorities within a class and between all classes of authorities. We describe a number of possible methods for identifying outlying and influential observations as well as techniques for improving robustness to the presence of such observations. Whilst these methods are technical in nature, they would only need to be carried out when a new SSA framework was established and require only those techniques available on standard statistical packages. The most useful practical response to the identification of outliers, however, will be to find further data (in the form of new indicators, etc), which can successfully explain the existence and performance of outliers. In general, the presence of outliers is an indication that in some respects the model does not adequately explain the behaviour of certain authorities; often this can be due to the omission of important explanatory variables from the model.

Methodological changes which could be considered over a longer time-scale include the following:

(i) One option which has been suggested, but which has not been discussed in detail in this report, is the possible use of ***multi-level modelling*** techniques, in place of the current single focus on spending differences between authorities. Data on sub-authority needs indicators and spending patterns could be employed to investigate how the allocation of spending within an individual authority is affected by the pattern of needs across wards or other sub-areas within the authority. This might be an attractive approach for a number of reasons. One is that it might be reasonable to suppose that the same quality of service is provided (or at least, is intended to be provided) throughout an authority, which makes it possible to limit the estimation problems which would arise in comparing spending in local authorities where the quality of service provision was very different. Another attraction is that analysis of sub-authority spending allocations might still generate meaningful information, even when a local authority's spending was subject to an overall cap. The allocation of this fixed total of spending between sub-areas might provide a way of discovering the relationship between needs and spending, even in capped authorities.

Potential drawbacks of sub-authority data can, of course, be envisaged, and would need to be assessed carefully. One is the extent to which the spending data at sub-authority level is accurately allocated between areas. Some spending items may be difficult (or impossible) to allocate between sub-authority areas; these could include the fixed costs of central administration, and other costs unattributable to local areas. For the analysis of sub-authority data to be meaningful it is vitally important that this data is generated on the basis of actual sub-authority accounting, rather than by statistical apportionment of the authority's aggregate spending between sub-authority areas on the basis of area population or other characteristics. If the data is produced in the latter way, then regression analysis of the relationship between spending and population characteristics will simply re-discover the apportionment formula.

(ii) ***Panel data techniques*** hold out, in principle, the hope of more accurate estimation of the relationship between needs and spending, through the elimination of authority-specific "fixed effects". These techniques would use data for more than one year. With data for two years, for example, it would be possible to model the relationship between *spending changes* over the period, and *changes in the needs indicators*. Such panel data approaches have become widely used in econometric analysis in recent years, and would potentially be an attractive development of the current SSA approach.

The main obstacle to implementing panel data techniques in the SSA context, however, is the lack of comparable data on spending and local authority characteristics for more than one time period. Many of the indicators of need derive from the ten-yearly Census, and a study of the effects of changes in the needs indicators using two data points for each authority would thus need to compare data from the most recent Census with data from the Census conducted ten years previously. In the current context this would require data from 1991 to be compared with data from 1981 - now 15 years ago. Ensuring that the spending data for 1981 are wholly comparable with the spending data for 1991 would be extremely difficult, but would be critical to ensuring that the measured changes in expenditure did in fact reflect changes in spending, and not simply changes in the classification of spending items, or in other methodological aspects. Despite these reservations, however, we think that, as part of a longer-term research agenda, it would be worth exploring the scope for greater use of panel data techniques in the SSA methodology, and for developing indicators and spending data comparable over a run of years.

Capping and the future of SSAs

These technical and conceptual issues are somewhat overshadowed by the problems posed by the present system of capping arrangements for any future recalibration of the SSA system. These are the subject of Section 5 of the report. Capping undermines the possibility of identifying a meaningful relationship between spending and needs indicators through the use of regression analysis. Moreover, even if there were a return to some more selective form of capping at some stage in the future, SSAs could only be calculated accurately if existing procedures were modified.

There are two main issues concerning the assessment of SSAs when capping is in operation. One concerns the inclusion of capped authorities in the analysis of the needs/spending relationship. Since the constraints on their spending differences are radically different from the constraints facing uncapped authorities, including capped authorities in the model, as if they were uncapped, is illegitimate. Also, since capping depends on criteria which may be correlated with the needs indicators, simply omitting capped authorities is also problematic, since it may introduce problems of sample selection bias into the estimates. Techniques for correcting for this bias are discussed in the report, but their application might require judgements likely to generate substantial controversy. How many authorities are capped will be an important consideration in assessing the need for, and scope for, these techniques. Where very few authorities are capped, and where the capped authorities differ little in terms of their needs and other characteristics from uncapped authorities, it may not matter much if the estimation simply drops the capped authorities (the resulting formula could still be used to calculate their grant entitlements). Where a higher proportion of authorities are capped, or where the capped authorities have characteristics which differ greatly, on average, from those of uncapped authorities, it may be necessary to consider the available correction techniques. Where all authorities are capped, or where all authorities of a particular type (eg high-needs authorities) are capped, the available correction techniques will not help rescue the current approach.

The second principal issue concerns the scope for needs estimation in the presence of wholesale capping. Capped authorities face a different set of constraints on their decisions; specifically, they must decide how to allocate a given budget total between different services. Some information about their relative needs is likely to be provided by their decisions on the relative priority to give to different services. An authority that devotes a higher budget share to education, for example might be inferred to have higher needs for education spending, relative to other spending needs in that authority. The techniques which would be needed to assess

needs in such a context would need to involve estimation of a single inter-related system explaining an authority's budget allocation for all services (since an authority's needs for spending on each service depends on its needs for all services, and therefore service-by-service regression, as at present, is not an option). Considerable further research would be needed to develop whether a workable framework for SSA estimation could be devised along these lines.

If it is not possible to estimate new SSA models on current data, due to the extensive use of capping, the question arises as to how long the existing SSAs might be seen as sustainable. Can we live with the existing SSAs (despite their imperfections) for some time to come?

There is, of course, no difficulty in updating the information about individual authority entitlements as new information about the needs indicators becomes available. The problems concern the durability of the relationship between local authority needs indicators and the need for local authority spending to provide a standard level of service. It is a matter for judgement as to the point at which the information derived about the spending/needs relationship from 1990/91 data will become so outdated that its use becomes unacceptable. The relevance of an analysis of this relationship in 1990/91 to the current situation will depend on a number of factors - including the pace of change in relative prices (probably slow), in the technologies of service delivery (rather faster in some cases, and probably varying widely across service blocks) , and in the services which local authorities provide under each head (changes to local authority service responsibilities can lead to abrupt changes in spending needs). It is probable that the durability of the estimates based on 1990/91 data will vary considerably between service blocks, and that the SSA formulae for those spending areas characterised by reasonably stable technologies and little change in local authority responsibilities may not become outdated too rapidly.

1 Introduction

This report considers alternatives to the regression methods that are currently used to estimate the spending needs of local authorities within England. These are central to the calculation of Standard Spending Assessments (SSAs) which account for around £40 billion of local authority spending each year. Since the various forms of external support to local authorities (Revenue Support Grant, specific grants and revenues distributed from the National Non-Domestic Rate) make up, on average, roughly 80% of local authority revenues, it is desirable that any method of assessing local authority needs is seen to be technically sound, fair and transparent to the public.

In section 2, we describe the present system of assessing local authority needs, known as Standard Spending Assessments (SSAs). We then discuss how SSAs are used for the twin purposes of allocating grant to local authorities and for capping local authority expenditure. Section 2 also presents an introduction to the role of regression analysis, used extensively in the calculation of SSAs, setting out a number of criteria against which any modifications of the system used to calculate SSAs should be evaluated.

In section 3, we evaluate whether the present SSA methodology estimates the relationship between variations in local authority expenditure and various indicators of need in a manner which is defensible, given systematic differences in both the standards of service provided by local authorities and their organisational efficiencies. We suggest three methods which could, in principle, be used to do this. First, some objective measure of service standards could be included in the SSAs regressions. But such indicators are likely to be difficult to collect and highly contentious. Second, we suggest a number of factors such as input costs, local incomes, grants and political considerations which might influence the quality of local service provision and discuss the problems which might be involved in measuring the influence of each of these factors. Third, we consider the use of indirect indicators of service quality. We conclude that, whilst it is imperative to take account of variations in service levels and organisational efficiencies of local authorities in principle, it is extremely difficult to do this in practice.

In section 4, we focus on a number of statistical issues which would be likely to arise if the calculation of Standard Spending Assessments were to be updated at some stage in the future. These could be considered as methods of testing and improving the technical robustness of the

present SSA methodology. Whilst the subject matter of this section is intrinsically technical in nature, such procedures would only be used by statistical experts in the initial stages of recalibrating the SSA system. We consider how local authorities whose spending is poorly explained by existing models of expenditure or who appear to have considerable influence over the relationships estimated can be identified. We also consider tests for determining whether the relationship between needs and spending differs for particular groupings of authorities such as authorities within a given class, geographical area or with various characteristics in common. We also discuss improvements that would become possible with the availability of data covering more than one period.

In section 5, we consider whether, for the future, the use of a regression based approach to identifying spending needs can be defended given the extensive capping arrangements that are now a central part of the local government finance system. The justification for the current method of calculating SSAs assumes that local authorities freely choose their levels of spending on each major service which they provide. In practice, however, local authority discretion may now be heavily constrained. Over the last few years, the majority of authorities responsible for providing strategic local services such as police, fire, education, transport and personal social services, have set their overall budgets at the centrally determined cap. In addition, although capping does not dictate the allocation of total spending between service areas, a number of other developments within the local government finance system may have encouraged local authority spending on each major service to gravitate towards the SSA for that service.

We suggest that it is simply not possible to use the current form of regression analysis to identify any meaningful relationship between spending and the various needs indicators when a large percentage of all local authorities set their budgets at a centrally determined cap, as at present. When an authority's aggregate expenditure is capped, it is not possible to infer from a low level of expenditure observed on, say, under-5 education that the area has a lower-than-average need for such spending compared to other authorities; it is possible that it could have above average needs for under-5 education spending, but needs in other areas of spending which are even further above the average. When aggregate spending is capped, spending on each service block will be affected by the factors affecting spending on all other service blocks (since spending decisions become a matter of *relative* priorities), and it is not possible to obtain meaningful information from a service-by-service regression analysis of individual spending blocks. Even if a more selective form of capping were to replace the present arrangements at some stage in the future, present methods of calculating SSAs would still need to be modified

to prevent the existence of capping leading to biases in the estimated impact of the various indicators on local authority spending.

In section 6, we summarise our key findings and suggest areas which we have identified as meriting further investigation.

2 The present system of Standard Spending Assessments.

In this section, we briefly discuss the present system which central government uses to estimate the spending needs of each local authority in England. This system is general enough in scope to be applicable to the whole range of local authorities, including unitary authorities in London and the Metropolitan areas, shire county and district authorities and authorities responsible for a single service such as Police and Fire authorities. We consider both how SSAs are calculated and the uses to which they are put within the local government finance system. In addition, we provide a brief introduction to a statistical technique known as regression analysis which is used extensively in the calculation of SSAs and suggest a number of criteria which should be borne in mind when evacuating any modifications to the present SSA methodology.

2.1 The concept of Standard Spending Assessments.

Standard Spending Assessments (SSAs) are defined in the most recent official guide as "the amount that the Government considers appropriate for each authority to calculate as its budget requirement for a given year consistent with the amount that the government considers it would be appropriate for all authorities to incur" (SSA: Guide to Methodology 1996/97). Without further clarification of the meaning of the term "appropriate", this definition of SSAs is too vague for practical application.

Earlier publications refer to "the amount of revenue expenditure which it would be appropriate for the authority to incur to provide a standard level of service"[1] This is compatible with the more recent definition if provision of this "standard level of service" is compatible with authorities spending "the amount that the government considers it would be appropriate for all authorities to incur". This earlier definition is more practically useful in that, although it does not specify what the "standard level of service" is, it does specify that "level of service" is the benchmark for judging appropriateness.

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[1] Revenue Support Grant (1990/91). Association of County Councils. London.

In practice, aggregate SSAs are influenced by both identified changes in spending needs and central government's plans for aggregate public expenditure. Individual SSAs are a way of assessing relative spending need between local authorities given the aggregate total.

SSAs attempt to measure the *relative* spending "needs" of individual local authorities in order to distribute a fixed amount of public expenditure provision, and hence grant, between those authorities. SSAs are calculated separately for seven blocks of services (education, personal social services, highways, police, fire, other services and capital finance). Regression analysis is used to determine, at least part of six of these service blocks, the exception being the Capital Finance service block.

2.2 The Uses Made of Standard Spending Assessments.

Once SSAs have been calculated for each local authority, they are put to two quite distinct uses.

- **`A basis for distributing grant.** Each local authority receives sufficient grant from central government such that if it chose to set its budget requirement at SSA, it would be able to set its Band D council tax rate at a standard level, known as the Council Tax for Standard Spending (CTSS). For 1996/97, the CTSS for a Band D property within an English local authority is £569.

- **A basis for determining expenditure caps.** SSAs are used to determine the spending limits or "caps" which central government places on local authority expenditure, Typically, over the last few years, the greater a local authority's budget compared to its SSA, the smaller maximum increase (or, in some cases, the greater the minimum decrease) that the government will allow in its budget in the present year. In the current year, central government slightly relaxed the caps on those authorities responsible for the provision of education, personal social services, police or fire services, by reflecting in the caps the increase in the individual authority's SSA on these services.

SSAs are extremely important in influencing the budgetary decisions of individual local authorities since they influence both the level of grant which the authority receives from central government and the maximum Council tax yield which the authority can raise from local tax-payers.

2.3 The estimation and calculation of Standard Spending Assessments.

The basic method used in the calculation of SSAs for services such as education or personal social services for an individual authority is known as a "client group" approach. In this approach, the size of the client group for the service in question is multiplied by a "price per unit need" - the cost of delivering the service in question to a given client. The cost of delivering a service to an individual client such as educating a local child or providing social services to an elderly resident may differ for a number of reasons, including both the costs of inputs such as labour and capital, and the differing demands on local services placed by different classes of users. Thus, for example, a child with special needs may require more intensive interaction with teaching staff than other children of a similar age; an elderly person who is house-bound and living alone may require more attention from home-helps or the meals on wheels service than an able bodied elderly person who lives with their spouse.

These "prices per unit need" are estimated using a technique known as regression analysis. This approach attempts to explain variations in the level of spending between local authorities in terms of variations in a number of indicators of spending need. For example, the education of children whose first language is not English may require more attention than those for whom English is their mother tongue. Hence, the proportion of children from ethnic minority backgrounds is one indicator of need used in the calculation of SSAs.

To take an example, the client group for the provision of "under-5" (nursery) education is children in the local area between the ages of 0 and 4. The number of children in this client group is then multiplied by the predicted "price" of providing the service given the value of needs indicators in the area, which has been estimated using regression techniques. In this case, the "price" per unit of nursery education is taken to be a fixed cost applied to all local authorities plus an additional allowance for areas with high needs. In education, the Additional Educational Needs indicator is a composite indicator which takes account of the number of children in the area who come from ethnic minority backgrounds, whose parents are on income support or who come from single parent families. The result is then scaled by the Area Cost Adjustment (which targets additional grant at areas with high labour and property costs) and by the Scaling Factor which ensures that the sum of each of the SSAs adds up to the total pre-determined by central government.

Box 2.2: Calculating the SSA for Under 5 Education (1996/97).

Multiply *resident population aged 0-4* by:
£181.27 plus *£93.48* multiplied by the *Additional Educational Needs* indicator.
Multiply result by *Area Cost Adjustment for Education*
Multiply result by *scaling factor*.

Thus, regression analysis is used to identify the relative importance of those factors which might be good proxies for the levels of need in local communities and which may drive observed patterns of local authority expenditure. In addition, for the personal social services SSA, the size of the client group for particular local authority services such as residential care for the elderly are also estimated by regression analysis.

2.4 The place of regression analysis

Regression analysis is a term for a well-established class of statistical tools which are used to estimate the impact of changes in certain variables, called the "explanatory variables", on the expected value of another variable, called the "dependent variable. In the context of SSA calculations, the dependent variable is typically local authority expenditure per head on some service and the explanatory variables are indicators of need.

Regression analysis is used in calculation of SSAs on the premise that we can infer a causal relationship between variations in needs indicators and variation in spending required to reach a standard level of service. It is only given a number of assumptions that this interpretation will be possible and it is the purpose of this report to explore these assumptions. These assumptions include both behavioural assumptions about local authority decision making and statistical assumptions about the relationship of the data to this behavioural model. It is the purpose of this report to evaluate the use of regression procedures in identifying needs assessments. In so doing it is not our intention to assess the choice of variables taken as needs indicators but rather to assess broader issues about the possibility of identifying needs relationships at all given this approach.

The structural relationship which we aim to identify is the impact of needs indicators on the expenditure needed to provide services to a given standard. Under assumptions about linearity of the relationship, regressing actual expenditure on the needs indicators provides an estimate

of the impact of the needs indicators on the expected value of *actual* expenditure. In order to sustain an interpretation of this effect as the effect of needs on required spending a number of further assumptions are required. For instance, if we assume that actual expenditure is caused by these needs indicators and by other unobserved factors uncorrelated with needs then this estimate can also be justified as an estimate of a causal effect of the needs indicators on expenditure. Furthermore if we assume that all authorities spend the amount necessary to provide the given service level then this also estimates the impact of needs indicators on the expenditure needed to reach the given service level. Note that without making these or similar additional assumptions, the desired interpretation cannot be justified.

What is important to establish is whether the existing regression methods deliver estimates which do correspond to the relationship of interest between required spending and needs indicators under reasonable assumptions about the behaviour of local authorities and the collection of data. If they do not then regression techniques may still be appropriate, albeit with modifications. It may be that there are alternative relationships to which regression techniques *can* be applied so as to deliver estimates which *do* correspond to the relationships of interest. Alternatively it may be that there are ways to modify the regression techniques currently used so as to deliver estimates which do correspond to the relationships of interest. We therefore also discuss alternative regression methods.

Whichever methods are chosen in order to calculate SSAs, there are a number of criteria which they should satisfy.

- They should be technically sound and rely solely on well established statistical techniques.

- They should be intelligible to a reasonably informed member of the general public and not inhibit the making of informed judgements about the performance of individual local authorities.

- They should not be open to manipulation by either local authorities or central government.

- They should rely as little as possible on controversial or potentially controversial assumptions about the mechanisms which determine authority decisions.

3 Alternative methods of identifying standard spending assessments

In this section, we discuss the key assumptions which need to hold in order for the present method of calculating SSAs to be defensible. One important issue is the possibility that standard of service provided by each local authority may vary in ways correlated with needs indicators. This may lead to biased[2] estimates of the importance of the various needs indicators on local authority spending. We go on to discuss possible remedies for this problem.

In principle, a measure of local authority spending need would represent the minimum cost of providing a standard level of service. This will vary between authorities for a number of reasons. The *minimum cost* of providing given activities will vary because of differences in input costs such as prevailing local wage rates. The *activities* required to provide a *given level of service* will also vary because of differences in *needs* in terms of the number of households likely to require the service and the demands that each household will place on the service.

The *actual* cost of providing a *given level of service* will also vary because of efficiency of organisation. It would seem inappropriate to see differences in efficiency of organisation reflected in standard spending assessments since this would imply targeting more grant at the least efficient authorities.

The *actual levels of spending* on a given local service will vary between authorities not only because the cost of providing a given level of service varies but also because authorities may choose different levels of service. Such variation may occur because :

- **Different authorities have different relative costs.** The chosen service level may differ between authorities because of differences in the relative cost of providing one service as opposed to another. Thus, a rural authority with a sparsely distributed population might find the provision of schools far more expensive relative to providing parks, than would a densely populated urban authority. It could respond by providing a lower quality of schooling relative to parks. Relative costs could also differ as a consequence of past

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[2] "Bias" is not a perjorative term in this report, but a technical description. An estimator is said to be *biased* if it would not give the correct value of the coefficient of interest on average across reestimation on repeated samples.

decisions about the provision of public services: start-up costs may be incurred in providing a new service from scratch, and it may be less costly to continue providing services if the organisational infrastructure for their provision is already in place.

- **The cost of local government services differs relative to private goods.** The chosen service level may differ because the cost of local public sector goods such as education and libraries, relative to private goods such as books or household durables may differ. The local authority could respond by providing a lower quality of publicly provided services and lower local taxes. The main source of such differences in the relative prices of local government services and private goods is probably the impact of population density/sparsity on the costs of providing public services. Population density seems likely to affect the costs or attainable quality of a considerable number of local government services: for example, school sixth forms will be able to teach a wider range of subjects in urban areas than in sparsely populated rural areas without class sizes becoming uneconomic; refuse collection may be cheaper where the distances between properties are small; attaining the same response time for the fire service in rural areas as in urban areas would only be possible if many more fire stations are provided per head of population[3].

- **Different authorities have different resources.** The chosen service level may differ due to differences in the resources of both the local authority and of local tax-payers - grant plus tax base in the former case, and income in the latter. Although the system of equalisation employed to distribute grant between local authorities in England compensates local authorities for differences in their tax base, it does so at only one level of spending (SSA); local authority resources still affect the cost (in terms of the required change in council tax)

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[3] With each of these examples, it is suggested that costs of public services might be higher, relative to private goods, in rural areas than in urban areas; there may, however, be cases where the opposite is true. Urban density may, for example, involve much higher costs of providing a given standard of public protection through the police service (for example, if residents know less about their neighbours and are therefore less motivated to keep an eye on their property). The costs of certain production inputs (land and buildings, for example) may also be higher in urban areas, and local government services which require relatively large amounts of such inputs will then be more costly in urban areas, relative to private goods which do not involve a large amount of these inputs.

of changing spending from the standard level. Differences in need, if satisfied, will themselves cause differences in available resources.

- **Residents of different areas have different preferences.** The chosen service level may differ because of differences in tastes, political culture and so on between local authority areas. Thus, the residents of some areas may place a higher priority on nursery education compared to elderly residential accommodation than other areas, or may place a higher priority on publicly provided services relative to privately purchased goods. Past spending patterns may also lead to differences in preferences for current spending: residents may be reluctant to lose services which they have received in the past.

We will examine each of these issues in turn.

The estimation of local authority needs would be relatively straightforward if all local authorities provided services to a uniform standard and were equally efficient. Central government could simply gauge an authority's need to spend by observing how much the authority actually did spend. The relative costs which two local authorities incurred in providing a given level of service to their local populations could then be taken as an indicator of the relative needs of local authorities in those areas.

In practice, assessing local authority needs is highly complex. Local authorities may differ in both their relative efficiencies and their choice of service level (as noted by the Audit Commission, 1993, and by Senior, 1994). Under these conditions, simple comparisons of local authority costs may no longer be informative and the use of regression analysis to explain variations in either local authority costs or spending levels in terms of variations in the characteristics of local areas depends on restrictive assumptions. These involve the relationship of service level choices and relative efficiency levels to the characteristics of a local authority area.

One difficulty with the present SSA methodology, therefore, is that the regression equations which are used to estimate SSAs do not explicitly take account of variations in the quality of the services provided by local authorities. This would not create bias if variations in expenditure due to differences in policy choices or the relative efficiencies of local authorities were uncorrelated with needs indicators. However, this is an extremely strong assumption. It

means that there would be no association between the various indicators of economic or social deprivation and the quality of publicly provided services.

If we are unwilling to make these assumptions then we must attempt to control for differences in service level and differences in efficiency[4]. We consider the possibility of doing this in three ways.

- Inclusion of omitted factors influencing choice of service level in the SSA regressions
- Inclusion of direct measures of service level in the SSA regressions
- Inclusion of indirect indicators of service level in the SSA regressions

In sections 3.1 to 3.3, we consider each of these three methods in turn.

3.1 Omitted factors influencing choice of service level

There are a number of factors which might be expected to influence the standards of service provided by an individual local authority. These include the costs of providing each service in each area, the income available to both the local authority and to individuals living in the local area and political and cultural influences on spending. In this section, we consider how the influence of each of these factors on spending might be used within the SSA framework in order to calculate SSAs more accurately.

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[4] The objective of ignoring differences in "individual policies and efforts" has been referred to as "effort neutrality" in the context of design of the Australia system of grant allocation (see Australian Urban and Regional Development Review, 1994).

3.1.1 The impact of variations in costs between authorities.

At the most basic level, SSAs attempt to measure the variations in the cost of providing a given level of service between local authority areas. There is, however, a fundamental problem in using regression methods to measure the impact of variations in input costs on the level of spending on particular local services because input costs affect observed spending in two opposite directions. High input costs raise the costs of providing a given standard of service but may also affect the chosen quality of service[5]. The present SSA methodology aims to pick up the first of these two effects but ignores the affect of higher input costs on the choice of service level itself. Higher input costs may encourage authorities to choose lower standards of provision, such as a reduced provision of amenities in areas with sparse populations, and this will lead to lower measured spending. The present SSA methodology fails to take this indirect impact of input costs on spending into account and thus is likely to under-estimate the true needs of low cost areas and over-estimate the spending needs of high cost areas.

The costs of providing local authority services can be usefully divided into fixed and variable costs. Fixed costs are those that do not vary with the level of service provided that the service is offered at all. Such costs are often capital costs, and therefore outside the SSA system. For example, the construction and maintenance costs of a municipal library or museum are likely to vary very little, regardless of they are visited by several dozen individuals per day or several hundred. However, some current expenditures also have the character of fixed costs, in the sense that they are not affected (or very little affected) by the number of users. Thus, for example, the costs of planting municipal flower beds do not vary with the number of passers-by who enjoy them, and there are many fixed costs in the administration of services (only one clerk is needed to take council minutes, regardless of the population of the authority, etc). Variable costs are those costs that increase as activities are expanded. Labour costs often take this form. Thus, for example, providing care in the community services to a larger number of recipients is likely to require a greater number of social worker contact hours than providing to a smaller

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[5] The Area Cost Adjustment aims to offset some differences in input costs - specifically, differences between authorities in labour costs and business rates. However, even if differences in the cost of providing a standard level of service are compensated, the cost of choosing a level of service which differs from the standard level is still affected by input cost differences. As a result, input cost differences may still be liable to affect the chosen level of service.

number of recipients. The appropriate notion of variable costs is wider, however, than simply input prices. Any feature of an authority's circumstances that increases the expense required to expand service level by a given amount should be seen as increasing variable costs, including, for instance, needs-relevant features such as numbers of pupils lacking English as a first language.

Local authorities face a statutory requirement to carry out many of their most important functions. For services which are largely left to local discretion, it is conceivable that areas facing high fixed costs may be discouraged from providing the relevant services at all[6]. More commonly, since high fixed costs reduce the effective income of local residents this may also reduce the chosen level of service. However, given the nature of services provided and the small magnitude of the fixed costs relative to total income of residents it seems unlikely that either of these effects will be important and therefore offer, in our opinion, no major cause for concern.

Areas facing higher variable costs (relative to the prices of other goods) might be expected to choose lower levels of service since these services become more expensive relative to alternatives ways of spending residents' incomes. (Audit Commission, 1993). If providing any given level of service is more costly in one district than others then it is reasonable to think that the residents of that borough would prefer a lower level of service to those in areas of the country populated by similar individuals but where the service is cheaper to provide. Since local authorities are elected by residents, the political mechanism should largely reflect residents' preferences. One would then expect, other things being equal, to see a lower level of service in that borough. Given the differences in relative input or factor prices in different areas of the country, the current SSA methodology will only be valid under rather tightly defined and implausible conditions[7].

As a result, even if the characteristics of an areas which might give rise to differences in service costs, such as sparsity, are included in the regressions, the estimates of their effects on required

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[6] Even despite their receipt of revenue support grant. In cases where local authorities have full spending discretion they may decide that the grant would be better spent on other things.

[7] For the present SSA methodology to be valid, there would have to be no-substitution responses in local authority's behaviour. Thus, the relative quantity of different services such as nursery schools or sports centres purchased by a local authority would be independent of their relative prices.

spending cannot be inferred from the estimates of their effect on actual spending since, other things being equal, service levels will be lower in high cost areas. This would result in the grant system giving more money to areas where service provision is less costly at the expense of areas where it is more costly.

The extent of this problem depends upon how sensitive the chosen quality of local authority services are to variations in the relative costs of factor inputs such as labour, machinery and buildings. If an increase in variable costs such as a rise in prevailing wage rates in the local labour market were to induce greater than proportional reductions in the quantities of labour used in providing the service, i.e. local authority services were "price elastic" in the terminology of the economist, then areas facing higher costs would actually spend less than areas facing lower costs. The current SSA methodology would then lead to the perverse result that grant would be directed to areas with the lowest costs of providing a given standard of service. Information on the magnitude of such price or cost responses by local authorities is very limited and there is no justification for assuming them to be zero.

These points relate to relative rather than absolute costs. If the high cost of providing services in a particular area is mirrored in higher prices of all goods in that area (both local authority services and private goods) then substitution effects of the type under discussion would not be problematic. Disregarding other problems discussed below, spending regression might then pick up the impact of absolute cost differences accurately.

This conceptual problem of separating out the two impacts (direct and indirect) of input costs on local spending could be tackled if we could measure the quality of local service provision in some objective manner. It would then be possible to estimate the impact of input costs on local quality choice if there were reliable data on the input costs faced by each local authority or the factors driving those costs. However, if such service level data were reliably measured then, as argued below, it would make more sense to use the information directly in SSA regressions to capture differences in service levels.

In practice, measuring local authority input prices is far from a trivial matter. Direct information on variable costs is difficult to acquire. The most obvious data source is the Area Cost Adjustment (ACA), which is currently used to estimate input costs and yet this only considers two aspects of costs, labour and business rates. In addition, the ACA does not register any difference between the costs of providing service for any authorities outside of the

South East of England. There are some additional data sets which could be of interest. District level information on wage levels is available from the Survey of Personal Incomes, for example, although small sample sizes require the exercising of a considerable degree of caution in using these figures. In any case, wages are only one component of input costs, albeit the most important one for many local authority services.

Insofar as input costs can be reliably measured, data should be collected and used in SSA regressions.

3.1.2 Problems reflecting the influence of incomes on spending.

Individual demands for private goods such as restaurant meals or TV dinners are influenced by individual incomes. Thus, we might expect the type and amount of goods which richer individuals purchase to differ from those which poorer individuals might purchase. Similarly, the income of residents in a local authority area may influence both the types of demands which they place on their local authority and their capacity to pay for services above the level funded by central government.

It is possible that those authorities with more affluent resident populations may, other things being equal, choose higher standards of provision for some services and lower standards of provision for other services (such as those in which private alternatives are available). Residents' incomes also determine their capacity to purchase other goods, some of which may be substitutes or complements to local authority services. Private education services, for instance, are a substitute for local authority education whereas books might be a complement. This may also influence their desired levels of service, though the direction of such effects is not clear *a priori*.

The distribution of income within an area may also influence the types and standards of services which local authorities provide. If the demands placed on local services differ between income groups, then the chosen quality of service in a given spending program may be influenced by the distribution of income in an area as well as the average level of income. In addition, it is possible that the incomes of politically more influential groups may have a heightened influence on spending decisions. What might matter, for instance, may not be the average level of income but the income of the decisive voters in local elections.

The relationship between average household incomes and the local authority's capacity to raise revenues through local taxes would be relatively straightforward if the local tax system was related to household incomes. However, the Council Tax has elements of an income tax (through Council Tax Benefit), a property tax (banded valuations) and an personal tax (single person discounts). (Giles and Ridge, 1993). The Council Tax Benefit system means that those on the lowest incomes do not pay any tax at all whilst tax payments increase with income until individuals do not receive any Council Tax benefit at all. As a result, the capacity for a local authority to raise revenues may depend on the distribution of income in the local area as much

as average incomes, since it is those households with incomes just above the level at which Council Tax Benefit is withdrawn who often face the highest local tax bills as a percentage of their household incomes.

Any link between either average income or the distribution of income within a local authority area and the standard of service which the local authority provides would generate significant problems for the present SSA methodology. Both average levels of income and other facets of the income distribution are clearly correlated with many of the needs indicators, many of which are measures of economic or social deprivation. This could be a source of serious bias in estimating SSAs under the present methodology. If, for example, residents in richer areas were likely to vote for higher levels of education spending, for a given level of need, then leaving income out of the regression equation would lead to under-estimates of the importance of any needs indicators which were negatively related to income, thus leading to too little grant flowing to needy areas and too great a level of grant flowing to more prosperous areas. Conversely, for services where the standard of provision is higher in poorer areas, the current SSA methodology will exaggerate the importance of any needs indicators which are negatively correlated to income levels, leading to too great a level of grant flowing to poorer areas.

For the present SSA methodology to be valid, one of two conditions therefore need to be met. There must either be no systematic correlation between local incomes and the needs indicator variables used in the existing SSA regression equations or the level of service provided by local authorities must not be related to either the average income of local residents or the nature of the income distribution in that area. Neither of these conditions are likely to be met in practice. First, one would expect to find a negative relationship between the average income per head of individuals living in an area and the values of some of the indicators presently used within the SSA formula such as the proportion of children whose parents receive Income Support. This will tend to reduce the estimated importance of these characteristics of an area on local spending. Second, there are a number of service blocks in which proxies for the incomes of local residents are statistically significant when added to existing SSA regression equations, often significantly influencing the estimated coefficients of one or more of the needs indicators. This suggests that income does have an influence on spending decisions.

One would expect that it would be *real* incomes - i.e. incomes adjusted for variations in price levels between districts- that would influence spending choices. If prices are generally lower in one part of the country than another, then districts in that part of the country should be

regarded as having a higher capacity to spend on local authority services unless local incomes are proportionately lower.

The problem here is not the same as with the influence of costs on service level if income does not in itself affect costs or needs and therefore does not in itself affect required spending. It would then be purely an omitted influence and it might be thought sensible to include it in SSA regressions to capture the omitted influence on service level choice. The idea of doing this would not be that grant formula would include any income related component but rather that the needs component of the regression would be better estimated with the addition of income terms.

The circumstances under which this procedure would adequately correct for differences in service level are limited. For instance, in the unlikely case that relative costs of local authority services do not matter or do not vary then adding income might capture service level choice well. In general, service level is jointly affected by available resources and relative costs.

We experimented with two sets of data on incomes in local authority areas. In the case of incomes, data is available on incomes at county level from the New Earnings survey. In order to use this measure of income, we had to assume that the income level of each district in a county simply equalled the mean level for the county as a whole. This is clearly problematic since it involves some measurement error (the difference between actual district income per head and average county income per head). Moreover, this measurement error is likely to be correlated with many included indicators of need. Thus, a district with lower income per head than the county average is also likely to have higher measured needs than the county average.

The second measure of income used was from the Survey of Personal Incomes which does provide information on average incomes at district level. This, however, leads to rather small sample sizes and hence rather large margins of error and would almost certainly be too inexact to be suitable for inclusion in the SSA methodology.

If the coefficient on income reflects a choice based on ability to pay, it would not be sensible to include it in the grant allocation system. We have concentrated on two equations in the education sub block. Table 5 of the Appendix shows the effect of including the logarithm of income, together with political control which we discuss below. The inclusion of these variables appears to have a significant impact in the nursery education equation in particular.

This should give grounds for concern that the coefficients from estimation without income do not result in an allocation of resources which accurately reflects the consequences for local authorities of differences in needs levels.

However, the inclusion of information on labour incomes is rather problematic since higher wages in an area may influence the demands placed on local authority services but they also influence the costs of the labour inputs which the local authority uses to produce those services. Thus, we need separate variables to measure incomes of local residents and the costs faced by local residents. We could not find satisfactory data in this respect.

3.1.3 Political and cultural influences on spending.

Other sources of discretionary choice made by local authorities may also be correlated with the pattern of measured needs. There are at least two further sources of such choices which have not yet been discussed :

- **Political and Cultural Preferences.** Voters in certain areas may tend to favour or to vote for political parties which favour high spending without this being connected in any way to needs.

- **Omitted Needs.** Higher spending might reflect genuine variations in needs which are not currently measured and included in SSA regressions.

The appropriate method for dealing with the impact of political choices on spending depends on what drives these choices - political preferences or omitted needs. First, if, for example, areas with high levels of social deprivation tended to have different preferences for discretionary public spending, independent of their needs, compared to other authorities, then the present method for calculating SSAs would lead to high grant to compensate these areas for having tastes which weigh heavily on the public purse. This argument gives a reason for correcting for the influence of political factors in SSA regression. The most obvious suggestion for doing so would be to add variables reflecting political control to SSA regressions as a means to correct the estimated effects of needs variables. Such estimated political effects would not be entered into SSA formulae.

However, secondly, political choices are a vehicle for needs-driven choices as well as a reflection of cultural differences. It is partly through choice of political party (as opposed to through influence on choices made by party representatives) that spending needs are translated into local authority spending decisions. If political control variables were added to SSA regressions then they would pick up any such effect due to omitted needs variables. Not allowing such an effect to enter the SSA formulae would result in authorities with these omitted needs being explicitly undercompensated in grant formulae (whereas simply omitting the political variable would give them, roughly speaking, the average paid to all authorities).

One method for testing whether unobserved "cultural" or political influences are responsible for some of the variations in local authority expenditure is to include variables reflecting the political complexion of the local authority in to the SSA calculations. If terms reflecting political complexion were found to affect spending levels then there would be at least two attitudes to take towards this. It could be regarded as indicative of an omitted cultural influence which it would be inappropriate to recognise in the standard spending formula or it could be regarded as picking up the expression of an omitted need which it would be appropriate to recognise. The appropriate responses are diametrically opposed.

It would be possible econometrically to separate such effects if one could find data on the characteristics of local areas which were correlated with the political culture of the area but not to any needs which are omitted from the SSA formula. We have no suggestions for what such variables might be.

The consequence of adding a variable reflecting party of control to the SSA equations for two education sub blocks can be seen in Table 5 and was discussed in the previous section.

3.1.4 The influence of grant on spending.

A further factor which may influence the pattern of local authority expenditure is the amount of grant which central government distributes to each local authority. In principle, one could argue that lump sum grants from central government to local authorities should affect spending decisions in exactly the same way as increasing the incomes of local residents if the political mechanism allows local residents to choose how much of their income they wish to allocate to local services and how much to their own private consumption. In this case, it would be appropriate to add lump sum grants to total local income, and grant would not need to appear separately in the SSA regression equations.

There are, however, a number of reasons why we might expect central government grants to have a different impact on local spending than that of the implicit impact on incomes of local residents. The phenomenon of higher grants having a larger effect on local spending than changes in local incomes is sometimes known as the "flypaper effect" (since the money sticks with the organisation which receives it).

- **Failure to perceive true costs of local services.** Local residents may misunderstand the operation of the grant system. If they believe themselves to be paying directly for a mistakenly high proportion of local expenditure then large grants may cause them to underestimate the true cost of local services in terms of their own taxes.

- **Operation of Council Tax Benefit.** The operation of the Council Tax Benefit system and the existence of 100% rebates for those on the lowest incomes means that grants used to increase spending raise the welfare of the poorest citizens whilst grants used to reduce tax rates would have no impact on their welfare. Indeed, some of the grant would flow straight back to central government in the form of lower Council Tax Benefit payments. Thus, the larger the fraction of any additional grant which the local authority uses to increase spending rather than to lower taxes, the larger the amount of extra resources which remain in the local area and more progressive the impact of the changes on individuals are likely to be.

- **Actions of those working for local authorities.** Actions of those working within local authority organisations who are more favourably disposed towards local spending than

ordinary residents may lead to extra grant being directed towards extra spending rather than lower taxes. (See King 1984 for a description).

- **The impact of capping.** The introduction of universal capping in 1991/92 has restricted the ability of local authorities to respond to changes in the preferences of their residents caused by such factors as incomes, costs or political tastes. However, since the capping criteria and grants are both based on SSAs, it is possible that a relationship may be observed between changes in grant and changes in spending (because of the change in the cap); changes in residents' incomes, on the other hand, cannot influence the spending of a capped authority unless the cap is relaxed.

If there are flypaper effects then it may be that estimated needs effects are actually reflections of the fact that high measured needs increase grants and thereby spending. Thus, grant may flow to areas which have high spending largely because they received large levels of grant under a previous local government finance system. This is one "circularity" problem which seems difficult to solve under the current system since adding grants to SSA regressions would be highly problematic. Grants to local authorities depend on local resources and SSAs. Since the SSAs are calculated using the needs indicators, then a grant term in the regression equation would be highly collinear with the needs indicators and hence it would not be possible to derive precise estimates of the importance of the needs variables in explaining variations in local authority spending.

One relatively straightforward approach to evaluating how important the impact of grant on spending might be in practice was to use evidence provided by the massive redistributions of grant between local authorities which occurred when the rating system was replaced by the Community Charge in 1990/91. Under the rating system there was full resource equalisation, with local authorities benefiting from the resource equalisation through grant contributions inversely related to aggregate rateable values. The need for these large resource equalisation flows ended with the move to the Community Charge since the tax base for the Community Charge was, of course, the adult population in the area, and the per capita (strictly, per adult) tax base was therefore constant across authorities. The switch in 1990/91 from the rating system to the Community Charge led to large losses of grant experienced by authorities with low property values per head of population and large gains in grant by authorities with high property values per head of population. We simply ranked local authorities by their ratios of rateable value per head in 1989/90 and compared this with a ranking of the percentage

spending increases by local authorities between 1987/88 and 1991/92. The correlation coefficient between authorities ranked in order of percentage increases in spending and loss of grant was small and *positive.*

Whilst this approach is clearly relatively simplistic, it does provide some limited evidence that the lack of concern over the impact of grants on spending within the present SSA methodology may not be a major weakness of the approach. This conclusion may not readily transfer from an era of selective capping to the present era in which the budgets of most local authorities are set at a centrally determined cap.

3.1.5 Organisational Efficiency

As well as variations in service quality between authorities, there may also be systematic differences between the organisational efficiencies of local authorities. This will lead to high costs in those areas but not to high values of required spending if these organisational inefficiencies are regarded as avoidable. This increase in costs will have effects of the sort discussed above, raising spending at any given level of service but also possibly encouraging residents to vote for lower levels of service.

If the efficiency of local authorities varied between areas of the country, perhaps due to variations in labour market conditions, in a way systematically correlated with values of some of the needs indicator variables, then again, high spending in these areas could be mistakenly attributed to the needs indicators in these areas. This would result in grant compensating some local authorities for their inefficiency and penalising others for being efficient.

In practice, it is by no means obvious how the organisational efficiency of local authorities could be measured in ways which would not prove to be highly controversial.

3.1.6 Summary

Throughout this discussion, we have argued that a weakness of the current SSA methodology is the omission of any information on the standard of service provided by local authorities and we have suggested a number of factors such as local incomes and input costs which may have an impact on service standards. It is important to note that including one of these omitted influences, such as income, costs or political choices alone is not the same as standardising for service level if the other factors remain important. The present SSA methodology, even adjusted to take account of variations in income levels between areas, would still fail to give unbiased estimates of the impact of the currently used needs indicators on a local authority's need to spend unless none of these other omitted variables was systematically related to any of the included needs variables. In particular, if service levels do vary with differences in the relative costs or faced by different local authorities or due to different political choices, then simply adding income levels to the SSA regressions will not eradicate bias in the estimation of SSAs. Correcting for the omission of only one of these influences on service levels within the SSA regressions is therefore not a simple solution to the problem of service level differences.

Nevertheless, we present in the Appendix some examples of the inclusion of variables relating to local incomes and political control in some of the SSA regression equations. In the case of the unit cost equation for primary and secondary education (Education SSA Regression 2), neither appears statistically significant and their combined impact on the coefficient on the Additional Educational Needs (AEN) indicator is limited. In comparison, both income and political control appear to be significant in influencing the level of spending on nursery education (Education SSA Regression 6), and the inclusion of these variables significantly lowers the size of the estimated coefficient on AEN. Whilst we can only draw limited conclusions from these examples, these results do suggest that further investigation of the impact of local characteristics which may influence service standards may be warranted.

We have considered a number of factors which, in principle, might have an influence on the standard of service which a local authority might choose to provide. These include input costs, the nature of the income distribution in the area, the level of grant received by the local authority, political and cultural influences and the organisational efficiency of the local authority. In practice, tackling many of these issues would be problematic and controversial. An alternative approach for dealing with variations in service levels between authorities is to

include some measure of the standard of service provided by the authority directly within the SSA calculations. We discuss the methods which could be used to provide measures of service quality below.

In the presence of capping, the current methodology would only estimate the impact of the needs indicators on spending if the cap equalised service levels across capped authorities or if differences in service level caused by differences in caps are uncorrelated with needs indicators. The impact of the capping arrangements within the local government finance system on the calculation of SSAs is discussed in more detail in section 6. The methods discussed below are not undermined by capping in the same way since correction is made for induced differences in service level.

3.2 Taking account of Service Quality Directly.

In order to correctly estimate the impact which the various needs indicators have on the standard spending needs of local authorities, we would need to incorporate an allowance for differences in local service quality within the regression method. This would not be used to allocate grant but simply to correct for any bias caused in the estimation of the impact of the various indicators of need on spending levels which are caused by variations in the quality of provision. We consider the possibility that the measure of service quality could either be made through direct observations of service standard levels or through the use of indirect proxies for the standard of service provided.

Adding such terms to regressions would generate technical concerns associated with the simultaneity in determination of expenditure levels and quality of service. That is to say, omitted influences on spending given quality are themselves likely to influence choice of quality. Service level is therefore likely to be correlated with the omitted influences in the spending equation and bias introduced into OLS estimates involving the relationship between the two. Dealing with these concerns would require use of, say, instrumental variables techniques in estimation. However, we believe that the omission of indicators of service quality from the present SSA methodology is a serious issue and that such methods merit further investigation.

3.2.1 The use of service quality indicators.

Inclusion of service level indicators would deal with the problem that service levels may vary because of political choices or because the costs of provision vary by actually including the quality of service provided directly within the regression equation. These sorts of methods have the advantage of not being undermined by capping since if a capped authority is forced to provide a lower standard of service than it would have ideally liked, the "service quality" indicator deals with this and leaves the conditional relationship between the needs indicators and the level of spending unbiased by the impact of the capping arrangements. We think there is a case for further investigation of the potential for direct measurement of service quality, and its use in the SSA analysis. However, we recognise that this will involve a longer-term research agenda, and that it may prove controversial, since there are considerable difficulties, of both a conceptual and practical sort, in the measurement of service quality.

"Service quality" indicators could take a number of possible forms, including the sort of performance indicators now published by the Audit Commission, or questionnaire surveys of local people, for example. Ideally, measures of output such as the outcomes generated by the education system or the support provided by personal social services would be used, but these are much less straightforward to measure accurately.

The use of any form of service quality measures is likely to prove controversial. First, there is plenty of room for fundamental disagreements over what the purpose of each local authority service is and therefore, what the relevant measure of performance is. Second, there is likely to be concern that local authorities might be able to direct their efforts at improving measurable aspects of their performance at the expense of non-measurable aspects which might be at least as important.

The meaning of "level of service" is ambiguous and there could be fundamental differences in opinion between local authorities over how service quality could be objectively defined and measured. For example, the service level of local education authorities could conceivably be measured by their exam results, in a similar way to that found in school "league tables" which are already published on an annual basis. Conversely, the improvement which a local education system generates in local children's educational attainment from variable starting levels might be seen as a more appropriate measure, though this could be argued to be more a measure of

performance than of output. These issues are interpretative rather than technical issues and lie beyond the scope of the current report. However, this ambiguity may render such methods rather controversial if implemented in practice.

In principle, under this system of treating the quality of local service provision, local authorities would *not* have any incentives to manipulate the quality indicators. The "quality indicators" would simply be used to estimate the SSA formula and would not, in themselves, be used in calculating the distribution of grant. If this were generally understood, then local authorities would have nothing to gain from seeking to manipulate the performance indicators.

However, in practice, if the publication of these indicators caused local authorities to concentrate on them anyway (or if misunderstandings were to arise) the usefulness of this method could be compromised. It is typically easier to generate performance indicators which measure particular minor aspects of service provision such as the time taken to answer phone calls or respond to correspondence without capturing more fundamental aspects of service quality. Local authorities are often able to target resources to improve their ratings on these performance indicators without having any impact on the underlying quality of service provision. Thus, the local education authority could make dramatic improvements in the time it takes to deal with correspondence without making any changes to the overall quality of the educational experience of young people in its area. One concern with the use of any form of performance indicators is therefore, the extent to which authorities could manipulate the indicator by focusing their energies on altering only the measurable aspects of performance. This would be a problem in that the performance indicators would then cease to measure service standards accurately, which would render them of little use within a regression formula.

In many cases, there are multiple service outputs and these would need to be aggregated in some way. One method would be some weighting imposed by central government judgement. In order to avoid accusations of arbitrariness it would be sensible to base the weighting on evidence of residents actual preferences, as might be garnered, for instance, through questionnaire surveys.

An alternative method would be to measure the quality of local services directly through the use of such questionnaire surveys. Although a promising area for further research, a questionnaire based approach to the measurement of quality would have to tackle a number of difficulties.

- It would be extremely costly to undertake regular representative surveys of the residents in local areas which had sufficiently large sample sizes in each area to generate the information needed for this purpose.

- Designing simple questions to measuring the quality of each service could give rise to controversy.

- Views on local services could be influenced by a wide range of factors, of which the actual quality of provision may only be one. Thus, the use of surveys would be a very noisy indicator of service quality.

- Views on quality may be expressed relative to expectations and therefore be recorded as less favourable where residents are more demanding rather than less well-served. Paradoxically, quality could be measured as lower in less well-off areas even if there were no real differences.

It would not be necessary to use observations on every authority in every year provided that the selection of authorities was not in any way linked to factors determining spending decisions. Hence the lack of availability of output indicators for certain authorities need not be problematic.

The drawbacks of these various approaches suggest that these methods of taking service quality in to account may be extremely problematic. They might also raise the level of controversy surrounding the annual grant settlement process. In any event, approaches based on the direct measurement of quality could not be implemented satisfactorily in a short time frame, and require considerable further study if reliable methods of measuring service quality are to be developed.

3.3 The use of indirect indicators.

In the absence of acceptable direct measures of service quality it could be that, under certain assumptions, there would exist other aspects of authority behaviour or performance which would be suitable proxies for overall service levels and could be used in their place.

For example, there might be some item of spending such that it would be legitimate to expect authorities sharing the same overall service level to pursue a similar standard for that particular item, irrespective of their needs relating to other items. For instance, authorities with similar standard of physical education facilities, say, could be taken to have the same overall standard of education, irrespective of differences in needs relating to other aspects of teaching. We may think of such an item of expenditure as an *indirect indicator* of overall service quality. If this were so then it would be possible to control for differences in service level by using information on standards of provision of the indirect quality indicator without using direct measures of overall service quality at all.[8] Intuitively, the sort of circumstances in which this approach could be reasonable would be if all differences in required spending on other items were due to fixed costs or fixed needs. If differences were due to differences in variable costs then one would expect these cost differences to encourage substitution between budgetary items in such a way as to make differences in standards of the indirect indicator an unreliable guide to overall service level. For instance, to take the example above, if two authorities had the same overall standard of education but classroom teaching were more expensive in one than the other then one would expect that authority to choose to pursue highest standards of teaching in subjects taught outside the classroom and therefore to acquire better physical education facilities. Physical education facilities would not then indicate overall education standard. Since much of the differences in required spending may be expected to be due to cost differences this makes the approach under consideration seem unattractive.

In general, even if differences in required spending are not due to these sorts of cost differences, the assumptions required to justify an approach like this are difficult to explain and difficult to test and this is not therefore an approach which can be currently recommended. While the idea is of academic interest it would fail the criteria of ready applicability and avoidance of controversial assumptions.

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[8] This idea is adapted from ideas used in the measurement of child costs. Spending on certain items in household budgets - called "adult goods" - such as adult clothing or alcohol may be indicative of overall living standard irrespective of the presence of children. The costs of children may then be indicated by the differences in overall spending of households with and without children but which spend the same amount on adult goods. The idea here is that some items of authority spending may be like adult goods in being unaffected by the relevant needs

indicators. For a detailed description of the assumptions required fr this approach to be justifiable in the context of child costs, see Blackorby and Donaldson (1994).

3.4 Conclusions

The validity of the models currently used in the SSA calculations depends on a number of fairly restrictive assumptions. In this chapter we have highlighted, in particular, the difficulties which are posed for the SSA methodology by variations in service quality between authorities. The main elements of the argument may be usefully summarised in a diagram (Figure 9). Observed levels of local authority spending on a particular block of services will reflect three things: the level of service (service "quality"), the costs of providing each unit of the service (in turn a function input costs and efficiency), and the level of needs for the service (reflecting the composition of the population and other factors). The objective of the SSA exercise is to assess the relationship between needs and spending, the bottom arrow in the diagram, (and, through the area cost adjustment, the effects of input cost differences on spending), whilst controlling for variations in spending due to differences in efficiency and service level.

Estimating the relationship between needs and spending would be straightforward if there were no variation between authorities in either costs or service level, or if costs and service level varied across authorities in a way that was wholly uncorrelated with needs. However we have argued that there are reasons to expect service levels to be a function of costs and needs as well as in tastes and attitudes. As a result, needs and costs will be found to affect spending through two channels, as Figure 9 shows: firstly, through their direct effect on spending, and secondly through their indirect effect on service level. Unless this second channel can be taken into account in the estimation, the estimated effects of needs differences on spending will be liable to include effects which arise through the determination of spending quality.

One response to this problem would be to include measurements of the quality of local services explicitly within the regression framework. However, whilst this may reduce the potential bias in the estimates of the impact of the various indicators on spending needs, the measurement of quality is problematic and likely to prove controversial. The indicators would need to be technically-sound measures of the quality of local services, immune from manipulation by local authorities, comprehensible and broadly accepted by participants in the process. In the longer term there may be some benefits from further research to develop a good set of indicators of service quality for use in the SSA analysis.

Given the difficulties in making direct measurements of local authority service quality, an approach which may be more immediately feasible would be to include in the analysis some indicators which might influence the quality of services which a local authority chooses to provide such as information on local incomes, costs and political factors. We have illustrated how this might be done with some examples. However, it is important to recognise that this may be an inferior method of controlling for variations in the level of service provided by local authorities, compared to direct quality measurement. This can be seen clearly in Figure 9. Although including indicators of income, preferences and costs in the analysis can control for their influence on spending, operating through the choice of service level, the link between needs and service level still remains unaccounted for, and cannot be separated from the direct effect of needs on spending, for a given level of service quality. The approach is therefore less effective, in principle, than the use of direct quality data. Also, although data on incomes, for example, may be more readily available than data on quality, it may be no less controversial in practice.

We have briefly noted the possibility of a third approach to the problem of service quality, base on the use of indirect indicators of service quality. However, this suffers from a lack of obvious candidates for good proxies of service quality. Even if these were to be found, it is unlikely that such a method would be either transparent to the general public or uncontroversial to local authorities.

4 Technical Issues in Estimation of Expenditure Regressions

In the previous sections, we discussed a number of fundamental weaknesses in the present approach to estimating SSAs which might suggest that it would be better, in principle, to estimate SSAs using alternative methods. In this section, we consider a number of specific statistical criticisms which have been made against the present methodology. In estimating models which are to be used for the purpose of distributing large amount of grants from central government to local authorities, it must be borne in mind that improvements to the technical merit of the regressions must be balanced against concerns of simplicity and transparency which are crucial to local authority accountability.

We outline some methods for evaluating whether models of local authority spending fail to explain the spending patterns of some individual authorities and whether some authorities exert disproportionate influence over the relationships which are estimated. We discuss whether the same models of spending can be applied to diverse classes of authorities. We consider the spatial dimension of local spending patterns and dynamic aspects. We also consider the possibility of controlling for unobserved characteristics of local areas which are relatively fixed over time. For earlier discussions of similar issues regarding the SSA regressions, see, for instance, University of Salford (1992) and Senior (1994).

The discussion in this section is, in places, necessarily more technical than in other parts of this report. Where possible, however, we endeavour to provide an intuitive explanation of the techniques discussed.

4.1 Outliers and influential observations

The present SSA methodology estimates a linear relationship between observed patterns of local authority spending and a variety of indicators of spending need. The technical points made in the current section would continue to apply if extra variables were introduced into SSA regressions or indeed if entirely different linear equations were to be estimated as a way of calculating SSAs.

Since simple regression models cannot explain all of the variation in spending between authorities, actual spending will differ from the level predicted by the model. The difference between actual and predicted spending is known as the residual. A consideration of the distribution of the residuals from a model can give us a number of insights into how well the model explains the observed pattern of variations in local authority spending.

The existence of large residuals is worrying since it indicates a failure of the model to explain the level of spending for a particular local authority accurately. This is of particular concern when that regression equation is used in the design of formula used for distributing grant to that particular local authority. Furthermore, if the residual is large enough, this can be sufficiently worrying as to be regarded as evidence of mis-specification in the expenditure relationship.

Influential observations occur when the inclusion of a given local authority in the regression equation has a large impact on the estimated importance of the needs indicators in explaining variations in local authority spending, i.e. when the inclusion or exclusion of a local authority has a large impact on the estimated coefficient on the needs indicators. This often occurs when the values taken by explanatory variables for these observations are substantially distant from those values taken by most other local authorities, as might be the case if, for example, the authority in question has an extremely high incidence of shared accommodation or individuals from ethnic minorities, compared to other local authorities. The consequence of the existence of these "outlying" observations is that they may consequently exert a high leverage on the resulting estimates. Such observations need not have a high residual since the high leverage which they exert tends to pull the estimated regression line towards the observation in such a way as to diminish the magnitude of its residual.

It is important to distinguish between authorities which simply have large residuals (and hence, whose observed spending patterns are poorly estimated by the model) and those which exert high leverage (which may have small residuals).

Local authorities with high residuals need not be particularly influential on the coefficient estimates, particularly if the explanatory variables take values which are not extreme. A hypothetical example of such a case is illustrated in Figure 10, where the authority does not exhibit extreme values of the explanatory variables; consequently it has a large residual, but has little impact on the slope of the estimated regression line. The existence of authorities with large residuals but little leverage does not necessarily seriously distort the estimate of needs effects applicable to other authorities. However, if the reason for its large residual is the presence of unobserved needs factors affecting that particular authority it may suggest that the estimated regression might not be wholly satisfactory as the basis for distributing grant to the high-residual local authority itself.

By contrast, the existence of authorities with high leverage may cause one to worry about distortion to the estimates of the relationship between spending and needs for other authorities. Such a case is illustrated in Figure 11. The outlying authority takes extreme values of the independent variable, and exerts a large influence on the slope of the estimated regression line. In such cases of high leverage, outliers cause much more fundamental problems than in the case shown in Figure 10.

4.2 Identification of Outliers and influential observations

The simplest and most direct method of identifying which local authorities have spending which is poorly explained by the SSA model is to inspect the residuals from the estimated SSA model. The larger the residual for a given local authority, the poorer the ability of the SSA model to explain the observed level of spending of that particular local authority.

The distribution of ordinary residuals is scale dependent in so far as their variance depends both on the scale of the dependent variable and on the values taken by explanatory variables. The variance of the residual for a given local authority observation depends on the true variance of the errors on the relationship and whether the needs indicator values for that authority are similar to those of most other authorities. If the explanatory variables (the needs indicators) take values which are substantially distant from those of other local authorities, then that local authority will exert a high leverage on the regression estimates and, as a result, the residual on that local authority is consequently pulled toward zero. The variance of the residuals on such observations is consequently lower. It is therefore sensible when looking for outliers to standardise the residuals by dividing by an estimate of their variance.

Annex 4.2 outlines a number of methods which can be used to identify which local authorities exert a high degree of leverage over the relationship between local authority spending and the various indicators of needs which have been estimated.

If there are prior grounds for suspecting a particular observation of being a potential outlier then the value of the (appropriately scaled) residual on that observation can be used to test formally whether the relationship between spending and needs for that authority differs from what is observed for all others. Equivalently one can test for the significance of a dummy variable for that particular observation.

Usually however candidate outliers are identified only after estimation. It is natural and unexceptional to find a few moderately large residuals in any data set of moderate size. What is important is to identify the magnitude at which the *largest* residuals in the sample become of concern. In so doing it must be recognised that these observations have been chosen on this basis and ought therefore to be expected to have a larger residual than would an arbitrarily chosen observation. It would therefore be appropriate to use a higher critical value than would

be appropriate for a single observation chosen on prior grounds. The derivation of the appropriate critical values for formal tests are discussed in Annex 4.2.

The basic method for identifying *influential* observations is to look at the impact that deleting the local authority in question from the sample would have on the estimated relationship between spending and the needs indicators. Cook (1977) has proposed a diagnostic statistic based on the (appropriately scaled) change in coefficient estimates after a local authority is deleted from the sample. This and related statistics are discussed in Annex 4.2.

Identification of influential groups of observations requires consideration of the effects of deleting groups of observations. This is extremely problematic. Pairs of observations which appear individually influential may together exert little influence in which case removing one but not the other, for instance, would be a dangerous practice.

It is a common and useful practice to look jointly at leverage and influence. This allows one to judge whether observations of apparently high influence arise because of high leverage (i.e. extreme values of explanatory variables) or to poor fit (i.e. departure from the typical relationship between the dependent and explanatory variables).

In the Annex to this section, we present plots of studentised residuals, Cook's influence statistic and leverage for two of the regression equations used to estimate the education SSA, one of which models unit costs for primary and secondary education, the other models nursery education Figures 1 and 5 show these values for the current regressions and Figures 3 and 7 for the same regressions with the dependent variables in logarithmic form.. The authorities with the highest five values of the statistics are identified by name, illustrating the frequency with which the names of certain Inner London authorities crops up. Thus, Newham and Hackney (5 times), Lambeth (4 times) and Kensington and Islington (3 times) are repeatedly identified as influential observations. This suggests that the processes which determine spending within Inner London are not wholly captured by a single regression across all authorities, with the functional form and the set of variables currently employed. One response, which would allow the analysis to take account of the possibility that the spending drivers differ between groups of authorities, could be to estimate the model using split samples: we discuss this issue of split samples in more detail in section 4.6 below.

4.3 Dealing with Outliers

There are a number of possible causes of outliers and these include the following

- **Data errors:** If there are grounds for suspecting particular outliers or influential observations to be due to faulty data then there is good reason for removing these observations from the data set. If there were prior grounds for believing the data to be faulty then there would be good reason for doing so independently of whether the data were found to be apparently outlying or not If there is a general but unspecified suspicion that some of the data may be faulty then this may give grounds for using regression techniques robust to the presence of outliers - these are discussed in the following section.

- **Non-constant variance** ("heteroscedasticity"): Large residuals could be caused by a large error variance on certain observations. The situation in which some observations have residuals with higher error variances is known as heteroscedasticity. Heteroscedasticity does not lead to bias in coefficient estimates and therefore need not lead to seriously misleading estimates. However, conventional standard errors are calculated under assumptions of its absence and inference about the coefficients may therefore be misleading i.e. faulty judgements may be made about which needs indicators are important in explaining expenditure. Ordinary least squares is also no longer the most efficient estimation method in the presence of heteroscedasticity and there is therefore the possibility of improving the precision of the estimates. Feasible generalised least squares is a two-step technique for estimation in the presence of heteroscedasticity and is discussed in Annex 4.3. For an example of the application of such a procedure to the SSA regressions see Mangan (1996). However since heteroscedasticity does not lead to bias and is difficult to deal with properly without being certain that the rest of the model is correctly specified we regard it as currently a concern of secondary importance.

- **Departure from normality in the error distribution:** It is possible that the appearance of occasional large residuals may be due to an error distribution which has substantially fatter tails (i.e. a higher probability of occasional large residuals) than the normal. Non-normality, like heteroscedasticity, does not create bias provided the errors remain uncorrelated with included explanatory variables. However it does imply that the small sample distribution of test statistics, such as the t values on individual coefficients, could be misleading. The

occurrence of occasional large residuals could also magnify the variance of coefficient estimates in a way which might make the adoption of robust regression techniques attractive.

- **Omitted variables:** Non-normality of the residuals might be attributable to the influence of an omitted variable, particularly if this were a dichotomous variable taking one of its values for relatively few observations. Thus, for example, if it were possible to divide authorities into two groups and the costs of the authorities in each group were different because of statutory provisions, for example, then the residuals might tend to be higher for one of the groups of authorities. The omission of such a variable would create bias only if this omitted variable were correlated with explanatory variables. If this seemed a plausible explanation then one should attempt to establish the identity of any such variable in case it were correlated with the included needs indicators. The issue of sample splits is discussed in more detail in section 4.6 below.

- **Mis-specification of functional form:** The assumption of a linear relationship between the dependent and explanatory variables, i.e. expenditure and needs indicators might be unjustified and could lead to the appearance of outliers in regions of the data where the fitted linear relationship offered a particularly poor approximation. It is possible to test linearity against appealing and interpretable alternatives such as a logarithmic relationship. This is discussed at greater length in Section 4.5.

4.4 Robust regression techniques

It has been argued above that the regression techniques used in estimating can be very sensitive to the presence of outlying observations of local authority spending. Alternative estimation methods, which minimise functions of the observations other than the sum of squared residuals may be more robust to the presence of outliers.

An example of such a procedure is "least absolute deviations" estimation which minimises the sum of the absolute values of residuals. This, therefore, gives less weight to outlying observations in estimating the relationship between observed patterns of spending and the needs indicators. Other "robust" regression techniques minimise the sum of other transformations of residuals, for instance by taking the squared residual where the residual is small but another transformation which effectively downweights large residuals where the residual is large. Commonly used "robust regression" techniques are discussed in Annex 4.4.

The application of one such robust technique to regressions in the Education sub-block can be seen in the Appendix in Tables 1 to 4. In none of the instances analysed does it make any great difference to estimated coefficients. This should not be taken to imply that it would not be worth investigating its use for other sub-blocks.

4.5. Transformations and functional form

The present SSA methodology is based on the assumption that the relationship between local spending and the various indicators of need is a linear one - by this we mean that a unit increase in the proportion of children from one parent families, for example, has the same impact on the level of local spending whatever the original proportion of lone parent families, be it 1%, 10% or 75%.

This is only one of a range of forms which the relationship between local spending and the various indicators of need could conceivably take. It might, for example, be more appropriate to specify a relationship in which a unit increase in population density leads to a constant x% increase in spending, or in which a 1% increase in population density leads to a constant x% increase in spending[9]. The estimation process can run in to a number of problems if the wrong functional form for this relationship is specified. The presence of outliers can be an indication that the most suitable form of the relationship between spending and the various indicators of need has not been chosen.

Since it is conceivable, and intuitively plausible in some instances, that the impact of a particular needs indicator on a local authority' spending should occur in a non-linear way, it is desirable to be able to test for the adequacy of linearity. One method is known as Ramsey's (1969) RESET test. This tests for whether added terms in higher powers of either the explanatory variables or of the fitted values of the dependent variable would be jointly significant if entered in to the SSA regression equation. This test is discussed further in Annex 4.5.

The RESET test effectively tests acceptability of the linear form within a class of transformations based on power series. Another method for testing the adequacy of linearity is to test its acceptability within a class of transformations including the possibly more attractive logarithmic transformation. The Box-Cox (1964) transformation offers one such class of transformations and is discussed further in Annex 4.5. This transformation could be applied to the dependent or explanatory variables or both. We do not suggest that transformations other

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[9] These latter examples of functional forms correspond to logarithmic transformations of the values of variables.

than the logarithmic be considered as possibilities for SSA formulae on grounds of public explicability. However, both the logarithmic and linear cases correspond to readily explicable mechanisms. Estimation here needs to be by maximum likelihood techniques.

For the education regressions chosen for illustrative investigation there do appear to be problems with the assumption of linearity. The results can be seen in Tables 1 and 3 of the Appendix. For primary and secondary education the RESET test suggests rejection of linearity at the 10% but not the 5% significance level. .Use of the Box-Cox transformation on the dependent variable suggests preferability of a logarithmic transformation. Adopting such a logarithmic transformation gives the results in Table 2 where it can be seen to improve markedly the performance on the RESET test (and Cook-Weisberg heteroscedasticity test). For nursery education the RESET test suggests a clear rejection of linearity. Use of the Box-Cox transformation on the dependent variable suggests linearity to be more acceptable however than a logarithmic transformation. In neither case have we tried applying the transformation to the explanatory variable.

Other alternatives might include explaining the share of total authority spending or the share of total income in the local area which is spent on particular local spending programs in terms of the needs indicators, rather than simply using levels of spending. However, the difficulties which are encountered in obtaining accurate measures of income, as discussed above, reduce the attractiveness of such a formulation and we do not recommend that it be considered.

4.6 Sample splits

One particular source of mis-specification could be that spending by different classes of authorities could conceivably be driven by different factors. For example, it is noticeable in the foregoing examples that observations with high leverage or poor fit are frequently London boroughs or Metropolitan district authorities. It has been suggested that the inclusion of the Inner London authorities in particular within the SSA regression equations may bias the estimated impact of certain needs indicators on spending for other local authorities. This may be because many of the Inner London authorities have very high values of certain needs indicators such as population density and ethnicity and high per capita levels of spending for reasons which may have little to do with these needs indicators.

Bias would occur only if the unobserved factors that drive spending need differ significantly between the Inner London authorities and authorities in other areas of the country. It would not be a problem if Inner London simply had high values of both spending and needs as long as the relationship was the same as for authorities in the rest of the country. Fortunately, this is something that can be tested using relatively straightforward statistical techniques.

A preliminary approach to addressing the issue of whether the models of spending used to estimate SSAs at present are able to explain the variations in spending between classes of authorities as well as between authorities within a particular class is simply to plot the residuals from the SSA regressions by class of authority.

Formal tests for structural dissimilarity such as the well-known Chow (1960) test are appropriate in this context, as discussed in Annex 4.6.. These tests are based on running regressions allowing coefficients to differ between classes of authority and then testing the hypothesis that coefficients can be constrained to be identical in different classes. These tests can be applied to all or only to a subset of coefficients.

If use of such test rejects identity of coefficients in different classes then separate regressions can be run on the different sub-samples. This is rather problematic in the context of designing grant formula, however, since there would be no economic rationale for the differences in grants for authorities with otherwise similar attributes in the two classes.

4.7 Weighting Regressions.

It is not always the case that we would wish to give equal treatment to all of the local authorities when calculating SSAs. In some cases, we may wish to give more *weight* to some local authorities than to others for one of at least two reasons

- **Allowing for unequal variances.** If we believe that the variance of expenditure conditional on the explanatory variables is different for different authorities (i.e. there is "heteroscedasticity") then it may be appropriate to use a weighting which reflects this when running regressions. This weighting may be adopted *a priori* on grounds of some prior argument or it may be driven by the data. If we believe, for instance, that the underlying economic model applies to individuals and the data which we are using are averages based on different population sizes then it may be appropriate to adopt a weighting scheme based on relevant population sizes to reflect the lower variance of averages based on larger samples.

- **Allowing for unequal importance.** It may be sensible to regard the consequence of mis-predicting the expenditures of large authorities as more serious than mis-predicting those of small authorities. This could be justified if the objective of the regression exercise is felt to relate to the welfare of individual clients and large authorities have more clients who will be affected by the outcome of the exercise. This would justify some scheme of weighting by client group size.

Following this approach, the regression equations used to calculate SSAs for police, fire, social services, highways and other services all assign weights to each local authority according to the size of the client group in that area. Thus, for example, the Other Services Block SSA regressions weight each authority by a measure of its *enhanced population* which takes in to account resident population, commuters and day and over-night visitors, all of which place demands of local authority services.

The exception to the weighting procedure is the SSA which is used to distribute the greatest amount of grant : the education SSA. The absence of weighting within the education SSA means, for example, that inner London boroughs have twelve times the weight in the regression if they each provide education than they would if they were grouped under the former Inner

A more satisfactory approach would be to investigate and identify which factors are responsible for the different pattern of spending which occurs between classes of authorities. Finding that the relationship between spending and the currently used needs indicators differs between classes of authorities should really be interpreted as identifying a problem rather than providing a solution. The appropriate response is to determine what additional indicators could be introduced to the model which could adequately explain the observed pattern of local authority expenditure between classes of authorities as well as within them.

Similarity of coefficients in and out of Inner London is rejected very strongly for nursery education but only at the 10% level for the primary and secondary education sub-block, as can be seen in Tables 1 and 3. For both sub-blocks similarity of the constant alone can be rejected at the 5% level. For primary and secondary education, however, these rejections disappear if a logarithmic transformation is taken as in Table 2, whereas the rejections are much stronger and more resilient to transformation of the dependent variable in the case of nursery education. There appears to be some substance behind the criticisms that have been made concerning the inclusion of Inner London authorities in the general regression equation.

The present methodology does make some concessions to the demands placed on a local authority's services by residents of other areas. Within the Other Services SSA, for example, indicators of the number of over-night visitors, day visitors and the net inflow of commuters were found to help explain variations in the pattern of spending between local authorities. The inclusion of a measure of coast-line in the Fire service SSA also takes in to account the extra provision which authorities may have to make if it is more difficult to call on back-up from other authorities in emergencies, although it is far from obvious that the length of coastline is the only relevant factor here. Nevertheless, the basic thrust of this approach seems to be sensible.

A second possible source of spatial effects is the influence that one local authority's spending choices may have on the spending levels in neighbouring authorities. The theoretical literature on competition between local authorities tends to focus on tax levels, but it is equally likely that some element of competition exists between neighbouring authorities in the provision of local services. This may be particularly marked in the Metropolitan districts and London boroughs where the major strategic services such as education and personal social services are carried out by geographically small entities.

If there are spatial interactions in local authority expenditure decisions then regression of expenditure levels on needs indicators as currently practised will tend to recover the effects of needs on spending after all spatial interaction. If these spatial interactions are felt to reflect aspects of spending decision which do not embody responses of required spending to needs then this is undesirable. Spatial econometric techniques exist for identifying spatial interaction terms but application is complex. Essentially they involve the addition of neighbours' spending to regression equations but estimation is not straightforward since the nature of spatial interaction ensures correlation between unobserved influences on own spending and neighbours' outcomes. This necessitates the use of techniques such as instrumental variables or maximum likelihood (see Anselin (1988), Cressie (1991)).

There are many possible methods for calculating neighbours' values, differing according to both their technical merit and their complexity. Some possible methods are :

- **Averaged approach.** One approach would be to take a simple average of the spending levels of all authorities sharing a common border with a given authority. This would be simple but would fail to take in to account that some neighbouring authorities might have a

London Education Authority (ILEA). This would not be the case if the weight given to each education authority depended on pupil numbers, for example.

In the Appendix, we present a number of suggestions as to weights which could be used in each of the regression equations within the education SSA. We also present the results of this weighting procedure. It turns out that, whilst many of the coefficients estimating the impact of needs on spending change, none of them change by a statistically significant amount. Thus, within the education SSA, weighting would have made little difference to the results obtained.

4.8 Dynamic and Spatial Effects on Spending.

At present, the SSA methodology makes little allowance for the spatial or dynamic aspects of the provision of local services - spending may be affected not only by the needs of residents within the local area in a given year, but also by the spending and characteristics of surrounding areas and by the spending and characteristics of the same authority in previous years.

- Spatial effects arise if the level of spending by one local authority is affected by the characteristics or levels of service provision of neighbouring authorities. These may impact on the provision of local services in one of two ways. First, residents of one area may place demands on the services provided by local authorities in other areas. Some attempts are made to take account of this within current methods for estimating SSAs. Second, the policy decisions of one local authority may influence the choices made by neighbouring local authorities. We consider each of these spatial aspects of local spending decisions in turn.

- Dynamic effects arise if a local authority's expenditure or characteristics in previous years affect its level of expenditure in other years. This could arise if response to changes in need are sluggish and distributed over a number of years or if there is persistence in spending so that past years' spending levels have effects on choices in current periods. These effects could conceivably arise through past levels of either current or capital expenditure.

Spatial effects

much larger impact than others. In the case of Norfolk County Council, for example, Lincolnshire County Council's policies would be assumed to influence Norfolk's spending to the same degree as Suffolk, with which Norfolk has a much larger common border.

- **Common Border Length Approach.** To take in to account that authorities with a larger common border may have more impact, one could take a weighted average of the spending levels of neighbouring authorities, with the weights determined by the length of each common border. This would still be relatively straightforward but would ignore the distribution of populations and local services within local authority areas. In the case of Northumberland County Council, for example, Cumbria has a much larger Common border than the Metropolitan boroughs of Newcastle Upon Tyne or North Tyneside, but the population of Northumberland is concentrated in the areas of the county closer to these latter two authorities.

- **Population Concentration Approach.** To take in to account the importance of population centres just across an authority border, one would have to estimate a system based on some notion of distance weighted populations in neighbouring areas. However, the relationship between weights and distance between population centres would largely be a matter of judgement, which renders this procedure highly controversial. In any case, this method is likely to be rather opaque to the general public.

Since the possible solutions to these spatial issues seem to be either too simplistic or too opaque and judgmental to merit a place in the SSA methodology, we do not propose to investigate the spatial aspects of local spending further in this report.

Dynamic effects

For some services such as leisure facilities and nursery school provision, local authority current expenditure is directly constrained by past levels of capital expenditure. It simply may not be possible to provide a given service unless the necessary infrastructure is in place and controls on local authority capital expenditure may restrain the ability of local authorities to determine the provision of that infrastructure, at least in the short run. Thus, a local authority which had previously invested a lot of resources in the provision of infrastructure for nursery provision may have a much higher level of nursery expenditure than an authority with similar preferences which did not have such infrastructure available. To some extent, this may simply be a relative

cost effect - local authorities with the facilities available would be able to provide nursery education at a lower cost, although, in this case, compensating authorities for higher costs may generate perverse incentives over a longer time period.

If there are dynamic effects in local authority expenditure decisions then regression of expenditure levels on needs indicators in single cross sections will tend to recover the long run effects of needs on spending. In the short term, the key issue with the influence of past spending on current spending is the extent to which the omitted influence on current spending (in this case, past expenditure) is related to the needs indicators which are included within the SSA regressions. If, for example, declining areas with high needs tended to have higher than average levels of infrastructure for particular services, then omitting high levels of past spending from the regression equation could lead to the present methodology over-estimating the impact of the needs indicators on current levels of expenditure.

It has occasionally been suggested that capital expenditure on one service affects current expenditure on all other services because of the need to make debt repayments from an overall budget whose size is fixed by the present capping arrangements. The difficulties for the present SSA methodology which are posed by the present capping arrangements will be discussed in section 5, below. Aside from this, it is not obvious that levels of local authority debt could be taken in to account in the SSA regressions without affecting the incentives for local authorities to plan their current and capital expenditure optimally over time.

In practice, it is by no means obvious how significant these problems are, or to what extent the impact of historical spending on current spending differs between local services. It is impossible to investigate issues of dynamic specification without several years of data on dependent and explanatory variables under a reasonably stable system of local finance. Such data is not currently available. Thus, analysing this issue in depth is beyond the scope of this report.

4.9 Fixed effects and panel data techniques.

We argued above that local spending may be affected by several factors currently omitted from the SSA regression. These include several variables which may be intrinsically unobservable or currently poorly measured. This may be because such data would be difficult or costly to

collect, or simply that it is not easy to determine exactly what we would like to measure. Examples could be taste factors or certain sorts of social problems. In such cases there may be a serious worry that correlation between these omitted influences and included needs indicators may bias estimated needs effects.

When there is more than one year of information on dependent and independent variables then the use of panel data techniques may allow us to eliminate the bias caused by omitted **fixed** effects, i.e. omitted influences which are constant over time. What is required is effectively to use the spending and needs information from all years and to include *dummy* variables for each individual authority to pick up the influence of the omitted factor. This local authority dummy variable will absorb the influence of any fixed characteristics of the area, whether measured by the needs indicators or not. (One would usually also include a set of year dummies to control for any developments which affected local spending in all authorities in a given year). A comparison of the results of such a regression with results as currently calculated which do not correct for fixed effects provides the basis for a test of the presence of any fixed effects bias.

Whilst this approach could conceivably lead to improvements to the present SSA methodology, the data requirements are considerable. To estimate models of local spending with a full set of authority fixed effects, one needs at least two years of data on dependent and explanatory variables under a reasonably stable system of local finance.

For some spending programs, as noted in the previous section, demands placed on local services may change gradually over time as the characteristics of an area change. For example, the growth of long term unemployment in an area may have a larger impact on the demand for local services after a few years than it does immediately. In such cases, more than two years of data would be appropriate. In addition, alternative methods of correcting for these unobservable fixed characteristics of areas may require even more years of data.

To make use of these sorts of models, it is imperative that changes in spending levels or area characteristics over time are measured accurately. This requirement poses some serious difficulties. It is not always easy to obtain consistent definitions of local spending over a number of years, given frequent changes to the precise range of local spending responsibilities and changes in accounting definitions. In addition, many of the characteristics of local areas which are used to explain spending are originally based on Census data. Since the Census only

provides information once every ten years, estimates of changes in local characteristics between census years can be highly misleading.

For this reason, we were unable to investigate the improvements to the present SSA methodology which could be obtained from the use of a "panel" of several years of spending and needs data in this study. Whilst we think this approach is promising, the data which would be required by such an exercise simply does not exist at present. It would seem likely that considerable work would have to be undertaken in terms of both consistent definitions of local spending, and on mid-term censuses or improved methods for forecasting trends between censuses.

4.10 Summary of Technical Issues.

This section has, by necessity, raised a number of relatively technical modifications which might be considered in any future recalibration of the SSA system. These are concerned with identifying those local authorities whose spending is poorly explained by any initial models, or who exert an undue influence over the SSAs which are then estimated. We also explored a number of alternative specifications of the nature of the relationship between variations in local spending and the various indicators of need. These methods all involve well established statistical techniques which are available on generally available statistical packages and hence should prove relatively uncontroversial. Whilst they may not be readily explicable to the general public, they would only be used during the initial estimation of SSAs and hence their use would only need to be understood by the experts who were designing the system.

We also discuss a number of influences on expenditure which are not addressed within the present SSA framework such as spatial and dynamic affects on local spending, and the impact of characteristics of local areas which are not readily measurable but which are relatively constant over time. Whilst each of these techniques might improve the technical accuracy of SSA calculations, this is likely to be largely at the expense of transparency in the local government finance system. In addition, the data requirements of using such methods are likely to be considerable.

Annex to Section 4 : Technical Issues in Estimation of Expenditure Regressions.

This annex is intended to be read in conjunction with section 4 of the text. It contains material which is too technical in nature to be included in the full body of the text, but is not intended to be self standing. The Annex is structured in the same format as the main text for ease of cross reference. Thus, Annex 4.2 contains technical material which relates to section 4.2 in the main body of the text

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Annex 4.2 Outliers and influential observations

Identification of outliers and influential observations

Let y denote the vector of observations on the dependent variable and X denote the matrix of observations on the explanatory variables. Suppose also that the assumptions of the normal linear model apply

$$y = X\beta + u$$
$$u|X \sim N(0,\sigma^2 I)$$

The OLS estimate of β is

$$\hat{\beta} = (X'X)^{-1}X'y.$$

The vector of predictions of the dependent variable is

$$\begin{aligned}\hat{y} &= X\hat{\beta} \\ &= X(X'X)^{-1}X'y \\ &\equiv Hy\end{aligned}$$

where H is the "hat matrix" $X'(X'X)^{-1}X$. The vector of residuals is

$$\begin{aligned}\hat{u} &= y - \hat{y} \\ &= (I - H)y\end{aligned}$$

The residuals are conditionally distributed normally but with a heteroscedastic variance covariance matrix

$$\hat{u}|X \sim N(0,\sigma^2(I - H)).$$

Residuals have a smaller variance on observations where the corresponding diagonal element of the "hat" matrix H is small. This value provides a measure of leverage and we denote it h_i for the ith observation.

Standardising the ith residual involves dividing by an estimate of its standard deviation and produces a value

$$r_i = \frac{\hat{u}_i}{s_i\sqrt{1-h_i}}$$

where $\hat{u}_i$ is the residual for the ith observation and s_i^2 is an estimate of σ^2.

Typically s_i is the residual mean square computed either with or without the ith observation. Values of r_i calculated on both bases are frequently calculated by standard statistical packages and are useful in identification of outliers and influential observations.

If s_i is the residual mean square computed *without* the ith observation then the resulting quantity has been given a variety of names including the "externally studentised residual", the "jack-knife residual", the "deletion residual" and RSTUDENT. Under the assumptions made above then r_i should have a Student's t distribution with n-k-1 degrees of freedom (conditional on X) where n is the number of observations and k is the number of coefficients estimated. If the ith observation is suspected on prior grounds of being an outlier then this offers the basis for an obvious test. This test is identical to the conventional t test for the inclusion of a dummy for the ith observation. Formally speaking, we can see it as a test against the mean shift model in which the intercept differs for this observation.

Usually however the candidate outliers are identified only after estimation - that is to say there are no prior candidates for outliers. Testing must therefore be done on the basis of the largest absolute values of r_i in the sample. It is natural and unexceptional to find a few moderately large residuals in any data set of moderate size. What is important is to note the magnitude at which the largest absolute values of r_i begins to be of genuine concern. Under the above assumptions, the appropriate critical value for a two tailed test of level α for a single outlier is that for the maximum of n such Student's t values. A conservative critical value, based on Bonferroni's inequality, would be the critical value for a two tailed test of level α/n for an individual Student's t distribution. Cook and Weisberg (1982) suggest that the power of such a

test can be improved upon by taking the critical value for a two tailed test of level $h_i\alpha/k$. Alternatively, simulation methods could provide a closer-to-exact critical value in particular instances.

There may be more than one possible outlier and none of these suggestions are helpful in providing critical values for the second and subsequent highest value of r_i.

Identification of influential observations is usually by looking at the effect on the coefficient estimates of deleting the observation in question. Cook (1977) has proposed a diagnostic statistic based on the appropriately scaled change in coefficient estimates after such deletion. His proposed statistic can be shown to have the form

$$C_i = r_i^2 \frac{h_i}{1-h_i}.$$

where the estimated standard error s_i used in calculation of r_i is the residual mean square computed *with* the *i*th observation (see Cook 1977, Cook and Weisberg 1982).

A related statistic can be calculated by using an estimated standard error s_i computed without the *i*th observation (see Atkinson 1981, Belsley et al 1980). The resulting statistic is the square of the DFFITS statistic of Belsley et al (*op cit*). Cook and Weisberg (1982, p124) argue that the different scaling applied to each observation makes this unsatisfactory for comparing influence on $\hat{\beta}$ and that if robustness is a concern then a single robust estimator be used for s_i for all observations.

Annex 4.3 Heteroscedasticity

Large residuals could also be caused by a large error variance on certain observations. The situation in which some observations have residuals with higher error variances is known as heteroscedasticity. Heteroscedasticity does not lead to bias in coefficient estimates and therefore need not lead to seriously misleading estimates. However, conventional standard errors are calculated under assumptions of its absence and inference about the coefficients may therefore be misleading. Ordinary least squares is also no longer the most efficient estimation method in the presence of heteroscedasticity and there is therefore the possibility of improving the precision of the estimates.

There exist tests for detecting heteroscedasticity. These tests typically work by looking for a systematic relationship between the squared residuals and powers or cross-products of the regressors or predicted values. An example is the Cook-Weisberg (1983) test which tests $\gamma=0$ in $\text{var}(u) = \exp(Z\gamma)\sigma^2$ where Z is either the regressors or predicted values. Such tests are conditional however on correct specification of the rest of the model and reliability of the data. If there exist outliers caused by mis-specification of the main relationship or by faulty data then the results of heteroscedasticity tests may be seriously misleading.

Feasible generalised least squares is a two-step technique for estimation in the presence of heteroscedasticity. The form of the heteroscedasticity is estimated by estimating some relationship between the squared OLS residuals and variables thought to be related to the variance. The main relationship of interest is then re-estimated by weighted least squares using weights based on the estimated variances. For an example of the application of such a procedure to the SSA regressions see Mangan (1996).

Annex 4.4 Robust regression techniques.

If there is concern about the reliability of the data for certain observations then estimation methods which are less sensitive to outliers may merit consideration. For instance, estimation methods which minimise functions of the observations other than the sum of squared residuals may be more robust to the presence of outliers.

An example of such a procedure is "least absolute deviations" estimation which minimises the sum of absolute values of residuals. Other "robust" regression techniques minimise the sum of other transformations of residuals, often taking the squared residual where the residual is small but otherwise taking a transformation which effectively downweights large residuals. One commonly used "robust regression" technique discussed by Huber (1964, 1981) minimises the sum of the following transformation of residuals

$$\rho(\hat{u}_i / \sigma) = \begin{cases} \frac{1}{2}(\hat{u}_i / \sigma)^2 \text{ for } |\hat{u}_i / \sigma| \le c \\ c|\hat{u}_i / \sigma| - \frac{1}{2}c^2 \text{ for } |\hat{u}_i / \sigma| > c \end{cases}$$

where σ is a parameter to be estimated. For $c=0$ this is the same as "least absolute deviations" estimation whereas for $c=\infty$ it is the same as ordinary least squares. Suitable values for c in this case are usually felt to be in the range 1.2 to 1.5.

Other versions of robust regression use alternative transformations such as the biweight function (see Beaton and Tukey 1974) which downweights large residuals and actually drops extreme outliers altogether, or iterate using more than one weighting scheme.

Annex 4.5 Transformations and functional form

A test for the adequacy of the linear representation is Ramsey's RESET test. This is an F test for the addition of higher powers either of the explanatory variables or of the predicted values of the dependent variable. (If there is only one explanatory variable then these alternatives are identical; if there are more than one then using powers of the fitted values is a natural way of avoiding loss of power by using up fewer degrees of freedom than would addition of powers of all explanatory variables).

If we confine attention to a parametric class of transformations then we might investigate the well-known Box-Cox transformation

$$z^{(\lambda)} = \begin{cases} \dfrac{z^{\lambda}-1}{\lambda} & \text{for } \lambda \neq 0 \\ \ln z & \text{for } \lambda = 0 \end{cases}$$

It would be usual to estimate models in this class by maximum likelihood under the assumption of normality for the error distribution. This transformation could be applied to the dependent or explanatory variables or both.

This transformation has the advantage of including the linear and logarithmic cases as particular instances. It can therefore be used for testing these two functional forms against other values for λ. It would not be envisaged that intermediate values for λ be considered as possibilities for SSA formulae on grounds of public explicability. However, both the logarithmic and linear cases correspond to readily explicable mechanisms.

Annex 4.6 Sample splits

Suppose we believe that there may be different coefficients in two subgroups of authorities i.e.

$$y_1 = X_1\beta_1 + u_1$$
$$y_2 = X_2\beta_2 + u_2$$

where subscripts denote the different subgroups. We can estimate the coefficients of this model by running separate regressions on the two subsamples. Equivalently the following unrestricted regression can be estimated on the pooled sample

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}.$$

In either case we can calculate an F-test for the hypothesis $\beta_1 = \beta_2 = \beta$. by comparing residual sums of squares in the restricted and unrestricted models.

If we wish to test change only in a subset of coefficients, such as the constant only, then an F-test is again appropriate. If we denote the explanatory variables on which the coefficient may differ X and on which it may not Z and the relevant coefficients β and γ then the relevant unrestricted regression takes the form

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} X_1 & 0 & Z_1 \\ 0 & X_2 & Z_2 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \gamma \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}.$$

Annex 4.7 Weighted least squares

Suppose that we adopt the following heteroscedastic linear model

$$y = X\beta + u$$
$$u|X \sim N(0, \sigma^2 W)$$

where W is a known diagonal matrix with all diagonal elements w_{ii} positive. It is possible to estimate this model by transforming to a homoscedastic form

$$y^* = X^*\beta + u^*$$
$$u^*|X \sim N(0, \sigma^2 I)$$

where

$$y^* = W^{-1/2} y$$
$$X^* = W^{-1/2} X.$$
$$u^* = W^{-1/2} u$$

Several of the diagnostic procedures listed above can then be applied to the transformed residuals (See Cook and Weisberg (1982))

$$\hat{u}^* = y^* - X^* \hat{\beta} .$$

5 Issues in Estimation of Expenditure Regressions under Capping

The present system of assessing local authority needs implicitly assumes that observed levels of local spending would be largely freely chosen, reflecting all of the information on local needs and preferences which are available to local authorities. However under the present capping arrangements, this is not always the case. The problem was present in 1990/91 (the year currently used for estimation) though only 20 authorities were capped. However, if the year in which SSAs are calculated is updated, to reflect more recent information on the nature of the demands placed on local services, then the problem would become more severe since numbers of authorities capped will have increased considerably. For example, in 1995/96 ten authorities were formally capped as a result of the budgets which they set, but a further 278 set budgets which respected *ex ante* capping limits, announced prior to their budget-setting process.

This reflects the change in the design of the capping arrangements in 1991/92 from a system of selecting a small number of authorities for capping once local budgets had been set to a system of announcing a set of provisional capping criteria before local authorities set their own budgets, effectively allowing local authorities to "cap themselves" by setting their budgets at the centrally determined cap. After 1992/93, these provisional capping criteria also applied to those shire districts with budgets of less than £15 million which had, until then, been excluded from the capping arrangements altogether. By 1995/96, for example, 278 local authorities in England set their budgets at the centrally determined cap, representing well over half of all local authorities. In addition, aggregate local authority expenditure was only 0.4% below the maximum allowable under the capping arrangements[10].

[10] Finance and General Statistics 1995/96. CIPFA.

5.1 The Circularity Argument.

If local spending levels are not freely chosen, then they may not provide sufficient information to allow meaningful estimates of the forces which drive local spending needs. If variations in spending levels between local authorities merely reflect variations in expenditure caps which are themselves based on SSAs, then any regression analysis based on these spending levels will essentially be estimating new SSAs based on variations in the SSAs within the previous system. In other words, caps determine spending, spending then determine SSAs which then determine the next set of expenditure caps. Effectively, this "circularity" could eventually result in estimates which are an almost perfect fit statistically yet which fail to reveal any meaningful new information at all.

This "circularity" argument may appear to be rather too pessimistic. Whilst it is true that the capping system may constrain overall levels of local spending close to overall SSAs, local authorities retain a considerable degree of autonomy in allocating budgets between the various budgetary heads, such as education and personal social services. Variations across authorities in spending levels on individual services such as primary schooling or residential care for the elderly rather than in aggregate budgets still contain useful information.

However, the constraints under which capped and uncapped authorities operate are different For a capped authority to increase spending on a particular item, it must reduce spending on some other item in order to stay within the cap. This constraint is not there for an uncapped authority and will not constrain its behaviour. The behaviour of capped authorities must therefore be different in kind to the behaviour of uncapped authorities.

Estimation of separate regressions for individual budgetary items and addition of predicted spending on the individual items would necessarily recover the capping formula if all authorities were capped and all regressors were included in all equations. Circularity therefore remains a problem even if regressions are run on individual items. If the information on the allocation of spending between items within the cap is to be used then it would be necessary to use some method other than total expenditure regressions, such as one of those discussed in section 3.

Other recent developments within the local government system may have increased the importance that the capping arrangements have on the present SSA methodology. For some services there are effectively caps on individual budgetary items. The single service Fire and Civil Defence authorities which took over the services which were provided by the former Metropolitan counties are a case in point. Nearly all their funding depends on a single service block SSA, and several of these authorities have argued that capping impinges unfairly on their budgets for this very reason. This is also true of the new "stand-alone" Police Authorities which were set up in April 1995. In addition, over 95%[11] of the overall SSAs of shire district authorities derives from the Other Services Block SSA, severely restricting the ability of these authorities to move funds between budgetary heads.

It seems likely that the growth of single service authorities and other recent reforms to the system of central-local relations in the UK may make this problem even more acute in the future, In particular, the extension of the Common Funding Formula, which funds Grant Maintained Schools at a level determined by the education SSA of the local authority area in which they are located may deter local authorities from spending below SSA on education, lest it lead to a wave of schools "opting out" of local authority control. Since education spending accounts for almost half of all local spending, the ability of local authorities to determine their relative priorities between budgetary heads could become highly constrained.

5.2 Estimating SSAs in the presence of capping arrangements.

Given that authorities subject to capping arrangements must behave differently to those that are not and that their inclusion in expenditure regressions introduces a sort of circularity into regression methods it would make sense to drop such authorities from the regression if one intended to continue with the current method and there were sufficient numbers of uncapped authorities to estimate SSAs. However, given that there are very few uncapped authorities amongst those authorities responsible for the provision of the bulk of local authority services and that even the budgets of those authorities which are not capped may be influenced by the announcement of the provisional capping criteria each year, this approach is simply not practical.

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[11] Standard Spending Indicators 1995/96. Society of County Treasurers.

In a future era in which capping only directly affects a small number of authorities, one could, in principle, drop capped authorities. However, merely omitting such authorities from the sample, without any adjustment to the estimation methods, is liable to lead to bias in the estimated coefficients, as Figure 12 illustrates. It would be desirable to use sample selection correction techniques, of the sort associated for instance with Heckman (1976), to control for the effects of the capping arrangements. Duncan and Smith (1995) have discussed the application of such procedures to local authority expenditure behaviour.

The details of such sample selection procedures are described in Annex 5.2. Models allowing for sample selection can be estimated either by maximum likelihood or by Heckman's two step procedure. This procedure has two stages. In the first stage, an equation describing selection into capped and uncapped authorities using all local authorities is estimated. The predicted value of the error on this equation is then calculated for each uncapped authority. In the second stage of the procedure, this predicted error from the first stage is entered as a separate regressor in estimates of the expenditure equation using only uncapped authorities. A significant effect on the added term would be indicative of a selection bias problem in the standard SSA methodology, whilst the estimated impact of the needs indicators on patterns of local authority spending will be consistent within this two-stage procedure[12].

For this method to be successful,[13] we would need to discover characteristics of an area which influenced the probability of the local authority being capped and yet which had no influence on the level of spending of authorities which were not capped. These requirements are rather demanding, given the importance of the SSA calculations in allocating grant to local authorities, and it is far from obvious what such variables might be or what basis there might be for discovering them.

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[12] The method relies on assumptions about joint normality of the errors in the selection equation and expenditure equations. If there were worries about the validity of this then higher powers of the added term could be entered as one informal method to enhance robustness. However, the details of this would add a further judgemental aspect to the technique which would be undesirable in the context of design of SSA formula.

[13] The method would deliver estimates even in the absence of such variables by virtue of the non-linear transformation involved in calculating the added term. However, the method would then be relying extremely strongly on assumptions about distribution of the errors.

In the second stage, the "added term" (which is the predicted error from the first stage) is included to correct for bias introduced by the exclusion of non-capped authorities from the second stage regression. Information on the importance of the needs indicators is still derived from the relationship between the needs indicators and the level of spending in the uncapped authorities so these two stage methods should *not* be seen as inferring extra information on the impact of the needs indicators from the behaviour of capped authorities. This should not be seen as a method for reliably recovering information on the importance of the needs indicators even if there are very few uncapped authorities.

We applied this method to the education sub-blocks considered in the earlier sections. The additional variable used to predict capping at the first stage was political control. This was consistently statistically significant as a predictor in the capping equation. However, its exclusion from the main equation could well be questioned, and to that extent these results can only be considered to be illustrative Applying the correction makes a noticeable difference to the estimated coefficient in the nursery education sub-block though not in the primary and secondary education block. In general the full maximum likelihood estimates are to be preferred though differences between the ML and two-step results of a magnitude such as seen in the nursery education case may be regarded as indicative of some problem with the underlying assumptions, particularly normality.

5.3 Implications for Policy.

These results suggest that the capping system may pose significant problems for the present method of estimating SSAs. Any up-dating of the present SSA methodology may run in to difficulties if SSAs continue to be used as the basis for capping local authority budgets. Simply using a more recent year of spending data to take in to account changes in the forces which drive local spending needs could lead to a great deal of unfairness in the allocation of grant because capping arrangements may prevent variations in local spending decisions fully reflecting variations in local needs. Alternatively, if an adjustment is made to spending levels in line with the methodology described in the Annex to this chapter, the local government finance system may lose a great deal of transparency and lead to suspicions about how easy it is for central government to manipulate the grant payments to individual local authorities.

In the longer run, if the present system of extensive capping arrangements were to remain in place, alternatives to regression may need to be considered in order to estimate local spending needs, to prevent either the present formula becoming increasingly out-dated or it being up-dated using methodologies which are either opaque to the general public or increasingly hard to justify statistically.

Annex to Section 5 : Technical Issues in Estimation of Expenditure Regressions.

This annex should be read in conjunction with section 5.3 in the text. It contains material which is rather too technical in nature to be included in the full body of the text, but it is not intended to be self standing.

Annex to Section 5.3 Selection correction

Suppose that the linear model above is modified so that expenditure y_i is capped at c_i if

$$Z_i\gamma + v_i > 0$$

$$\begin{pmatrix} u_i \\ v_i \end{pmatrix} | X, Z \sim IN(0, \begin{pmatrix} \sigma_{uu} & \sigma_{vv} \\ \sigma_{uv} & \sigma_{vv} \end{pmatrix})$$

Then the conditional expectation of expenditure given X_i and that the authority is uncapped is not $X_i\beta$ since

$$E(y_i | X_i, Z_i, Z_i\gamma + v_i \leq 0) = X_i\beta + E(u_i | Z_i\gamma + v_i \leq 0) \neq X_i\beta \,.$$

Hence a regression of expenditure on the explanatory variables using only the uncapped authorities will give a biased estimate of the coefficient β. This problem can be solved by estimating the model using maximum likelihood or by applying the Heckman two-step procedure.

The Heckman two-step procedure is to first estimate γ by probit maximum likelihood using the capped and uncapped authorities. The expected value of v_i given that the authority is uncapped, λ_i, is then calculated for each uncapped authority using the formula

$$\lambda_i = \frac{\phi(Z_i\hat{\gamma})}{\Phi(Z_i\hat{\gamma})}$$

where $\phi(.)$ and $\Phi(.)$ are the standard normal density and cumulative distribution functions and $\hat{\gamma}$ is the probit estimate of γ. It is then possible to estimate β consistently by adding λ_i as an additional regressor in the OLS regression of expenditure on the explanatory variables. The

OLS standard errors are, however, an incorrect basis for inference and require correction according to formulae suggested by Heckman(1979)[14].

[14] See Greene (1993 ch 22) for a good summary.

6 Conclusions and Practical Implications.

This report has explored the validity of the methods which are currently used in the calculation of Standard Spending Assessments (SSAs). SSAs are estimates made by central government of the costs which local authorities would incur if they provided a similar level of service to their local populations; they attempt to control for differences in the number and needs of potential clients for a particular service in each local area and differences between areas in the costs of providing the service to each potential client. SSAs are used as the basis for the distribution of revenue support grant to local authorities, and as the basis for capping local authority budgets.

We have identified a number of conceptual and technical issues which may need to be addressed in any future recalibration of the SSA system, and we discuss the problems which the present capping arrangements pose for the identification of spending needs.

Standard Spending Assessments are estimated using a statistical tool known as regression analysis. In principle, the aim of the analysis is to assess how differences between local authorities in the *needs* for local authority services affect the level of *expenditure* which they must incur in meeting these needs. Local authority expenditures may differ for a range of reasons - including differences in efficiency and in political preferences regarding local public spending, as well as differences in the characteristics of the population and other factors which give rise to different needs. Regression analysis is, in principle, a tool which makes it possible to assess the separate contribution to expenditure of needs differences from the effects of the other influences on spending. However, its ability to assess the effects of needs on spending depends on some key requirements:

First, there are technical requirements about the methods employed. The aim of the exercise is to uncover the relationship between needs and spending, uncontaminated by other possible effects on spending arising from sources such as efficiency differences and political preferences. How far any particular regression technique will be able to estimate the "pure" effect of needs differences will depend on the particular ways in which efficiency and political factors affect spending. This will determine whether relatively straightforward techniques can get close to the needs/spending relationship, or whether more complex techniques are required. It is a matter for judgement in particular cases, how much technical sophistication is needed in

the estimation of the relationships underlying SSAs. A considerable part of our report has been devoted to assessing this issue.

Second, for the approach to be in any way meaningful, a much more fundamental condition must be met. The method requires that local authorities should have the ability to vary their spending levels to reflect differences in need; if they are not able to do this, then we cannot discover anything about needs by looking at their spending levels. The SSA calculations currently in operation are based on local authority spending data for a period (1990/91) when only a small number (20) of local authorities were capped. Capping now affects many more local authorities, and there is an important issue about its implications for any future updating of the SSA analysis. The second main element in our report has been an assessment of the implications of capping for the estimation of local authority needs through regression analysis.

We set out our main conclusions below. In assessing their implications it is important to bear in mind the relatively limited number of real alternatives which exist to the current approach. Whilst there may be scope for methodological refinement, there are, in our view, few plausible radical alternatives to statistical analysis of past spending patterns as the basis for assessing the relationship between needs indicators and spending. If it remains the intention to have a system of revenue support grants which aims to compensate local authorities for differences in spending needs, something like the present arrangements may continue to be needed.

Many of the important issues for policy will then involve assessing the balance of advantage between the current, admittedly imperfect, arrangements, and other, also imperfect, alternatives. Where greater methodological sophistication might be considered, for example, it will be necessary to assess whether it makes any significant difference to the results, and whether it does not also incur costs in terms of poorer understanding or acceptance of the approach, greater instability in the results, or the need to employ less accurate, or more controversial, data. Likewise, when assessing the desirability of updating the analysis to use more recent data than the 1990/91 spending data currently employed, it will be necessary to weigh up the disadvantages of an analysis based on old data against the difficulties of employing the regression approach to analyse a period when capping has been more widespread.

The methods currently employed

We have reviewed a range of issues concerning the methods currently used to estimate the relationship between needs indicators and local authority spending. These fall into three groups:

- The underlying validity of the approach as a way of discovering the true relationship between needs and spending.

- Some technical issues concerning possible "short-term" refinements to the current methodology, not involving major new data, or a new approach.

- Some more fundamental changes to methodology which could be considered, involving new data, or wholesale changes to the estimation techniques.

Regarding the first question, our analysis concludes that the validity of the models currently used in the SSA calculations depends on a number of fairly restrictive assumptions. It is far from obvious that these conditions are met, either conceptually or technically. In particular, we highlight the role played in the analysis by *variations in service quality* across authorities. The validity of the approach currently used to estimate SSAs requires *either* that local authorities should all be providing a common standard of service, *or* that variations in service standards between authorities should be unrelated to the pattern of needs across authorities. Neither of these conditions seems likely to be met in practice. There is no evidence for the former. The latter also seems unlikely: for example, if the political control of an authority carries with it any implications for service quality and if political control is partly a function of some of the needs variables (such as socio-economic conditions) then service standards will be correlated with the needs variables.

We discuss three approaches which could be suggested as means to correct the present SSA methodology for variations in service standards.

- Firstly, service standards could be measured directly and these measures used as additional regressors. In the longer term we think there would be considerable gains from developing a set of good indicators of service quality for local authorities for use in the analysis, although we accept that the choice of quality indicators is unlikely to avoid controversy.

- Secondly, information on the characteristics of local areas which might be expected to influence the quality of services provided by local authorities could be included as additional

regressors. These omitted influences could include incomes, grant and the costs of delivering particular services. These methods make quite stringent data requirements and are unlikely to capture variations in service standards with a great deal of accuracy. More fundamentally, for certain factors which influence both choice of service level and cost of provision, it is difficult to see how these effects could be separated by this method. We illustrate the use of this approach by applying it to some of the regression equations, and show the difference that it makes to the results. Further research would be needed to assess whether more general application of these methods would result in major changes to the SSA results.

- Thirdly, indirect indicators of service quality could be included, although it is far from obvious what these might be in practice or how particular choices might be justified without inviting controversy.

We assess secondly the scope for "short-term" technical refinements to the methodology. Regardless of whether SSAs continue to be estimated along the lines used at present or not, we describe a number of technical improvements which could be made to calculation of SSAs. These would allow a consideration of whether estimated SSAs can explain variations in spending across all authorities within a class and between all classes of authorities. We describe a number of possible methods for identifying outlying and influential observations as well as techniques for improving robustness to the presence of such observations. Whilst these methods are technical in nature, they would only need to be carried out when a new SSA framework was established and require only those techniques available on standard statistical packages. The most useful practical response to the identification of outliers, however, will be to find further data (in the form of new indicators, etc), which can successfully explain the existence and performance of outliers. In general, the presence of outliers is an indication that in some respects the model does not adequately explain the behaviour of certain authorities; often this can be due to the omission of important explanatory variables from the model.

Other methodological changes could be considered over a longer time-scale:

(i) One option which has been suggested, but which has not been discussed in detail in this report, is the possible use of ***multi-level modelling*** techniques, in place of the current single focus on spending differences between authorities. Data on sub-authority needs indicators and spending patterns could be employed to investigate how the allocation of spending within an

individual authority is affected by the pattern of needs across wards or other sub-areas within the authority. This might be an attractive approach for a number of reasons. One is that it might be reasonable to suppose that the same quality of service is provided (or at least, is intended to be provided) throughout an authority, which makes it possible to limit the estimation problems which would arise in comparing spending in local authorities where the quality of service provision was very different. Another attraction is that analysis of sub-authority spending allocations might still generate meaningful information, even when a local authority's spending was subject to an overall cap. The allocation of this fixed total of spending between sub-areas might provide a way of discovering the relationship between needs and spending, even in capped authorities.

Potential drawbacks of sub-authority data can, of course, be envisaged, and would need to be assessed carefully. One is the extent to which the spending data at sub-authority level is accurately allocated between areas. Some spending items may be difficult (or impossible) to allocate between sub-authority areas; these could include the fixed costs of central administration, and other costs unattributable to local areas. For the analysis of sub-authority data to be meaningful it is vitally important that this data is generated on the basis of actual sub-authority accounting, rather than by statistical apportionment of the authority's aggregate spending between sub-authority areas on the basis of area population or other characteristics. If the data is produced in the latter way, then regression analysis of the relationship between spending and population characteristics will simply re-discover the apportionment formula.

(ii) ***Panel data techniques*** hold out, in principle, the hope of more accurate estimation of the relationship between needs and spending, through the elimination of authority-specific "fixed effects". These techniques would use data for more than one year. With data for two years, for example, it would be possible to model the relationship between *spending changes* over the period, and *changes in the needs indicators*. Such panel data approaches have become widely used in econometric analysis in recent years, and would potentially be an attractive development of the current SSA approach.

The main obstacle to implementing panel data techniques in the SSA context, however, is the lack of comparable data on spending and local authority characteristics for more than one time period. Many of the indicators of need derive from the ten-yearly Census, and a study of the effects of changes in the needs indicators using two data points for each authority would thus need to compare data from the most recent census with data from the census conducted ten

years previously. In the current context this would require data from 1991 to be compared with data from 1981 - now 15 years ago. Ensuring that the spending data for 1981 are wholly comparable with the spending data for 1991 would be extremely difficult, but would be critical to ensuring that the measured changes in expenditure did in fact reflect changes in spending, and not simply changes in the classification of spending items, or in other methodological aspects. Despite these reservations, however, we think that, as part of a longer-term research agenda, that it would be worth exploring the scope for greater use of panel data techniques in the SSA methodology, and for developing indicators and spending data comparable over a run of years.

Capping and the future of SSAs

These technical and conceptual issues are somewhat overshadowed by the problems posed by the present system of capping arrangements for any future recalibration of the SSA system. Capping undermines the possibility of identifying a meaningful relationship between spending and needs indicators through the use of regression analysis. Moreover, even if there were a return to some more selective form of capping at some stage in the future, SSAs could only be calculated accurately if existing procedures were modified. Techniques for achieving this are discussed in the report, but their application might require judgements likely to generate substantial controversy.

If it is not possible to estimate new SSA models on current data, due to the extensive use of capping, the question arises as to how long the existing SSAs might be seen as sustainable. Can we live with the existing SSAs (despite their imperfections) for some time to come?

There is, of course, no difficulty in updating the information about individual authority entitlements as new information about the needs indicators becomes available. The problems concern the durability of the relationship between local authority needs indicators and the need for local authority spending to provide a standard level of service. It is a matter for judgement as to the point at which the information derived about the spending/needs relationship from 1990/91 data will become so outdated that its use becomes unacceptable. The relevance of an analysis of this relationship in 1990/91 to the current situation will depend on a number of factors - including the pace of change in relative prices (probably slow), in the technologies of service delivery (rather faster in some cases, and probably varying widely across service blocks) , and in the services which local authorities provide under each head (changes to local

authority service responsibilities can lead to abrupt changes in spending needs). It is probable that the durability of the estimates based on 1990/91 data will vary considerably between service blocks, and that the SSA formulae for those spending areas characterised by with reasonably stable technologies and little change in local authority responsibilities will not yet have experienced undue "deterioration".

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Table 1: Primary and Secondary education, levels

	OLS	Robust regression	OLS, Uncapped only	Heckman ML	Heckman 2 Step
Additional educational needs	0.187 (11.75)	0.178 (10.70)	0.187 (9.68)	0.197 (8.38)	0.207 (7.42)
Constant	1.138 (58.13)	1.148 (56.02)	1.136 (51.37)	1.133 (50.51)	1.130 (49.67)
Sample size	106	106	87	106	106
R^2	0.570		0.524		0.530
Root MSE	0.080		0.078		0.078
Joint significance	$F_{1,104}$=138.09 p=0.000	$F_{1,104}$=114.45 p=0.000	$F_{1,85}$=93.74 p=0.000	χ^2_4=17.09 p=0.002	$F_{2,84}$=47.44 p=0.000
RESET	$F_{3,101}$=2.32 p=0.080		$F_{3,82}$=1.21 p=0.312		
Hetero	χ^2_1=7.77 p=0.005		χ^2_1=7.49 p=0.006		
Box Cox: λ=1	χ^2_1=4.28 p=0.039		χ^2_1=3.20 p=0.070		
Box Cox: λ=0	χ^2_1=0.38 p=0.539		χ^2_1=0.30 p=0.581		
$Chow_1$	$F_{1,103}$=4.53 p=0.036		$F_{1,103}$=8.68 p=0.004		
$Chow_2$	$F_{2,102}$=2.79 p=0.066		$F_{2,102}$=4.40 p=0.015		
$Select_1$				t=2.778 p=0.005	t=2.589 p=0.010
$Select_2$				t=0.750 p=0.453	t=1.032 p=0.305

Notes:
Asymptotic t statistics in parentheses below coefficients
RESET is a Ramsey RESET test using fitted powers of the dependent variable
Hetero is a Cook-Weisberg test for heteroscedasticity using fitted powers of the dependent variable
Box Cox is a test of hypotheses on λ for the Box Cox transformation of the dependent variable
$Chow_1$ is a Chow test for similarity of the constant in and out of inner London
$Chow_2$ is a Chow test for similarity of all coefficients in and out of inner London
$Select_1$ is a test for significance of regressors included in the capping equation but excluded from the main equation. (In the case of the 2 step procedure this is the t statistic on thevariable in the associated probit).
$Select_2$ is a test for no correlation between errors on the selection equation and the main equation. (In the case of the 2 step procedure this is the t statistic on the included Mills ratio).

Table 2: Primary and Secondary education, logs

	OLS	Robust regression	OLS, Uncapped only	Heckman ML	Heckman 2 Step
Additional educational needs	0.132 (11.33)	0.129 (10.53)	0.133 (9.32)	0.148 (7.82)	0.141 (6.80)
Constant	0.147 (10.29)	0.152 (10.08)	0.144 (8.79)	0.140 (8.07)	0.142 (8.37)
Sample size	106	106	87	106	106
R^2	0.553		0.505		0.507
Root MSE	0.058		0.058		0.058
Joint significance	$F_{1,104}$=128.44 p=0.000	$F_{1,104}$=110.90 p=0.000	$F_{1,85}$=86.84 p=0.000	χ^2_4=17.12 p=0.002	$F_{2,84}$=43.23 p=0.000
RESET	$F_{3,101}$=1.44 p=0.235		$F_{3,82}$=0.90 p=0.443		
Hetero	χ^2_1=1.60 p=0.205		χ^2_1=2.09 p=0.148		
$Chow_1$	$F_{1,103}$=3.20 p=0.077		$F_{1,103}$=5.85 p=0.018		
$Chow_2$	$F_{2,102}$=1.75 p=0.180		$F_{2,102}$=2.90 p=0.061		
$Select_1$				t=2.612 p=0.009	t=2.589 p=0.010
$Select_2$				t=0.750 p=0.453	t=0.561 p=0.576

Notes:
RESET is a Ramsey RESET test using fitted powers of the dependent variable
Hetero is a Cook-Weisberg test for heteroscedasticity using fitted powers of the dependent variable
$Chow_1$ is a Chow test for similarity of the constant in and out of inner London
$Chow_2$ is a Chow test for similarity of all coefficients in and out of inner London
$Select_1$ is a test for significance of regressors included in the capping equation but excluded from the main equation. (In the case of the 2 step procedure this is the t statistic on thevariable in the associated probit).
$Select_2$ is a test for no correlation between errors on the selection equation and the main equation. (In the case of the 2 step procedure this is the t statistic on the included Mills ratio).

Table 3: Nursery education, levels

	OLS	Robust regression	OLS, Uncapped only	Heckman ML	Heckman 2 Step
Additional educational needs	0.103 (6.09)	0.105 (6.03)	0.116 (5.33)	0.078 (3.20)	0.019 (0.66)
Constant	0.223 (10.67)	0.219 (10.21)	0.206 (8.27)	0.225 (8.32)	0.232 (10.14)
Sample size	106	106	87	106	106
R^2	0.263		0.251		0.410
Root MSE	0.085		0.088		0.079
Joint significance	$F_{1,104}$=37.09 p=0.000	$F_{1,104}$=36.40 p=0.000	$F_{1,85}$=28.44 p=0.000	χ^2_4=28.54 p=0.000	$F_{2,84}$=29.30 p=0.000
RESET	$F_{3,101}$=8.25 p=0.000		$F_{3,82}$=6.68 p=0.000		
Hetero	χ^2_1=0.40 p=0.528		χ^2_1=0.09 p=0.769		
Box Cox: λ=1	χ^2_1=0.63 p=0.427		χ^2_1=2.38 p=0.123		
Box Cox: λ=0	χ^2_1=8.94 p=0.003		χ^2_1=3.86 p=0.050		
$Chow_1$	$F_{1,103}$=10.47 p=0.002		$F_{1,103}$=8.73 p=0.004		
$Chow_2$	$F_{2,102}$=5.21 p=0.007		$F_{2,102}$=4.50 p=0.014		
$Select_1$				t=4.020 p=0.000	t=2.589 p=0.010
$Select_2$				t=3.633 p=0.000	t=4.780 p=0.000

Notes:
Asymptotic t statistics in parentheses below coefficients
RESET is a Ramsey RESET test using fitted powers of the dependent variable
Hetero is a Cook-Weisberg test for heteroscedasticity using fitted powers of the dependent variable
Box Cox is a test of hypotheses on λ for the Box Cox transformation of the dependent variable
$Chow_1$ is a Chow test for similarity of the constant in and out of inner London
$Chow_2$ is a Chow test for similarity of all coefficients in and out of inner London
$Select_1$ is a test for significance of regressors included in the capping equation but excluded from the main equation. (In the case of the 2 step procedure this is the t statistic on thevariable in the associated probit).
$Select_2$ is a test for no correlation between errors on the selection equation and the main equation. (In the case of the 2 step procedure this is the t statistic on the included Mills ratio).

Table 4: Nursery education, logs

	OLS	Robust regression	OLS, Uncapped only	Heckman ML	Heckman 2 Step
Additional educational needs	0.328 (6.01)	0.308 (5.45)	0.374 (5.34)	0.272 (3.64)	0.063 (0.70)
Constant	-1.493 (22.30)	-1.460 (21.05)	-1.559 (19.40)	-1.514 (17.89)	-1.476 (19.95)
Sample size	106	106	87	106	106
R^2	0.258		0.251		0.409
Root MSE	0.273		0.285		0.254
Joint significance	$F_{1,104}$=36.17 p=0.000	$F_{1,104}$=29.69 p=0.000	$F_{1,85}$=28.53 p=0.000	χ^2_4=27.60 p=0.000	$F_{2,84}$=28.99 p=0.000
RESET	$F_{3,101}$=9.11 p=0.000		$F_{3,82}$=7.03 p=0.443		
Hetero	χ^2_1=4.02 p=0.045		χ^2_1=0.91 p=0.341		
$Chow_1$	$F_{1,103}$=8.99 p=0.003		$F_{1,103}$=7.64 p=0.007		
$Chow_2$	$F_{2,102}$=4.46 p=0.014		$F_{2,102}$=3.82 p=0.026		
$Select_1$				t=3.987 p=0.000	t=2.589 p=0.010
$Select_2$				t=3.948 p=0.000	t=4.722 p=0.000

Notes:
RESET is a Ramsey RESET test using fitted powers of the dependent variable
Hetero is a Cook-Weisberg test for heteroscedasticity using fitted powers of the dependent variable
$Chow_1$ is a Chow test for similarity of the constant in and out of inner London
$Chow_2$ is a Chow test for similarity of all coefficients in and out of inner London
$Select_1$ is a test for significance of regressors included in the capping equation but excluded from the main equation. (In the case of the 2 step procedure this is the t statistic on thevariable in the associated probit).
$Select_2$ is a test for no correlation between errors on the selection equation and the main equation. (In the case of the 2 step procedure this is the t statistic on the included Mills ratio).

Table 5: Inclusion of political and economic variables

	Primary and secondary education	Nursery education
Additional educational needs	0.181 (9.80)	0.074 (4.44)
Ln Income	0.053 (0.92)	-0.105 (2.02)
Poliical control	0.008 (0.44)	0.082 (4.83)
Constant	0.643 (1.199)	1.199 (2.48)
Sample size	106	106
R^2	0.574	0.480
Root MSE	0.080	0.072
Joint significance	$F_{3,102}$=45.81 p=0.000	$F_{3,102}$=31.33 p=0.000
RESET	$F_{3,99}$=2.44 p=0.069	$F_{3,99}$=2.70 p=0.050
Hetero	χ^2_1=7.85 p=0.005	χ^2_1=0.01 p=0.912
Box Cox: λ=1	χ^2_1=4.38 p=0.036	χ^2_1=1.83 p=0.177
Box Cox: λ=0	χ^2_1=0.48 p=0.487	χ^2_1=8.08 p=0.004
$Chow_1$	$F_{1,101}$=4.15 p=0.044	$F_{1,101}$=2.43 p=0.122
$Chow_2$	$F_{4,98}$=3.22 p=0.016	$F_{4,98}$=1.76 p=0.144

Notes:
RESET is a Ramsey RESET test using fitted powers of the dependent variable
Hetero is a Cook-Weisberg test for heteroscedasticity using fitted powers of the dependent variable
$Chow_1$ is a Chow test for similarity of the constant in and out of inner London
$Chow_2$ is a Chow test for similarity of all coefficients in and out of inner London

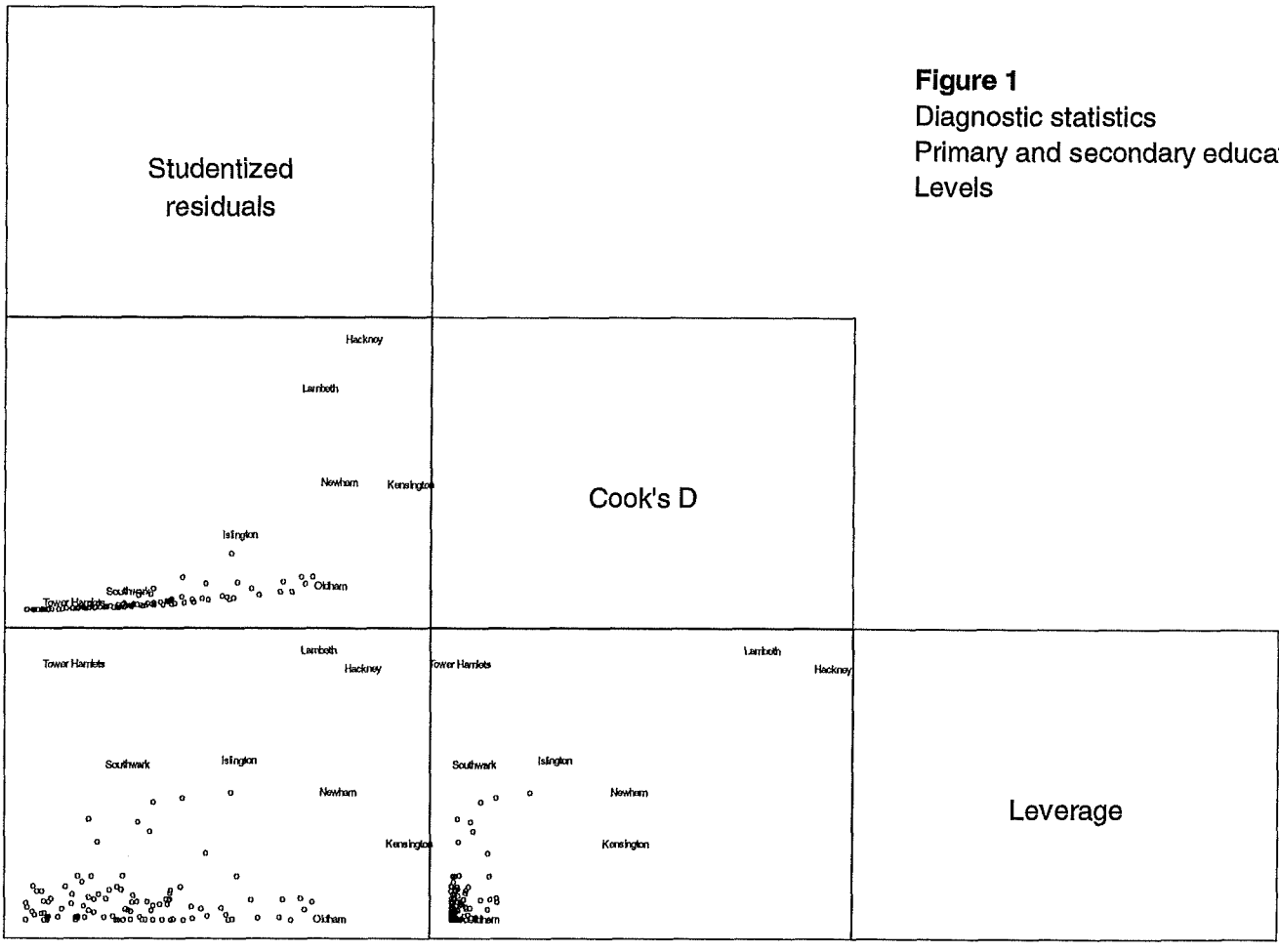

Figure 1
Diagnostic statistics
Primary and secondary educatio
Levels

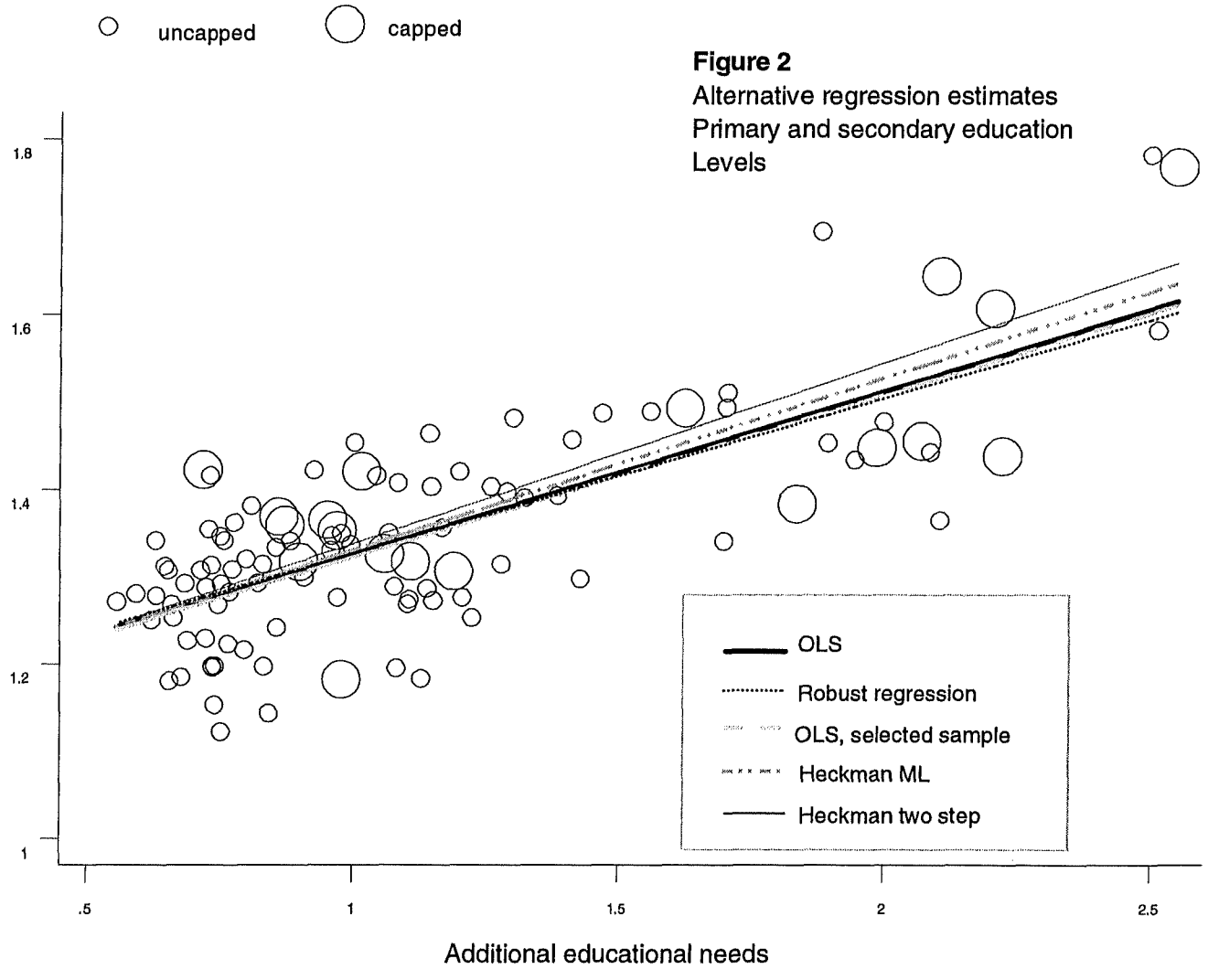

Figure 2
Alternative regression estimates
Primary and secondary education
Levels

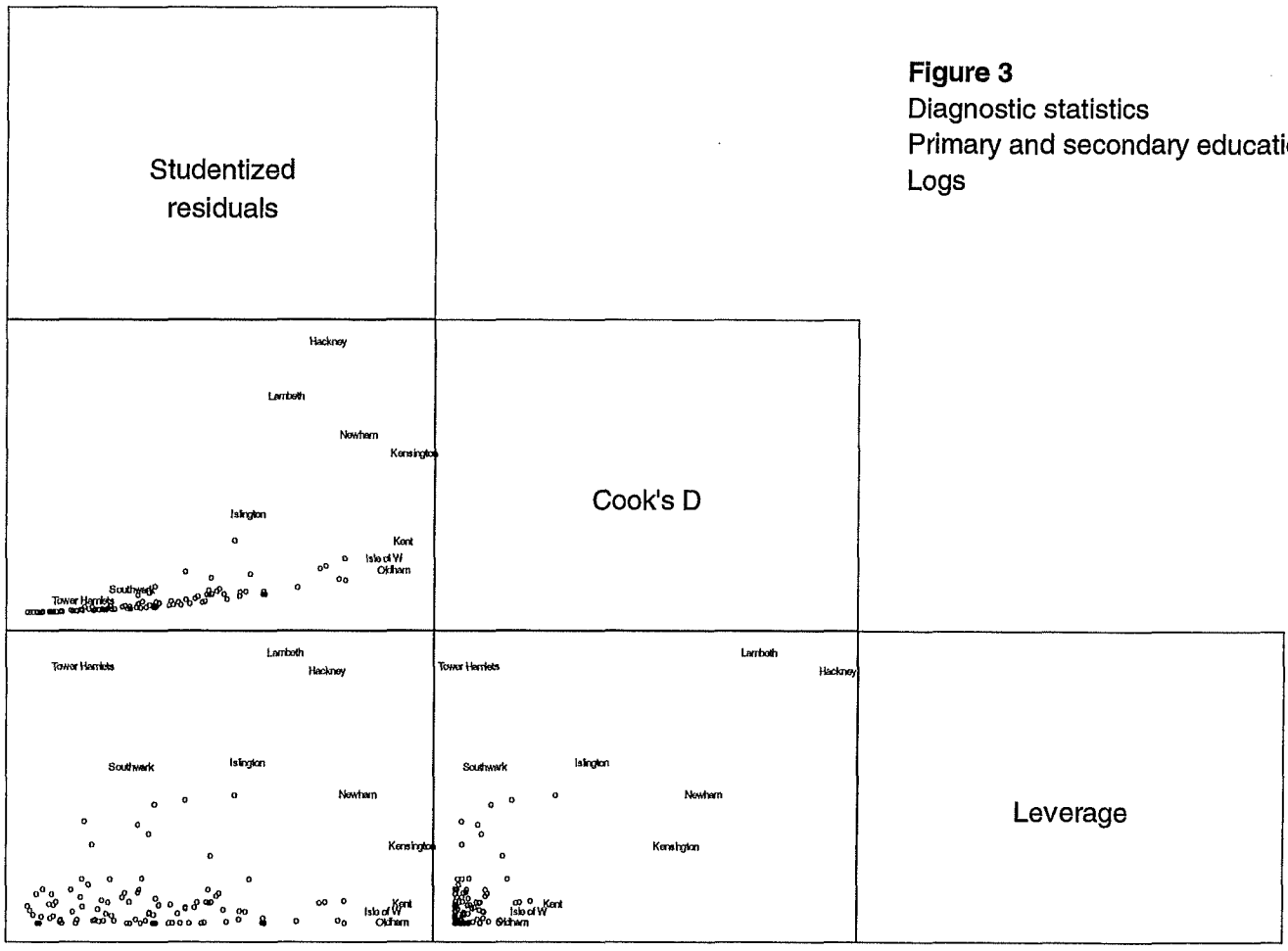

Figure 3
Diagnostic statistics
Primary and secondary education
Logs

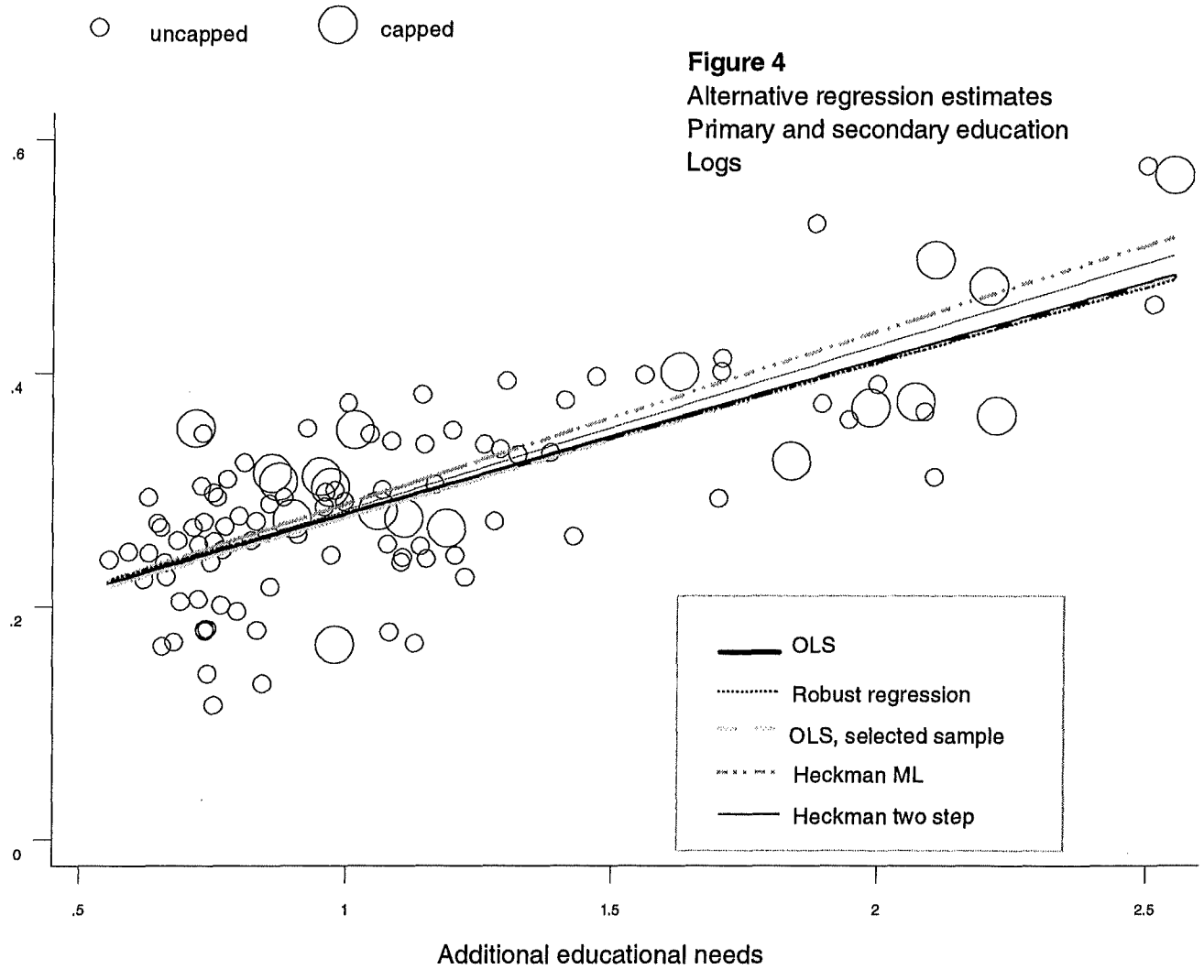

Figure 4
Alternative regression estimates
Primary and secondary education
Logs

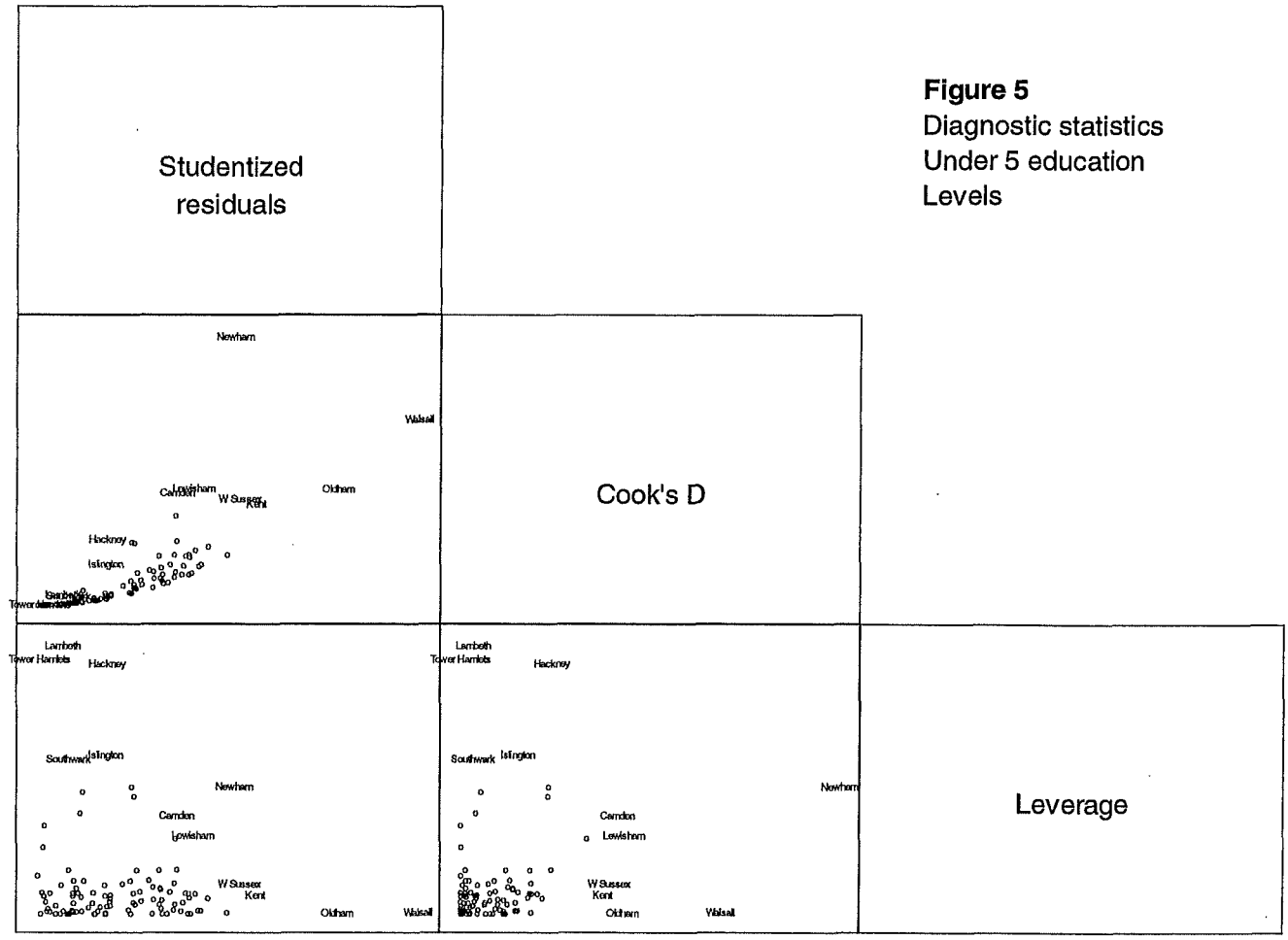

Figure 5
Diagnostic statistics
Under 5 education
Levels

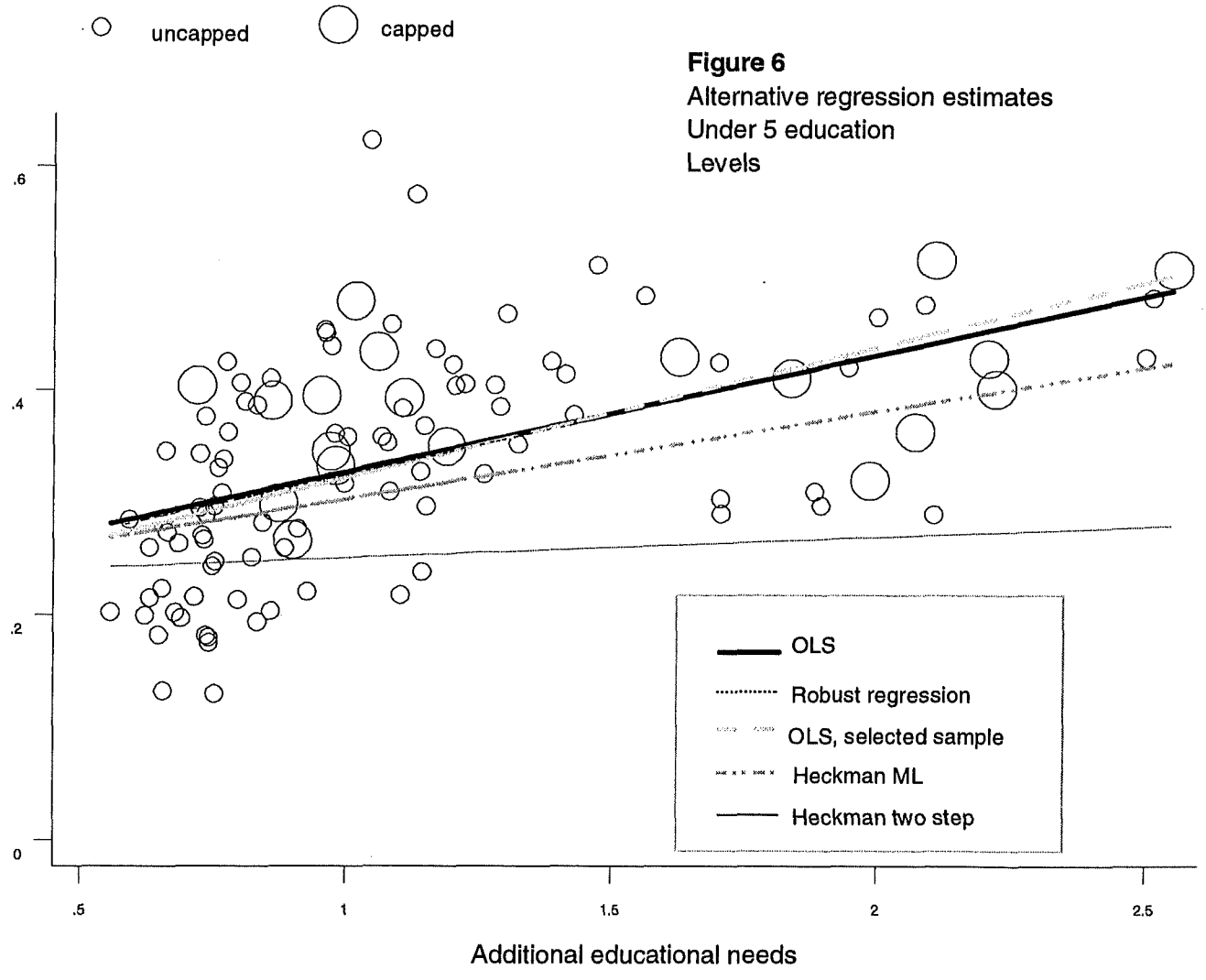

Figure 6
Alternative regression estimates
Under 5 education
Levels

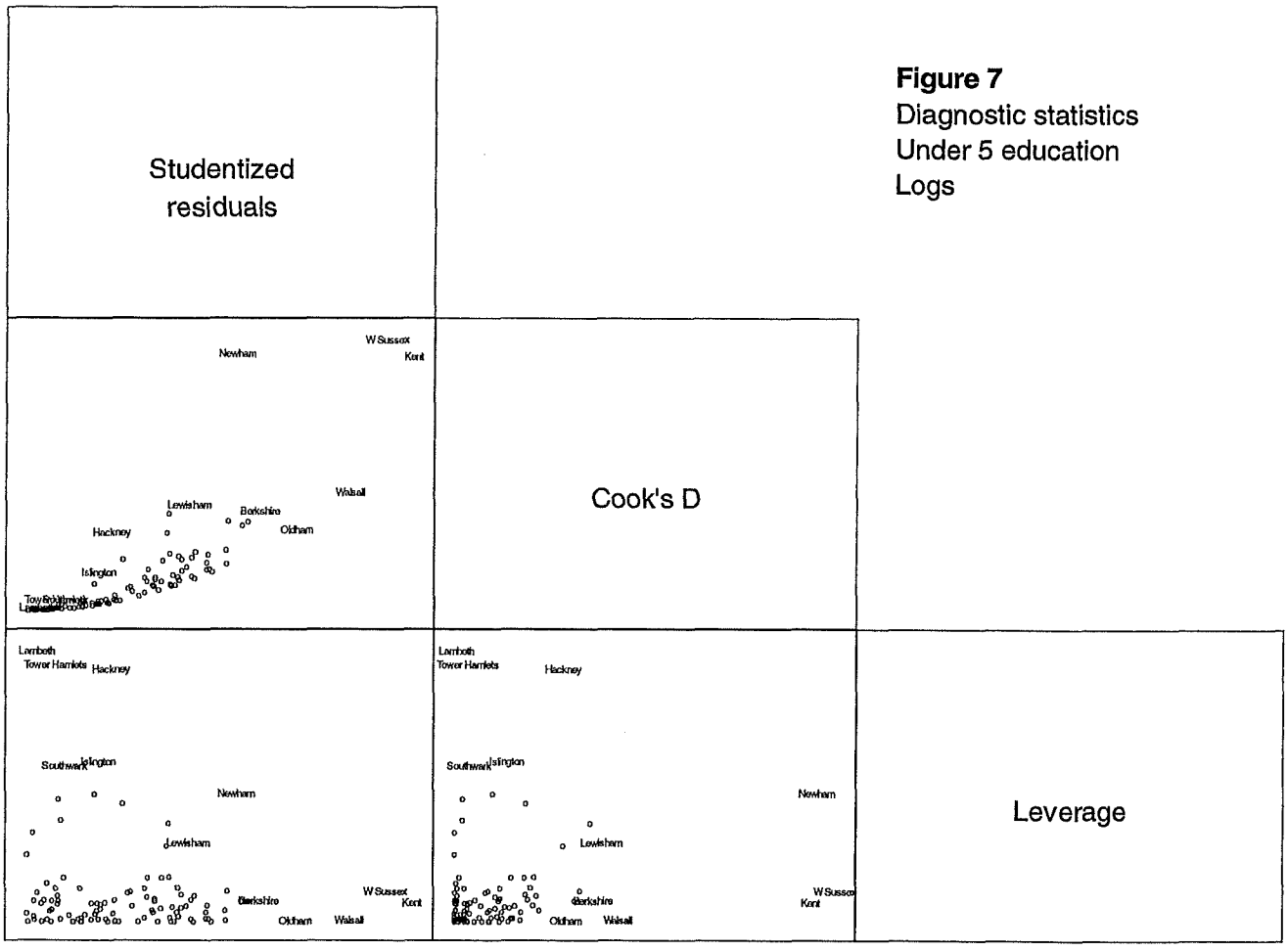

Figure 7
Diagnostic statistics
Under 5 education
Logs

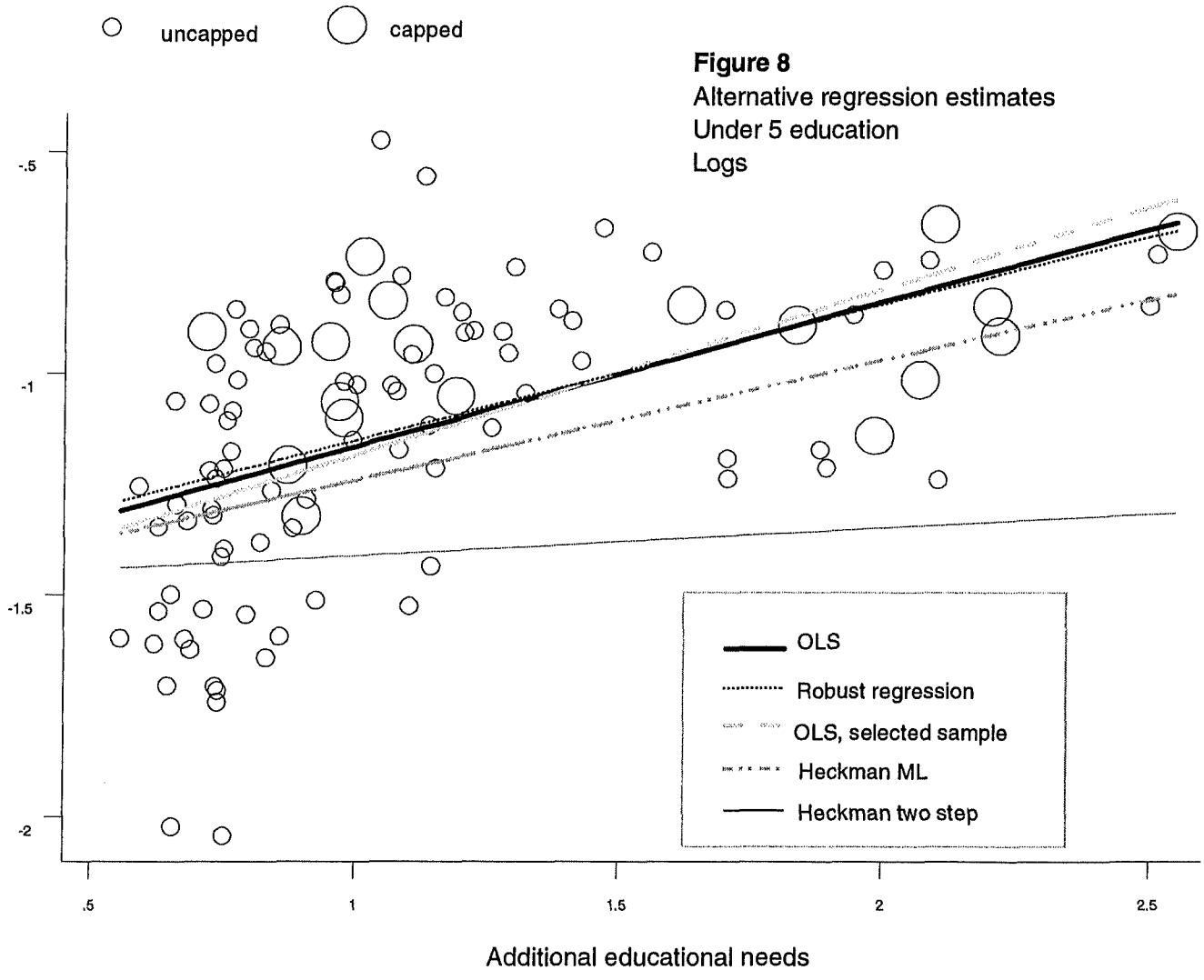

Figure 8
Alternative regression estimates
Under 5 education
Logs

Figure 9.

A schematic illustration of the relationship between the determinants of local authority spending and service levels

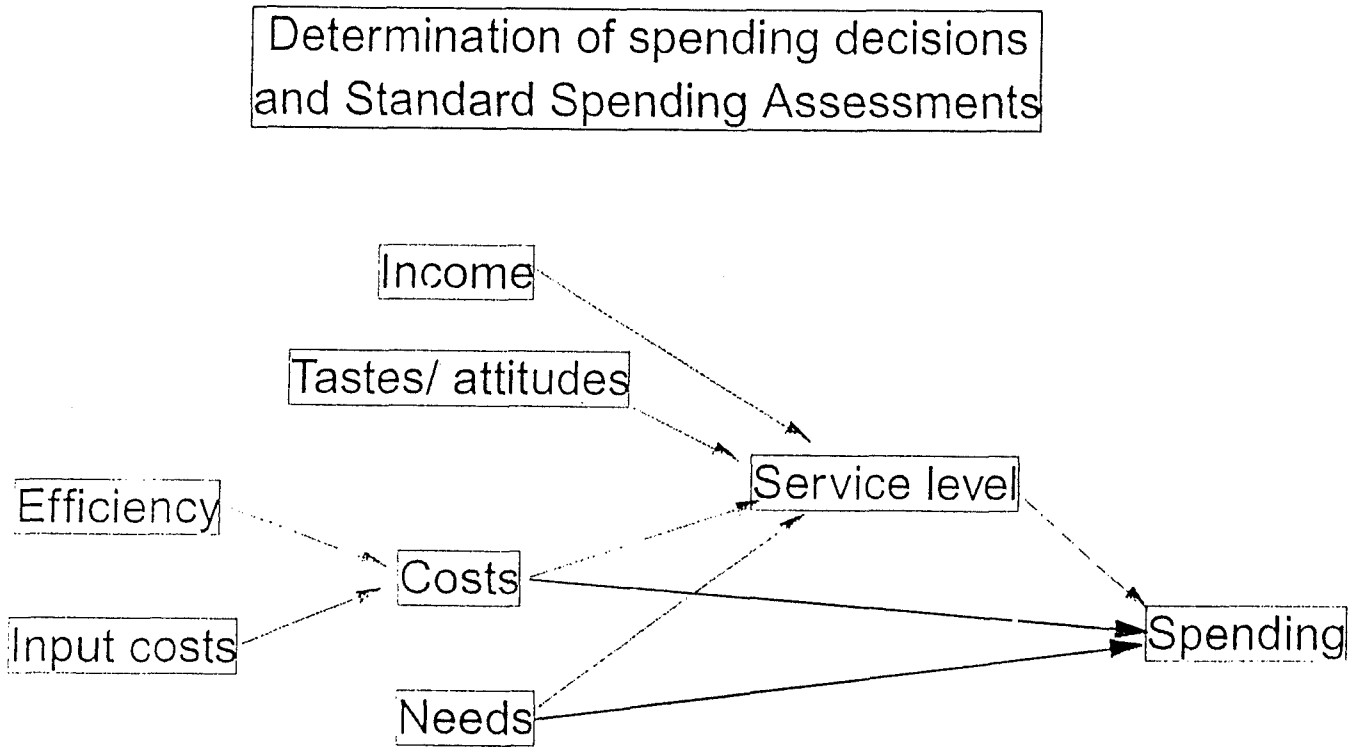

Figure 10.

Illustration of the effect on a hypothetical SSA regression of an authority with a large residual, but average needs. The authority exerts no influence on the estimated slope of the relationship.

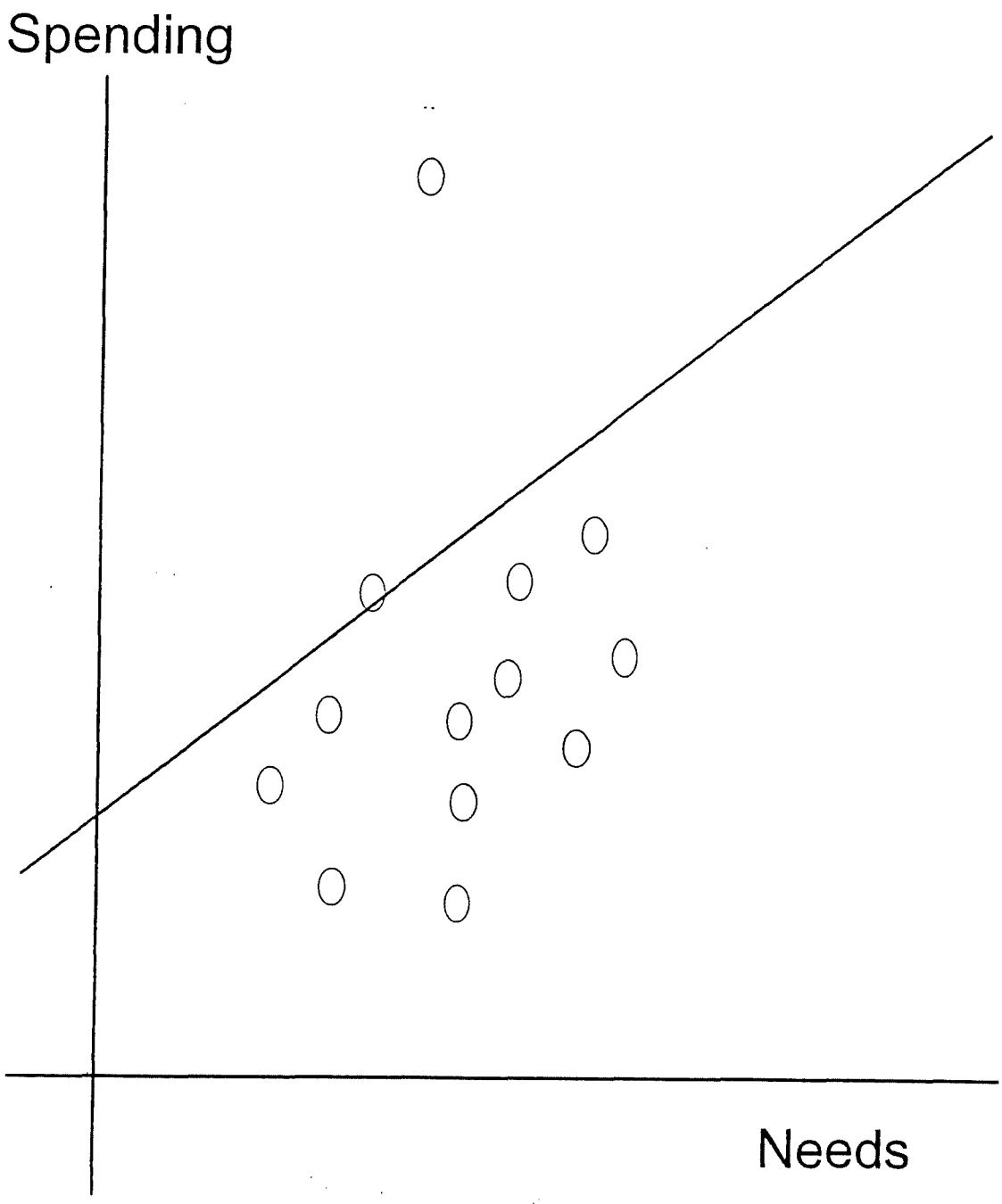

Figure 11.

Illustration of the effect on a hypothetical SSA regression of an authority with high leverage but a low residual. The authority has an extreme value for the needs indicator, and exerts a large effect on the slope of the estimated relationship.

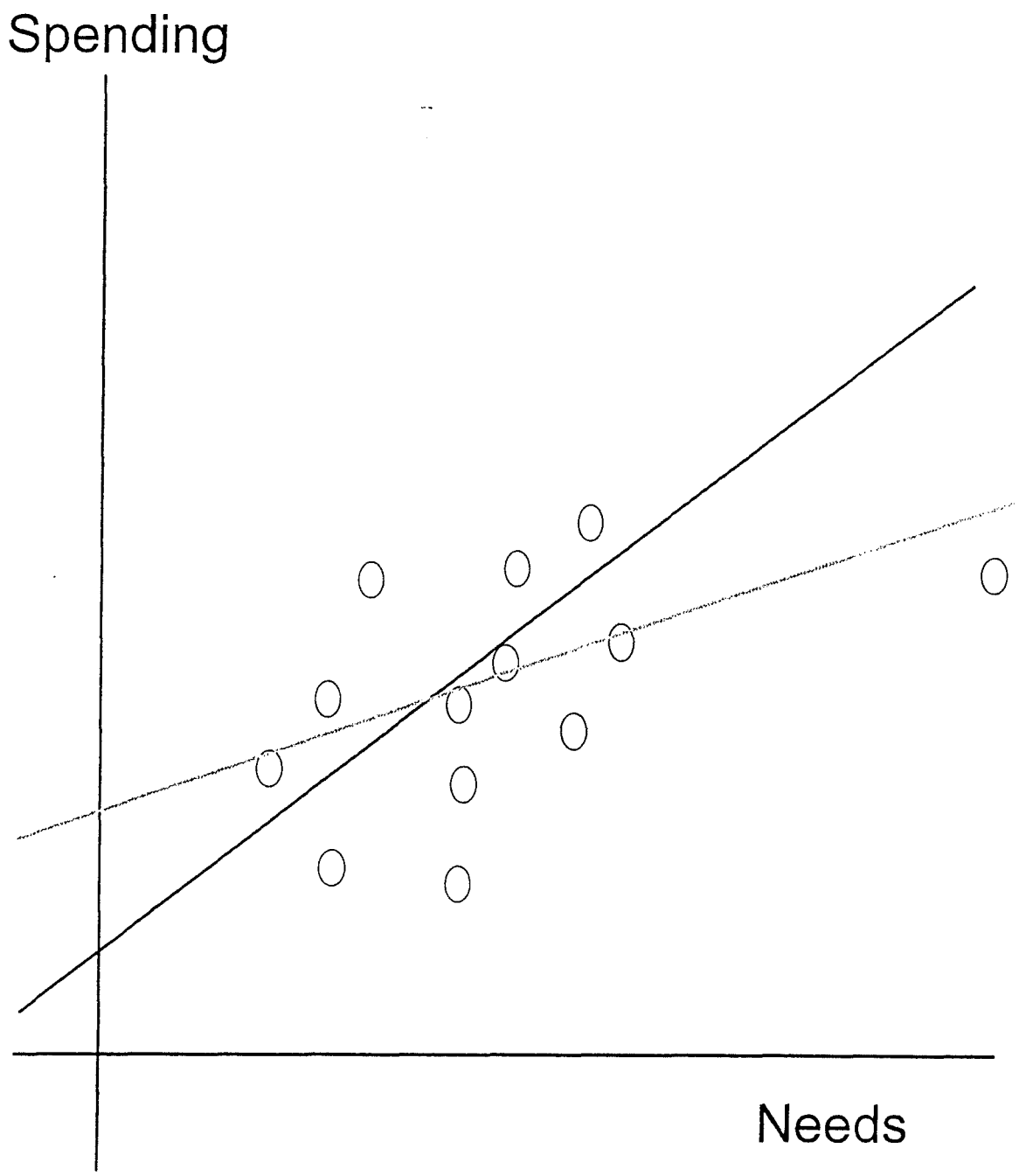

Figure 12.

Illustration of the sample selection bias caused by omitting capped authorities from a hypothetical SSA regression. The capped authorities (those with desired spending above the level of the cap, C) lie, on average above the "true" regression line. Their omission from the sample biases downwards the gradient of the estimated regression line.