# Spousal Matching with Children, Careers and Divorce*

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**Abstract**

We present a model in which couples match in the marriage markets and, on the basis of their relative spousal endowment levels, decide whether to have children, specialize in market production or home work and determine their intra-household allocations. After couples' marriage match qualities are revealed, better-matched couples stay married and poorly-matched couples separate. While higher household incomes raise the number of children, a narrower gender earnings gap lowers fertility among middle-income but relatively unequal couples. The propensity of divorce is a decreasing function of total household income, and when divorce produces a distance between children and at least one of the spouses, it is also decreasing in the number of children. Thus, a narrow gender earnings gap could lead to higher divorce rates through its depressing effect on fertility. In turn, higher divorce rates feed back into lower fertility and investment in children because they reduce expected marital surplus. In contrast, investment in human capital early on and anticipated levels of high investment in human capital later encourage higher levels of fertility, thereby lowering the divorce likelihood and producing a larger inter-temporal marital surplus. If childrearing at home involves costly labor force separation, fertility rates are lower and divorce rates higher primarily among the lower-income or higher inequality couples.

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# 1 Introduction

The one-to-one assignment model with transferable utility was spearheaded by Shapley and Shubik (1972) and applied to the marriage markets by Becker (1981). In it various theoretical as well as empirical incarnations, the transferable utility assignment model has been instrumental for linking the marriage market dynamics to intra-household allocations and outcomes. A drawback of the spousal assignment model, however, has been its static nature. This has become more of an impediment in light of the staggering evolution of marriage and families during the last half century in various dimensions, such as the rise of dual-career couples, declines in fertility and childbearing, and increases in the rates of divorce.

In this paper, we extend the spousal assignment model to cover endogenous spousal specialization, divorce and investment in marriage-specific goods, such as children. The main ingredients of our model are as follows: There is a continuum of men and women who live for two periods. Each agent is characterized by a single attribute, income (or human capital), with continuous distributions of incomes on both sides of the marriage market, so that each agent has a close substitute. In the first period, individuals who wish to get married match in the marriage market. Once matched, couples decide on their number of children, the optimal levels of a variety of other public goods which they share, as well as their individual private consumption levels. The economic gains from marriage arise from joint consumption of various public goods, children and a non-monetary common factor that is match specific. This match quality for each couple is revealed ex post and those with poor matches may divorce.

In our baseline model, couples only invest in the quantity of their children, although investment in children can be interpreted more broadly—as we do in an extension below—to encompass other kinds of investment in offspring, such as human capital. In either case, investment in children is a form of couple-specific public good which is distinct from the other maritally public goods for two reasons: One, unlike other marital public goods, which need to be purchased on the market at prevailing market prices, children can also be reared at home.[1] If the couple decides to send

[1] As we shall clarify later, the key distinction is not whether marital public goods other than children could be produced at home, but whether the market and home production options exist for

their children to daycare, then both spouse work full time in the labor market. But if the couple decides to rear for their children at home, then it devotes part of its time endowment to this pursuit, with one spouse committing at least part of his or her time to childrearing in the first period. Second, investment in children is durable; the decision to have kids is made by couples in the first period, although the utility each spouse derives from their presence lasts into the second period. This is essentially why both spouses will derive utility from their children even after divorce.

By embedding endogenous spousal specialization, investment in children and divorce into a standard assignment model, we illustrate how the dynamics of spousal competition and matching in the marriage markets come to bear on the equilibrium outcomes in spousal specialization, investment in children and divorce. One of our central findings is that spousal matching in the marriage markets plays an important role in mapping income inequalities among men and women into spousal inequality. The latter, in turn, determines intra-marital outcomes, such as fertility, spousal time use and allocations, as well as the likelihood of divorce. More specifically, while higher household incomes raise the number of children, a narrower gender earnings gap lowers fertility among middle-income but relatively unequal couples. For all couples, the propensity of divorce is a decreasing function of total household income, and when divorce produces a distance between children and at least one of the spouses, it is also decreasing in the number of children. Thus, a narrow gender earnings gap could lead to higher divorce rates through its depressing effect on fertility. In turn, higher divorce rates feed back into lower fertility and investment in children because they reduce the expected marital surplus. Somewhat analogously, if childrearing at home involves costly labor force separation, fertility rates would be lower and divorce rates would be higher primarily among the lower-income or higher inequality couples.

Accounting for the role of marriage market competition in influencing spousal allocations and intra-marital outcomes also enables us to identify that the gender gap in spousal allocations would be bounded from above by the gender gap in incomes or endowments. The reason for this is that, while lower spousal endowments do reduce

childrearing. In other words, while we make a strong assumption by ruling out the production of marital public goods other than children at home, the qualitative nature of our main results would go through more generally when all public goods could be produced at home or purchased on the market.

marital surplus, the marginal contribution to a marriage of a spouse who commits to childrearing at home is decreasing in his or her opportunity cost, which is lost labor income. Given the competition in the marriage markets, which helps to pin down spousal allocations over time and within each of the two periods, this maps into more generous allocations for the childrearing spouses, the lower their income or labor endowments, ceteris paribus.

Finally, we find that the conventional quantity-quality tradeoff is a conditional and primarily intra-temporal one: investment in human capital early on and anticipated levels of high investment in human capital later in life encourage higher levels of fertility among couples, thereby lowering the divorce likelihood and producing a larger inter-temporal marital surplus.

# 2 The Model

Individuals live two periods. In the first period, all men and women who are willing to be married match in the marriage market based on their incomes. At the end of the first period, the quality of each marital match is revealed and the poor-quality marriages dissolve.

There exists a continuum of men and a continuum of women. The measure of men is normalized to unity and the measure of women is denoted by $r$, where $r \gtrless 1$.

## 2.1 Endowments

Each man has an amount $y$ at the beginning of each period. Individual incomes $y$ are distributed over the support $[a, A]$, $0 < a < A$, according to some distribution $F$. Similarly, each woman has an income $z$ at the beginning of each period, and the $z$'s are distributed over the support $[b, B]$, $0 < b < B$ according to the distribution $G$.

Following divorce, there can be income transfers between the ex-spouses. We assume here that these transfers are fully determined by law although, as we elaborate later, voluntary transfers between ex-spouses could be made. In any event, legal redistribution corresponds to an approach where property incomes or spousal earnings are treated as a common resource and each spouse has some claim on the income of the other. Specifically, if a man with income $y$ marries a woman with income $z$, her income following divorce is $z' = \beta(y+z)$ and his income is $y' = (1-\beta)(y+z)$. Note that the net income of a divorced person is generally different from what his or her income would have been had he or she not married. Therefore, marriage in the first period is associated with a potential cost (benefit) that depends on the identity of the prospective spouse. Consequently, the distribution of incomes among divorcees can differ from the distribution of income in the whole population. The special case in which all incomes are considered private, implying no redistribution, is represented by a $\beta$ that is couple-specific, namely $\beta \equiv \frac{z}{y+z}$ .

In our baseline model, we shall assume that spousal labor incomes are constant over the individuals' lifetimes or that, more specifically, engaging in home production has no bearing on labor market productivity. But, in an extension, we shall consider the potential adverse impact of labor market detachment on incomes $y$ and $z$.[2]

## 2.2 Preferences

In each period, individuals derive utility from the consumption of $m$ private goods, $q^1, ...q^m$, $M$ public goods $Q^1, ..., Q^M$ and the number of their children, $N$. Let $p^1, ...p^m$ and $P^1, ..., P^M$ denote the corresponding prices, with the normalization $p^1 = 1$. Married people also derive satisfaction from the quality of their match, $\theta$. The husbands' and wives' individual utilities take the form

$$U_i = u_i(q_i, Q, N) + \theta, \qquad i = h, w, \tag{1}$$

where $q_i = (q_i^1, ...q_i^m)$ is the vector of private consumption of member $i$, $Q = \left(Q^1, \ ..., \ Q^M\right)$ is the vector of public consumption by the couple, $N$ represents the number of kids they choose to have, and $\theta$ is the quality of the couple-specific match.

We assume that preferences of married individuals are a variant of the *generalized quasi-linear* ($GQL$) form (see Bergstrom, 1989):

$$u_i\left(q_i, Q, N\right) = A\left(Q\right) q_i^1 + B_i\left(Q, q_i^{-1}\right) + C(N) + \theta, \tag{2}$$

where $Q = \left(Q^1, ..., Q^M\right)$ and $q_i^{-1} = (q_i^2, ...q_i^m)$. Here, $A$, $C$, and $B_i$, $i = h, w$, are positive, increasing, concave functions such that $A(0) = 1$ and $B_i(0) = C(0) = 0$.

[2] For simplicity, we do not allow savings or human capital investments during marriage so that human capital or income is constant. Labor supply is also assumed fixed.

Investment in children is a form of couple-specific public good and it is distinct from other public goods, $Q$, for two reasons: One, unlike other marital public goods, which need to be purchased on the market at the prices $P^j$, $1 \leq j \leq M$, children can reared at home or they can be sent to daycare. If the couple decides to utilize daycare, then it pays a price of $P^N$ per child for childrearing. But if the couple decides to rear for their children at home, then it devotes a fraction $\tau$ of its time endowment to this pursuit, taking away some time from employment and sacrificing wage income. Second, investment in children is durable; the decision to have kids is made by couples in the first period, although the utility each spouse derives from their presence lasts into the second period, when couples will not have to invest in their children any further.[3] Precisely due to this feature, spouses derive utility from their children even after they divorce.

If a man with income $y$ is matched with a woman with income $z$, they can pool their incomes. Their intra-temporal sum of utility levels is thus given by

$$u_h + u_w = A(Q)\left(q_h^1 + q_w^1\right) + B_h\left(Q, q_h^{-1}\right) + B_w\left(Q, q_w^{-1}\right) + 2[C(N) + \theta] \qquad (3)$$

where $u_h$ and $u_w$ are the attainable per-period utility levels that can be implemented by the allocations of the private good $q^1$ between the two spouses, given the efficient consumption levels of all other goods, private and public.

For all couples, match quality $\theta$ is drawn from a fixed distribution $\Phi$ with a mean $\bar{\theta} \geq 0$. Upon marriage, both spouses expect to derive the same non-monetary match utility from marriage, $\bar{\theta}$. At the end of the first period, the match quality is revealed and the realized value of $\theta$ may be either above or below it's expected value, $\bar{\theta}$. A realized value of $\theta$ that is below $\bar{\theta}$ constitutes a negative surprise that may trigger divorce.

If a couple divorces at the end of the first period, the divorced individuals' preferences take the *strictly quasi-linear* form:

$$u_i^s(q_i, Q, N) = q_i^1 + B_i^s\left(Q, q_i^{-1}\right) + C(N), \qquad (4)$$

where again, $C$ and the $B_i^s$, $i = h, w$, are increasing concave functions, with $B_i^s(0) = C(0) = 0$, and where the number of children $N$ was pre-determined in the first

[3]In Section 7, we shall relax this restriction and consider a quantity-quality tradeoff: a case in which parents choose the number of their offspring in the first period, but they invest in the human capital of their offspring in both periods.

period when $i$ was married. This utility is quasi-linear; in particular, the optimal consumptions of public goods and private goods other than goods 1 are given by the conditions:

$$\frac{\partial B_i^s\left(Q, q_i^{-1}\right)}{\partial Q^j} = P^j, \ \ 1 \leq j \leq M \qquad \text{and} \qquad \frac{\partial B_i^s\left(Q, q_i^{-1}\right)}{\partial q_i^k} = p^k, \ \ 2 \leq k \leq m \quad (5)$$

Neither these conditions nor the optimal levels of any private and public consumption (except for good 1) depend on income. Let the latter be denoted $\left(\bar{Q}, \bar{q}_i^{-1}\right) = \left(\bar{Q}^1, ..., \bar{Q}^M, \ \bar{q}_i^2, ...\bar{q}_i^m\right)$. To simplify notation, we choose units such that $B_i^s\left(\bar{Q}, \bar{q}_i^{-1}\right) = \sum_{j=1}^M P^j\bar{Q}^j + \sum_{k=2}^m p^k \bar{q}_i^k$, $i = h, w$. Then, the indirect utility of a divorced, single person equals his or her income in the second period, $y'$ or $z'$, plus the utility from his or her children born in the first period, $N$.

On this basis, the sum of the divorced couples' utility levels in the second period is given by:

$$u_h^s\left(q_h, Q, N\right) + u_w^s\left(q_w, Q, N\right) = t + 2C(N) \ . \quad (6)$$

## 2.3 Expected Lifetime Utilities

Due to the fact that utility is transferable within and without marriage, based on the quasi-linearity property of individual utilities we defined above, couples make all of their choices efficiently. In particular, couples choose the optimal levels of the $M$ public goods, the number of their offspring, $N$, as well as the $2(m-1)$ private goods by maximizing the sum of their expected lifetime utilities. Then, on the basis of the marriage market equilibrium and their outside reservation utility levels, they determine the allocations of the private good $q^1$ between the husband and the wife. The allocation of $q^1$ is determined for the first period and, conditional on the fact that the marriage continues, for the second period as well.

Let $\hat{\theta}$ represent the match quality draw below which a couple would divorce at the end of the first period and let $\alpha$, $0 \leq \alpha \leq 1$, denote the divorce probability associated with $\hat{\theta}$. Both $\hat{\theta}$ and $\alpha$ are endogenous and we will show that, in our baseline version, $\hat{\theta}$ and $\alpha$ will both be independent of the choices of all private and public goods, including the number of children. Hence, for $\hat{\theta}$ and $\alpha$ that are independent of $Q$, $N$ and $q_i$, $i = h, w$, and for a couple whose total income equals $y + z \equiv t$, the optimal choices of the $M$ public goods, fertility $N$, and the $2(m-1)$ private consumption allocations would be given by the following problem:

$$\max_{\left(N,Q,q_h^{-1},q_w^{-1}\right)} \left\{ \begin{array}{c} A\left(Q\right)q^1 + B_h\left(Q,q_h^{-1}\right) + B_w\left(Q,q_w^{-1}\right) + 2[C(N)+\bar{\theta}] + \alpha(t+2C(N)) \\ + \\ (1-\alpha)[A\left(Q\right)q^1 + B_h\left(Q,q_h^{-1}\right) + B_w\left(Q,q_w^{-1}\right) + 2C(N) + 2E(\theta \left| \theta \geqslant \hat{\theta} \right.)] \end{array} \right\} \tag{7}$$

where $q^1 = q_h^1 + q_w^1$. All of the terms on the first line of (7) except the final one represent the couples' aggregate utility in the first period; the last term on that line, $\alpha(t+2C(N))$, denotes the couple's expected second-period utility in case of divorce; and the entire second line represents the sum of its expected second-period utility in case their marriage continues.

Given the quasi-linearity of preferences, utility is *transferable* between spouses both within marriage and without. As a result, there is a unique efficient level for the consumption of each of the public goods, $M$, the number of children, $N$, and each of the private goods 2 to $m$.

## 2.4 The Optimal Level of Marital Public Goods

The optimal consumption of public goods and private goods other than good 1 are given by the following two conditions:

$$\frac{\partial B_h\left(Q,q_i^{-1}\right)}{\partial Q^j} + \frac{\partial B_w\left(Q,q_i^{-1}\right)}{\partial Q^j} = P^j, \quad 1 \leq j \leq M, \tag{8}$$

and

$$\frac{\partial B_i\left(Q,q_i^{-1}\right)}{\partial q_i^k} = p^k, \quad i = h, w \text{ and } 2 \leq k \leq m \ . \tag{9}$$

None of these $M + 2(m-1)$ conditions depend on income and they are solely pinned down on the basis of relative prices. Moreover, given that all of the $M$ public good and $2(m-1)$ private good choices need the be made in both periods, the first-order conditions apply in the first period and, conditional on the continuation of the marriage, in the second period as well.

## 2.5 Investment in Children

As we stated above, investment in children is a unique marital public good, in part because it is durable and the number of children chosen by the couple in the first period, $N$, is sustained in the second period without necessitating any additional investment by the couple.

For this reason, the optimal choice of $N$ has an inter-temporal element, unlike the optimal levels of the $M$ other marital public goods and the $2m$ private goods. Taken together with the fact that there are two alternative modes of investment in offspring and that the other public and private good choices depend solely on relative prices and not household income, we can note that

$$q^1 = q_h^1 + q_w^1 = t - \sum_{j=1}^{M} P^j Q^j - \sum_{k=2}^{m} p^k \left(q_h^k + q_w^k\right) - \min[P^N, \tau \min(y,z)]N \ . \qquad (10)$$

where the terms $\sum_{j=1}^{M} P^j Q^j$ and $\sum_{k=2}^{m} p^k \left(q_h^k + q_w^k\right)$ represent the total expenditures on the $M + 2(m-1)$ public and private goods, and where the final term $\min[P^N,$ $\tau \min(y,z)]\ N$ denotes the couple's total expenditure on childrearing, depending on the choice between childrearing at home and daycare.

Given the maximization problem defined by equation (7) and the budget constraint defined by (10), the couple $(y,z)$ decides on their optimal number of offspring as:

$$\bar{N}_1 = C'^{-1}\left(\frac{A(Q)\min[P^N, \tau \min(y,z)]}{4}\right) \ . \qquad (11)$$

As (11) implies, the optimal number of offspring is decreasing in the cost of childrearing and it also does not directly depend on the income of the couple, $t = y + z$. Together with the fact that neither of the other public goods nor the private goods except $q^1$ depends on income $t$, we can rewrite (7) as follows:

$$\begin{aligned} S(t) \ \equiv \ & \eta(t - \min[P^N, \tau \min(y,z)]\bar{N}_1) + 2[C(\bar{N}_1) + \bar{\theta}] + \alpha(t + 2C(\bar{N}_1)) \\ & +(1-\alpha)[\eta(t) + 2C(\bar{N}_1) + 2E(\theta \,|\, \theta \geqslant \hat{\theta})] \end{aligned} \qquad (12)$$

Assuming, as is standard, that the optimal public consumption levels are such that $A(Q)$ is increasing in $Q$, we see that $\eta(.)$ is *increasing and convex* in $t$.[4] Due to the existence of marital public goods including children, the two individual traits $y$ and $z$ of a married couple are *complements* within the household. This complementarity generates positive economic gains from marriage in the sense that the material output $\eta(.)$ the partners generate together exceeds the sum of the outputs that the partners can obtain separately. Specifically, the marital surplus $\eta(.) - t$ rises with the total income of the partners, $t$, and equality holds only when both partners have no income. Note that $\eta(t) + 2C(N)$ denotes the intra-temporal marital value associated with the marriage of spouses whose total income equals $t$. Hence, as we shall illustrate soon, the assortative nature of matching in the marriage markets will be determined by whether or not $S(t)$ itself is increasing and convex in $t$ and not $\eta(.)$.

## 2.6 Spousal Specialization

Given the optimal number of children for the couple $(y, z)$, we can now identify the mode through which the couple rears for its children. In particular, the total cost of childrearing would be given by

$$TC^N = \begin{cases} \tau z \bar{N}_1 & \text{if } \min(y,z) = z < \frac{P^N}{\tau} \ , \\ \tau y \bar{N}_1 & \text{if } \min(y,z) = y < \frac{P^N}{\tau} \ , \\ P^N \bar{N}_1 & \text{if } \min(y,z) \geqslant \frac{P^N}{\tau} \ . \end{cases} \tag{13}$$

In words, if either of the spouses potential income, $y$ or $z$, is relatively low or home productivity in childrearing, $\tau$, is relatively high, then the spouse with lower labor income would commit to childrearing, while the other with the higher labor productivity would work full time on the market. This would be the case when $\min(y, z) < P^N/\tau$. But if purchasing childcare on the market is cheaper, which would be the case when $\min(y, z) \geqslant P^N/\tau$, then both spouses would work on the market full-time while sending the children to childcare.

[4]By the envelope theorem, the derivative $\eta'(t)$ is equal to $A(Q)$. Therefore, $\eta$ is increasing in $t$ and, if $A(Q)$ is increasing in $t$ as well, then $\eta$ is convex. Note that a sufficient (but by no means necessary) condition is that public consumptions are all normal.

# 3 Marriage Market Outcomes

In the first period, all men and women wish to marry because the expected economic and non-monetary gains from marriage are positive. However, if $r \neq 1$, then either some women (when $r > 1$) or some men (when $r < 1$) will have to remain single.

## 3.1 Matching

**Who Marries Whom?** Given the results of transferable utility and the complementarity of individual incomes in generating marital surplus, a stable assignment must be characterized by *positive assortative matching.* That is, if a man with an endowment $y$ is married to a woman with an endowment $z$, then the mass of men with endowments above $y$ must exactly equal the mass of women with endowments above $z$. This implies the following marriage market clearing condition:

$$1 - F(y) = r[1 - G(z)]. \tag{14}$$

As a result, we have the following, spousal matching functions:

$$y = F^{-1}\{1 - [r1 - G(z)]\} \equiv \phi(z) \tag{15}$$

and

$$z = G^{-1}\left\{1 - \frac{1}{r}[1 - F(y)]\right\} \equiv \psi(y). \tag{16}$$

For heuristic purposes, take the case when $> 1$. In that case, all men are married and women with incomes below $z_0 = G^{-1}(1 - 1/r)$ remain single. Women with incomes exceeding $z_0$ are then assigned to men according to $\psi(y)$ which indicates positive assortative matching.

A simple illustration of the spousal matching patterns can be provided under the assumption that the sex ratio equals unity, $r = 1$, and the income distribution of women can be derived from that of men by a *linear* transformation such that

$$G(z) = F(\lambda z + \delta), \tag{17}$$

for some fixed $\lambda$ and $\delta$. This is the case, for instance, when both distributions are lognormal for $y \geq a$ and $z \geq b$ with mean and variance $(\mu_m, \sigma_m)$ for men and $(\mu_f, \sigma_f)$

for women, if we assume that $\sigma_m = \sigma_f$. Then, $\lambda = \exp\left(\mu_m - \mu_f\right)$ and $\delta = a - \lambda b$. If $\lambda \geq 1$ and $\delta \geq 0$, the distribution of men dominates that of women in the first degree. Then, for $r \geq 1$, each married man is wealthier than his spouse. In the special case with $r = 1$, the assignment function is linear, given by $\phi(z) = \lambda z + \delta$. We shall refer to this property as a linear shift or $LS$.

## 3.2 Divorce

At the end of the first period, the true value of match quality is revealed and each partner of a couple $(y, z)$ can decide whether or not to stay in the marriage, based on the continuation of the realized $\theta$. Because utility is transferable within marriage and upon divorce, the Becker-Coase theorem applies and divorce occurs whenever the total surplus generated outside the relationship is larger than what can be achieved within it.[5]

With $t \equiv y + z$, the couple $(y, z)$ divorces at the end of the first period whenever

$$\eta(t) + 2[C(\bar{N}_1) + \theta] < t + 2C(\bar{N}_1) \ , \tag{18}$$

or, equivalently,

$$\eta(t) + 2\theta < t \Leftrightarrow \theta < \hat{\theta}(t) = -\frac{1}{2}[\eta(t) - t] \ . \tag{19}$$

On this basis, the ex-ante probability of divorce for a couple with endowments of $y$ and $z$ is

$$\alpha(t) \ \equiv \ \Phi[\hat{\theta}(t)] \ . \tag{20}$$

Note that the threshold $\hat{\theta}(t)$ falls with the income of the couple, $t$, and consequently the probability of divorce $\alpha(t)$ declines with higher incomes. Because of the complementarity of individual incomes in household production, the economic loss generated by divorce is higher for wealthier couples.

In this baseline version, the number of children $N$ has no bearing on divorce. This, however, is due to the fact that neither spouse's utility from children differs in divorce than in marriage. As we shall show below, if utility from children in divorce

[5] See Chiappori, Iyigun and Weiss (2007) for a detailed investigation of the transferability in the presence of public goods.

differs than that in marriage for either spouse due to, say, the non-custodial parent's distance from his or her children, then divorce probability would be influenced by the number of children, $N$, as well.

The expected marital output generated over the two periods of a married couple with incomes $y$ for the husband and $z$ for the wife is given by equations (12) and (20). On this basis, we can now note that $S(t) > 2t$, because $\eta(t) \geq t$ and $E[\theta \mid \theta \geq \hat{\theta}(t)] > \bar{\theta} \geq 0$. Hence, all individuals prefer to get married rather than stay single. Secondly, $S(t)$ is increasing in $t$, hence in each partner's income. In particular, whenever women strictly outnumber men so that $r > 1$, women belonging to the bottom part of the female income distribution remain single.[6] Finally, individuals will sort positively into marriage because utility is transferable and $S(t)$ is convex so that the traits of the two partners are *complements*, even after the risk of divorce is taken into account. This last result follows from the fact that the gains from marriage depend only on total family income (see Appendix).

### 3.3 Stability Conditions

The allocations which support a stable assignment must be such that the implied expected lifetime utilities of the partners satisfy

$$U_h(y) + U_w(z) \geq S(t) \; ; \qquad \forall \; y, \; z, \tag{21}$$

where $U_h(y)$ and $U_w(z)$ respectively represent the expected lifetime utilities of the husband and the wife over the two periods. For any stable marriage, equation (21) is satisfied as an equality, whereas for a pair that is not married, (21) would be satisfied as an inequality. In particular, we have

$$\begin{aligned} U_h(y) &= \max_z \left[ S(t) - U_w(z) \right] \; , \\ U_w(z) &= \max_y \left[ S(t) - U_h(y) \right] \; . \end{aligned} \tag{22}$$

While the stability conditions above constrain the total (two-period) expected utilities $U_h$ and $U_w$, they have no implication for the distribution of utility inter-

[6] Analogously, whenever men strictly outnumber women so that $r < 1$, men belonging to the bottom part of the male income distribution remain single.

temporally over the two periods. Nonetheless, as we shall discuss shortly, marital participation constraints as well as the spousal allocation levels mandated by divorce laws, together with the sum of expected utilities we identified above, would pin down intra-temporal spousal allocations within each of the two periods.

## 3.4 Determination of Expected Lifetime Allocations

Condition (3.3) leads to an explicit characterization of the intra-household allocations. The envelope theorem applied to these conditions yields the differential equations :

$$U_h{}'(y) \quad = S'[y + \psi(y)], \tag{23}$$

and

$$U_w{}'(z) \quad = S'[\phi(z) + z] \ . \tag{24}$$

To derive the expected spousal allocations over the two periods and along the assortative marital order, we integrate the expressions in (23) and (24). Hence, the surplus share of a *married man* with income $y$ is

$$U_h(y) \ = \ k^h + \int_a^y U_h'(x)\, dx \ , \tag{25}$$

and the surplus share for a *married woman* with income $z$ is

$$U_w(z) = k^w + \int_b^z U_w'(x)\, dx \ , \tag{26}$$

for some constants $k^h$ and $k^w$ which we determine next.

**Pinning Down the Constants** The constants $k^h$ and $k^w$ are pinned down by two conditions. First, for all married couples, the total output is known as expressed by equations (25) and (26). Hence,

$$k^h \ + \ k^w \ = S[y + \psi(y)] - \int_a^y U_h'(x)\, dx - \int_b^{\psi(y)} U_w'(x)\, dx \ , \tag{27}$$

where the left-hand side, by construction, does not depend on $y$. Secondly, it must be the case that the last married person is just indifferent between marriage and singlehood. In the case with more women than men, $r > 1$, we have

$$U_w(z_0) \ = 2z_0 \qquad \Leftrightarrow \qquad k^w = 2z_0 - \int_b^{z_0} U_w'(x)\,dx\ , \tag{28}$$

with $z_0 \equiv \Phi(1-r)$. Consequently,

$$k^h = S[\phi(z_0) + z_0] - 2z_0\ ,$$

$$U_w(z) = 2z_0 + \int_{z_0}^{z} U_w'(x)\,dx\ , \tag{29}$$

$$U_h(y) \ = S[y + \psi(y)] - U_w[\psi(y)] = S[y + \psi(y)] - \left(2z_0 + \int_{z_0}^{\psi(y)} U_w'(x)\,dx\right).$$

In sum, conditions on the marriage market determine the allocation of lifetime utilities between spouses: since many perfect substitutes exist for each spouse due to competition, a wife would not agree to marry a husband who would provide less than the equilibrium utility, and neither would a husband.

## 3.5 Second-period Utilities

Let $u_h^2(y)$ and $u_w^2(z)$ denote the *monetary* components of utility derived from the intra-marital allocations respectively of husband with endowment $y$ and wife with endowment $z$ in the second period should they continue with their marriage. Hence, the husband's (wife's) total second-period utility is $u_h^2(y)+C(\bar{N}_1)+\theta_h$ (resp. $u_w^2(z)+C(\bar{N}_1)+\theta_w$) if the marriage continues. Feasibility constraints require that

$$u_h^2(y) + u_w^2(z) = \eta(t)\,. \tag{30}$$

Under unilateral divorce, each spouse can walk away with the share of family income determined by law; $\beta t$ for the wife and $(1-\beta)t$ for the husband, where $t = (y+z)$ is total family income. Individual rationality implies that these outside options cannot exceed the utility payoffs if the marriage continues. Therefore, it must be the case that

$$u_h^2(y)+C(\bar{N}_1)+\theta \geq (1-\beta)\,t+C(\bar{N}_1) \qquad \text{and} \qquad u_w^2(z)+C(\bar{N}_1)+\theta \geq \beta t+C(\bar{N}_1), \tag{31}$$

which we shall hereafter refer as the *individual rationality constraints* $(IR)$. Note that these conditions jointly imply that

$$u_h^2(y) + u_w^2(z) = \eta(t) \ , \tag{32}$$

or equivalently that $\theta \geq \hat{\theta}(t)$, so that divorce occurs efficiently. Any allocation that satisfies (31) and (32) can be implemented as part of a feasible marital contract.[7]

## 3.6 First-period Utilities

For each choice of $k$, we can now recover the first-period allocations. The expected two-period utilities equal

$$U_h(y) = u_h^1(y) + C(\bar{N}_1) + \bar{\theta} + \alpha(t)\left[(1-\beta)t + C(\bar{N}_1)\right]$$

$$+ \left[1-\alpha(t)\right]\left\{u_h^2(y) + C(\bar{N}_1) + E\left[\theta \mid \theta \geq \hat{\theta}(t)\right]\right\} \ , \tag{33}$$

$$U_w(z) = u_w^1(z) + C(\bar{N}_1) + \bar{\theta} + \alpha(t)\left[\beta t + C(\bar{N}_1)\right]$$

$$+ \left[1-\alpha(t)\right]\left\{u_w^2(z) + C(\bar{N}_1) + E\left[\theta \mid \theta \geq \hat{\theta}(t)\right]\right\} \ , \tag{34}$$

where $\alpha(t) = \Pr(\theta < \hat{\theta})$ is the divorce probability as given by (19) and (20). These utilities must coincide with the equilibrium values derived above. Therefore, for $r > 1$,

$$u_w^1(z) = z_0 + \int_{z_0}^{z} S'\left[\phi(x) + x\right] dx \ - \alpha(t)\left[\beta t + C(\bar{N}_1)\right]$$

$$- \left[1-\alpha(t)\right]\left\{u_w^2(z) + C(\bar{N}_1) + E\left[\theta \mid \theta \geq \hat{\theta}(t)\right]\right\} \ , \tag{35}$$

[7] A natural question is whether the material allocation $\left(u_h^2, u_w^2\right)$ can be contingent upon the realization of $\theta$. Contingent allocations raise specific problems. For instance, depending on the enforcement mechanism, they may require that the quality of the match be verifiable by a third party. Whether such verifiability is an acceptable assumption is not clear. It turns out, however, that under our assumption of common $\theta$, verifiability is not an issue because the *same* allocation satisfies the incentives compatibility constraints for *all* $\theta$. For more details, see Chiappori, Iyigun and Weiss (2008).

$$u_h^1(y) = S[y+\psi(y)] - z_0 - \int_{z_0}^{\psi(y)} S'[\phi(x)+x]\,dx - \alpha(t)[(1-\beta)t + C(\bar{N}_1)]$$

$$+ [1-\alpha(t)]\left\{u_h^2(y) + C(\bar{N}_1) + E\left[\theta \mid \theta \geq \hat{\theta}(t)\right]\right\} .$$

# 4 The Effects of Custody & Distance in Divorce

So far we abstracted from the possibility that the parents' utility from children could be different in divorce than it is in marriage, for at least one of the spouses. But it may be the case that, due to physical custody—or lack thereof—and the commensurate distance that divorce creates between the children and one or both of the parents, the utility parents derive from their children is altered in divorce.[8] This is a straightforward but important extension of our framework because, as we shall show next, such structural differences in utility from children in marriage versus divorce will have a bearing not only on the likelihood of divorce, but also on the optimal number of children as well as spousal allocations during marriage.

With that, consider the case in which the parents' utility from $N$ children becomes $\Delta_i N$, $0 \leq \Delta_i < 1$, $i = h, w$, if the couple separates at the end of the first period. In general, it could be the case that the impact of divorce on the parents' utility is asymmetric with $\Delta_h \neq \Delta_w$. But for heuristic purposes, assume for now that $\Delta_h = \Delta_w \equiv \Delta < 1$. Revising equations (18) through (20) with this change in effect, we establish that a couple $(y, z)$ divorces at the end of the first period whenever

$$\eta(t) + 2[C(N) + \theta] < t + \Delta C(N) , \tag{18$'$}$$

or, equivalently,

$$\theta < \hat{\theta}(t, N) = -\frac{1}{2}[\eta(t) - t + (1-\Delta)C(N)] . \tag{19$'$}$$

Thus, the couple's ex-ante probability of divorce is now a decreasing function of its total income $t$ as well as their number of offspring $N$:[9]

[8] For related discussion about custody and child support payments in divorce, see Chiappori and Weiss (2003, 2006, 2007).

[9] In fact, such a result is consistent with the empirical findings on marriage and divorce patterns

$$\alpha(t,N) \equiv \Phi[\hat{\theta}(t,N)] \ . \tag{20'}$$

Given the new threshold divorce match quality, $\hat{\theta}(t,N)$, and the associated likelihood of divorce for the couple, $\alpha(t,N)$, the maximization problem would now be given by the following:

$$\max_{\left(N,Q,q_h^{-1},q_w^{-1}\right)} \left\{ \begin{array}{c} A(Q)\,q^1 + B_h\left(Q,q_h^{-1}\right) + B_w\left(Q,q_w^{-1}\right) + 2[C(N)+\bar{\theta}] \\ +\alpha(t,N)(t+\Delta C(N)) \\ +[1-\alpha(t,N)] \left[ \begin{array}{c} A(Q)\,q^1 + B_h\left(Q,q_h^{-1}\right) + B_w\left(Q,q_w^{-1}\right) \\ +2C(N) + 2E(\theta \,\big|\, \theta \geqslant \hat{\theta}(t,N)) \end{array} \right] \end{array} \right\} \tag{36}$$

where $q^1 = q_h^1 + q_w^1$.

Accordingly, while the optimal choices of the $M$ marital public goods and the $2m$ $q_i$, $i = h, w$, private goods would not be altered, the optimal number of offspring would be. When $\hat{\theta}(t,N)$ and $\alpha(t,N)$ are given by (19′) and (20′), and (10) still applies, the first-order condition for $N$ is:

$$\left. \begin{array}{c} [2-\alpha(t,N)(1-\Delta)]2C'(N) - A(Q)\min[P^N, \tau\min(y,z)] \\ +2[1-\alpha(t,N)]\frac{\partial E(\theta|\theta\geqslant\hat{\theta}(t,N))}{\partial N} \\ -\alpha_N(t,N) \left\{ \begin{array}{c} A(Q)\,q^1 + B_h\left(Q,q_h^{-1}\right) + B_w\left(Q,q_w^{-1}\right) \\ +(1-\Delta)C(N) + 2E(\theta \,\big|\, \theta \geqslant \hat{\theta}(t,N)) - t \end{array} \right\} \end{array} \right\} = 0 \tag{37}$$

The optimal number of children for the couple $(y,z)$, which we shall now define as $\bar{N}_2$, is implicitly defined by equation (37), with $\bar{N}_2 \lesseqgtr \bar{N}_1$. First, given (19′) we already established that, $\forall\ t,\ N >> 0,\ \alpha_N(t,N) < 0$. Moreover, $A(Q)\,q^1 + B_h\left(Q,q_h^{-1}\right) + B_w\left(Q,q_w^{-1}\right) + 2E(\theta \,\big|\, \theta \geqslant \hat{\theta}(t,N))] > t$ due to the fact that, $\forall\ t > 0$, $\eta(t) \equiv A(Q)\,q^1 + B_h\left(Q,q_h^{-1}\right) + B_w\left(Q,q_w^{-1}\right) > t$. Hence, the last term in (37), which represents the

by schooling: individuals sort positively into marriage based on schooling and individuals with more schooling and more children are less likely to divorce. See Browning, Chiappori, Weiss (in progress, ch. 1).

extra benefit of having more number of children on marital surplus—with the impact of the former on the latter working via the effect of children on divorce likelihood—is strictly positive. This extra beneft is only partially offset by two factors: One, given that $\hat{\theta}_N(t, N) < 0$, the third term in equation (37) is strictly negative. Two, the sum of the first two terms of (37) is strictly negative evaluated at $\bar{N}_1$, because the first-order condition which determines $\bar{N}_1$ is given by those two terms evaluated at $\Delta = 1$. Hence, with $\Delta < 1$, the difference is strictly negative at $\bar{N}_1$.

To sum up, the impact of $\Delta < 1$ on the optimal number of children can be ambiguous. Regardless of the impact of $\Delta < 1$ on the level of $\bar{N}_2$, however, couples with more children would divorce with a lower likelihood when $\Delta < 1$. Taken together with the fact that children are normal goods, we establish that there would be two complementary effects of higher household incomes, $t$, on divorce: On the one hand, there would be a direct *income effect*, which would lower the divorce likelihood for higher-income couples. On the other hand, there would be an indirect *marital public goods effect,* according to which higher-income couples would have more children and, as a consequence, divorce less.

As we shall discuss next, however, the optimal specialization mode would also matter. In particular, although higher incomes would raise the number of children, changes in the income of the childrearing spouse could lower $N$, thereby raising the propensity of divorce.

Recall the participation constraints which bound the second-period allocations in marriage given by equation (31). Rewriting them for the case in which $\Delta_h \neq \Delta_w$ with $\Delta_h$, $\Delta_w < 0$, we have

$$u_h^2(y) + \theta \geq (1-\beta)\, t - (1-\Delta_h)C(\bar{N}_2) \qquad \text{and} \qquad u_w^2(z) + \theta \geq \beta t - (1-\Delta_w)C(\bar{N}_2). \tag{38}$$

An interesting implication of (38) is that the spouse who would be worse off in divorce due to his or her distance to children (i.e., the one with the smaller $\Delta_i$) is the spouse whose allocations and welfare would be adversely affected within marriage too. In particular, the participation constraints above show that the second-period allocations of the spouse whose $\Delta_i$ is smaller would be lower, ceteris paribus. And taken together with the determination of lifetime allocations and first-period allocations, which we discussed in subsections 3.4 and 3.6, we see that, all else equal, the expected lifetime allocations as well as first-period allocations of the spouse with a lower $\Delta_i$ would be more depressed than one whose $\Delta_i$ is higher.

## 5 Gender Inequality, Specialization & Divorce

Regardless of whether $\Delta_i$ is less than unity or not, both the level of household income, $t$, and the inequality of spousal incomes, $y$ and $z$, affect the decision to invest in children as well as the means through which to rear children. Those choices, in turn, influence the intra-household allocations and the propensity to divorce.

In what follows, we shall elaborate on these findings in some detail, focusing our attention to the special case in which the sex ratio equals unity ($r = 1$) and the income distributions of men and women can be fully defined by the linear shift represented by equation (17). That is, $\forall\ z \in [b, B] \wedge G(z) = F(y)$, and $y = \lambda z + \delta$ with $\lambda \geqslant 1$ and $\delta \geqslant 0$.

Under these assumptions, we can express the total potential income of a couple $(y, z)$ as $t = (1+\lambda)z + \delta$ and with all wives potential incomes being weakly below those of their husbands'. Namely, $\forall\ z \in [b, B]$, note that $\min(y, z) = z$. Also relevant is the fact that, when $y = \lambda z + \delta$, spousal inequality, as measured by the ratio of the wives' potential earnings as a share of their husbands' income, $z/y$, decreases as household income $t$ rises.[10]

Under the assumption that $P^N/\tau > \min(a, b)$,[11] we can now summarize the equilibrium choices of spousal career choice and number of offspring as follows:

- Couples for whom $t < (1+\lambda)P^N/\tau - \delta$ represent the lower-income households. For all such couples, the wives' labor income potential is low enough that $z < P^N/\tau$ and the mothers rear for their children at home, taking time away from

[10] In particular, we have

$$\frac{\partial(z/y)}{\partial z} = \frac{\partial(z/\lambda z + \delta)}{\partial z} = \frac{\delta}{(\lambda z + \delta)^2} > 0\ .$$

[11] This assumption ensures that, in equilibrium, there will always be some couples who rear for their children at home and some who work full time in both periods and rely on daycare for childrearing.

the labor market. For all couples in this group, the cost of children is positively related to the wives' labor market income, $z$. Hence, ceteris paribus, higher household income, $t$, leads to higher fertility but a higher level of $z$ lowers $\bar{N}_2$ among these couples.[12] Consequently, among couples with $t < (1+\lambda)P^N/\tau - \delta$, higher household incomes lower the likelihood of divorce, whereas a smaller gender gap tends to raise it.

- Couples with $t \geqslant (1 + \lambda)P^N/\tau - \delta$ represent the higher-income households. For these couples, the wives' labor income potential, $z$, is high enough that $z \geqslant P^N/\tau$. Thus, the mothers work full time in the labor market with the couples using private market-based care for childrearing. For all couples in this group, the cost of children is simply the market price $P^N$. Therefore, spousal inequality has no influence on the decisions of households within this category and the number of their children, $\bar{N}_2$, is just an increasing function of their income $t$, with increases in $z$ having unambiguously positive effects on $\bar{N}_2$. In turn, increases in $\bar{N}_2$ lead to lower divorce probabilities in this segment of the marriage market, with higher-ranked couples within this category having relatively more children and divorcing less.

Taken together with the determination of spousal allocations over the lifetime and intra-temporally, these optimal patterns of spousal specialization cum gender endowment inequalities have some important implications for how income inequality between spouses maps into allocative inequalities. To see this, note that lower spousal endowments, $y$ or $z$, do lower the total marital surplus. But the marginal contribution to a marriage of a mother who commits to childrearing at home would be higher in childrearing the lower is her income or endowment $z$. And given the competition in the marriage markets, which helps to pin down spousal allocations over time and within each of the two periods, this would translate into more generous allocations for the childrearing wives, ceteris paribus. On this basis, the dynamics of marriage market competition would help to narrow the gender gap in spousal consumption beyond the gender gap observed in endowments.

[12] Examining (37), one can see that increases in $z$ raises the cost of children when $z < \bar{P}/\tau$. In contrast higher $t$ raises the benefit of $N$ by lowering the divorce probability; recall that the choices of $Q$, $q_h^{-1}$ and $q_w^{-1}$ are function of prices only and are all independent of household income.

In terms of the relationship between household income and fertility, we essentially might have a U-shaped relationship, when the source of higher household income is a closing of the gender income/endowment gap: Among couples for whom $t < (1+\lambda)P^N/\tau - \delta$, mothers rear for their children at home, whose opportunity cost rises with increases in $z$. For these couples, fertility falls with a closing of the gender gap, provided that the income effect of higher $z$ does not exceed the cost effect.[13] Once household income reaches a level $t \geqslant (1+\lambda)P^N/\tau - \delta$, however, higher household income leads to more fertility. Hence, the fertility rate troughs at $t = (1+\lambda)P^N/\tau - \delta$.

# 6 Costly Specialization

An interesting extension of our model involves costly labor market separation; the case in which childrearing at home is costly in terms of future labor income. In particular, consider this: we have already established that, when the spouse whose labor income potential is lower than the other spouse and $\min(y,z) < P^N/\tau$, the lower-income spouse devotes a $\tau$ fraction of his or her time per child to childrearing. Suppose that, in that case, the labor income of the childrearing spouse erodes by a factor $\delta$, $0 < \delta < 1$. Thus, when the spouse who deals with childrearing in the first period returns to full time employment in the second period, let his or her endowment become $(1-\delta)x_i$, $x_i = y, z$. Assume further that the divorce laws do not fully take this into account when allocating spousal income after divorce and divorce does not create distance between the children and at least one of the parents, as we discussed in Section 4.

Clearly, couples for whom $\min(y,z) \geqslant P^N/\tau$ won't be affected by costly labor market separation because neither spouse commits to home childcare and takes time away from labor employment in the first period. However, among all couples who still choose to undertake childrearing at home,[14] divorce would occur whenever

$$\eta(t - \delta\min(y,z)) + 2[C(N) + \theta] < t - \delta\min(y,z) \ + 2C(N) \ , \qquad (18'')$$

[13] This would more likely occur when $\tau$ is relatively large for exmaple.

[14] As we shall argue next, the condition $\min(y,z) < \bar{P}/\tau$ no longer suffices to identify whether a couple decides to undertake childrearing at home as opposed to private childcare.

or, equivalently, when

$$\eta(t - \delta \min(y, z)) + 2\theta < t - \delta \min(y, z)$$
$$\Leftrightarrow \tag{19''}$$
$$\theta < \hat{\theta}(t - \delta \min(y, z)) = -\frac{1}{2}\{\eta(t - \delta \min(y, z)) - [t - \delta \min(y, z)]\} \ .$$

On this basis, the ex-ante probability of divorce for a couple with endowments of $y$ and $z$ would be

$$\alpha(t - \delta \min(y, z)) \ \equiv \ \Phi[\hat{\theta}(t - \delta \min(y, z))] \ . \tag{20''}$$

A comparison of (19″) with (19) indicates that, because labor market separation reduces marital surplus, the divorce threshold match quality would now be higher. Hence, the divorce likelihood would be higher too when labor force separation is costly.

When labor force detachment is costly, notice that the condition $\min(y, z) < P^N/\tau$ no longer suffices to ascertain that the couple will rear for their children at home, with the lower labor income spouse being solely responsible for child care. In particular, while all couples with $\min(y, z) \geqslant P^N/\tau$ would still choose to acquire childcare on the private market with both spouses working full time in both periods, not all couples for whom $\min(y, z) < P^N/\tau$ would choose to specialize. All such couples would have to weigh the benefit of the cost effective means of childrearing in the first period with the additional cost of labor force detachment in the second period. Obviously, when the cost effective mode of childrearing is private childcare (i.e., $\min[P^N, \tau \min(y, z)] = P^N \Rightarrow \min(y, z) \geqslant P^N/\tau$), then there is no trade off in the couple's career choices. However, when the cost effective mode of childrearing is home care (i.e., $\min[P^N, \tau \min(y, z)] = \tau \min(y, z) \Rightarrow \min(y, z) < P^N/\tau$), then the couple would compare its expected marital surplus under the two alternative modes of childrearing to decide whether or not to undertake childcare at home.

The expected marital output generated over the two periods of a married couple who chooses to rear for their children at home would consequently equal

$$S(t) = \eta(t - \tau \min(y,z)\bar{N}_{1S}) + 2[C(\bar{N}_{1S}) + \bar{\theta}]$$

$$+ \alpha(t - \delta \min(y,z))[t - \delta \min(y,z) + 2C(\bar{N}_{1S})] \tag{39}$$

$$+ [1 - \alpha(t - \delta \min(y,z))][\eta(t - \delta \min(y,z)) + 2C(\bar{N}_{1S}) + 2E(\theta \left| \theta \geqslant \hat{\theta} \right.)] \ ,$$

with the optimal number of children given by

$$\bar{N}_{1S} = C'^{-1}\left(\frac{A\,(Q)\,\tau \min(y,z)}{4}\right) \ . \tag{11$'$}$$

Alternatively, the expected output when the couple works full time in both periods and their children are reared in private childcare is given by:

$$\begin{aligned} S(t) \ &= \ \eta(t - P^N\bar{N}_{1P}) + 2[C(\bar{N}_{1P}) + \bar{\theta}] + \alpha(t)[t + 2C(\bar{N}_{1P})] \\ &+[1 - \alpha(t)][\eta(t) + 2C(\bar{N}_{1P}) + 2E(\theta \left| \theta \geqslant \hat{\theta} \right.)] \ , \end{aligned} \tag{40}$$

with the optimal number of children given by

$$\bar{N}_{1P} = C'^{-1}\left(\frac{A\,(Q)\,P^N}{4}\right) \ , \tag{11$''$}$$

where $\hat{\theta}$ and $\alpha(t)$are defined by (19) and (20), respectively.

First, note that $\bar{N}_{1P} < \bar{N}_{1S}$ if $\min(y,z) < P^N/\tau$. For the same reason, we also have $\eta(t-\tau \min(y,z)\bar{N}_{1S}) > \eta(t-P^N\bar{N}_{1P})$ and $\alpha(t-\delta \min(y,z)) > \alpha(t)$, as a result of which we have $2E(\theta \left| \theta \geqslant \hat{\theta} \right.)$ in equation (40) exceed that in (39). All of these discrepancies arise due to the fact that the couple would be choosing a cost ineffective method of childrearing if they choose to employ private childcare when in fact $\min(y,z) < P^N/\tau$.

When $\min(y,z) < P^N/\tau$ so that home childcare is more cost effective and $\delta = 0$ so that labor force detachment is costless in terms of future labor income, these are the only effects of choosing private care over home childrearing and (39) would strictly exceed (40). As a corollary, note that when $\min(y, z) = P^N/\tau$ and $\delta = 0$, (39) and (40) would yield the same valuations, so as to make the couple indifferent between private care and home care.

When $\delta > 0$, however, a tradeoff arises do the fact that the couple's second-period output would be strictly higher if it chooses private childcare over home childcare when $\min(y, z) < P^N/\tau$. That is, $\eta(t) > \eta(t - \delta \min(y, z))$. On this basis, we can conclude that, for $\delta > 0$, $\exists\, (y, z)$ such that $\min(y, z) \leqslant P^N/\tau$ and (40) strictly exceeds (39).

Costly labor market separation would impact spousal allocations within marriage too. To see how, note that the second-period marital participation constraints, which help to identify intra-temporal spousal allocations in both periods, would also need to be modified to account for costly labor market separation. In particular, if $\min(y, z)$ $= y$ and the husband rears for the children at home, then (31) would become

$$\begin{aligned} u_h^2(y) + C(\bar{N}_{1S}) + \theta &\geq (1-\beta)\, t - \delta y + C(\bar{N}_{1S}) \ , \\ u_w^2(z) + C(\bar{N}_{1S}) + \theta &\geq \beta t + C(\bar{N}_{1S}) \ . \end{aligned} \tag{31$'$}$$

If, in contrast, $\min(y, z) = z$ and the wife rears for the children at home, then (31) would be

$$\begin{aligned} u_h^2(y) + C(\bar{N}_{1S}) + \theta &\geq (1-\beta)\, t + C(\bar{N}_{1S}) \ , \\ u_w^2(z) + C(\bar{N}_{1S}) + \theta &\geq \beta t - \delta z + C(\bar{N}_{1S}) \ . \end{aligned} \tag{31$''$}$$

The central implication of equations (31′) and (31″) is that the second-period allocations of the spouse who devotes time to childcare in the first period would be depressed, whereas his or her spouse's second-period allocations would not be affected. In the transferable utility framework we employ here, this implies that there would have to be some compensating differential allocations made in the first period to the spouse who commits to home childcare. On net, because labor force separation adversely effects total household production as well, the economic burden of labor force detachment would be borne by both spouses to some extent.

In summary, costly labor force detachment would have no effect on higher-income or more equal couples with $\min(y, z) > P^N/\tau$. Among the lower-income or higher inequality couples with $\min(y, z) \leqslant P^N/\tau$, however, costly labor force separation would have various effects. First, all such couples would have fewer children because the cost of children would be higher when home childrearing exacts a labor force detachment cost. Second, due to the lower level of effective household income and fewer number of children in this scenario, the divorce likelihood would be higher among the lower-income or higher inequality couples. Third, some couples who would have went with spousal specialization and home childrearing if labor force detachment weren't costly would prefer to acquire private childcare and work full time in the labor market in both periods.

# 7 Investment in Quality of Offspring

Although we abstracted from investment in the quality of children or the offspring's human capital thus far, incorporating a quantity-quality tradeoff into this model would require some minor modifications in the utility specifications and the budget constraints. To that end, consider a version of our model where the couples make their fertility choices in the first period only, but they invest in the human capital of their offspring in both periods. Also take a case in which divorce involves distance between the parents and the offspring, along the lines we presented in section 4.

When individuals derive utility from the quality of their children as well, let's assume that the husbands' and wives' individual utilities take the following form

$$U_i = u_i(q_i, Q, N, H) + \theta, \qquad i = h, w, \tag{1'}$$

where $q_i$, $Q$, $N$ continue to represent the same goods in the baseline model, $\theta$ is still the couple-specific match quality, and $H$ represents the human capital level per offspring.

The preferences of married individuals are now given by

$$u_i\left(q_i, Q, N\right) = A\left(Q\right) q_i^1 + B_i\left(Q, q_i^{-1}\right) + C(N)D(H) + \theta, \tag{2'}$$

where $A$ and $B_i$, $i = h, w$, are positive, increasing, concave functions such that $A(0) = 1$ and $B_i(0) = 0$. For heuristic purposes and without loss of generality, let $C(N) = N^\gamma$ and $D(H) = H^\rho/2$ where $0 < \gamma$, $\rho < 1$.

The revisions introduced in $(1')$ and $(2')$ would then be reflected in equations (3), (4), (6) and $(18')$ in the following fashion:

$$u_h + u_w = A\left(Q\right)\left(q_h^1 + q_w^1\right) + B_h\left(Q, q_h^{-1}\right) + B_w\left(Q, q_w^{-1}\right) + N^\gamma H^\rho + 2\theta \tag{3'}$$

$$u_i^s\left(q_i, Q, N\right) = q_i^1 + B_i^s\left(Q, q_i^{-1}\right) + \frac{\Delta N^\gamma H^\rho}{2}, \tag{4'}$$

$$u_h^s\left(q_h, Q, N\right) + u_w^s\left(q_w, Q, N\right) = t + \Delta N^\gamma H^\rho \ , \tag{6'}$$

with divorce occurring whenever

$$\eta\left(t\right) + N^\gamma H^\rho + 2\theta < t + \Delta N^\gamma H^\rho \ , \tag{36'}$$

where recall that $\Delta$ represents the drop in utility from children as a result of distance created by divorce.

The optimal levels of the $M$ public goods and the $2m$ private goods would not be altered in this case, with (5), (8) and (9) defining the levels of private as well as public goods consumed in marriage and divorce. Suppose that investment in a unit of human capital can be undertaken at the cost $P^H$. Revising (10) in accordance, we have

$$\begin{aligned} q^1 &= q_h^1 + q_w^1 = \\ & t - \sum_{j=1}^{M} P^j Q^j - \sum_{k=2}^{m} p^k\left(q_h^k + q_w^k\right) - \{\min[P^N, \tau\min(y,z)] + P^H H\}N \ . \end{aligned} \tag{10'}$$

Because couples can invest in the human capital of their offspring in both periods, but they decide how many children to have in the first, we first need to determine human capital investments in the second period conditional on the number of kids and marital status. On that basis, we can then identify the optimal number of children and human capital investments in the first period, reflecting the optimal investment patterns in the second.

Let's define the number of children in this case as $\bar{N}_3$. Conditional on $\bar{N}_3$, the optimal investment level per child might differ, depending on whether or not the couple $(y, z)$ divorces in the second period. When $(y, z)$ decide to stay together in the second period and they have $\bar{N}_3$ children, they decide the optimal amount of investment in their children according to

$$\max_{H_{2M}} A\left(Q\right)\left(q_h^1 + q_w^1\right) + B_h\left(Q, q_h^{-1}\right) + B_w\left(Q, q_w^{-1}\right) + \bar{N}_3^{\gamma} H_{2M}^{\rho} + 2\theta \tag{41}$$

where $q_h^1 + q_w^1$ is defined by $(10')$. We thus have

$$\bar{H}_{2M} = \left[\frac{1}{\bar{N}_3^{1-\gamma}}\left(\frac{\gamma}{A(Q)P^H}\right)\right]^{\frac{1}{1-\rho}} . \tag{42}$$

If $(y, z)$ divorce at the end of the first period, they decide the optimal amount of human capital investment in their children in the second period according to[15]

$$\max_{H_{2D}} t - P^H \bar{N}_3 H_{2D} + \Delta \bar{N}_3^{\gamma} H_{2D}^{\rho} , \tag{43}$$

where $q_h^1 + q_w^1$ is again defined by $(10')$. In turn, we get

[15] Custody laws and the determination of child support payments between the ex-spouses has important repercussions, in general.

Specifically when utility is not transferable between spouses following divorce, the appropriate method for determining child support payment would involve a non-cooperative, two-stage solution: The custodial parent would make investment in the children without taking into account the positive externality the investments generate for the non-custodial parent. Hence, it would be in the interest of the latter to take into account how child support would affect investment in children before deciding his or her child support payments. Of course, it is possible that this optimal child support payment might be lower than that mandated by law.

In any case, since we chose to employ a transferable utility specification even in divorce, the outcome of a two-stage, non-cooperative solution would be identical to the cooperative investment level which is implicit in what we present.

$$\bar{H}_{2D} = \left[ \frac{1}{\bar{N}_3^{1-\gamma}} \left( \frac{\gamma \Delta}{P^H} \right) \right]^{\frac{1}{1-\rho}} \tag{44}$$

Note that $\bar{H}_{2D} \gtrless \bar{H}_{2M}$ depending on two parameters: the distance created between parents and children as a result of divorce, $\Delta$, and the marginal cost burden of human capital investment, which changes in divorce due to the presence of other public goods. That is, $A(Q)P^H$ in marriage versus $P^H$ in divorce. The lower is $\Delta$, ceteris paribus, the more likely it is that $\bar{H}_{2M} > \bar{H}_{2D}$, with parents investing more in the human capital of their offspring if they remain married.

Taken together, the human capital investment schedules defined by (42) and (44) establish that the divorce probability of a couple $(y, z)$ would depend on their income as well as the the quality and quantity of their offspring:

$$\eta(t) + \bar{N}_3^{\gamma} \bar{H}_{2M}^{\rho} + 2\theta < t \ + \Delta \bar{N}_3^{\gamma} \bar{H}_{2D}^{\rho} \ , \tag{18'''}$$

or, equivalently,

$$\eta(t) + \bar{N}_3^{\frac{\gamma-\rho}{1-\gamma}} \left( \frac{\gamma}{A(Q)P^H} \right)^{\frac{\gamma}{1-\gamma}} + 2\theta < t \ + \bar{N}_3^{\frac{\gamma-\rho}{1-\gamma}} \left( \frac{\Delta\gamma}{P^H} \right)^{\frac{\gamma}{1-\gamma}}$$
$$\Leftrightarrow \tag{19'''}$$
$$\theta < \hat{\theta}(t, N) = -\frac{1}{2} \left\{ \eta(t) - t + \bar{N}_3^{\frac{\gamma-\rho}{1-\gamma}} \left[ \left( \frac{\gamma}{A(Q)P^H} \right)^{\frac{\gamma}{1-\gamma}} - \left( \frac{\Delta\gamma}{P^H} \right)^{\frac{\gamma}{1-\gamma}} \right] \right\} \ .$$

On this basis, the ex-ante probability of divorce for a couple with endowments of $y$ and $z$ would be given by (20′) in Section 4. Provided that the parameter values satisfy $\gamma > \rho$, the likelihood of divorce would decline with higher fertility. And depending on the value of $\Delta$, ceteris paribus, investment in both quantity and quality of offspring would complement each other to reduce the likelihood of divorce provided that $\gamma > \rho$ and $\Delta$ is relatively low.

With that, we can now turn to the first-period choices regarding the quantity and quality of offspring. Since, conditional on being married, optimal investment is independent of income and dependent only on the price of human capital investment $P^H$ and the number of children $\bar{N}_3$, we have $\bar{H}_1 = \bar{H}_{2M}$. Moreover, $\hat{\theta}(.)$, $\alpha(.)$, $\bar{H}_1$, $\bar{H}_{2M}$ and $\bar{H}_{2D}$ are established as function of $\bar{N}_3$. Thus, we can derive the expected marital surplus as in equation (7), but with (1′) through (6′) as the underlying utility specifications, $(19''')$, $(20')$ providing the divorce threshold match quality, $\hat{\theta}$, and the probability of divorce, $\alpha$, and after substituting in for the optimal human capital levels specified in (42) and (44):

$$
\begin{aligned}
S(t,N) \;=\;& A(Q)\left[\begin{array}{c} t-\sum_{j=1}^{M}P^{j}Q^{j}-\sum_{k=2}^{m}p^{k}\left(q_{h}^{k}+q_{w}^{k}\right) \\ -\min[P^{N},\tau\min(y,z)]N-P^{H}\left(\frac{\gamma}{A(Q)P^{H}}\right)^{\frac{1}{1-\rho}}N^{\frac{\gamma-\rho}{1-\rho}} \end{array}\right]+2\bar{\theta} \\
&+\alpha(t,N)\left[t+\left(\frac{\Delta\gamma}{P^{H}}\right)^{\frac{\rho}{1-\rho}}N^{\frac{\gamma-\rho}{1-\rho}}\left(1-\frac{\Delta\gamma}{P^{H}}\right)\right] \qquad (45) \\
&+[2-\alpha(t,N)]\left\{B_{h}\left(Q,q_{h}^{-1}\right)+B_{w}\left(Q,q_{w}^{-1}\right)+\left(\frac{\gamma}{A(Q)P^{H}}\right)^{\frac{\rho}{1-\rho}}N^{\frac{\gamma-\rho}{1-\rho}}\right\} \\
&+[1-\alpha(t,N)]\left\{\begin{array}{c} A(Q)\left[\begin{array}{c} t-\sum_{j=1}^{M}P^{j}Q^{j}-\sum_{k=2}^{m}p^{k}\left(q_{h}^{k}+q_{w}^{k}\right) \\ -P^{H}\left(\frac{\gamma}{A(Q)P^{H}}\right)^{\frac{1}{1-\rho}}N^{\frac{\gamma-\rho}{1-\rho}} \end{array}\right] \\ +2E(\theta\,\big|\,\theta\geqslant\hat{\theta}\,(t,N)\,) \end{array}\right\}
\end{aligned}
$$

The first, third and final lines in (45) represent the couple's total utility in the state of marriage (in both periods) and the second line in that equation represents their utility sum in case of divorce in the second period. Note that, conditional on the fact that the couple stays married, their human capital investments do not change from the first period to the next. That is, $\bar{H}_1 = \bar{H}_{2M}$. Moreover, utility derived from the $B_i,\ i = h, w$ terms remain unchanged in the second period as well, provided that the couple stays married. The third line in equation (45) reflects these facts, weighing the couples' aggregate utility from their children and the $B_i$'s by the expected duration of their marriage, which equals $2 - \alpha(t, N)$. In contrast, the couples' utility sum from the component of $A(Q)\,q^1$ does change in the second period even if their marriage continues. This is because the total expenditure on children drops in the second period, due to the fact that only human capital investment needs to be funded and no resources need to be allocated to childrearing in the second period. Of course, since expenditures on all other marital public goods depend purely on relative prices, the couples' utility from the $A(Q)$ component would be identical in both periods, but their sum of the private consumption of good 1 would be strictly lower in the first period than in the second.

Then, maximizing the couple's expected marital surplus $S(t, N)$ with respect to $N$, we get the first-order condition:

$$\left.\begin{array}{c} -A(Q)\min[P^N, \tau\min(y,z)] \\ \\ [2-\alpha(t,N)]\frac{(\gamma-\rho)(1-\gamma)}{1-\rho}\left(\frac{\gamma}{A(Q)^{\rho}(P^H)^{\rho}}\right)^{\frac{\rho}{1-\rho}} N^{\frac{\gamma-1}{1-\rho}} + 2[1-\alpha(t,N)]\frac{\partial E(\theta\left|\theta\geqslant\hat{\theta}(t,N)\right.)}{\partial N} \\ \\ \alpha(t,N)\frac{(\gamma-\rho)(1-\Delta\gamma)}{1-\rho}\left(\frac{\Delta\gamma}{(P^H)^{\rho}}\right)^{\frac{\rho}{1-\rho}} N^{\frac{\gamma-1}{1-\rho}} \\ \\ -\alpha_N(t,N)\left\{\begin{array}{c} A(Q)\,q^1 + B_h\left(Q, q_h^{-1}\right) + B_w\left(Q, q_w^{-1}\right) \\ \\ +N^{\frac{\gamma-\rho}{1-\gamma}}\left[\left(\frac{\gamma}{A(Q)P^H}\right)^{\frac{\gamma}{1-\gamma}} - \Delta\left(\frac{\Delta\gamma}{P^H}\right)^{\frac{\gamma}{1-\gamma}}\right] \\ \\ +2E(\theta\left|\theta\geqslant\hat{\theta}(t,N)\right.) - t + \left[\frac{\Delta\gamma}{(P^H)^{\rho}}\right]^{\frac{1}{1-\gamma}} N^{\frac{\gamma-\rho}{1-\rho}} \end{array}\right\} \end{array}\right\} = 0 \tag{46}$$

An interior solution to this problem would exist as long as $0 < \gamma$, $\rho < 1$—as we assumed at the outset—and $\gamma > \rho$. The first line in (46) is the marginal cost of raising $N$ incurred through childrearing in the first period. The second line is the net marginal benefit of $N$ conditional on staying married in the second period; this term is net of the human capital investment cost of $N$ in the state of marriage. The third line in (46) is the analog of the second line, conditional on getting divorced in the second period. Finally, the last term in equation (46) represents the marginal benefit of more children accrued via a lower divorce probability.

According to (46), the impact of the quantity and quality of children on divorce plays an important role in determining fertility. In terms of the direct marginal utility benefit of an increase in $N$, represented by the second and third terms in (46), we see that there exists a quantity-quality complementarity: for $\Delta$ low enough and $\gamma > \rho$, investment in children's human capital is higher in marriage and if the cost of investment is children is low to justify a relatively high level of investment, then the direct utility of having more children is higher. But provided that $\Delta$ is low enough and $\gamma > \rho$, the same complementarity has an indirect effect too, as suggested by the last term on the LHS of (46): if more human capital investment is warranted and staying married encourages a higher level of human capital investment, then higher fertility would reduce the divorce probability, thereby raising marital surplus even further. In these dynamics, our assignment model identifies that the conventional quantity-quality tradeoff is a conditional one. Put differently, investment in human capital early on and anticipated levels of high investment in human capital later in life encourage higher levels of fertility among couples, thereby lowering the divorce likelihood and producing a larger inter-temporal marital surplus. Of course, the conventional tradeoff between quantity and quality still exists at the intra-temporal level.

Finally, it is worthwhile to make note of the fact that changes in the spousal income gap would affect the quantity-quality tradeoff through its impact on fertility: That is, among couples who raise their children at home, if higher household incomes accrue on the back of a narrower spousal gap in incomes, then increases in household income would be accompanied by deceases in fertility and increases in investment in quality.

# 8 Conclusion

This paper complements our earlier work on incorporating some dynamic aspects of the lifecycle as well as marriage into assignment models. In some of our earlier papers, our focus had been on analyzing matching and intra-household allocations in the presence of pre-marital investments or when divorce and remarriage were possible. In this paper, our main goal was to consider the role of durable, marriage-specific investments exemplified by children in an assignment model with possible divorce. The presence of children fundamentally alters spousal interactions, necessitating a long-term connection between spouses even after divorce. Moreover, while investment in the quantity of offspring is generally conditional on the maintenance of a spousal relationship, investment in the quality of offspring is typically not.

With such considerations in mind, we embedded spousal specialization, investment in children and divorce into an assignment model. By doing so, we were able to illustrate how the dynamics of spousal competition and matching in the marriage markets come to bear on the equilibrium outcomes in spousal specialization, investment in children and divorce. The central theme is that spousal matching in the marriage markets plays an important role in mapping income inequalities among men and women into spousal inequality, which subsequently determines intra-marital outcomes, such as fertility, spousal time use and allocations, as well as the likelihood of divorce. More specifically, while higher household incomes raise the number of children, a narrower gender earnings gap lowers fertility among middle-income but relatively unequal couples. For all couples, the propensity of divorce is a decreasing function of total household income, and when divorce produces a distance between children and at least one of the spouses, it is also decreasing in the number of children. Thus, a narrow gender earnings gap could lead to higher divorce rates through its depressing effect on fertility. In turn, higher divorce rates feed back into lower fertility and investment in children because they reduce the expected marital surplus. And if childrearing at home involves costly labor force separation, fertility rates would be lower and divorce rates would be higher primarily among the lower-income or higher inequality couples.

Accounting for the role of marriage market competition in influencing spousal allocations and intra-marital outcomes enabled us to identify that the gender gap in spousal allocations would be bounded from above by the gender gap in incomes or endowments. The reason for this is that, while lower spousal endowments do reduce marital surplus, the marginal contribution to a marriage of a spouse who commits to childrearing at home is decreasing in his or her opportunity cost, which is lost labor income. Given the competition in the marriage markets, which helps to pin down spousal allocations over time and within each of the two periods, this maps into more generous allocations for the childrearing spouses, the lower their income or labor endowments, ceteris paribus.

Perhaps most interestingly, our model has helped us to identify that the conventional quantity-quality tradeoff is a conditional one. Investment in human capital early on and anticipated levels of high investment in human capital later in life encourage higher levels of fertility among couples, thereby lowering the divorce likelihood and producing a larger inter-temporal marital surplus. Nevertheless, we also saw that the conventional tradeoff between quantity and quality still exists at the intra-temporal level.

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# A Assortative matching

It is sufficient to show that the expected *surplus* of a marriage, given by $\tilde{S}(t) = S(t) - 2t$, is increasing and convex in $t$, where $t = y + z$.

$$\begin{aligned}\tilde{S}(t) &= \eta(t) + N^{\gamma} + 2\bar{\theta} + \alpha(t)(t + N^{\gamma}) - 2t \\ &\quad + [1 - \alpha(t)]\left(\eta(t) + N^{\gamma} + 2E\left[\theta \mid \theta \geq \hat{\theta}(t)\right]\right) \qquad \text{(A.1)} \\ &= \eta(t) + N^{\gamma} + 2\bar{\theta} + \int_{\hat{\theta}}^{\infty} [\eta(t) + N^{\gamma} + 2\theta] f(\theta) d\theta + (t + N^{\gamma}) \int_{-\infty}^{\hat{\theta}} f(\theta) d\theta - 2t,\end{aligned}$$

with

$$\hat{\theta}(t) = -\frac{1}{2}[\eta(t) - t]. \qquad \text{(A.2)}$$

Recall that $\eta(t)$ is strictly convex and $\eta'(t) > 1$. Therefore,

$$\begin{aligned}\tilde{S}'(t) &= \eta'(t)\left(1 + \int_{\hat{\theta}}^{\infty} f(\theta)\, d\theta\right) + \int_{-\infty}^{\hat{\theta}} f(\theta) d\theta + f(\hat{\theta})[-\eta(t) - 2\hat{\theta} + t]\hat{\theta}'(t) - 2 \qquad \text{(A.3)} \\ &= \eta'(t) + \eta'(t)\int_{\hat{\theta}}^{\infty} f(\theta)\, d\theta + \int_{-\infty}^{\hat{\theta}} f(\theta) d\theta - 2 > 0.\end{aligned}$$

and

$$\begin{aligned}\tilde{S}''(t) &= \eta''(t)\left(1 + \int_{\hat{\theta}}^{\infty} f(\theta)\, d\theta\right) + f(\hat{\theta})[-\eta'(t) + 1]\hat{\theta}'(t) \qquad \text{(A.4)} \\ &= \eta''(t)\left(1 + \int_{\hat{\theta}}^{\infty} f(\theta)\, d\theta\right) + f(\hat{\theta})\frac{[\eta'(t) + 1]^2}{2} > 0.\end{aligned}$$

Hence, $S(t)$ is convex in $t$, implying that $z$ and $y$ are complements.